Lecture IV

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Cargèse, 03/08/2007

Phenomenological requirements on ~~SUSY~~

Soft SUSY breaking terms in the MSSM

- For each term in the superpotential

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

- we can have the "A-terms" and "B-term"

$$A_u^{ij} Y_u^{ij} Q_i u_j^c H_u + A_d^{ij} Y_d^{ij} Q_i d_j^c H_d + A_l^{ij} Y_l^{ij} L_i e_j^c H_d + B\mu H_u H_d$$

- scalar masses for all scalars

$$m_{Qij}^2 \tilde{Q}_i^* \tilde{Q}_j + m_{uij}^2 \tilde{u}_i^* \tilde{u}_j + m_{dij}^2 \tilde{d}_i^* \tilde{d}_j + m_{Lij}^2 \tilde{L}_i^* \tilde{L}_j + m_{eij}^2 \tilde{e}_i^* \tilde{e}_j + m_{H_d}^2 |H_d|^2 + m_{H_u}^2 |H_u|^2$$

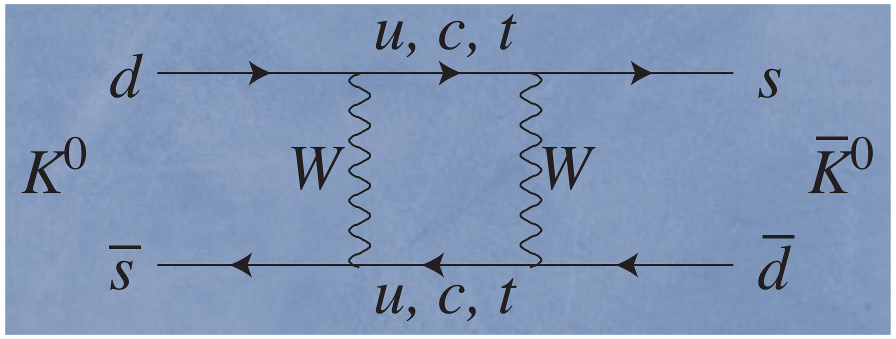
- gaugino mass for all three gauge factors

$$M_1 \tilde{B} \tilde{B} + M_2 \tilde{W}^a \tilde{W}^a + M_3 \tilde{g}^a \tilde{g}^a$$

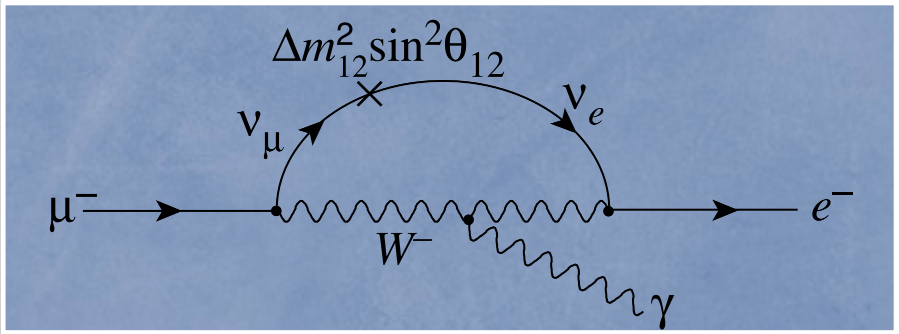
- A(18x3)+B(2)+m(9x5+2)+M(2x3)+μ(2)=111
  $U(1)_R \times U(1)_{PQ}$ removes only two phases
  cf. SM has two params in the Higgs sector
  107 more parameters than the SM!

Flavor-Changing Neutral Current

- There is no tree-level vertex such as $\bar{s}\gamma^\mu d Z_\mu$
- In the Standard Model, FCNC is highly suppressed
- e.g.,



$$\sim \frac{1}{16\pi^2} G_F^2 m_c^2 (V_{cd}^* V_{cs})^2$$



$$\sim \frac{e}{16\pi^2} G_F^2 m_\mu \Delta m_{12}^2 \sin^2\theta_{12}$$

SUSY flavor violation

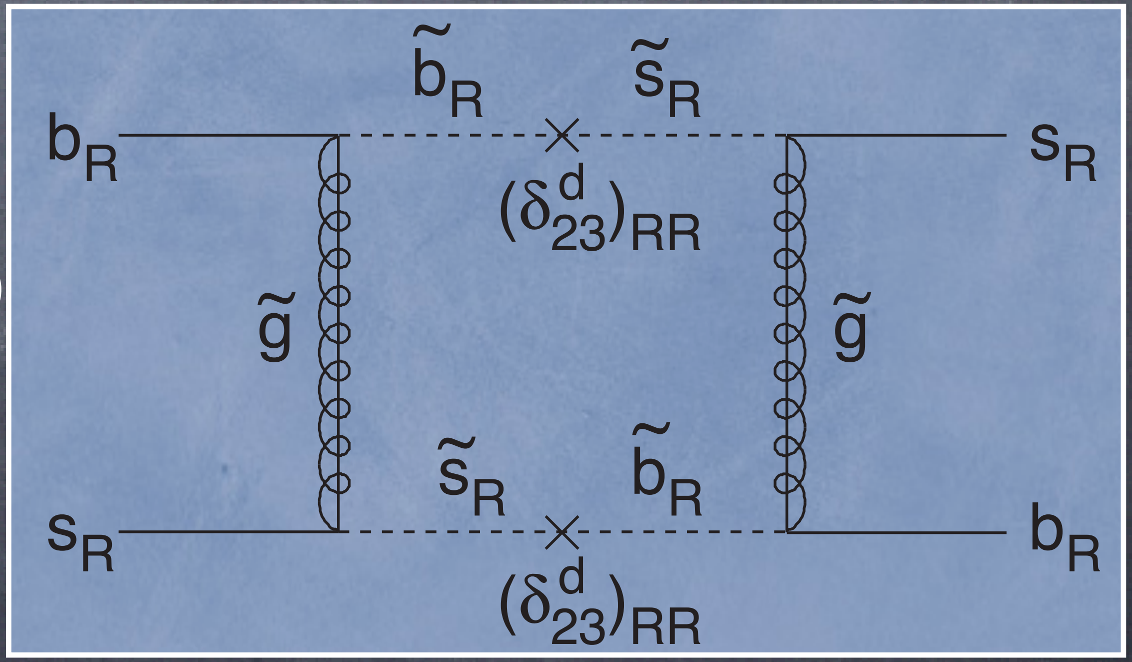
- soft SUSY breaking parameters can violate flavor

