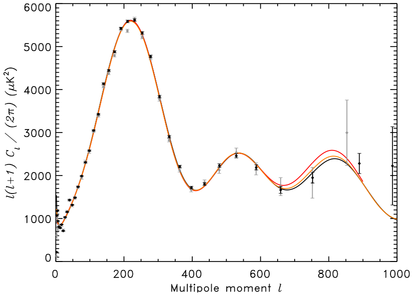
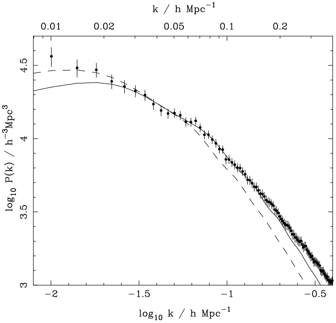
How much of the inflaton potential do we see?

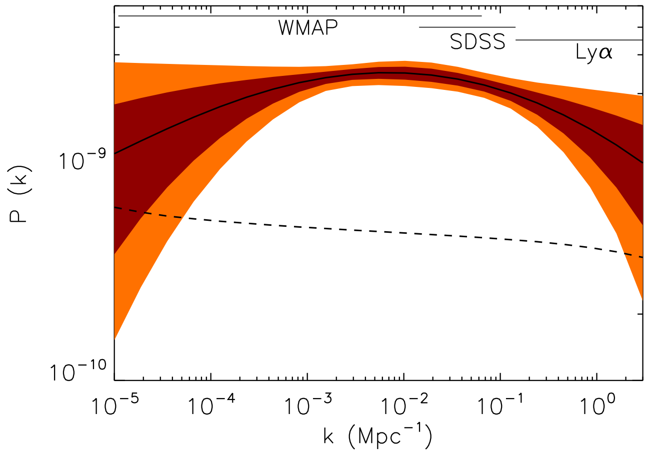
Wessel Valkenburg
8 August, 2007

astro-ph/0703625, Phys.Rev.D75:123519, 2007





WMAP3, from Spergel et al, astro-ph/0603449
SDSS-LRG5, from Percival et al, astro-ph/0608636



Taken from Easther & Peiris, astro-ph/0609003.

CMB+LSS

$\downarrow$

Primordial power spectrum

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$\downarrow$

$H(\eta)$

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$\downarrow$

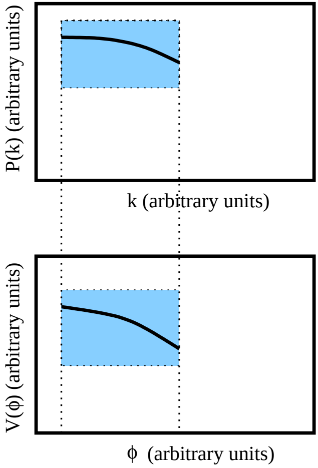
Primordial power spectrum

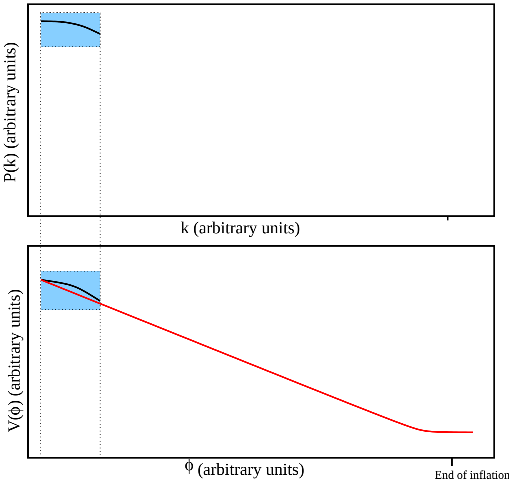
$\downarrow$

$H(\eta)$

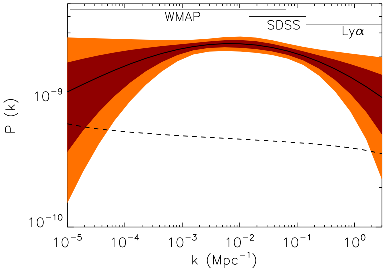
$\downarrow$

$V(\phi)$

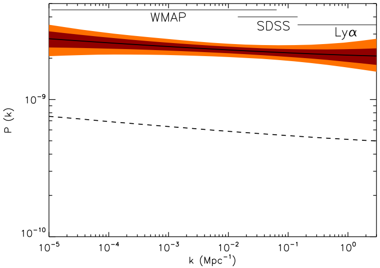

P(k) (arbitrary units)
k (arbitrary units)
V(ϕ) (arbitrary units)
ϕ (arbitrary units)


