Boost factors – a hazardous kind of magic

Pierre Salati – Université de Savoie & **LAPTH**

1) Motivations for a statistical approach

2) Computing the odds of the galactic lottery

3) The example of mini–spikes around IMBHs

Cosmology and Particle Physics Beyond the Standard Models – Cargese – August 7, 2007

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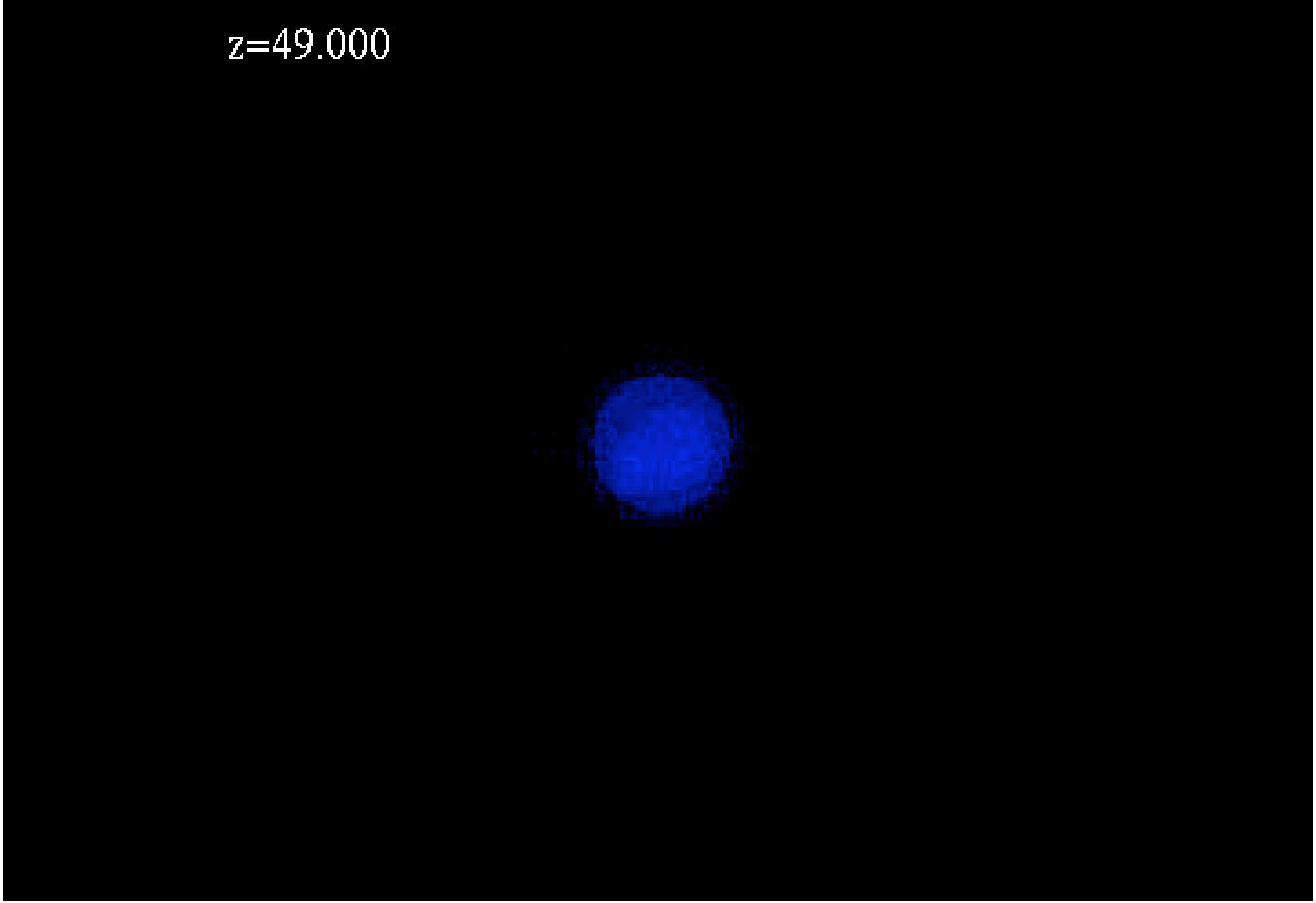
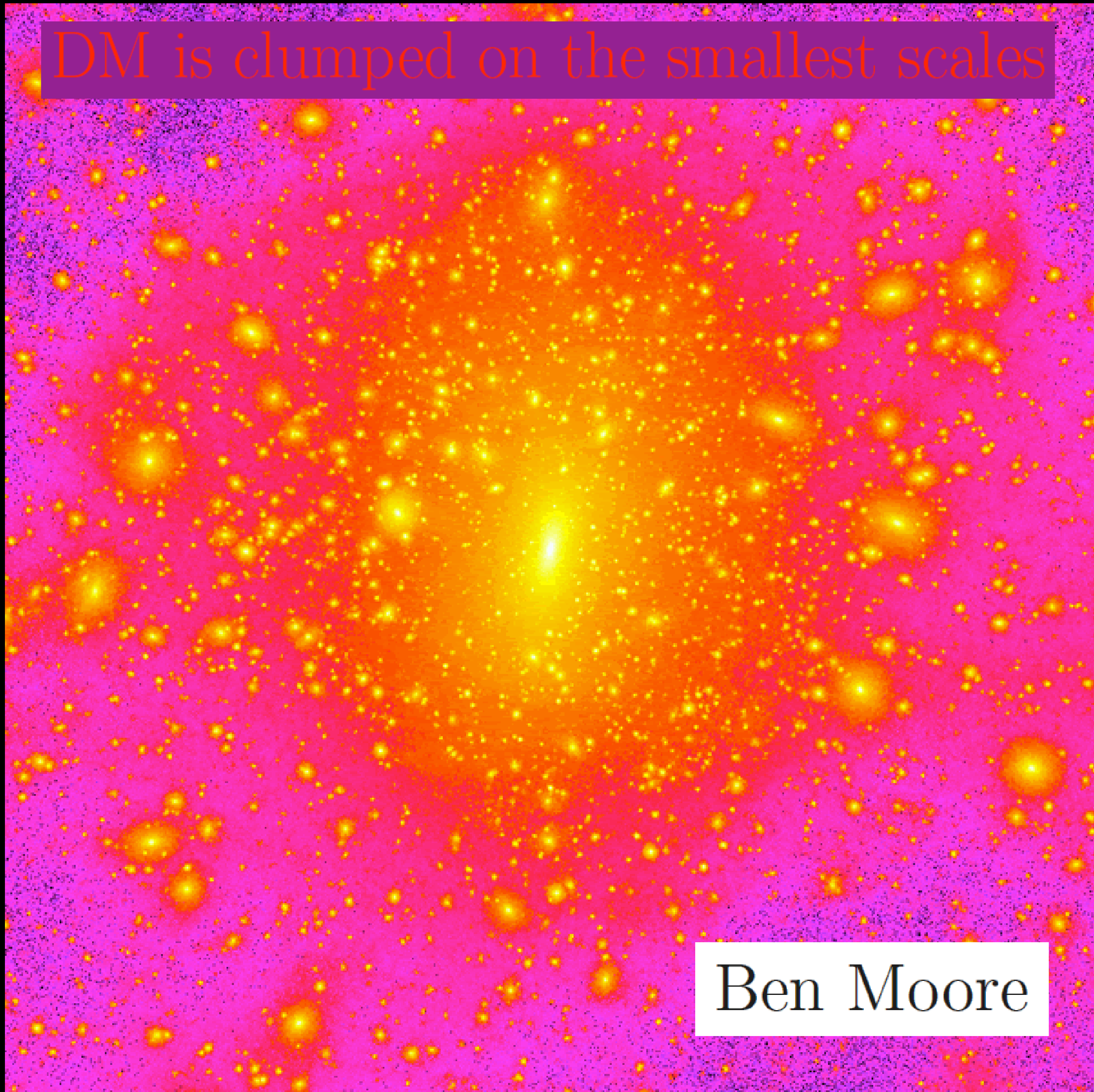
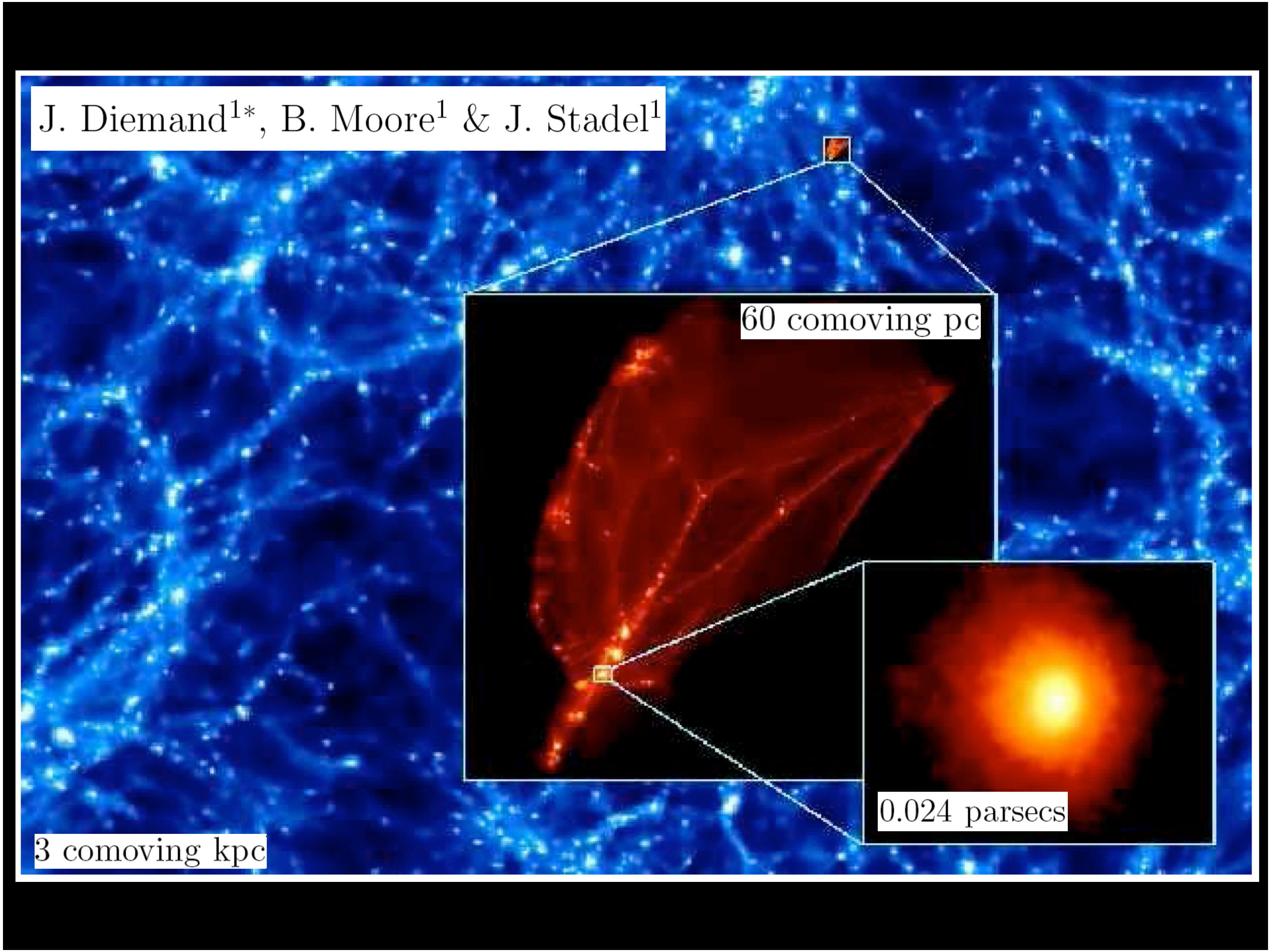
3) The example of mini-spikes around IMBHs

Numerical simulations and the formation of galaxies



DM is clumped on the smallest scales




J. Diemand1*, B. Moore1 & J. Stadel1
60 comoving pc
0.024 parsecs
3 comoving kpc

Jürg Diemand[1,2], Michael Kuhlen[1,3], & Piero Madau[1,4]