$$(\tilde{d},\, \tilde{s},\, \tilde{b}) \begin{pmatrix} m^2_{11} m^2_{12} m^2_{13} \\ m^2_{21} m^2_{22} m^2_{23} \\ m^2_{31} m^2_{32} m^2_{33} \end{pmatrix} \begin{pmatrix} \tilde{d} \\ \tilde{s} \\ \tilde{b} \end{pmatrix}$$

$$(\delta^d_{12})_{RR} \equiv \frac{m^2_{12}}{m_{11} m_{22}} < 0.04 \frac{m_{SUSY}}{500\text{GeV}}$$

$$\sqrt{(\delta^d_{12})_{RR} (\delta^d_{12})_{LL}} < 0.001 \frac{m_{SUSY}}{500\text{GeV}}$$

K^0



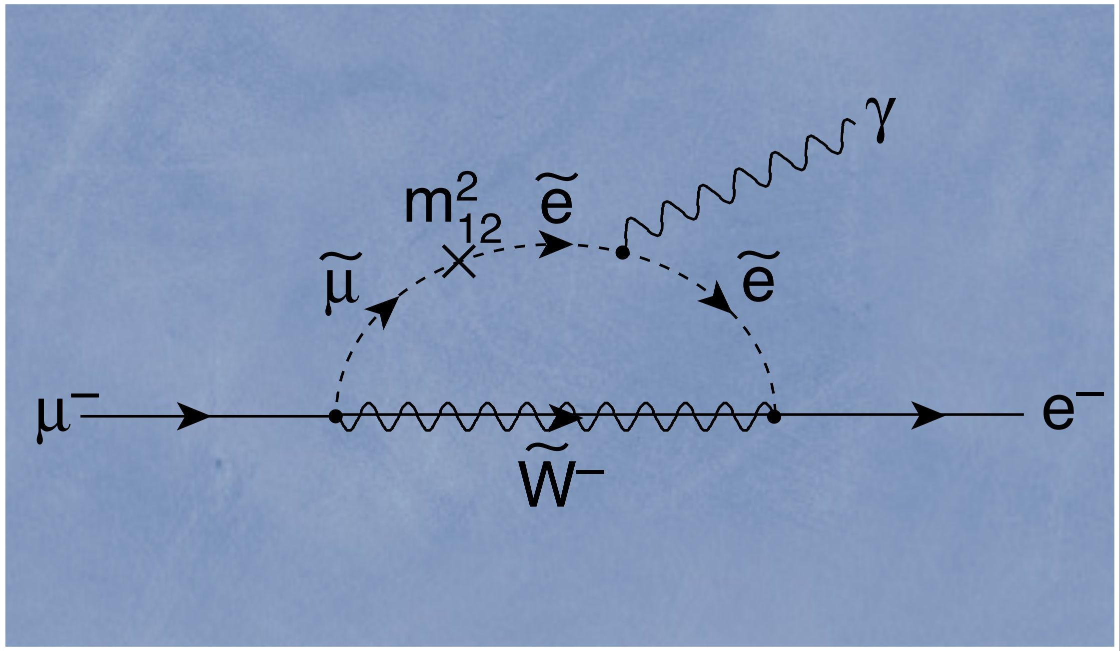
$\overline{K}^0$

SUSY flavor violation

- soft SUSY breaking parameters can violate flavor

$$(\tilde{e}, \tilde{\mu}, \tilde{\tau}) \begin{pmatrix} m_{11}^2 m_{12}^2 m_{13}^2 \\ m_{21}^2 m_{22}^2 m_{23}^2 \\ m_{31}^2 m_{32}^2 m_{33}^2 \end{pmatrix} \begin{pmatrix} \tilde{e} \\ \tilde{\mu} \\ \tilde{\tau} \end{pmatrix}$$