P(k) (arbitrary units)
k (arbitrary units)
V(ϕ) (arbitrary units)
ϕ (arbitrary units)
End of inflation

Approximations:

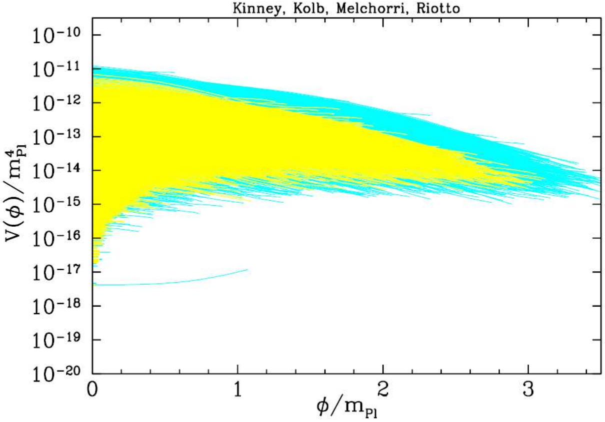


Fitting $P(k) = k^{(n_S - 1 + \ldots)}$.



Fitting $P(k) = k^{(n_S - 1 + \ldots)}$, selecting SR-inflationary models with $N > 30$. In this case WMAP1 already constrains V''''. (Caprini et al. 2002)

Taken from Easther & Peiris, astro-ph/0609003.



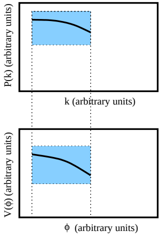
Taken from Kinney et al., astro-ph/0605338.

Directly fit the inflaton potential, numerically, using CosmoMC[I] and our own freely available module[II].

CBM + LSS

$\updownarrow$

$V(\phi)$



[I] Lewis & Bridle, 2002
[II] see astro-ph/0703625

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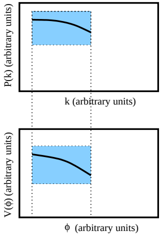


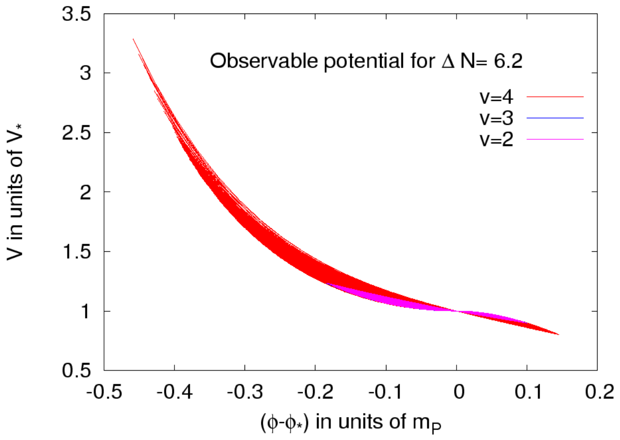
Result applies to any theory of inflation which, during the observable window, has effectively one scalar degree of freedom.

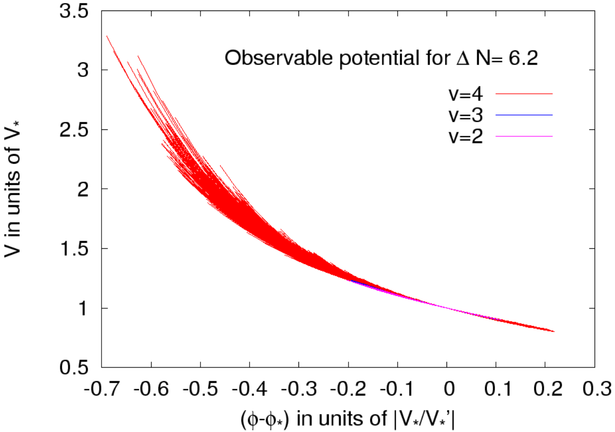
[I] Lewis & Bridle, 2002
[II] see astro-ph/0703625