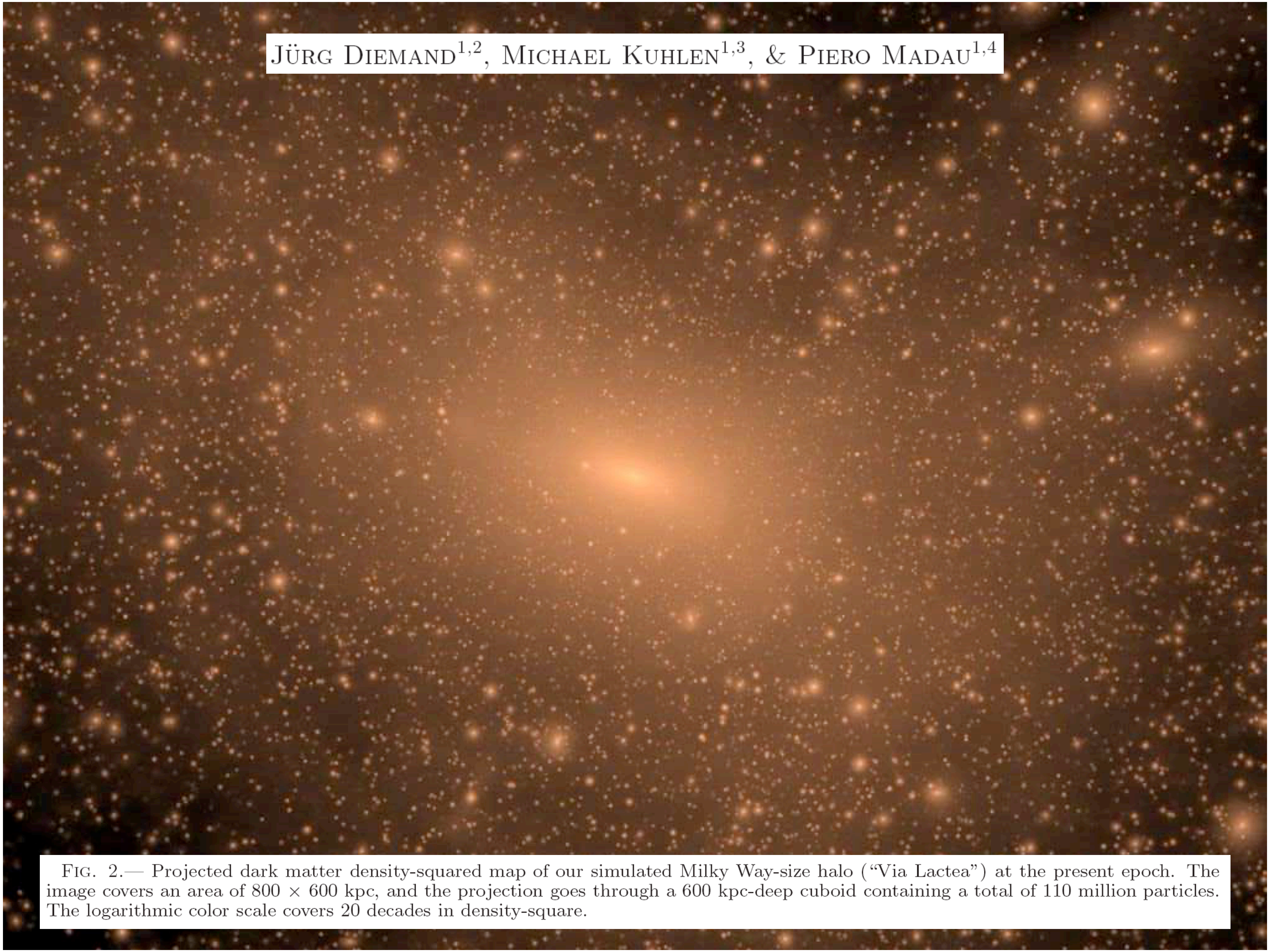
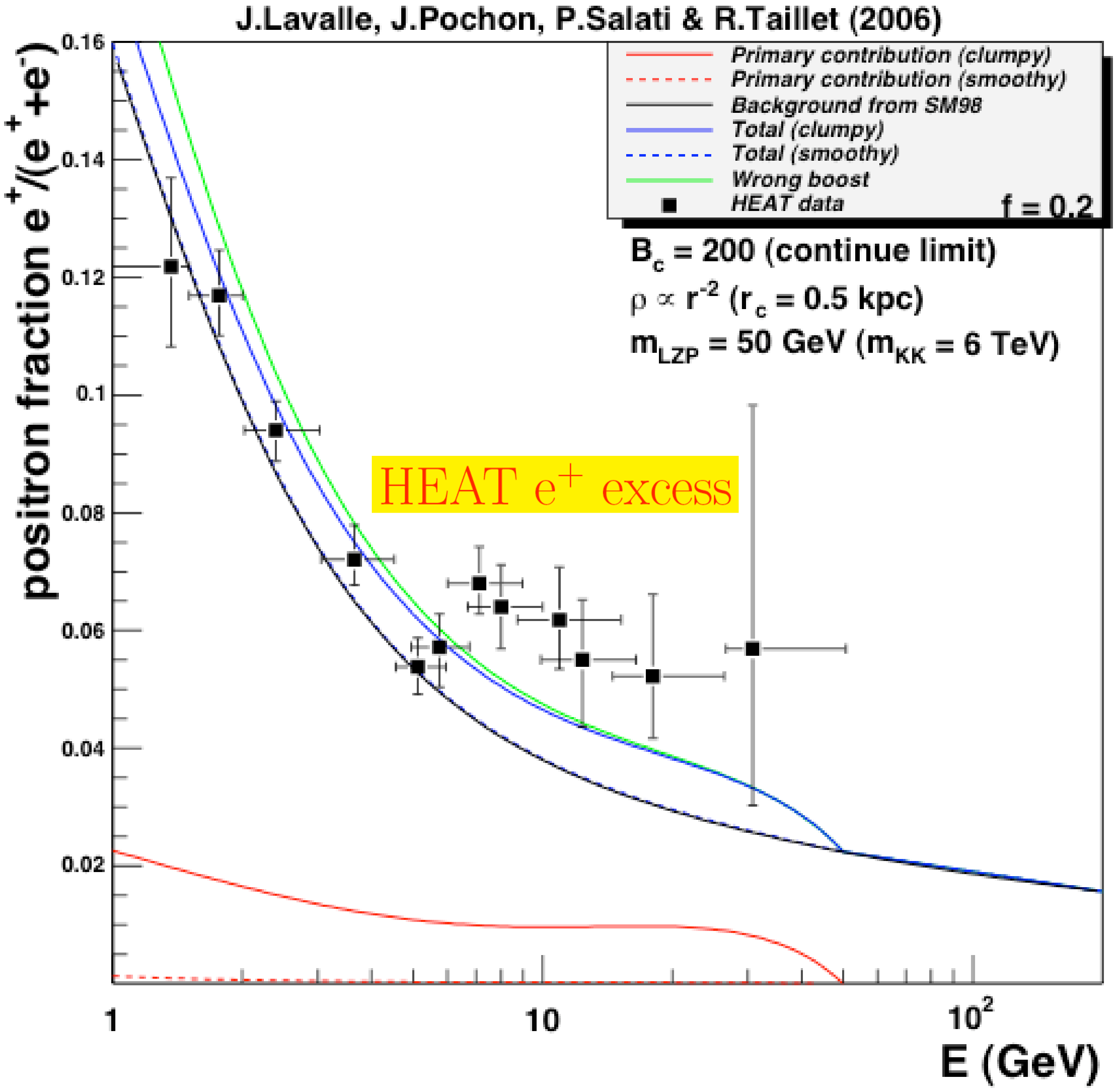
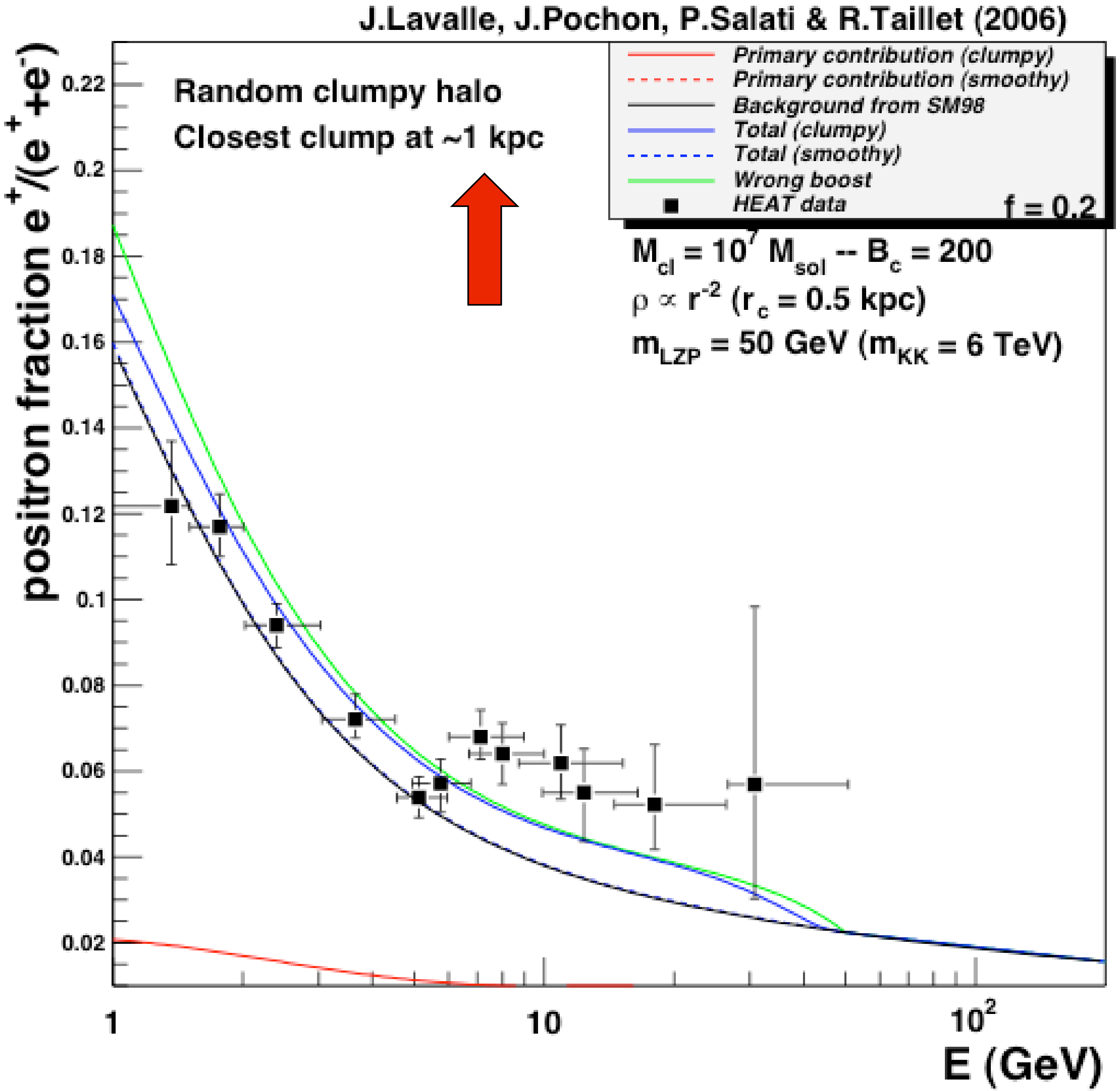


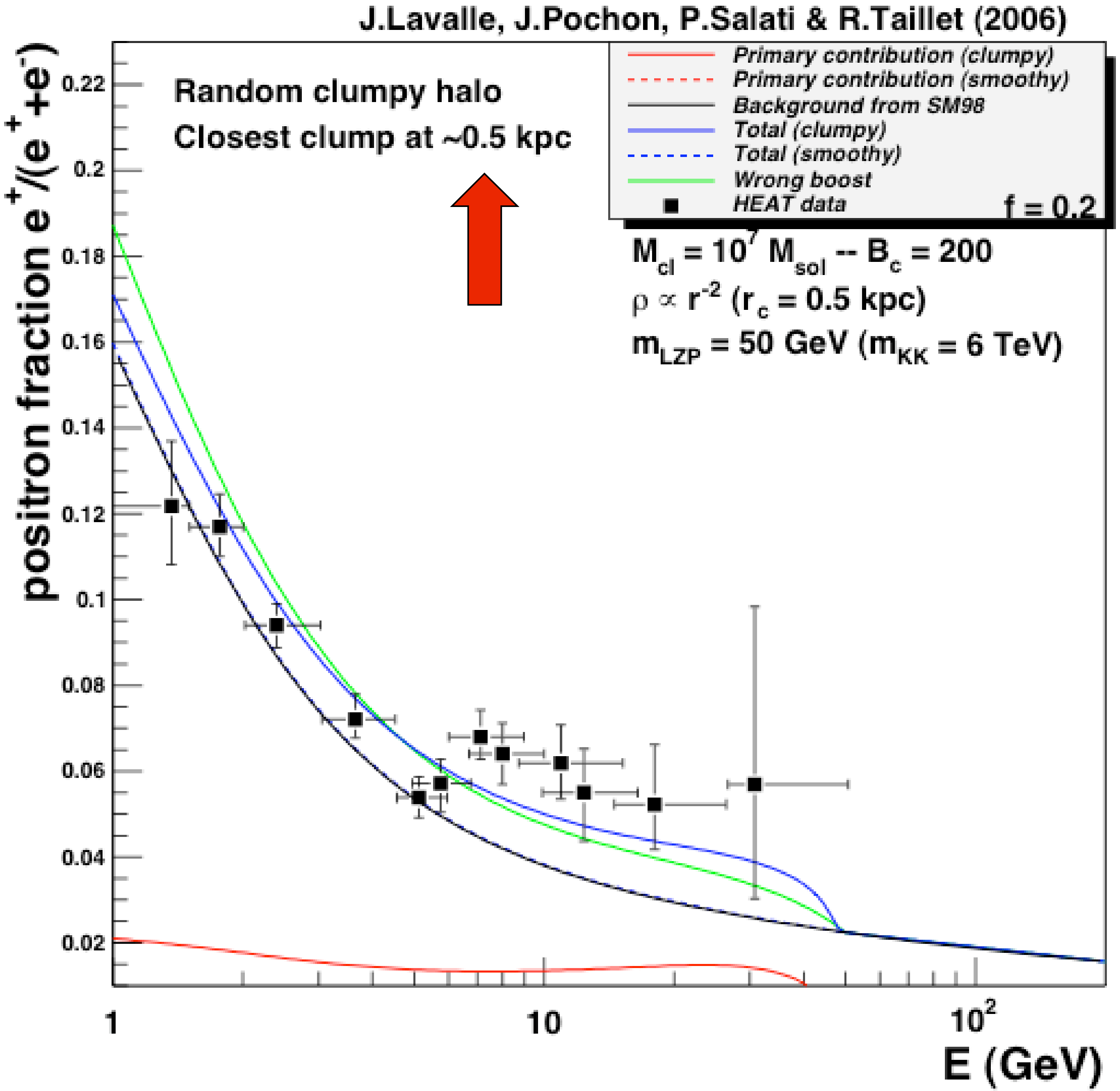
Fig. 2.— Projected dark matter density-squared map of our simulated Milky Way-size halo ("Via Lactea") at the present epoch. The image covers an area of 800 × 600 kpc, and the projection goes through a 600 kpc-deep cuboid containing a total of 110 million particles. The logarithmic color scale covers 20 decades in density-square.

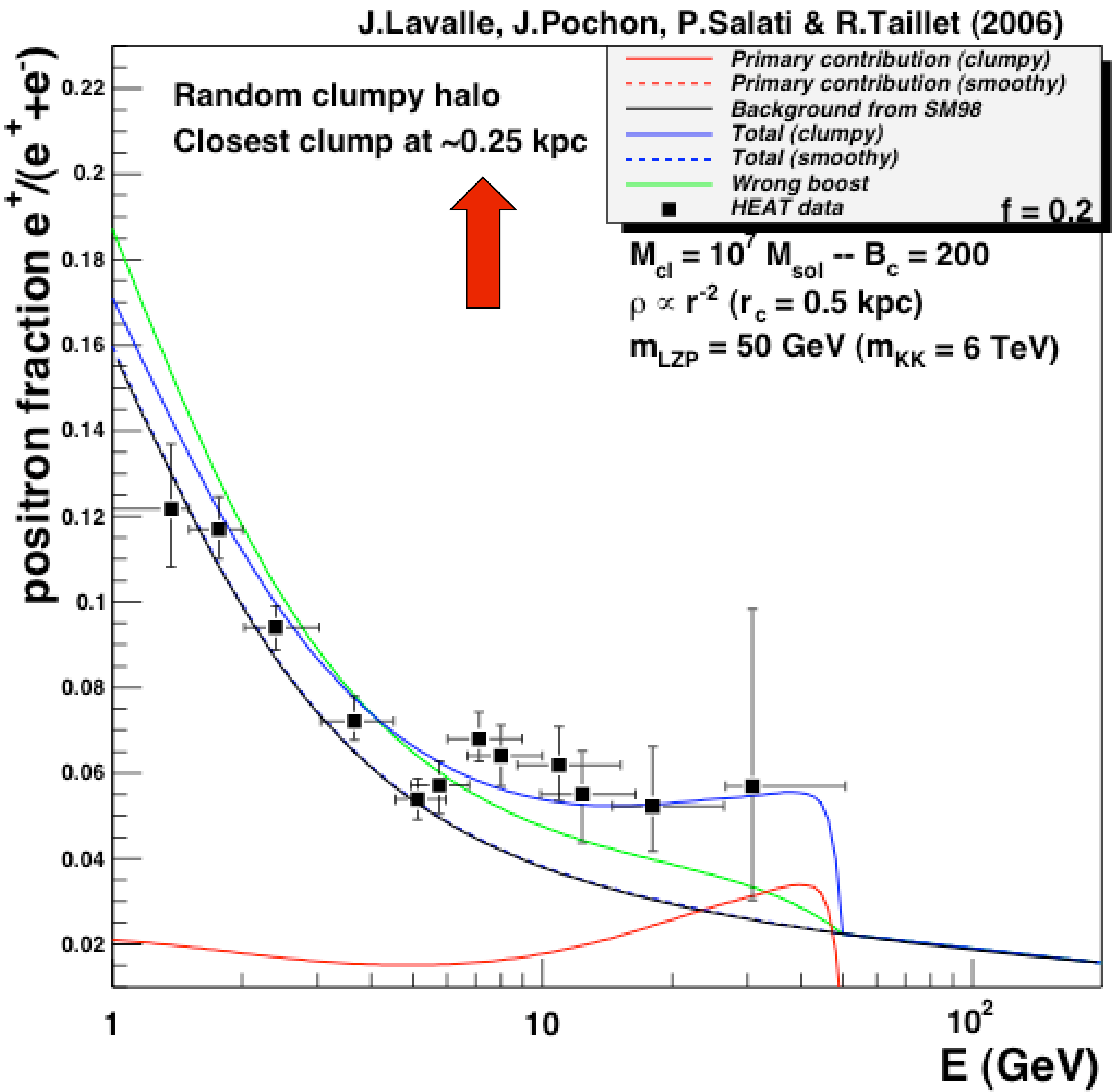


J.Lavalle, J.Pochon, P.Salati & R.Taillet (2006)
Primary contribution (clumpy)
Primary contribution (smoothy)
Background from SM98
Total (clumpy)
Total (smoothy)
Wrong boost
HEAT data
f = 0.2
B_c = 200 (continue limit)
$\rho \propto r^{-2}$ (r_c = 0.5 kpc)
m_{LZP} = 50 GeV (m_{KK} = 6 TeV)
HEAT e^+ excess
positron fraction $e^+/(e^+ + e^-)$
E (GeV)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
1
10
10^2