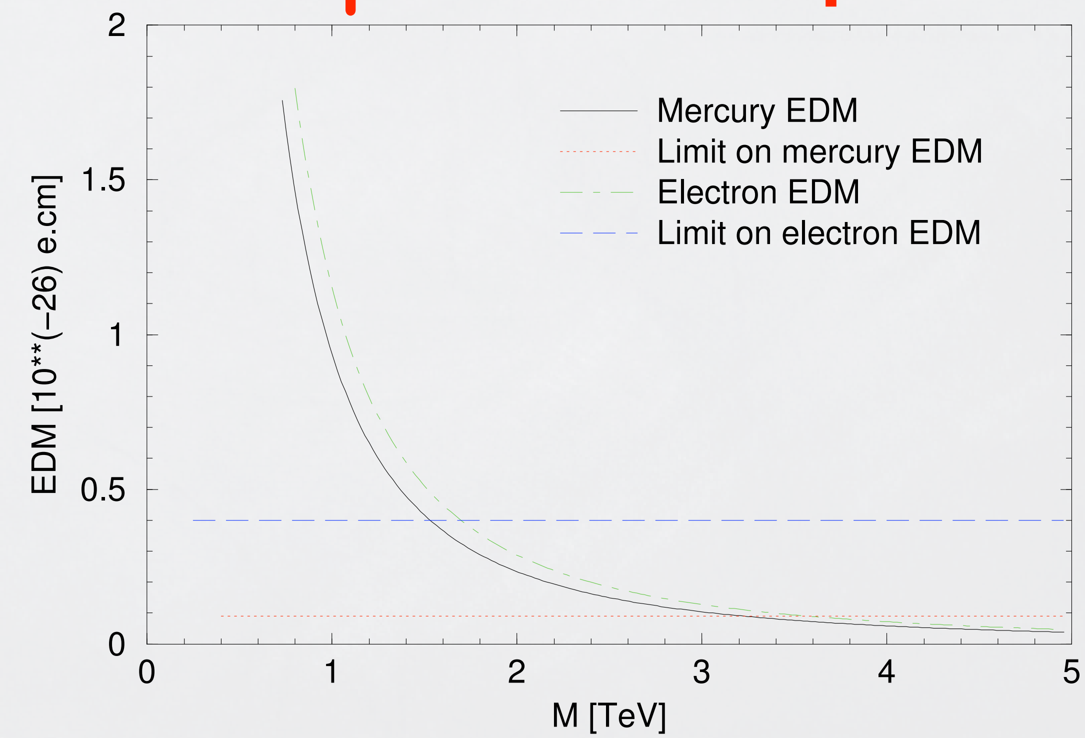
$$(\delta^l_{12})_{RR} < 3.9 \times 10^{-3}$$



Supersymmetric CP problem

- The relative phases of μ and $M_{1,2,3}$ are physical
- induces electric dipole moments $H \propto \vec{s} \cdot \vec{E}$
- stringent limits on electron, neutron, and Hg atom
- either m_{SUSY}>TeV or phase~10^{-2}

phase of μ



Falk, Olive, Pospelov, Roiban

“minimal supergravity”

- At the GUT-scale 2×10^{16} GeV
- assume all scalar masses are equal m_0^2
- assume all gaugino massses are equal $M_{1/2}$
- assume all trilinear couplings are equal A_0
- in addition, B, $B\mu$
- calculate all SUSY breaking terms via RGE down from the GUT-scale
- fix m_Z: leaves four parameters (and sign(μ))

one-loop RGE

- GUT prediction of gaugino masses

$$\frac{d}{dt}\frac{M_i}{g_i^2} = 0$$

$$M_1 : M_2 : M_3 \approx 1 : 2 : 7 \text{ at } m_Z$$

- gauge interaction boosts scalar masses

$$\frac{d}{dt}m^2 = -\frac{1}{16\pi^2} 8 C_F g^2 M^2$$

- Yukawa interaction suppresses scalar masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2$$

$$16\pi^2 \frac{d}{dt} m_{\tilde{t}_R}^2 = 2X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2$$