Directly fit the inflaton potential, numerically

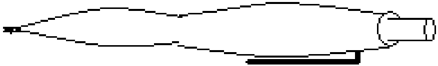
+ self-consistent tensor parameters:
$n_{\mathrm{T}} = -r/8$,
$\alpha_{\mathrm{T}} = n_{\mathrm{T}}[n_{\mathrm{T}} - n_{\mathrm{S}} + 1]$

Slow Roll	Numerical potential fit
$\Omega_b h^2$	$\Omega_b h^2$
$\Omega_{cdm} h^2$	$\Omega_{cdm} h^2$
θ	θ
τ	τ
$\ln[10^{10}\mathcal{P}_{\mathcal{R}}^{k_*}]$	$\ln\left[\frac{128\pi 10^{10} V_*^3}{3 V_*'^2 m_P^6}\right]$
r	$\left(\frac{V_*'}{V_*}\right)^2 m_P^2$
n_{S}	$\frac{V_*''}{V_*} m_P^2$
α_{S}	$\frac{V_*'''}{V_*}\frac{V_*'}{V_*} m_P^4$
β_{S}	$\frac{V_*''''}{V_*}\left(\frac{V_*'}{V_*}\right)^2 m_P^6$

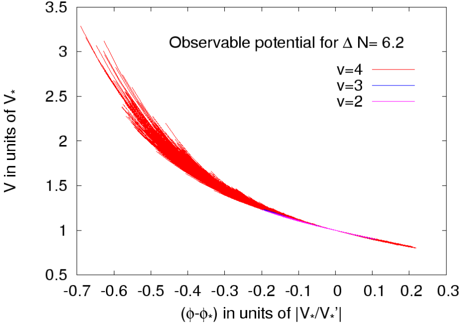

Observable potential for Δ N= 6.2
v=4
v=3
v=2
3.5
3
2.5
2
1.5
1
0.5
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
V in units of V*
(ϕ-ϕ*) in units of mP


Observable potential for Δ N= 6.2
v=4
v=3
v=2
V in units of V*
(ϕ-ϕ*) in units of |V*/V*'|
3.5
3
2.5
2
1.5
1
0.5
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3

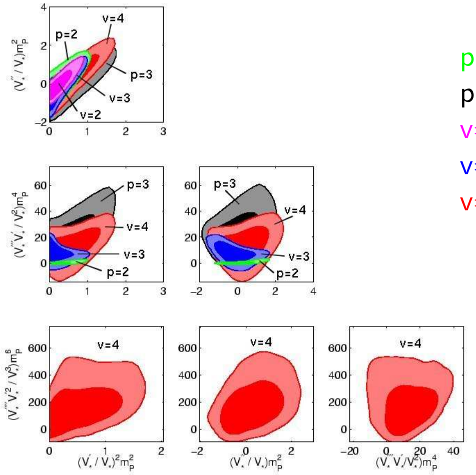
Did anyone see my pencil? (Parker..)



Conclusion



- The data allows departure from Slow Roll
- Previously obtained info on $V(\phi)$ depends on strong assumptions.
- Conservative analysis now gives $V(\phi)$ up to $V'''(')$
- Hint to go to one order higher in SR



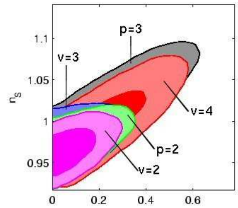
p=2 - A_S, n_S

p=3 - A_S, n_S, α_S

v=2 - V_*', V_*''

v=3 - V_*', V_*'', V_*'''

v=4 - V_*', V_*'', V_*''', V_*''''