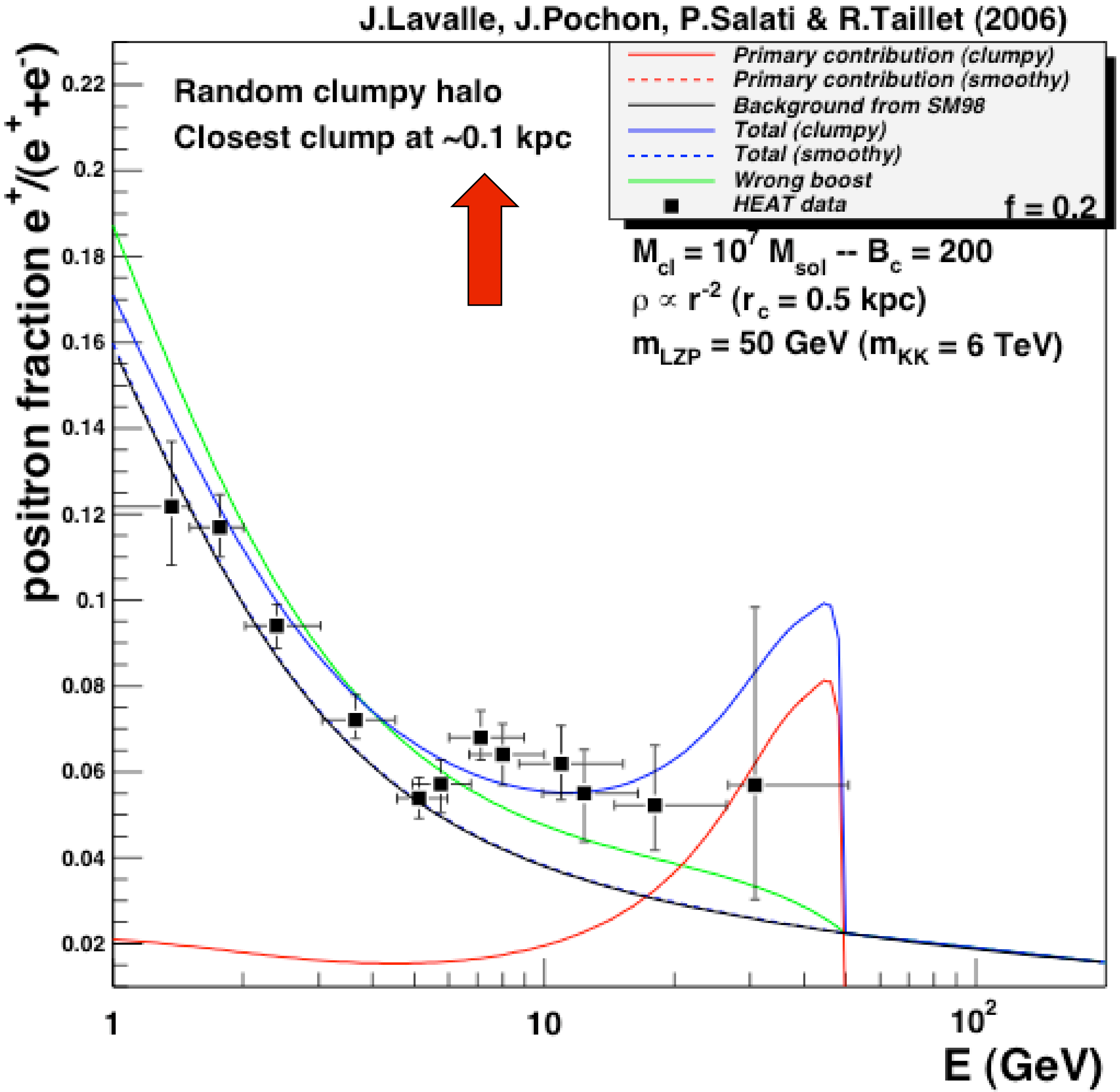


J.Lavalle, J.Pochon, P.Salati & R.Taillet (2006)
Random clumpy halo
Closest clump at ~1 kpc
Primary contribution (clumpy)
Primary contribution (smoothy)
Background from SM98
Total (clumpy)
Total (smoothy)
Wrong boost
HEAT data
f = 0.2
$M_{cl} = 10^7\ M_{sol}$ -- $B_c = 200$
$\rho \propto r^{-2}$ (r_c = 0.5 kpc)
m_{LZP} = 50 GeV (m_{KK} = 6 TeV)
positron fraction $e^+/(e^+ + e^-)$
E (GeV)


J.Lavalle, J.Pochon, P.Salati & R.Taillet (2006)
Random clumpy halo
Closest clump at ~0.5 kpc
Primary contribution (clumpy)
Primary contribution (smoothy)
Background from SM98
Total (clumpy)
Total (smoothy)
Wrong boost
HEAT data
f = 0.2
$M_{cl} = 10^7$ M_{sol} -- $B_c = 200$
$\rho \propto r^{-2}$ (r_c = 0.5 kpc)
m_{LZP} = 50 GeV (m_{KK} = 6 TeV)
positron fraction $e^+/(e^++e^-)$
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
1
10
10^2
E (GeV)

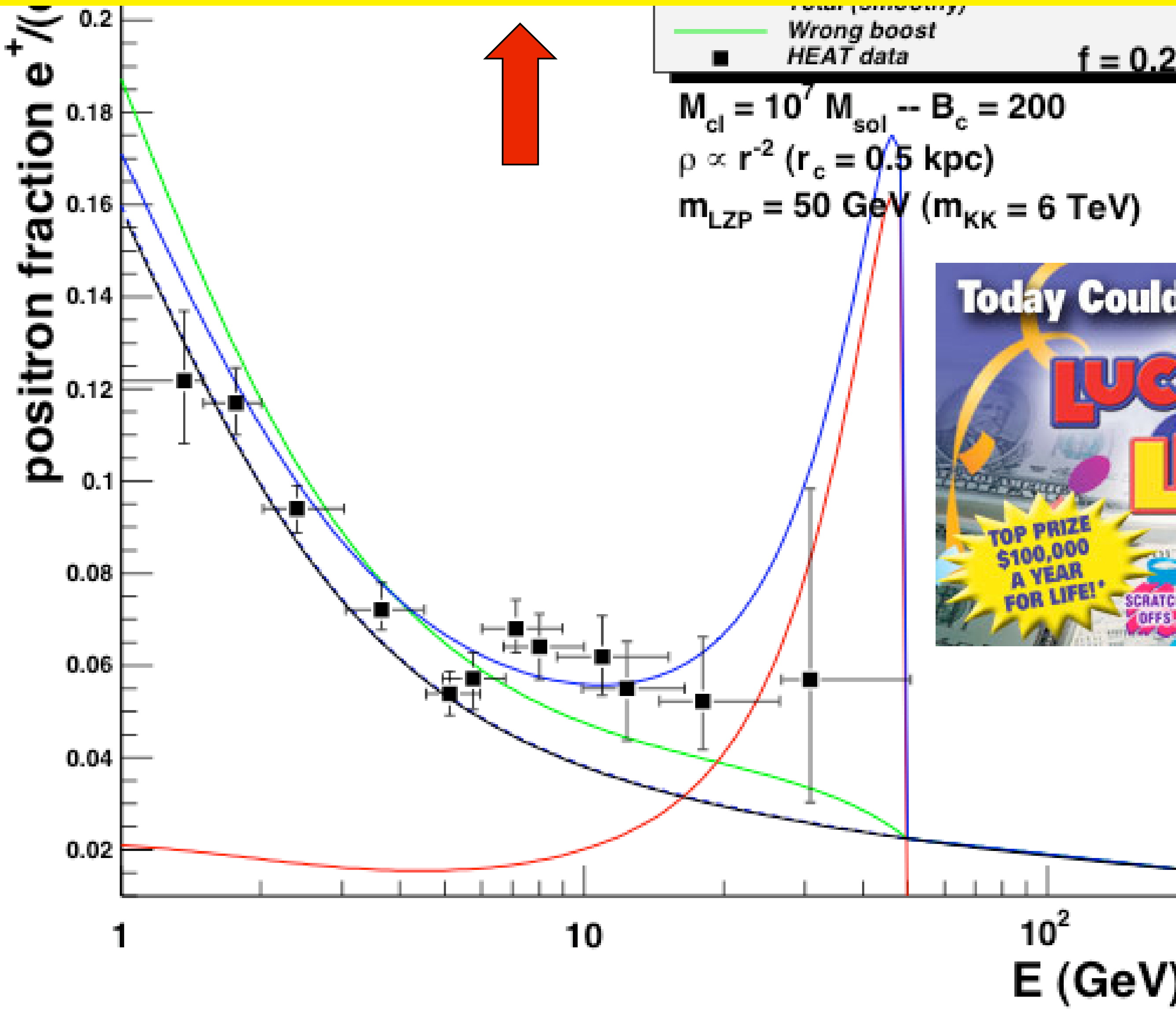


J.Lavalle, J.Pochon, P.Salati & R.Taillet (2006)
Random clumpy halo
Closest clump at ~0.25 kpc
Primary contribution (clumpy)
Primary contribution (smoothy)
Background from SM98
Total (clumpy)
Total (smoothy)
Wrong boost
HEAT data
f = 0.2
$M_{cl} = 10^7\ M_{sol}$ -- $B_c = 200$
$\rho \propto r^{-2}$ (r_c = 0.5 kpc)
m_{LZP} = 50 GeV (m_{KK} = 6 TeV)
positron fraction $e^+/(e^+ + e^-)$
E (GeV)
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
1
10
10^2



J.Lavalle, J.Pochon, P.Salati & R.Taillet (2006)
Random clumpy halo
Closest clump at ~0.1 kpc
Primary contribution (clumpy)
Primary contribution (smoothy)
Background from SM98
Total (clumpy)
Total (smoothy)
Wrong boost
HEAT data
f = 0.2
$M_{cl} = 10^7\ M_{sol}$ -- $B_c = 200$
$\rho \propto r^{-2}$ (r_c = 0.5 kpc)
m_{LZP} = 50 GeV (m_{KK} = 6 TeV)
positron fraction $e^+/(e^+ + e^-)$
E (GeV)

How probable is that ?



Boost factors – a hazardous kind of magic

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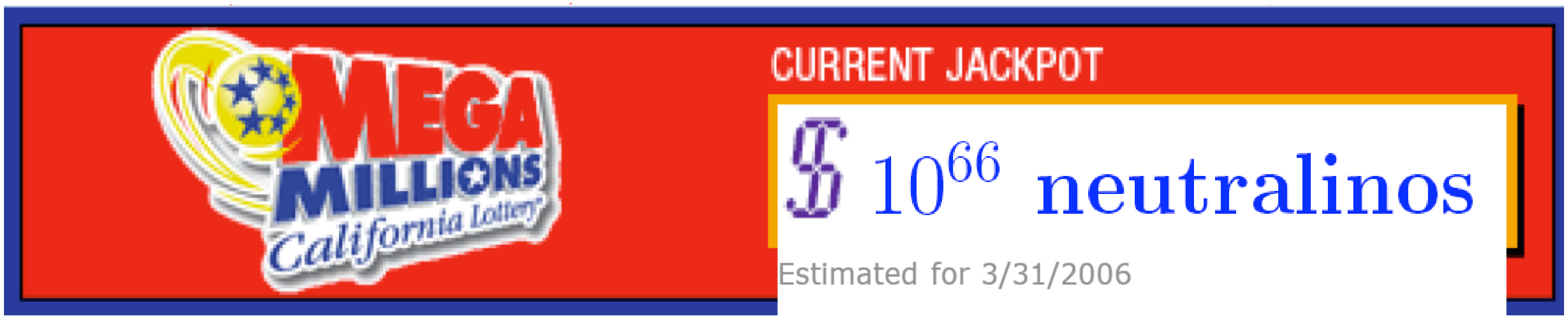


- Without clumps – with the smooth DM distribution ρ_s

$$\phi_s \;=\; \mathcal{S} \int_{\rm DZ} G(\mathbf{x}) \, \frac{\rho_s^2(\mathbf{x})}{\rho_\odot^2} \, d^3\mathbf{x}$$

- With clumps – with the DM distribution $\rho = \rho'_s + \delta\rho$

$$\phi = \phi'_s + \left(\phi_r = \sum_i \varphi_i \right)$$

$$\varphi_i \;=\; \mathcal{S} \times G(\mathbf{x}_i) \times \left\{ \xi_i = \frac{B_i \, M_i}{\rho_\odot} = \int_{\rm ith\,clump} \frac{\delta\rho^2(\mathbf{x})}{\rho_\odot^2} \, d^3\mathbf{x} \right\}$$

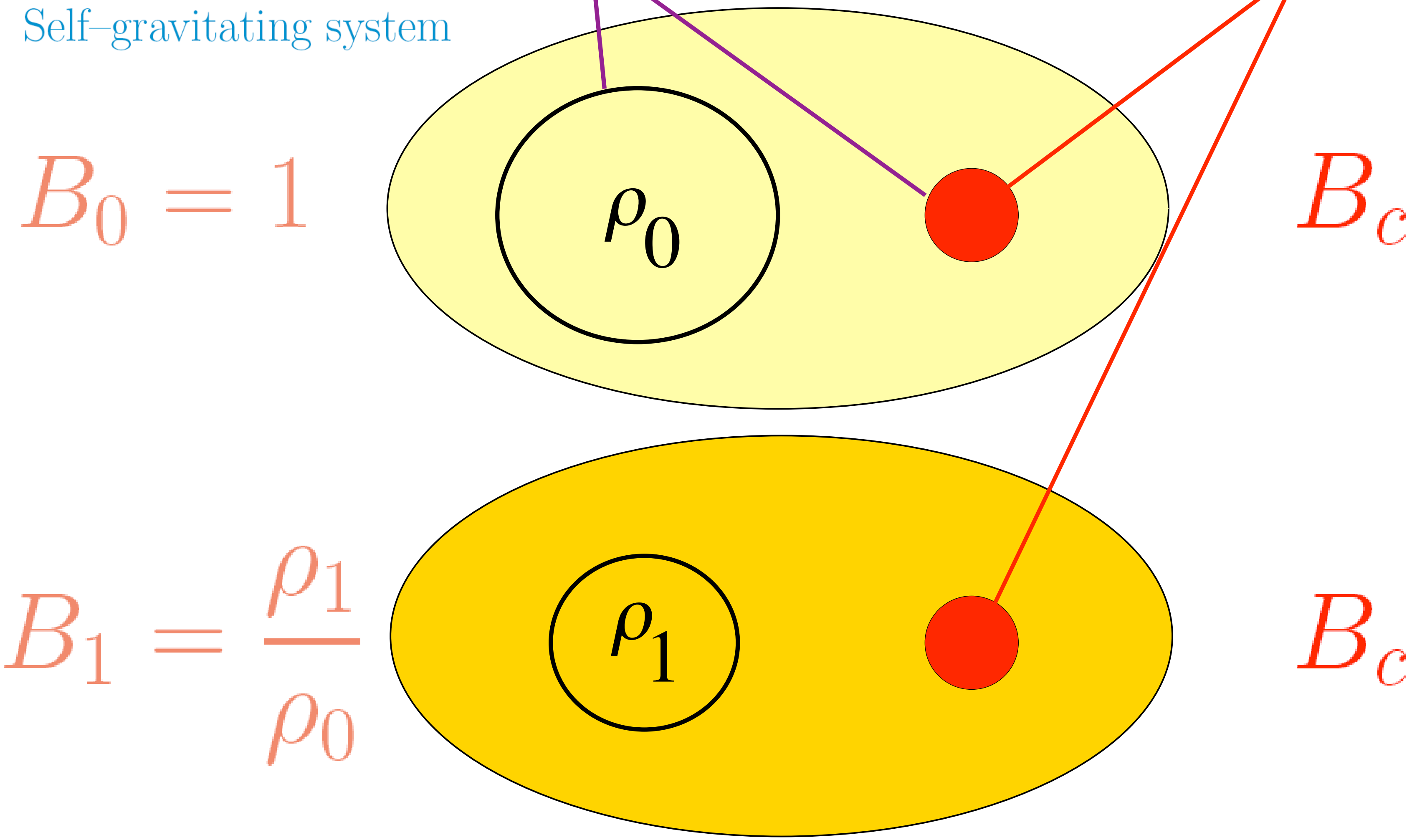
random behaviour !

$$\Downarrow$$

Boost factor $B \equiv \dfrac{\phi}{\phi_s}$

The clump boost factor B_c

$$\int_{\text{clump}} d^3\vec{x}\;\; \delta\rho(\vec{x}) \;=\; \boxed{M_c} \quad \text{and} \quad \int_{\text{clump}} d^3\vec{x}\;\; \delta\rho^2(\vec{x}) \;=\; \boxed{B_c\,\rho_0}\, M_c$$



Statistical analysis of the signal for a 100 GeV positron line

(i) To simplify the discussion – without loss of generality – we assume that clumps are identical.