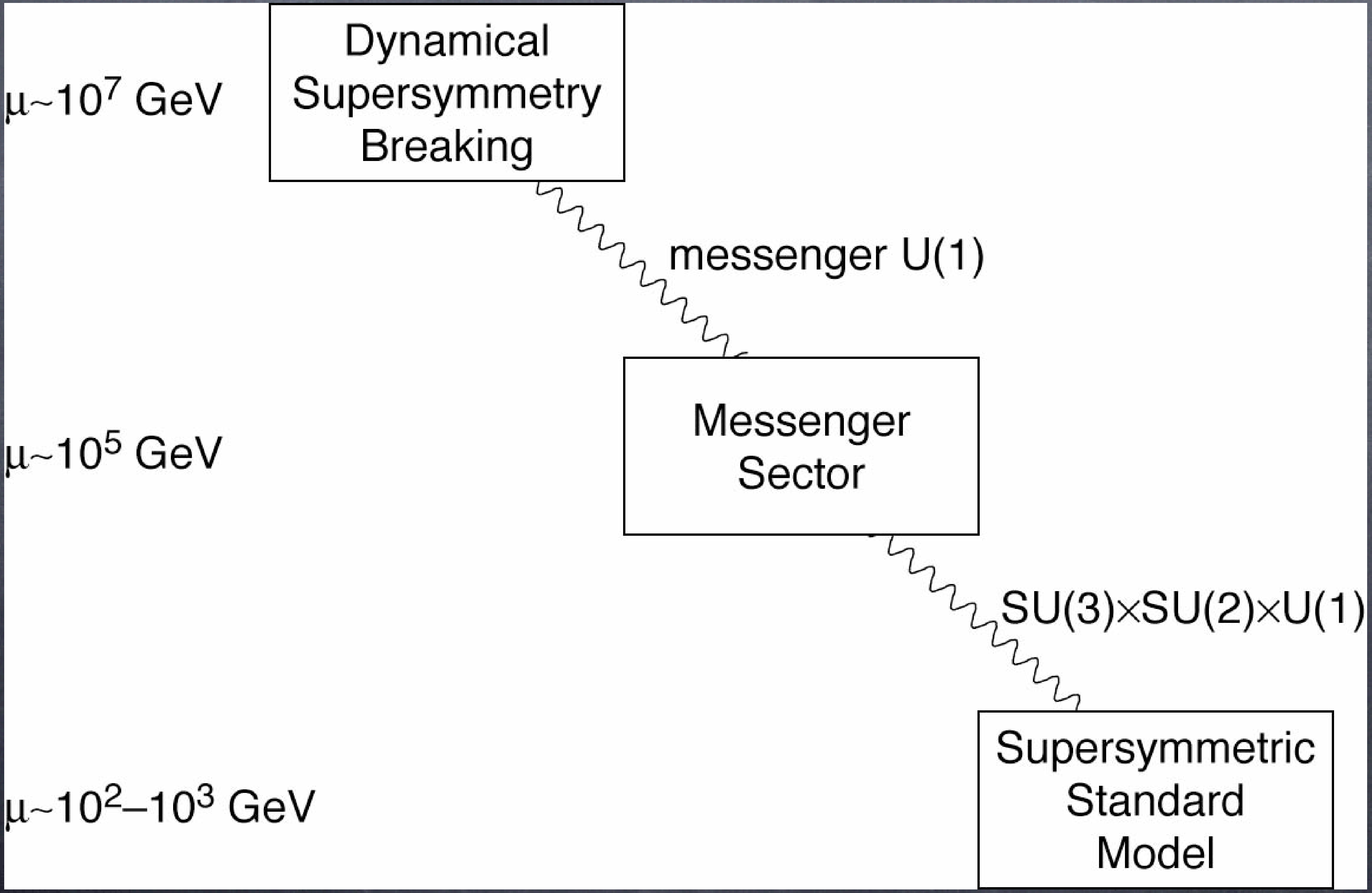
$$16\pi^2 \frac{d}{dt} m_{\tilde{t}_L}^2 = X_t - \frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2$$

$$X_t = 2 Y_t^2 (m_{H_u}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{t}_L}^2 + A_t^2)$$

- H_u mass-squared most likely to get negative!

Flavor-blind Mediation Mechanisms

Gauge Mediation (GMSB)