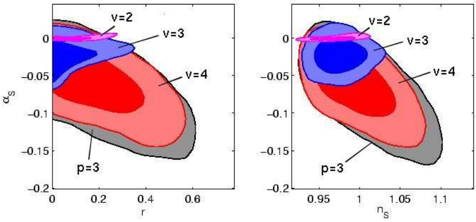
p=2 - A_S, n_S

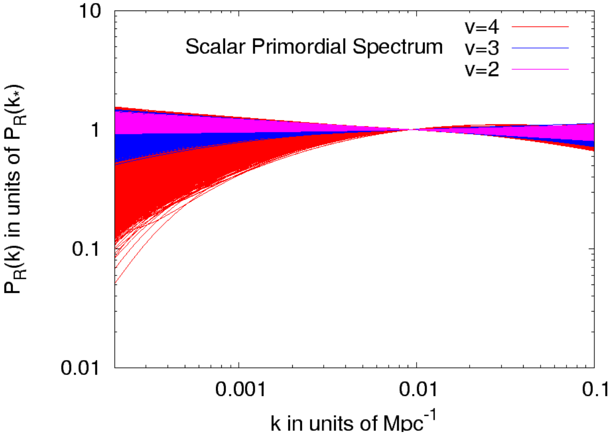
p=3 - A_S, n_S, α_S

v=2 - V'_*, V''_*

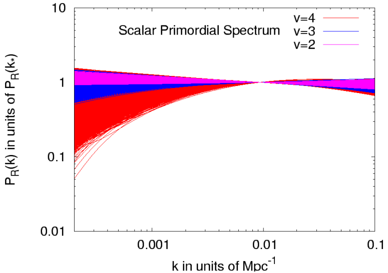
v=3 - V'_*, V''_*, V'''_*

v=4 - V'_*, V''_*, V'''_*, V''''_*

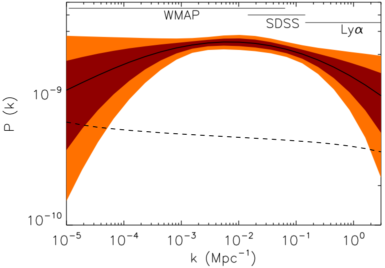



Scalar Primordial Spectrum
v=4
v=3
v=2
10
1
0.1
0.01
0.001
0.01
0.1
$P_R(k)$ in units of $P_R(k_*)$
k in units of Mpc^{-1}

Approximations:

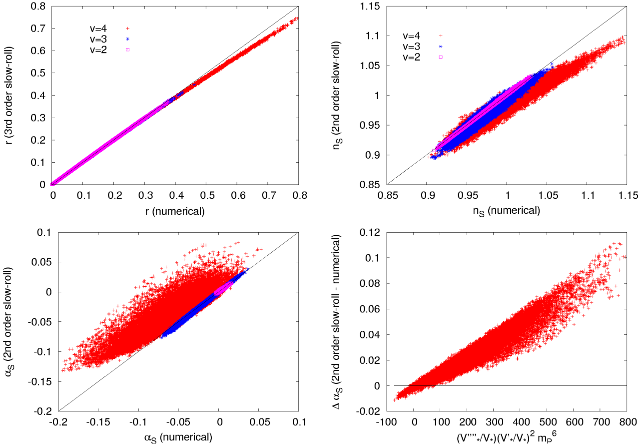


Fitting $V(\phi - \phi_*)$.



Fitting $P(k) = k^{(n_S - 1 + \ldots)}$.[a]

[a] Taken from Easther & Peiris, astro-ph/0609003.


r (3rd order slow-roll)
r (numerical)
v=4
v=3
v=2
n_S (2nd order slow-roll)
n_S (numerical)
α_S (2nd order slow-roll)
α_S (numerical)
$\Delta\, \alpha_S$ (2nd order slow-roll - numerical)
$(V''''_*/V_*)(V'_*/V_*)^2\, m_P^6$

$$\dot{\phi} = -\frac{m_P^2}{4\pi} H'(\phi)$$

$$\left[H'(\phi)\right]^2 - \frac{12\pi}{m_P^2} H^2(\phi) = -\frac{32\pi^2}{m_P^4} V(\phi)$$

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$$\partial_\eta^2 \mu_{\mathrm{S,T}} + \left[k^2 - \frac{\partial_\eta^2 z_{\mathrm{S,T}}}{z_{\mathrm{S,T}}}\right] \mu_{\mathrm{S,T}} = 0$$

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{8\pi^2} \left| \frac{\mu_{\mathrm{S}}}{z_{\mathrm{S}}} \right|^2$$

$$\mathcal{P}_h(k) = \frac{2k^3}{\pi^2} \left| \frac{\mu_{\mathrm{T}}}{z_{\mathrm{T}}} \right|^2$$

Approximations:

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Primordial power spectrum of curvature perturbations:

$$\ln \frac{\mathcal{P}_{\mathcal{R}}(k)}{\mathcal{P}_{\mathcal{R}}(k_*)} = (n_S - 1)\ln \frac{k}{k_*} + \frac{\alpha_S}{2}\ln^2 \frac{k}{k_*} + \frac{\beta_S}{6}\ln^3 \frac{k}{k_*} \ldots,$$