$$\phi = \phi'_s \; + \; \left(\phi_r = \sum_i \varphi_i = \mathcal{S} \times \frac{B_c \, M_c}{\rho_0} \times \sum_i G_i \right)$$

(ii) The actual distribution of DM substructures is one particular realization $\in$ statistical ensemble of all the possible **random** distributions.

$$\langle \phi_r \rangle \quad \text{and} \quad \sigma_r^2 = \langle \phi_r^2 \rangle \; - \; \langle \phi_r \rangle^2$$

$$B_{\text{eff}} = \langle B = \phi / \phi_s \rangle \quad \text{and} \quad \sigma_B = \sigma_r / \phi_s$$

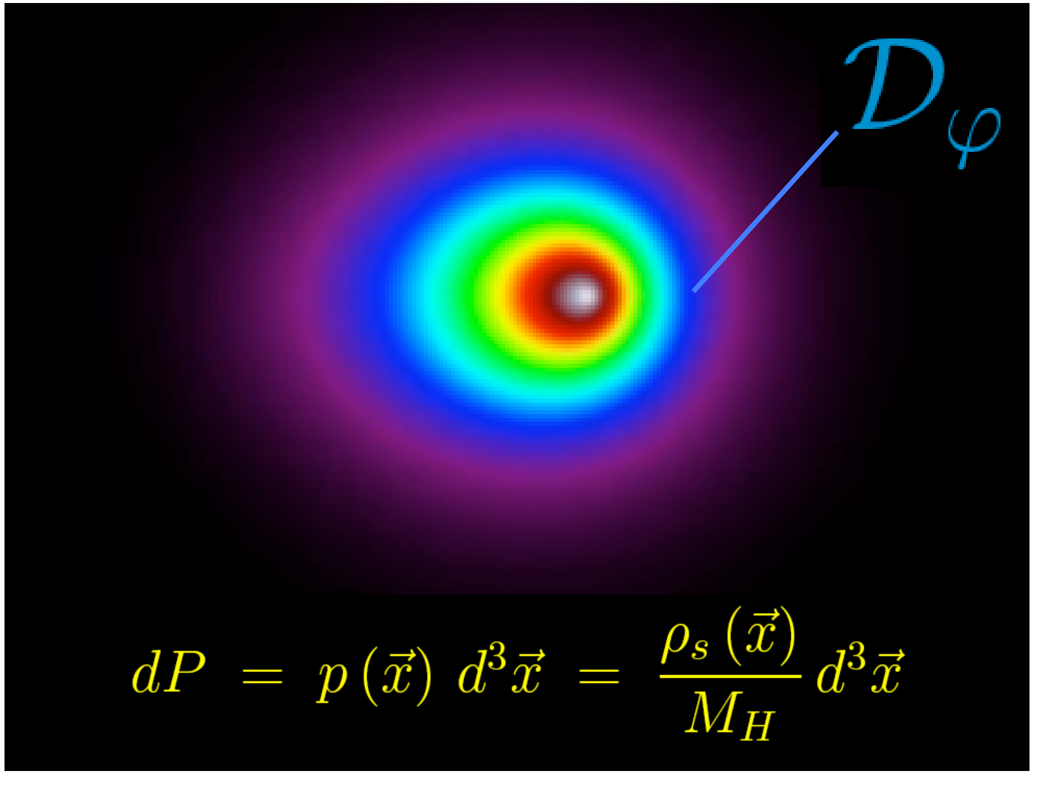
(iii) Clumps are distributed independently of each other. Therefore, we just need to determine how a single clump is distributed inside the galactic halo in order to derive the statistical properties of an entire constellation of N_H such substructures.

$$\langle \phi_r \rangle = N_H \, \langle \varphi \rangle \quad \text{and} \quad \sigma_r^2 = N_H \, \sigma^2 = N_H \left\{ \langle \varphi^2 \rangle \; - \; \langle \varphi \rangle^2 \right\}$$

(iv) The set of the random distributions of one single clump inside the domain $\mathcal{D}_H$ forms the statistical ensemble $\mathcal{T}$ which we need to consider. An event from that ensemble consists in a clump located at position $\vec{x}$ within the elementary volume $d^3\vec{x}$.

$$\mathcal{P}(\varphi)\, d\varphi \;=\; dP \;=\; \int_{\mathcal{D}_\varphi} p(\vec{x})\, d^3\vec{x}$$

$$\langle \mathcal{F} \rangle \;=\; \int \mathcal{F}(\varphi)\, \mathcal{P}(\varphi)\, d\varphi \;=\; \int_{\mathcal{D}_H} \mathcal{F}\{\varphi(\vec{x})\}\, p(\vec{x})\, d^3\vec{x}$$



(v) In the following analysis we have furthermore assumed that clumps trace the smooth DM distribution. **This is not generally correct – see IMBHs !**

$$dP \;=\; p\,(\vec{x})\;d^3\vec{x} \;=\; \frac{\rho_s\,(\vec{x})}{M_H}\,d^3\vec{x}$$

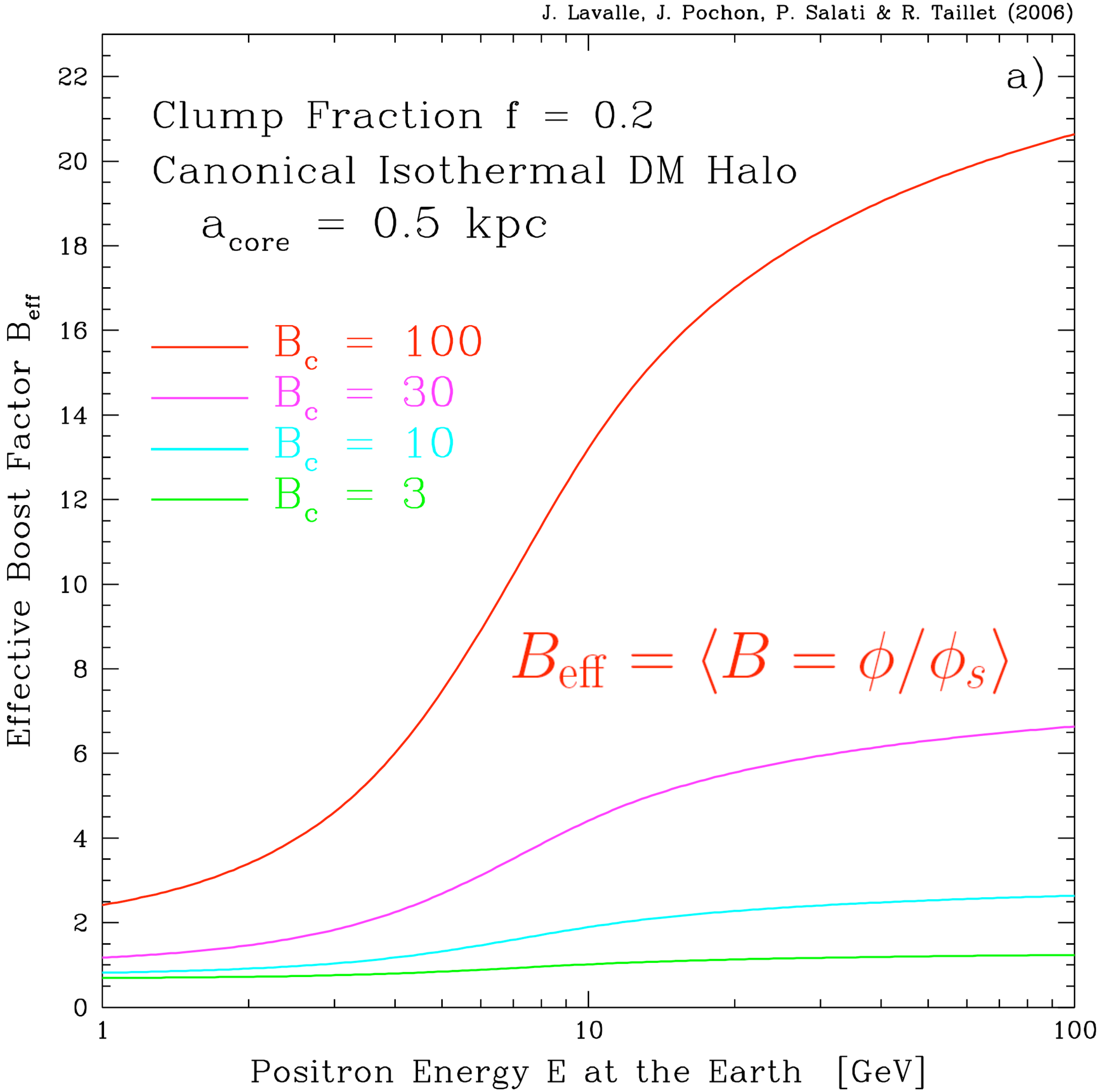
(vi) We have finally assumed that $\rho'_s \equiv (1-f) \times \rho_s$ so that

$$\phi = (1-f)^2\,\phi_s \;+\; \left(\phi_r = \mathcal{S} \times \frac{B_c\,M_c}{\rho_0} \times \sum_i G_i\right)$$

$$\Downarrow$$

Analytical determination of B_{eff} and σ_B

Comparison between analytical and Monte-Carlo

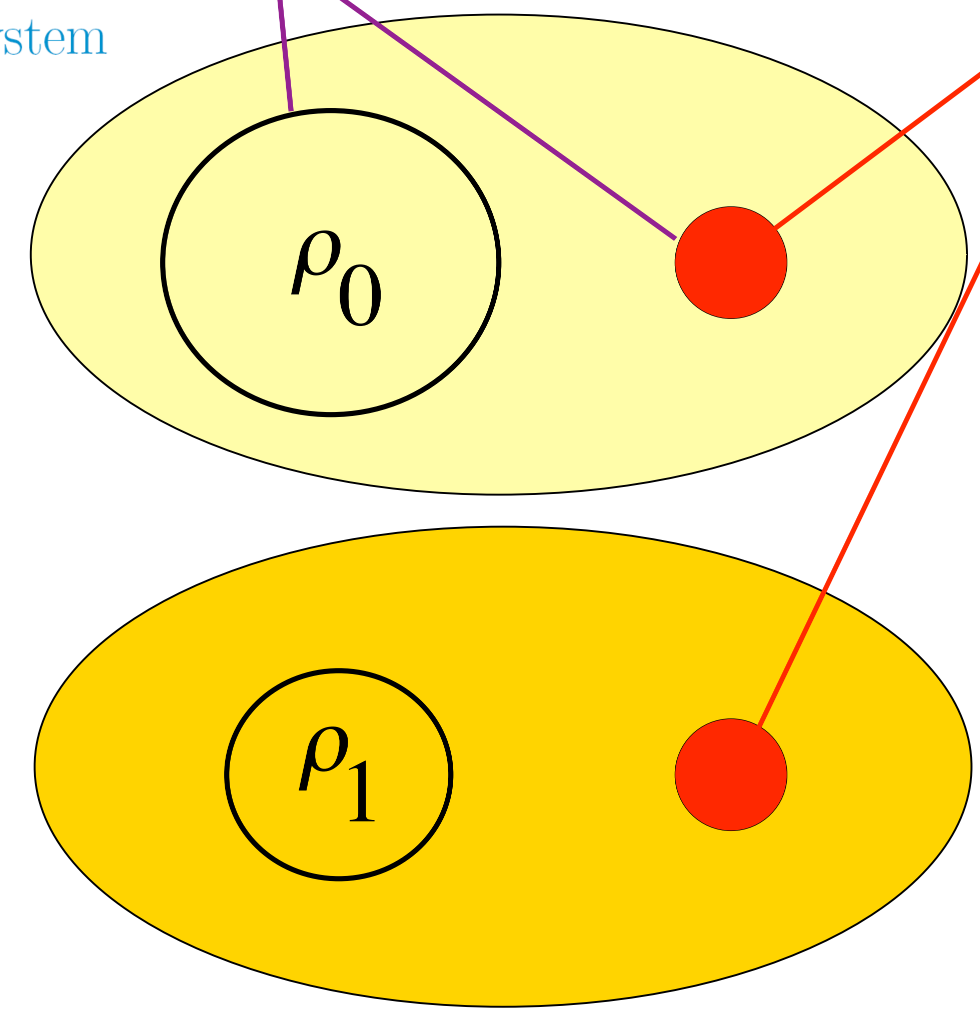


J. Lavalle, J. Pochon, P. Salati & R. Taillet (2006)
a)
Clump Fraction f = 0.2
Canonical Isothermal DM Halo
a_{core} = 0.5 kpc
B_c = 100
B_c = 30
B_c = 10
B_c = 3
$B_{\rm eff} = \langle B = \phi/\phi_s \rangle$
Effective Boost Factor B_{eff}
Positron Energy E at the Earth [GeV]
0
2
4
6
8
10
12
14
16
18
20
22
1
10
100

The clump boost factor B_c

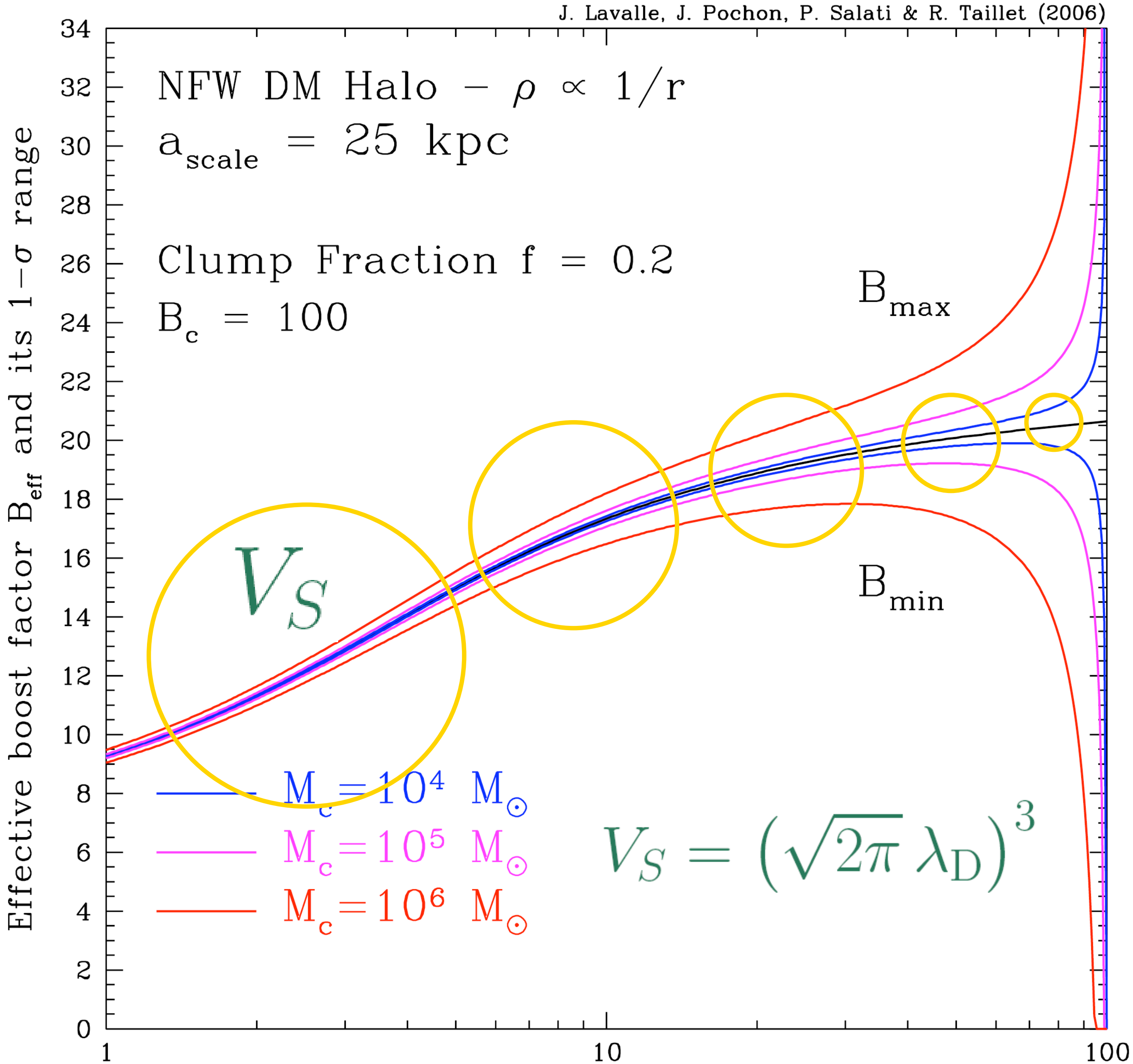
$$\int_{\text{clump}} d^3\vec{x} \;\; \delta\rho(\vec{x}) \; = \boxed{M_c} \quad \text{and} \quad \int_{\text{clump}} d^3\vec{x} \;\; \delta\rho^2(\vec{x}) \; = \boxed{B_c \, \rho_0} \, M_c$$