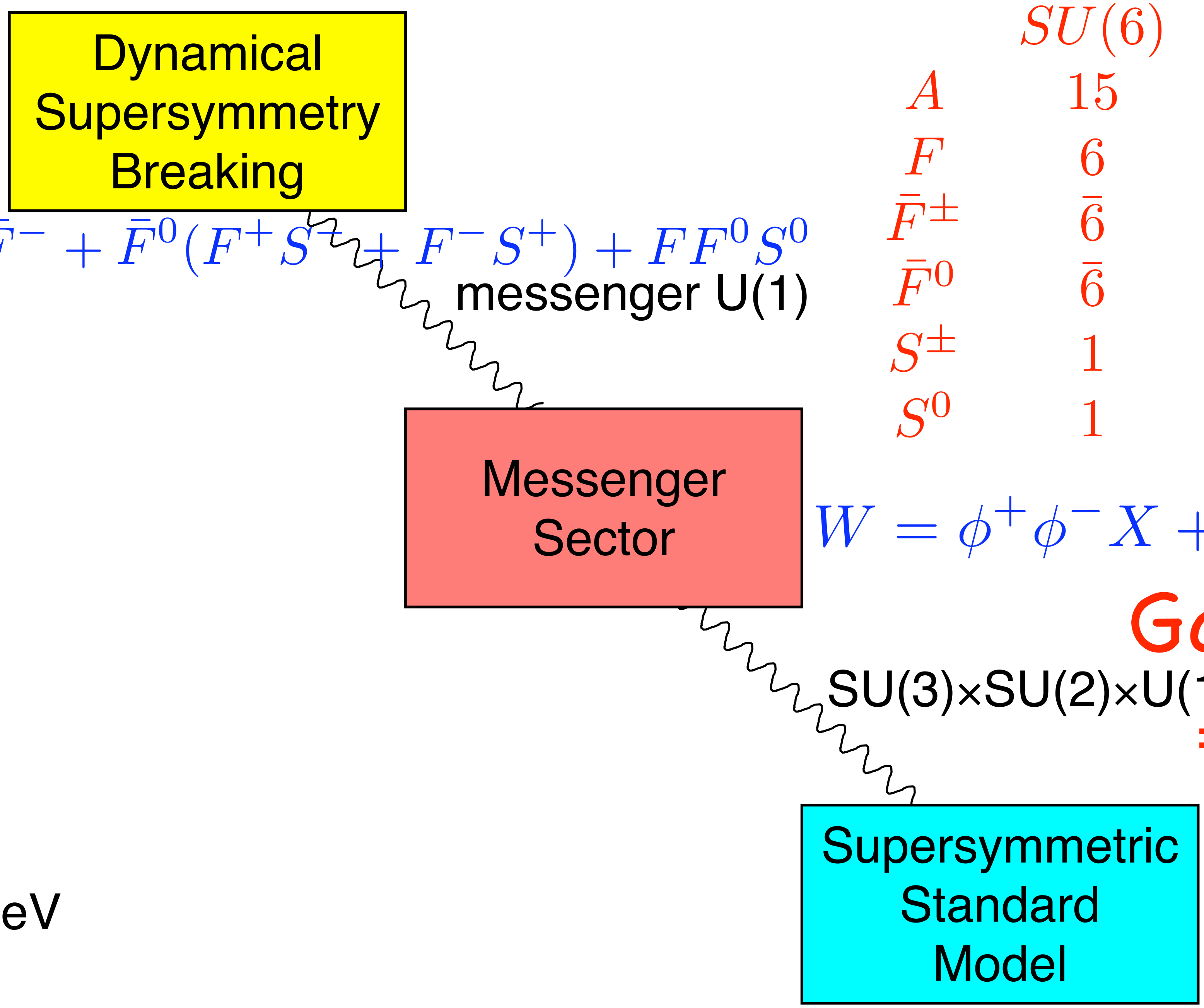


Special Model II
Mediation Mechanism

μ~10^7 GeV

μ~10^5 GeV

μ~10^2–10^3 GeV



$$W = A\bar{F}^+\bar{F}^- + \bar{F}^0(F^+S^- + F^-S^+) + FF^0S^0$$

	$SU(6)$	$U(1)$	$U(1)_m$	$U(1)_R$
A	15	$+2$	0	$-\frac{18}{7}$
F	6	-5	0	$-\frac{18}{7}$
$\bar{F}^\pm$	$\bar{6}$	-1	± 1	$\frac{16}{7}$
$\bar{F}^0$	$\bar{6}$	-1	0	$\frac{16}{7}$
$S^\pm$	1	$+6$	± 1	$\frac{16}{7}$
S^0	1	$+6$	0	$\frac{16}{7}$

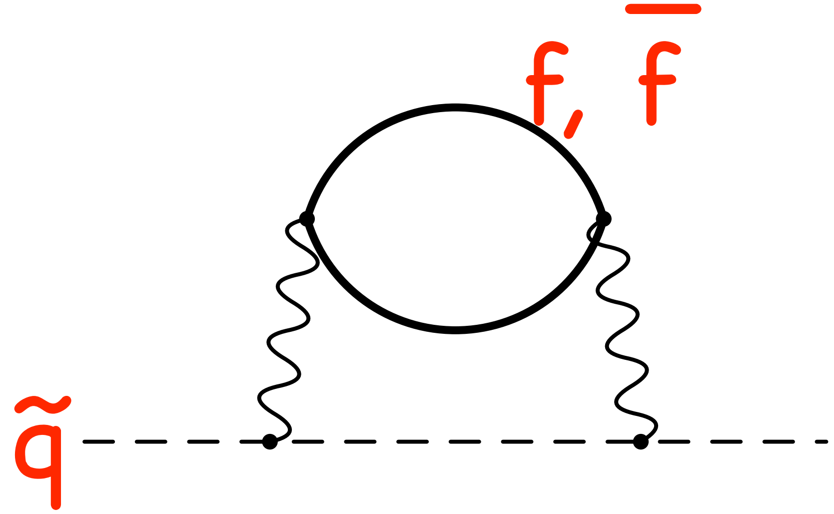
$$W = \phi^+\phi^- X + X^3 + X\bar{f}f$$