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Slow-roll approximation of inflation:

$$\begin{aligned} n_{\mathrm{S}} - 1 &= -2\epsilon_1 - \epsilon_2 - 2\epsilon_1^2 - (2C + 3)\epsilon_1\epsilon_2 - C\epsilon_2\epsilon_3, \\ \alpha_{\mathrm{S}} &= -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3 \\ \epsilon_0 &\equiv H(N_{\mathrm{i}})/H(N) \\ \epsilon_{n+1} &\equiv \frac{\mathrm{d} \ln |\epsilon_n|}{\mathrm{d}N}, \qquad n \geq 0. \end{aligned}$$

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Slow-roll approximation of inflation:

$$\epsilon_n = \epsilon_n(n_S, \alpha_S, \ldots)$$

$$V = \frac{3 m_{\mathrm{Pl}}^2 H^2}{8\pi} \left(1 - \frac{\epsilon_1}{3}\right)$$

$$V' = -\frac{3 m_{\mathrm{Pl}} H^2}{(4\pi)^{1/2}} \epsilon_1^{1/2} \left(1 - \frac{\epsilon_1}{3} + \frac{\epsilon_2}{6}\right)$$

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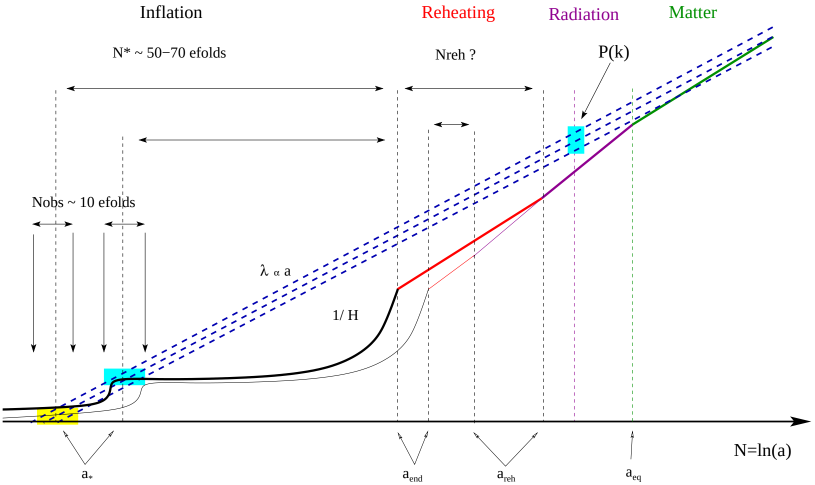
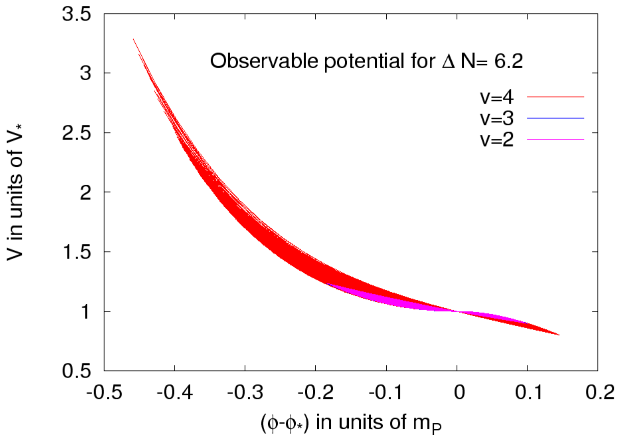
- Fit to data using an MCMC.



[II] taken from Ringeval, astro-ph/0703486


Observable potential for Δ N= 6.2
v=4
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3.5
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-0.3
-0.2
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0.2
V in units of V*
(ϕ-ϕ*) in units of mP

$$\lim_{k/aH\to\infty} \mu_{\rm S,T}(\eta) = \frac{4\sqrt{\pi}}{m_{\rm Pl}} \frac{e^{-ik(\eta-\eta_{\rm i})}}{\sqrt{2k}}$$