Self–gravitating system



$$B_0 = 1 \qquad B_c$$

$$B_1 = \frac{\rho_1}{\rho_0} \qquad B_c$$



V_S increases as E decreases

The hard–sphere approach

The signal originates from the volume $V_S = \left(\sqrt{2\pi}\,\lambda_{\rm D}\right)^3$

Binomial density of probability for one clump

$$\mathcal{P}(\varphi) \;=\; p\,\delta(\varphi - \varphi_{\max}) \;+\; (1-p)\;\delta(\varphi)$$

n clumps inside

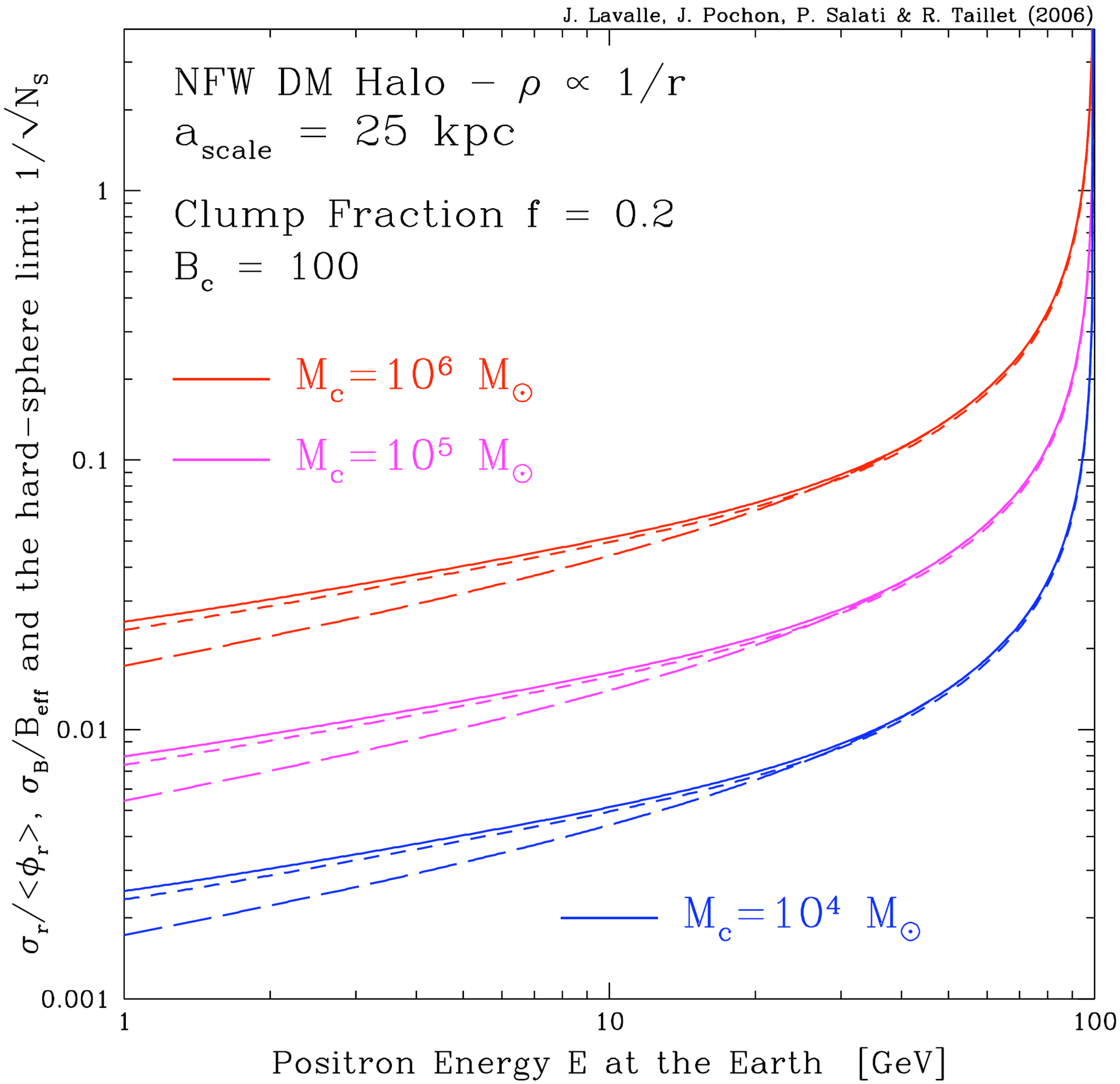
$$p = \frac{M_S}{M_H} = \frac{V_S\,\rho_s(\odot)}{M_H} \qquad (1-p)$$

Poisson distribution

$$P(n) \;=\; \frac{{N_S}^n}{n!}\;\exp\left(-N_S\right) \text{ where } N_S \equiv p\,N_H$$

On average $\langle n \rangle$ clumps generate the signal

$$\langle n \rangle = N_S = \frac{V_S\,f\,\rho_s(\odot)}{M_c} \text{ and } \frac{\sigma_r}{\langle \phi_r \rangle} = \frac{\sigma_n}{\langle n \rangle} = \frac{1}{\sqrt{N_S}}$$


J. Lavalle, J. Pochon, P. Salati & R. Taillet (2006)
NFW DM Halo − $\rho \propto 1/r$
a_{scale} = 25 kpc
Clump Fraction f = 0.2
B_c = 100
$M_c = 10^6\ M_\odot$
$M_c = 10^5\ M_\odot$
$M_c = 10^4\ M_\odot$
$\sigma_r/\langle\phi_r\rangle$, σ_B/B_{eff} and the hard−sphere limit $1/\sqrt{N_S}$
Positron Energy E at the Earth [GeV]
0.001
0.01
0.1
1
1
10
100

The large N_S limit

The Poisson statistics becomes Maxwellian

$$P\,(\delta \equiv n - N_S) \;=\; \frac{1}{\sqrt{2\pi N_S}}\,\exp\left(-\,\delta^2/2N_S\right)$$

The 1–clump distribution $\mathcal{P}(\varphi)$ is continuous !

The theorem of the central limit

$$\mathcal{P}\left\{\phi_r = \sum_i \varphi_i\right\} \;=\; \frac{1}{\sqrt{2\pi\sigma_r^2}}\,\exp\left\{-\,\frac{\left(\phi_r - \langle\phi_r\rangle\right)^2}{2\sigma_r^2}\right\}$$

$$\mathcal{P}\{B \equiv \phi/\phi_s\} \;=\; \frac{1}{\sqrt{2\pi\sigma_B^2}}\,\exp\left\{-\,\frac{\left(B - B_{\mathrm{eff}}\right)^2}{2\sigma_B^2}\right\}$$

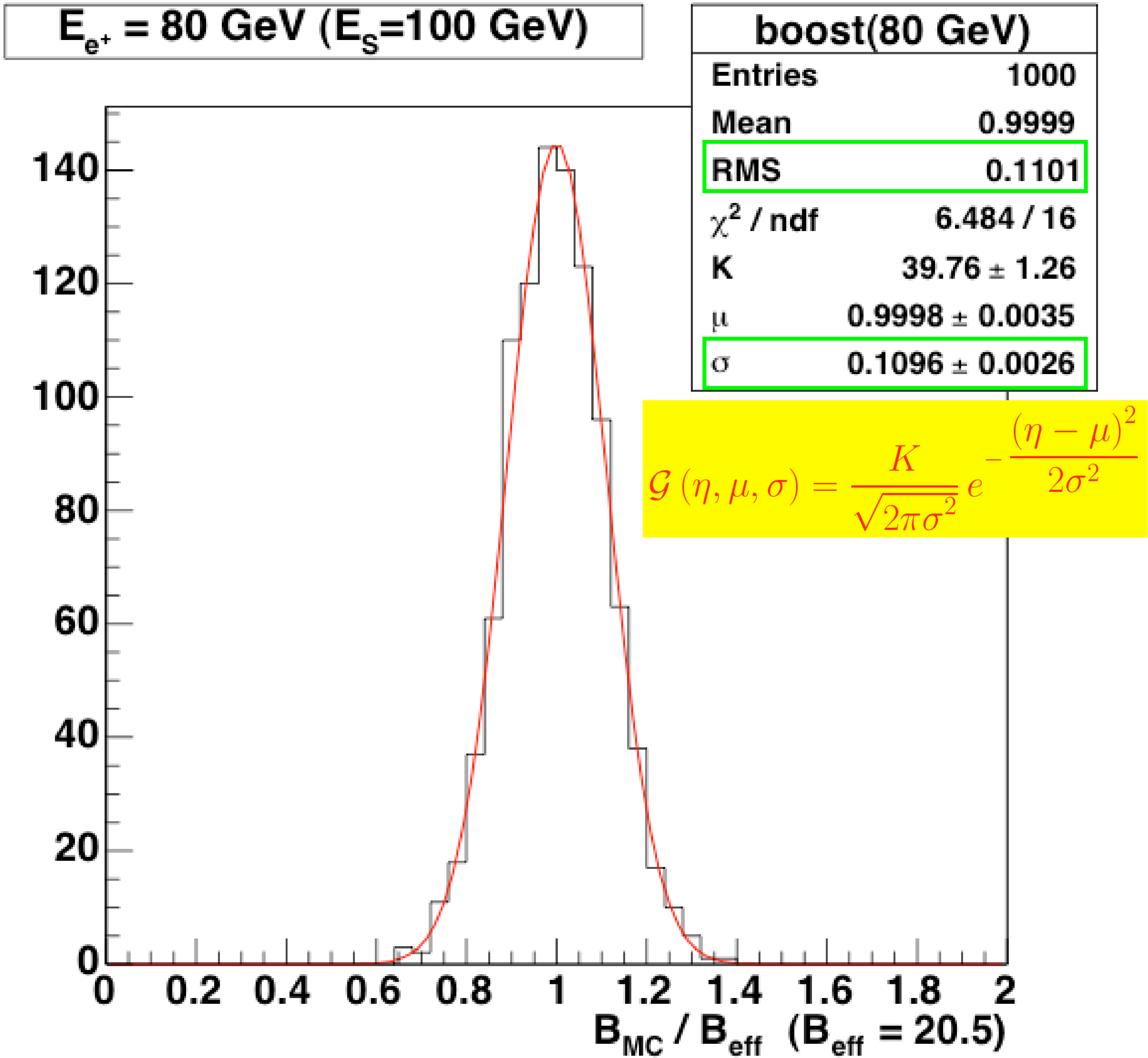
$$\mathcal{P}\{\eta \equiv B/B_{\mathrm{eff}}\} \;=\; \frac{1}{\sqrt{2\pi\sigma_\eta^2}}\,\exp\left\{-\,\frac{\left(\eta - 1\right)^2}{2\sigma_\eta^2}\right\}$$

Analytical versus Monte Carlo

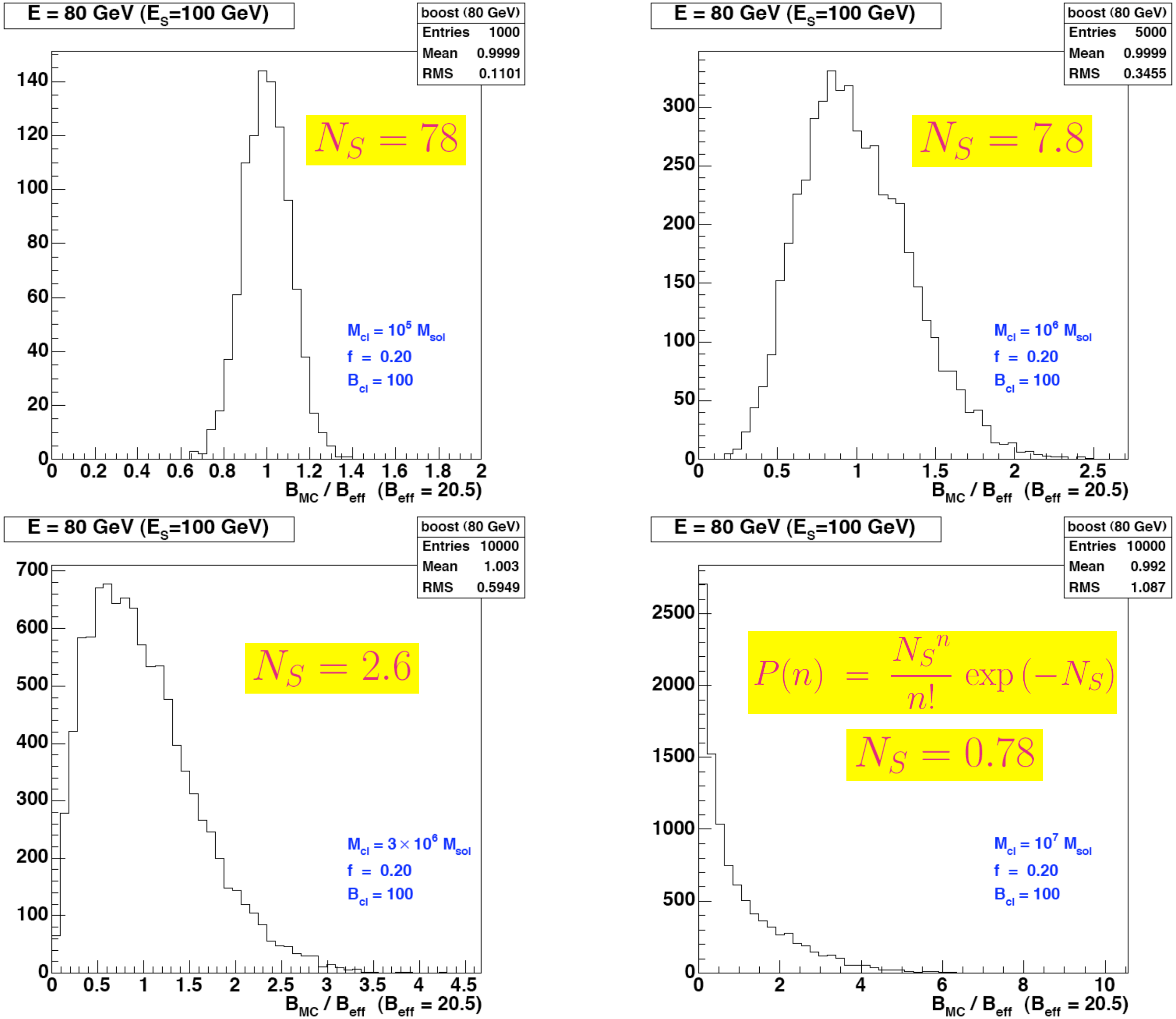
- Substructures contribute a fraction $f = 0.2$ to the mass of the Milky Way dark matter halo.
- A NFW profile is assumed with typical scale 25 kpc.
- Each clump has a mass of 10^5 $M_\odot$.
- 10^3 different Monte–Carlo realizations of the distribution of substructures.
- Each Monte–Carlo realization involves 271,488 clumps.
- The positron injection energy is $E_S = 100$ GeV.