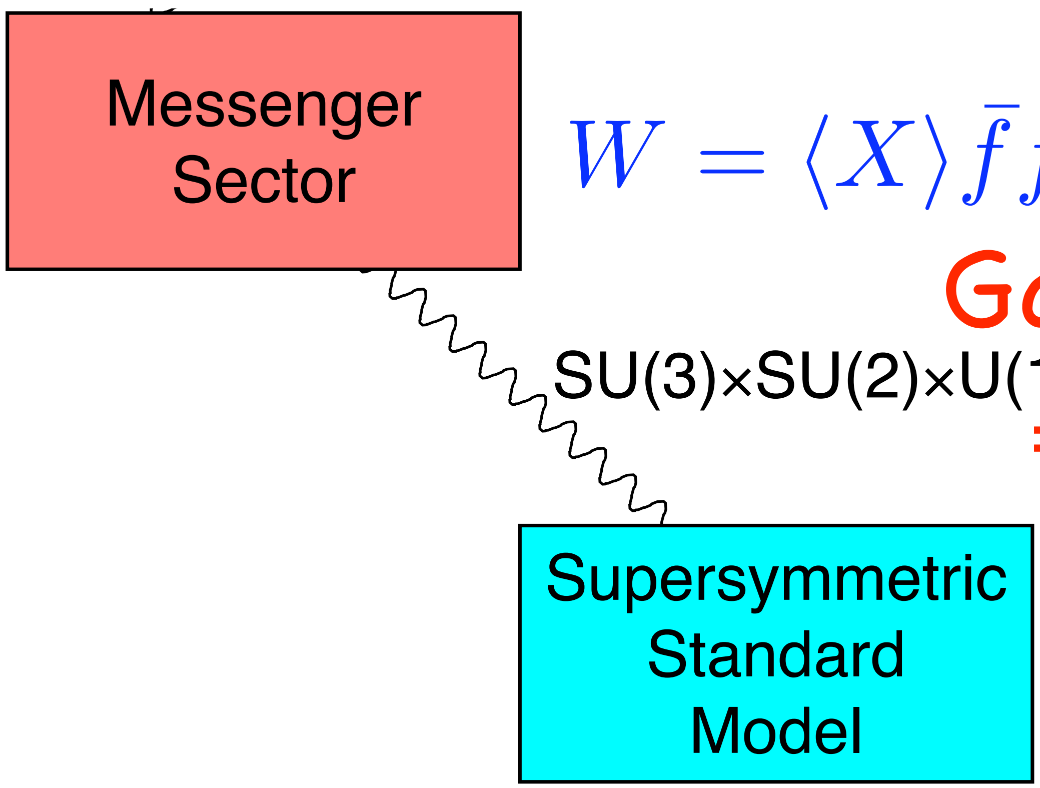
Gauge Mediation

⇒flavor blind

Dine–Nelson–Nir–Shirman

Special Model II
Mediation Mechanism





$$W = \langle X \rangle \bar{f} f$$

Gauge Mediation
⇒flavor blind

Dine–Nelson--Nir–Shirman

New Generic Scheme

$$\frac{1}{M_{Pl}}\bar{Q}Q\bar{f}f$$

HM, Nomura

SUSY QCD

SU(Nc), SO(Nc), Sp(Nc)

$m_Q\bar{Q}Q$

$M\bar{f}f$

SUSY SM

no $U(1)_R$ symmetry imposed

most general superpotential

wide choice of gauge groups, matter content

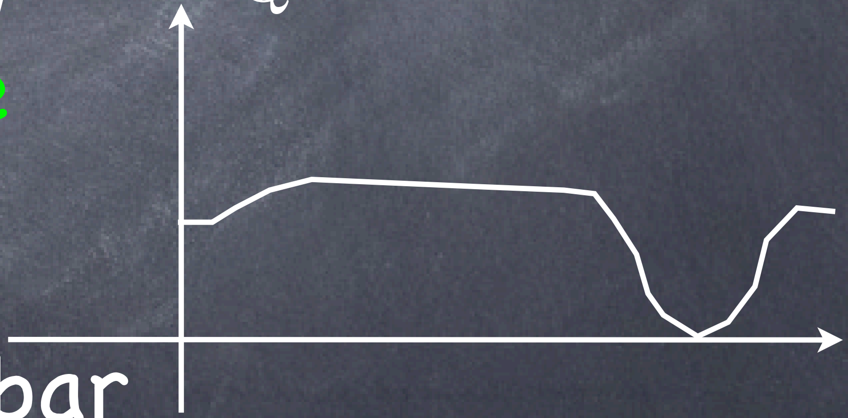
$$N_c < N_f < \frac{3}{2}N_c$$

How it works

- SUSY SU(N_c) QCD $N_c<N_f<3N_c/2$ $\quad W = m_Q^{ij}\bar{Q}_i Q_j$
- low-energy free magnetic theory ($m_Q<\Lambda$)

  $$W = m_Q^{ij}\Lambda M_{ij} + M_{ij}\bar{q}^i q^j$$
- SUSY breaking @ $M_{ij}=0, \quad \frac{\partial W}{\partial M_{ij}} = m_Q^{ij} \neq 0$
- Local minimum with long lifetime

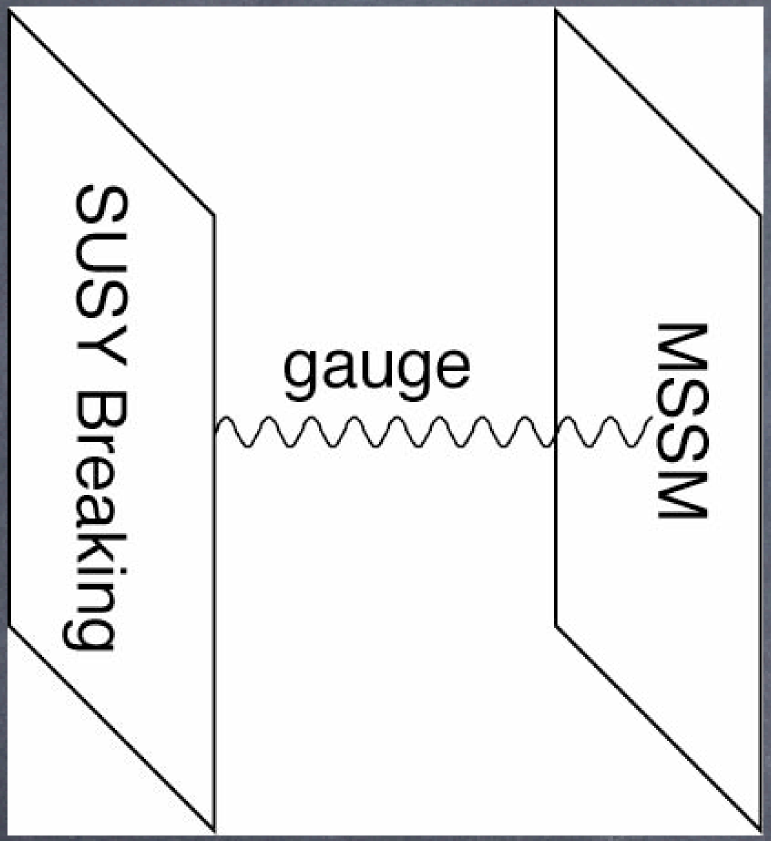
  $$W = \frac{1}{M_{Pl}}\bar{Q}Q\bar{f}f$$