$$\mu_{\rm S}(\eta) \equiv 2 z_{\rm S} \mathcal{R}$$

$$\mu_{\rm T}(\eta) \equiv z_{\rm T} h$$

$$z_{\rm S} \equiv a\sqrt{2 - aa''/a'^2}$$

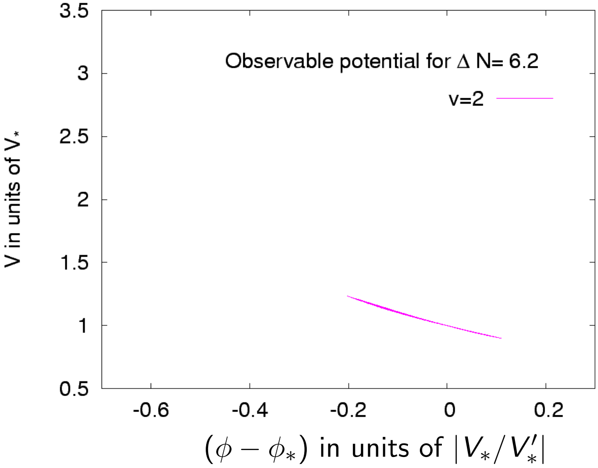
$$z_{\rm T} = a$$

$$\frac{k_*}{a_I H_I} = \frac{k_*}{a_0 H_0}$$

$$\frac{a_0}{a_I} = \frac{H_I}{H_0}$$

$$2N \equiv \ln \frac{a_0}{a_I} = \ln \frac{H_I}{H_0}$$

$$N \simeq 60 + \tfrac{1}{2} \ln \frac{H_I}{10^{13}\mathrm{GeV}}$$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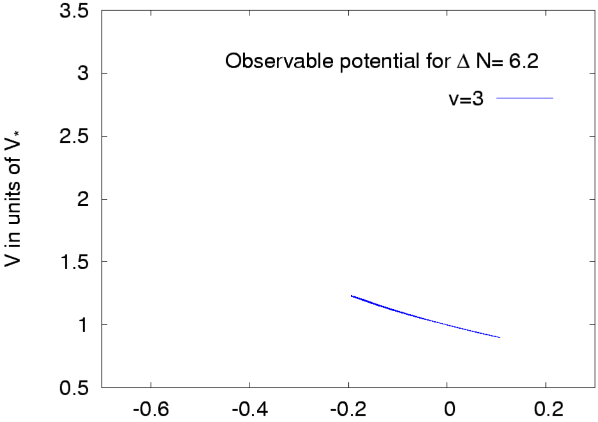
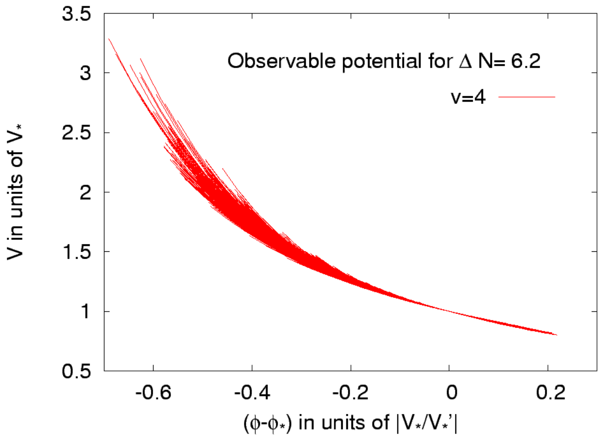
V in units of V_*
$(\phi - \phi_*)$ in units of $|V_*/V'_*|$



$(\phi - \phi_*)$ in units of $|V_*/V'_*|$


Observable potential for Δ N= 6.2
v=4
V in units of V*
(φ-φ*) in units of |V*/V*'|
3.5
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$$V' = -\frac{3m_{\rm Pl} H^2}{(4\pi)^{1/2}}\,\epsilon_1^{1/2}\left(1 - \frac{\epsilon_1}{3} + \frac{\epsilon_2}{6}\right)$$

$$\frac{V''}{3H^2} = 2\epsilon_1 - \frac{\epsilon_2}{2} - \frac{2\epsilon_1^2}{3} + \frac{5\epsilon_1\epsilon_2}{6} - \frac{\epsilon_2^2}{12} - \frac{\epsilon_2\epsilon_3}{6}$$

$$V''' = \frac{12m_p^2 H^2\sqrt{\pi}}{\sqrt{\epsilon_1}}\left(2\epsilon_1^2 - \frac{3\epsilon_2\epsilon_1}{2} + \frac{\epsilon_2\epsilon_3}{4}\right).$$