To be compared to the analytical results

E	$B_{\rm eff}$	$\sigma_\eta = \sigma_B / B_{\rm eff}$	N_S
50	20.09	0.04338	498.0
65	20.32	0.06472	223.5
80	20.48	0.10966	78.0
90	20.57	0.19680	24.2



E_{e^+} = 80 GeV (E_S=100 GeV)
boost(80 GeV)
Entries 1000
Mean 0.9999
RMS 0.1101
χ^2 / ndf 6.484 / 16
K 39.76 ± 1.26
μ 0.9998 ± 0.0035
σ 0.1096 ± 0.0026
$\mathcal{G}(\eta,\mu,\sigma) = \frac{K}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\eta-\mu)^2}{2\sigma^2}}$
0
20
40
60
80
100
120
140
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
B_{MC} / B_{eff} (B_{eff} = 20.5)



E = 80 GeV (E_S=100 GeV)
boost (80 GeV)
Entries 1000
Mean 0.9999
RMS 0.1101
$N_S = 78$
$M_{cl} = 10^5 M_{sol}$
f = 0.20
$B_{cl} = 100$
B_{MC} / B_{eff} (B_{eff} = 20.5)
E = 80 GeV (E_S=100 GeV)
boost (80 GeV)
Entries 5000
Mean 0.9999
RMS 0.3455
$N_S = 7.8$
$M_{cl} = 10^6 M_{sol}$
f = 0.20
$B_{cl} = 100$
B_{MC} / B_{eff} (B_{eff} = 20.5)
E = 80 GeV (E_S=100 GeV)
boost (80 GeV)
Entries 10000
Mean 1.003
RMS 0.5949
$N_S = 2.6$
$M_{cl} = 3 \times 10^6 M_{sol}$
f = 0.20
$B_{cl} = 100$
B_{MC} / B_{eff} (B_{eff} = 20.5)
E = 80 GeV (E_S=100 GeV)
boost (80 GeV)
Entries 10000
Mean 0.992
RMS 1.087
$P(n) = \frac{N_S{}^n}{n!} \exp(-N_S)$
$N_S = 0.78$
$M_{cl} = 10^7 M_{sol}$
f = 0.20
$B_{cl} = 100$
B_{MC} / B_{eff} (B_{eff} = 20.5)

The small N_S limit

The distribution of probability is the product of convolution

$$\mathcal{P}_{N_H} = \mathcal{P}_1 * \mathcal{P}_1 * \ldots * \mathcal{P}_1$$

$$\mathcal{P}_{N_H}\left\{0 \leq \varphi \leq \varphi_{\max}\right\} \simeq N_H \times \mathcal{P}_1\left(\varphi\right)$$

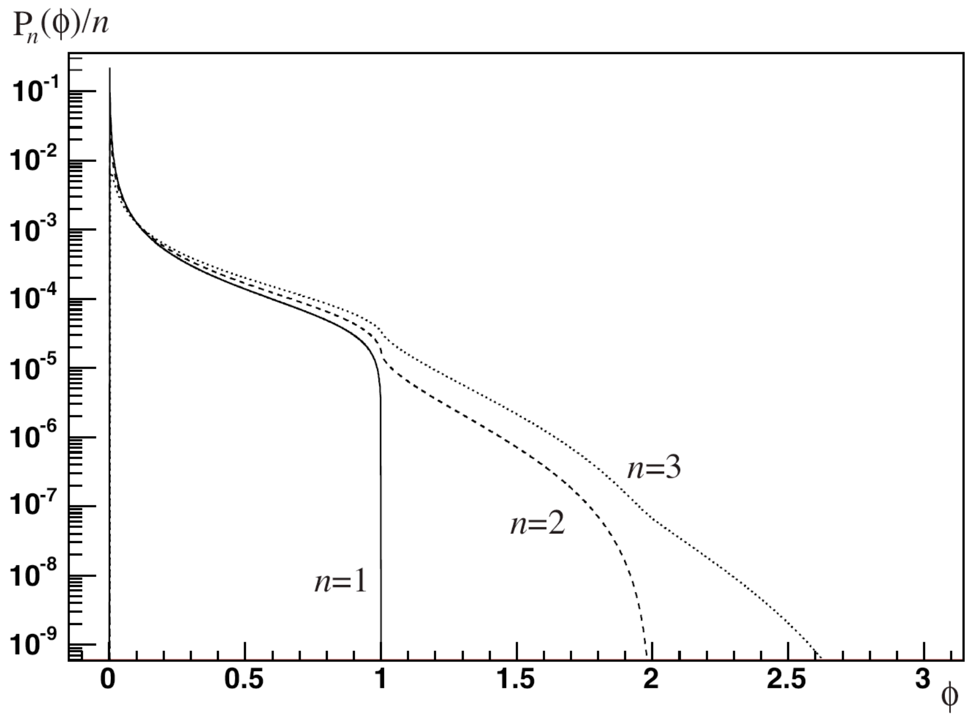
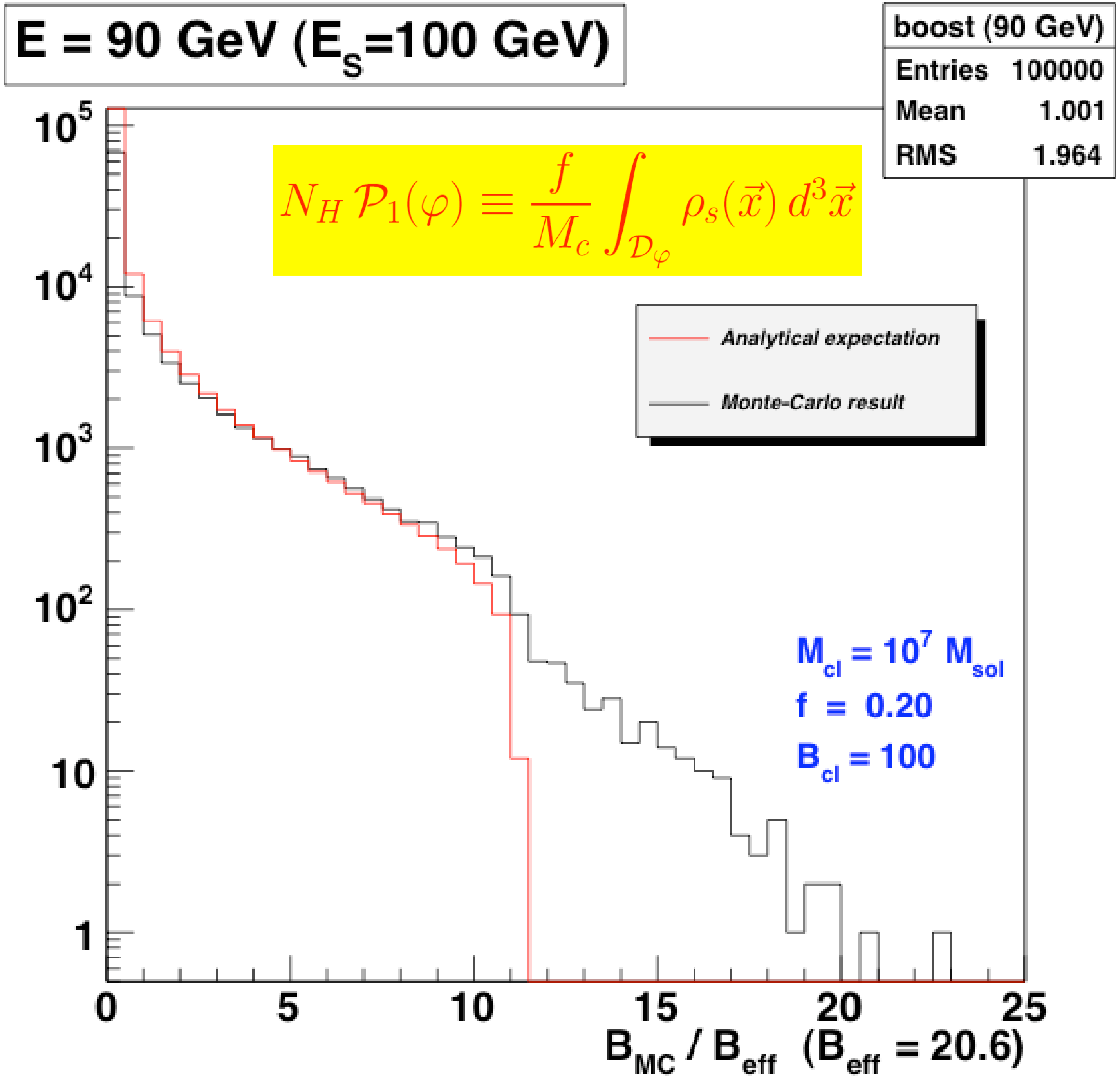


Fig. 10. Probability distribution $\mathcal{P}_n(\phi)/n$ for $n = 1$, 2 and 3, obtained by consecutive convolutions of $\mathcal{P}_1(\phi)$.



E = 90 GeV (E_S=100 GeV)
boost (90 GeV)
Entries 100000
Mean 1.001
RMS 1.964
$N_H\,\mathcal{P}_1(\varphi) \equiv \frac{f}{M_c}\int_{\mathcal{D}_\varphi}\rho_s(\vec{x})\,d^3\vec{x}$
Analytical expectation
Monte-Carlo result
M_cl = 10^7 M_sol
f = 0.20
B_cl = 100
B_MC / B_eff (B_eff = 20.6)

Substructures, Cosmic Rays and Statistics

(i) The position $\mathbf{x}_i$ and annihilation volume ξ_i of the ith clump are **not** known and should be treated as random variables. In some cases, neither the number N_H of MW substructures is well determined – see IMBHs for instance.

$$\phi = \phi'_s \; + \; \left(\phi_r = \sum_{i=1}^{N_H} \varphi_i = \mathcal{S} \times \sum_{i=1}^{N_H} \xi_i \times G_i \right)$$

(ii) The average boost factor $B_{\rm eff}$ and its variance σ_B are **energy dependent**. The boost factor is not just a rescaling constant !

$$\langle \phi_r \rangle \quad \text{and} \quad \sigma_r^2 = \langle \phi_r^2 \rangle \; - \; \langle \phi_r \rangle^2$$

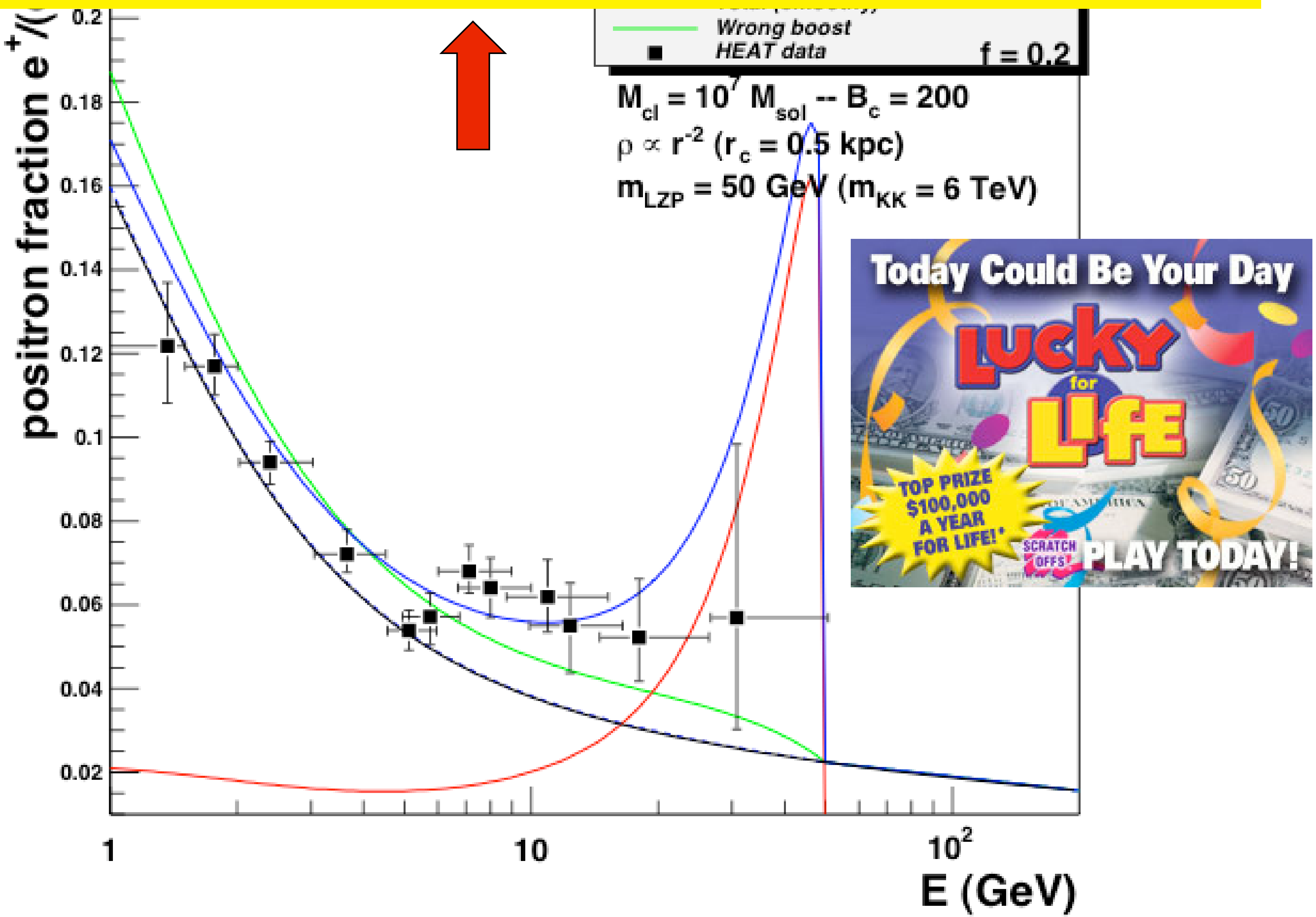
$$B_{\rm eff} = \langle B = \phi / \phi_s \rangle \quad \text{and} \quad \sigma_B = \sigma_r / \phi_s$$

(iii) Two statistical regimes appear depending on N_S – the average number of clumps that participate in the signal at the Earth.

$$\frac{\sigma_r}{\langle \phi_r \rangle} \simeq \frac{\sigma_B}{B_{\rm eff}} \simeq \frac{1}{\sqrt{N_S}}$$

- Large $N_S \Rightarrow$ maxwellian distribution.
- Small $N_S \Rightarrow \mathcal{P}_{N_H} \left\{ 0 \leq \varphi \leq \varphi_{\rm max} \right\} \simeq N_H \times \mathcal{P}_1 \left(\varphi \right)$

Probability of 2×10^{-4}



Boost factors – a hazardous kind of magic

Pierre Salati – Université de Savoie & **LAPTH**

1) Motivation for a statistical approach

2) Computing the odds of the cosmic lottery

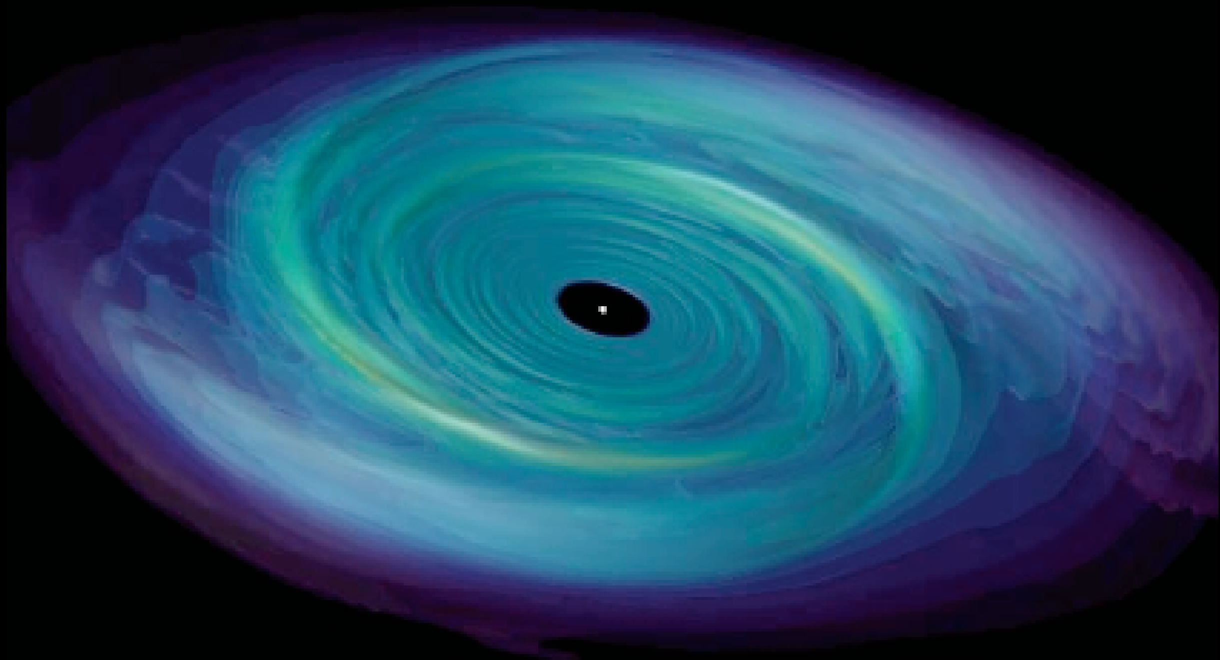
3) The example of mini–spikes around IMBHs