- Generates SUSY breaking in f, fbar
- their loops⇒gauge mediation

Intriligator, Seiberg, Shih

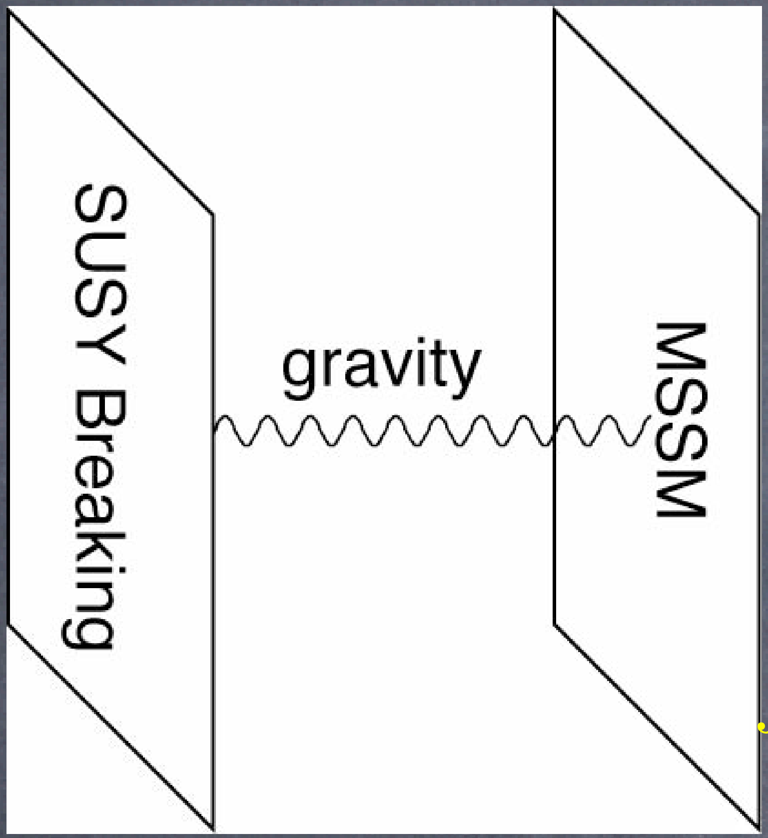
Gaugino Mediation (χMSB)



- DSB in another brane
- Gauge multiplet in the bulk
- Gauge multiplet learns SUSY breaking first, obtains gaugino mass
- MSSM at the compactification scale with gaugino mass only
- Scalar masses generated by RGE

Kaplan, Kribs, Schmaltz
Chacko, Luty, Nelson, Ponton

Anomaly Mediation (AMSB)



- no direct coupling between two sectors
- Supersymmetry breaking in the chiral compensator $\langle S \rangle = 1 + \theta^2 m_{3/2}$

$$\int d^4\theta S\bar{S}\phi^*\phi + \int d^2\left(S^3\lambda_{ijk}\phi_i\phi_j\phi_k + \frac{1}{g^2}W_\alpha W^\alpha\right)$$

- can be scaled away $\phi \to \phi/S$
- but the UV cutoff acquires S: $\Lambda_{UV} \to \Lambda_{UV} S$
- SUSY breaking through cutoff dependence: superconformal anomaly

Randall, Sundrum
Giudice, Luty, HM, Rattazzi

UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- Surprising result: depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, including flavor physics with O(1) generation-dependent couplings, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass

What's the catch?