Cosmology and Particle Physics Beyond the Standard Models – Cargese – August 7, 2007

Intermediate mass black holes (IMBHs) – an illustration

G. Bertone, A.R. Zentner & J. Silk, PRD **72** (2005) 103517

When the first DM halos form, gas cools and collapses as pressure supported disks

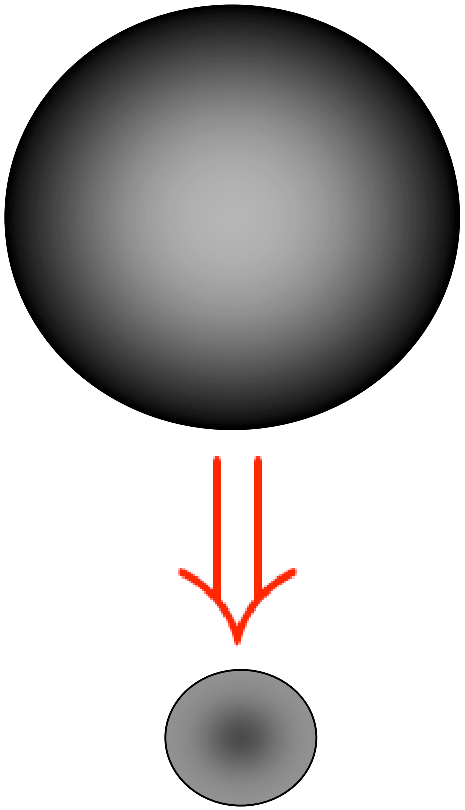


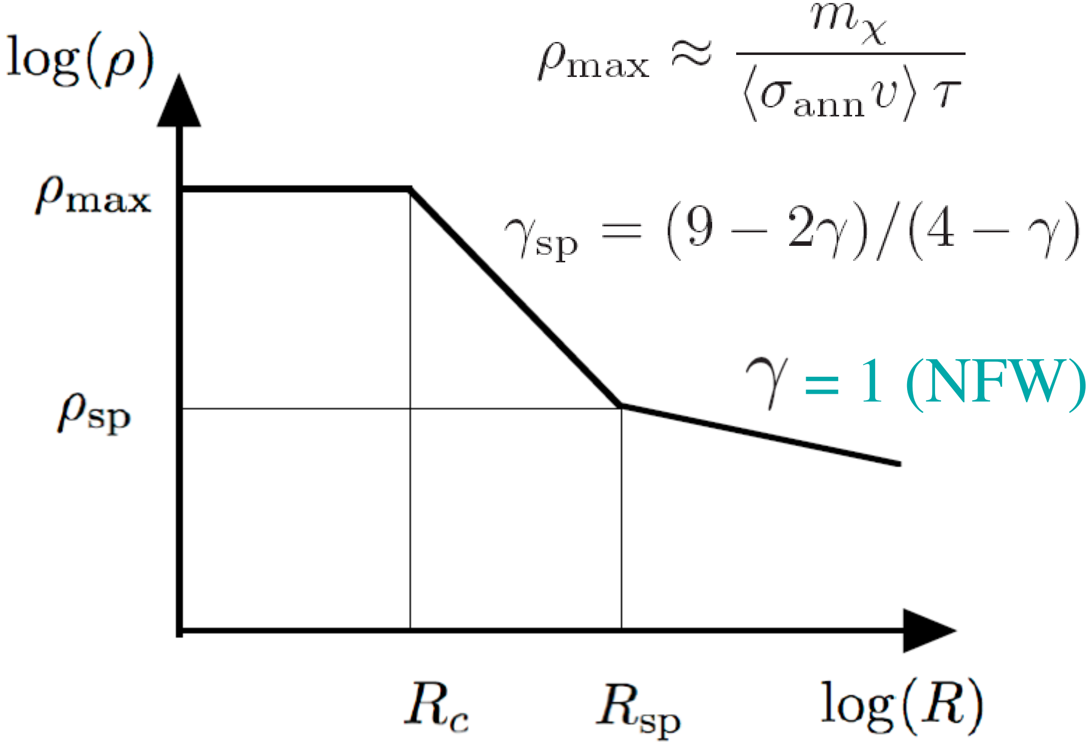
A baryonic mass of $\sim 10^5\ M_\odot$ looses its angular momentum

It is transferred at the center to form an Intermediate Mass Black Hole

During the process, DM is adiabatically compressed onto this central object

Adiabatic DM compression around the IMBH





$$\rho_{\rm max} \approx \frac{m_\chi}{\langle \sigma_{\rm ann} v \rangle \tau}$$

$$\gamma_{\rm sp} = (9 - 2\gamma)/(4 - \gamma)$$

Large annihilation volume .84 pc

pc^{-3}

DM mini-spike $\propto \langle \sigma_{\rm ann} v \rangle^{2/7} \, m_\chi^{-9/7}$

$\equiv \left\{ \frac{\rho_{\rm max}}{\rho_{\rm sp}} \right\}^{3/7}$

Compensation between $\rho_{\rm max}$ & $\langle \sigma_{\rm ann} v \rangle$

ξ eV

MC distributions of the annihilation volume ξ and position $\mathbf{x}$

$q(\xi)$

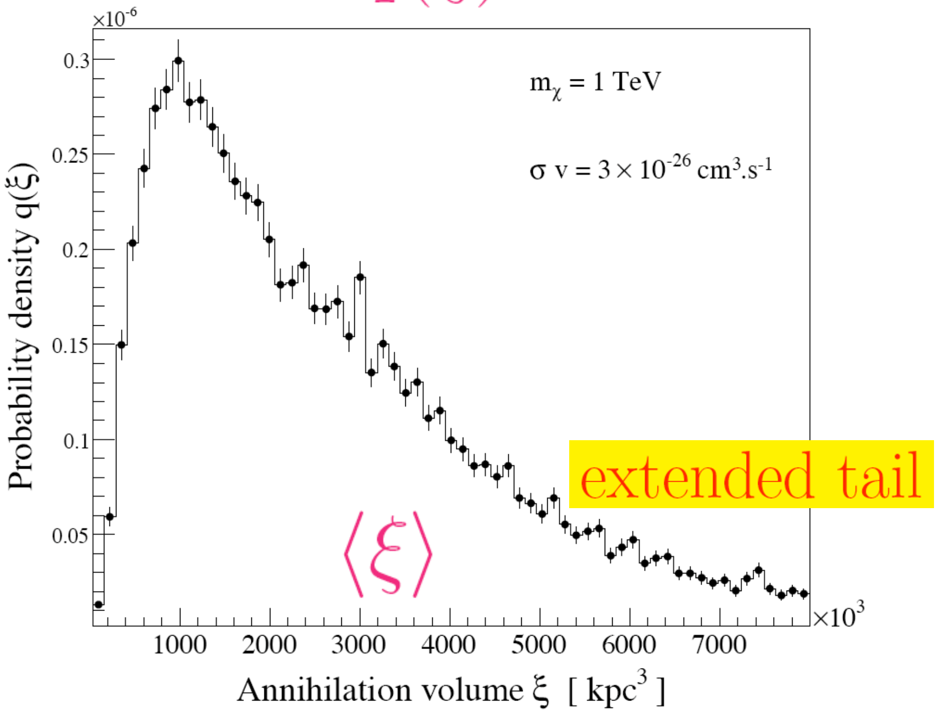


FIG. 3: The probability law $q(\xi)$ for the annihilation volume has been derived from the Monte-Carlo results of Ref. [40]. We found no correlation with the mini-spike position.

$p(\mathbf{x})$

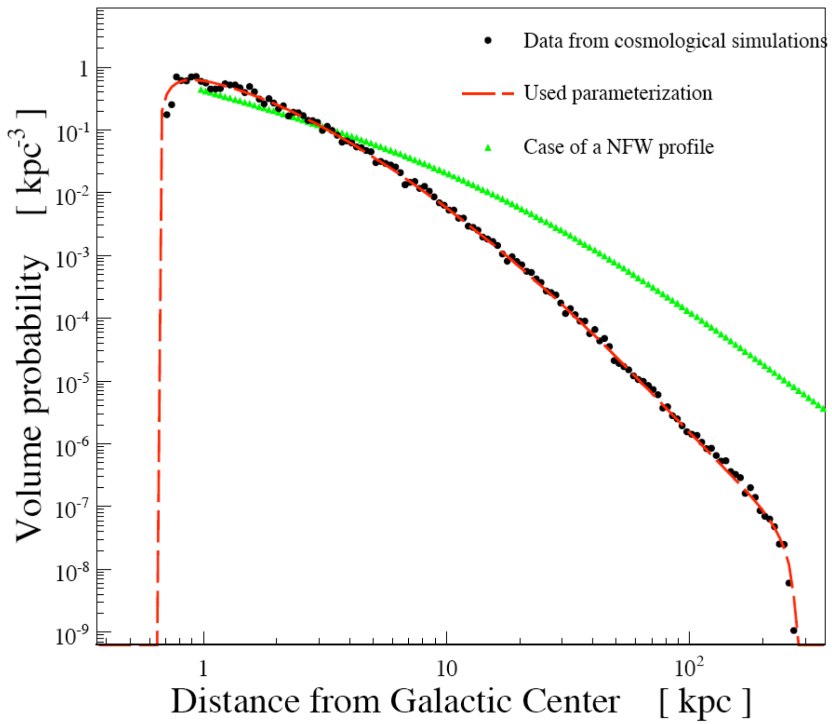
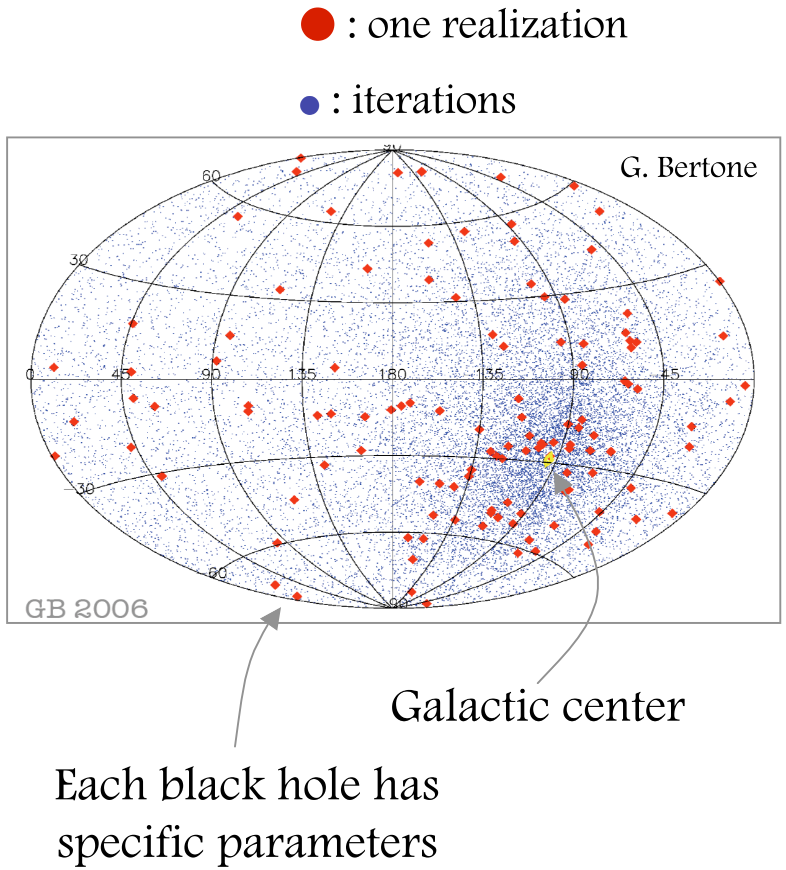


FIG. 4: Radial distribution of the mini-spikes, as extracted from the numerical results of Ref. [40].

$$\langle \xi^n \rangle = \int_0^{+\infty} \xi^n \, q(\xi) \, d\xi \quad \& \quad \langle G^n \rangle = \int_{\mathrm{DZ}} \{G(\mathbf{x})\}^n \, p(\mathbf{x}) \, d^3\mathbf{x}$$



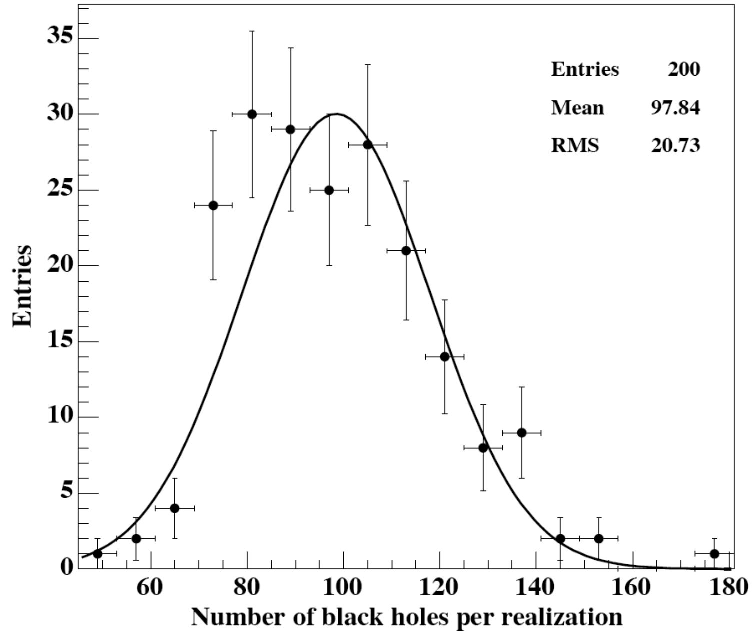


FIG. 1: Distribution of the Monte-Carlo realizations of the galactic mini-spike population – extracted from Ref. [40] – as a function of the number $N_{\rm BH}$ of objects within a galactocentric radius of 100 kpc.

$$B_{\rm eff} = 1 + \frac{\langle\phi_r\rangle}{\phi_s} = 1 + N_{\rm BH}\,\frac{\langle\xi\rangle\langle G\rangle}{\mathcal{I}}$$

$$\frac{\sigma_B}{B_{\rm eff}} = \frac{\sigma_r/\phi_s}{1+\langle\phi_r\rangle/\phi_s} \simeq \frac{\sigma_r}{\langle\phi_r\rangle}$$

$$\frac{\sigma_r^2}{\langle\phi_r\rangle^2} = \frac{1}{\langle N_{\rm BH}\rangle}\left\{\frac{\langle\xi^2\rangle}{\langle\xi\rangle^2}\,\frac{\langle G^2\rangle}{\langle G\rangle^2} - 1\right\} + \frac{\sigma_N^2}{\langle N_{\rm BH}\rangle^2}$$

Boost distributions

Positrons

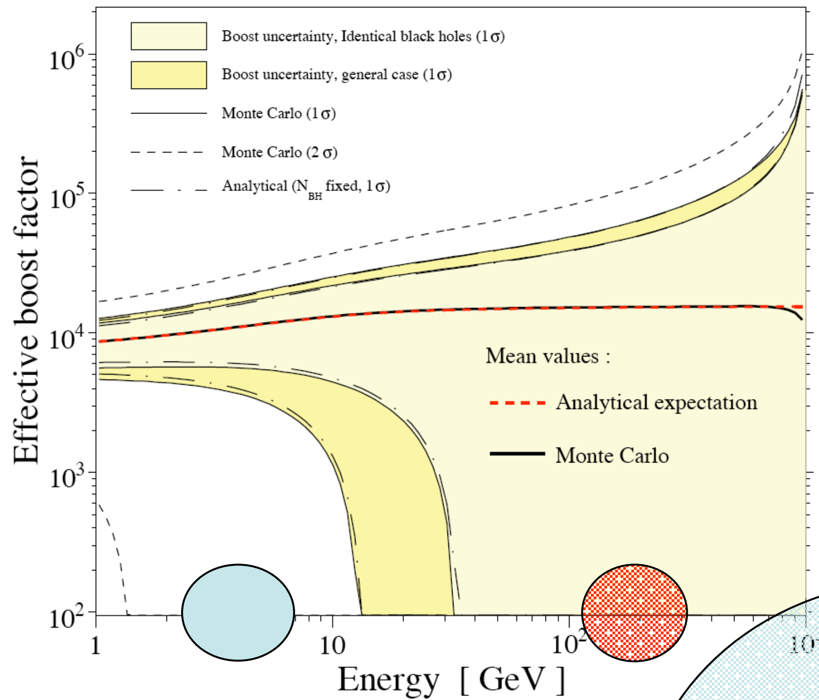


FIG. 5: Results from the Monte-Carlo simulations of the IMBHs population inside the Milky Way are compared to the analytical computations of the effective boost factor and its dispersion, for $m_\chi = 1$ TeV.

Antiprotons

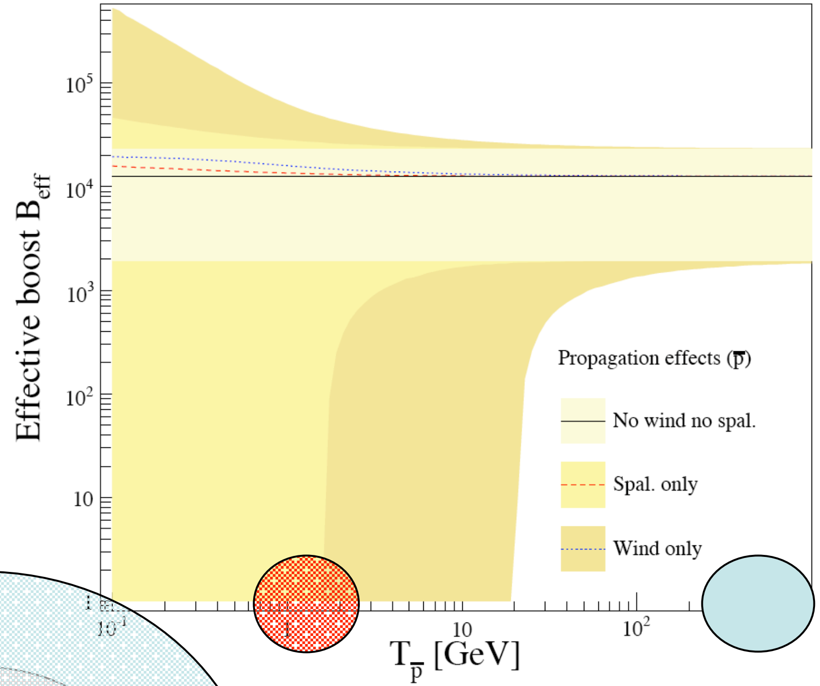
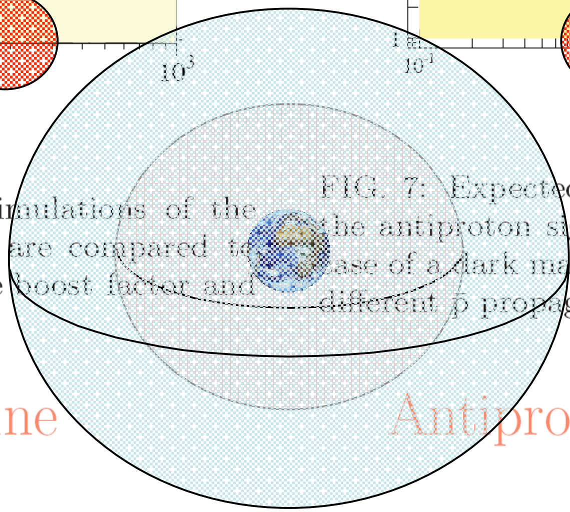


FIG. 7: Expected value and variance of the boost factor of the antiproton signal as a function of kinetic energy, in the case of a dark matter particle with mass $m_\chi = 1$ TeV and for different $\bar{p}$ propagation configurations.



1 TeV positron line

Antiprotons injected at $E_S \simeq E$

Boost distributions

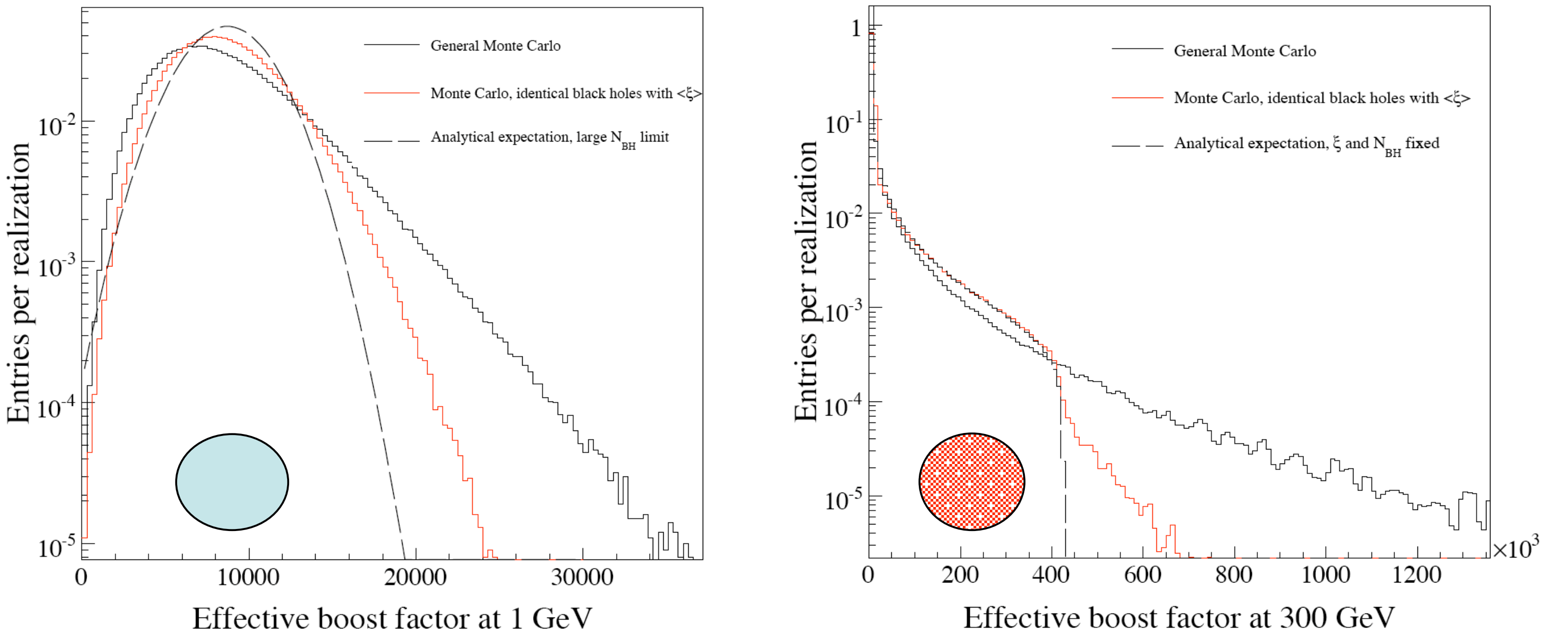
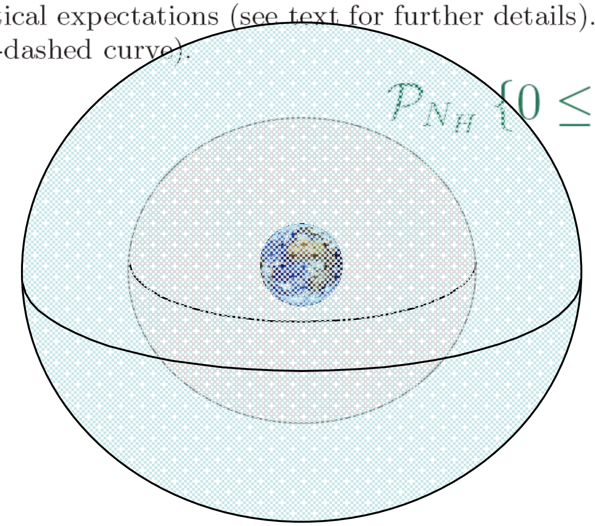


FIG. 6: Distribution of the boost factors at 1 GeV (left panel) and 300 GeV (right panel) obtained with the Monte Carlo simulations and comparison to analytical expectations (see text for further details). The gaussian distribution discussed in the text is plotted in the left panel (long-dashed curve).



$$\mathcal{P}_{N_H}\{0 \leq \varphi \leq \varphi_{\max}\} \simeq N_H \times \mathcal{P}_1(\varphi)$$

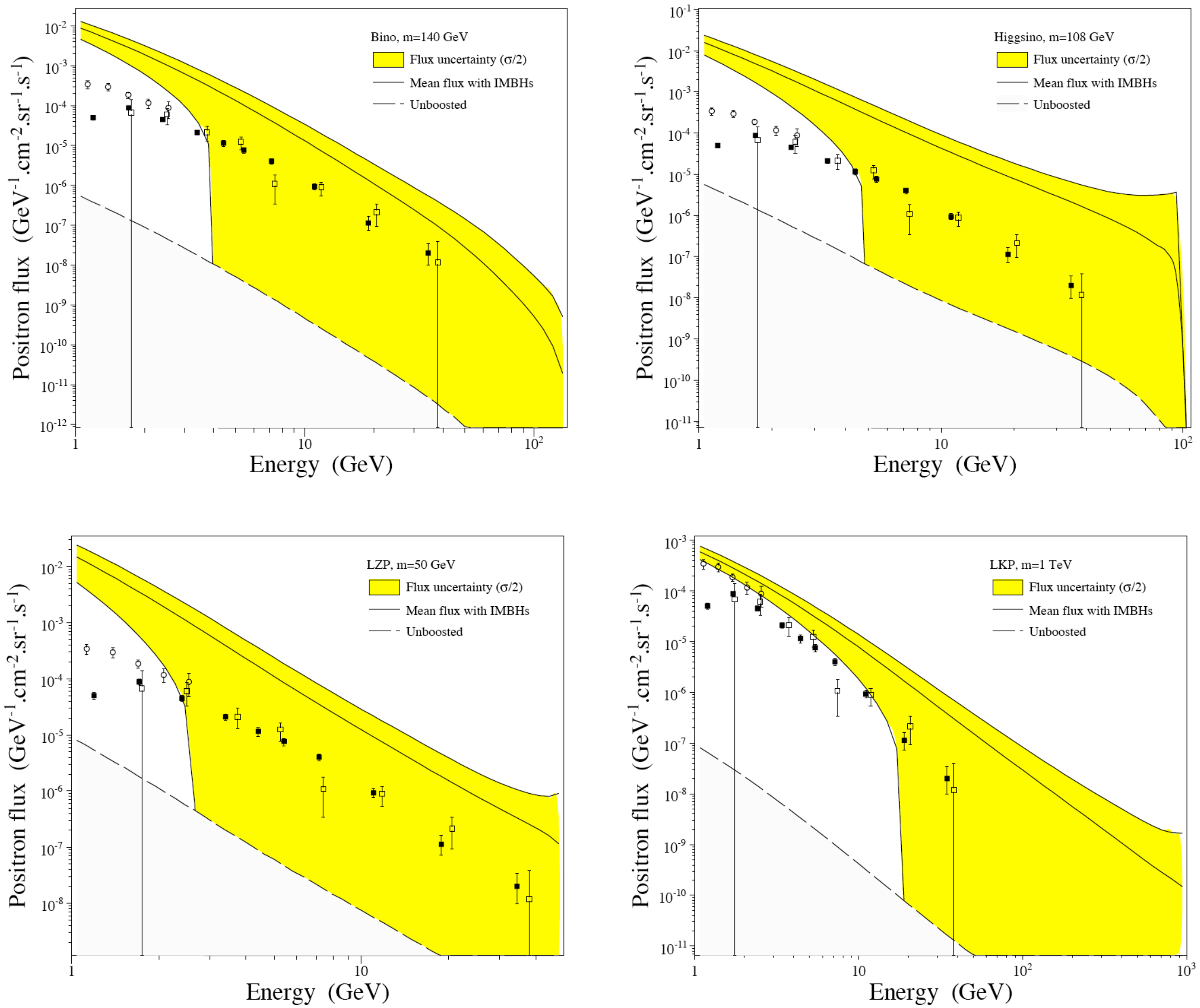


FIG. 8: Exotic positron fluxes in four particle physics models (top : supersymmetric DM, bottom : Kaluza-Klein DM) within IMBHs mini-spike scenario, and comparison to actual measurements.

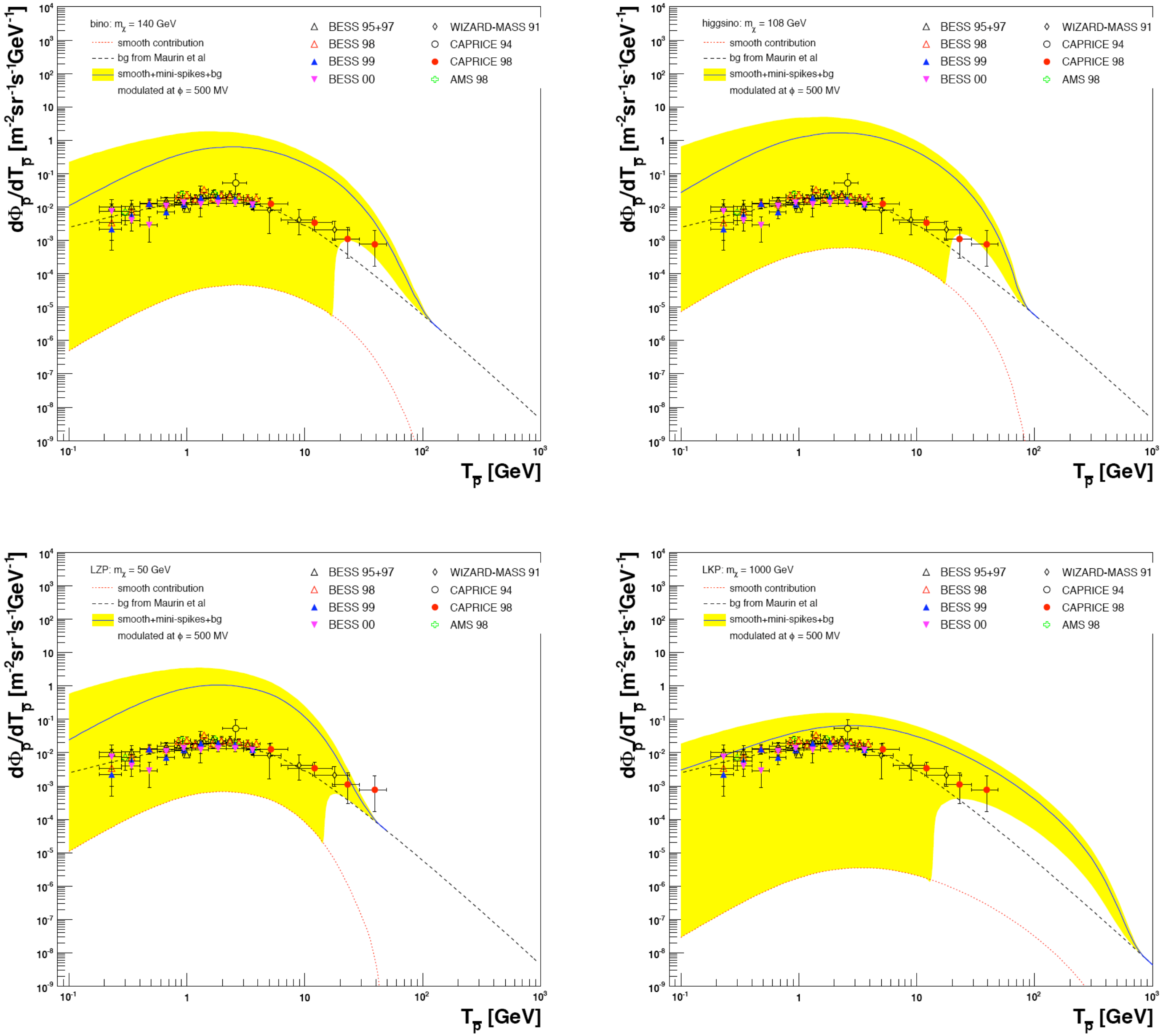


FIG. 9: Antiproton fluxes as a function of kinetic energy in the case of a bino dark matter particle with mass $m_\chi = 140$ GeV. The fluxes were computed without (upper panel) or with (lower panel) wind (with a value $V_c = 12$ km/s), without (left) or with (right) spallation.