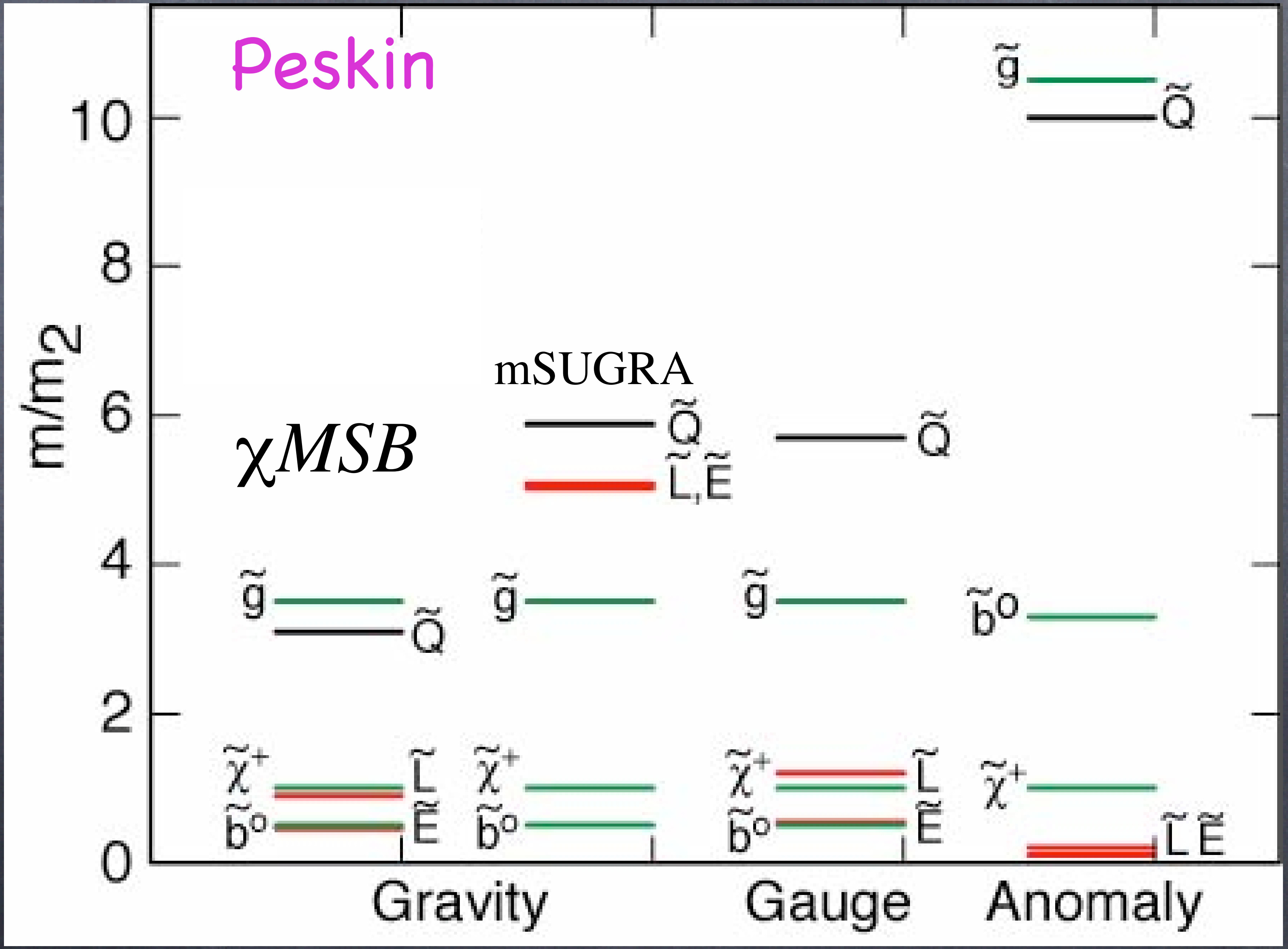
- Two problems
- negative slepton mass-squared
- can't have a light bulk moduli of $m \sim O(m_{3/2})$

cause additional terms of $O(m_{3/2}^2/m) \sim O(m_{3/2})$

- common fixes:
  - add m_0^2
  - add D_Y and D_{B-L}

$$
\begin{aligned}
m^2_{\tilde{l}} &= -0.344 M^2, \\
m^2_{\tilde{e}^c} &= -0.367 M^2, \\
m^2_{\tilde{q}} &= 11.6 M^2, \\
m^2_{\tilde{u}^c} &= 11.7 M^2, \\
m^2_{\tilde{d}^c} &= 11.8 M^2, \\
M &= \frac{m_{3/2}}{(4\pi)^2}
\end{aligned}
$$

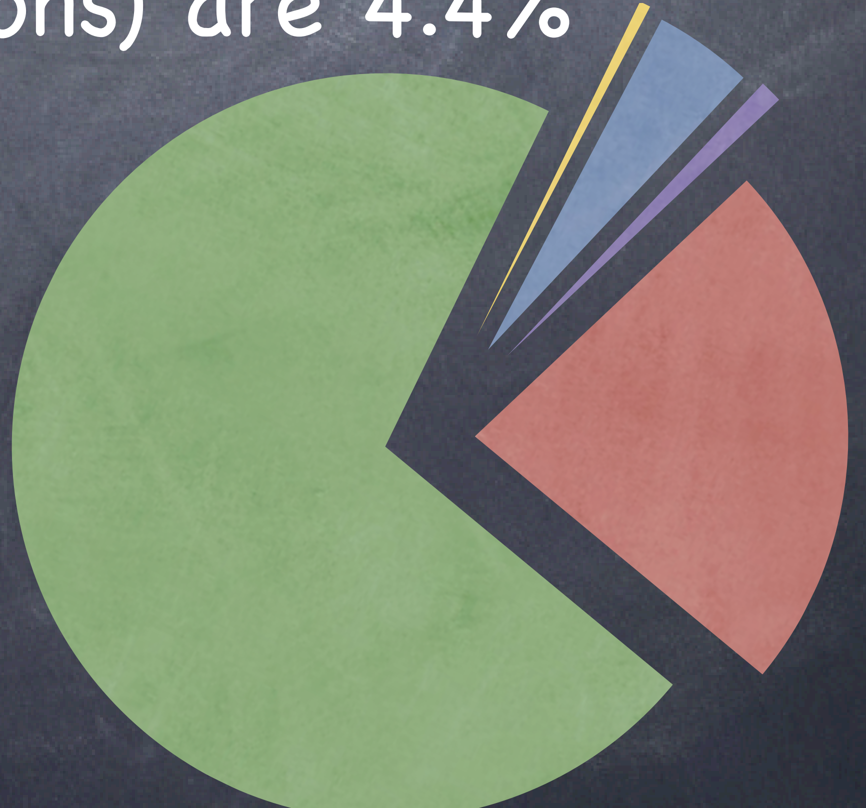
SUSY spectra



WIMP paradigm

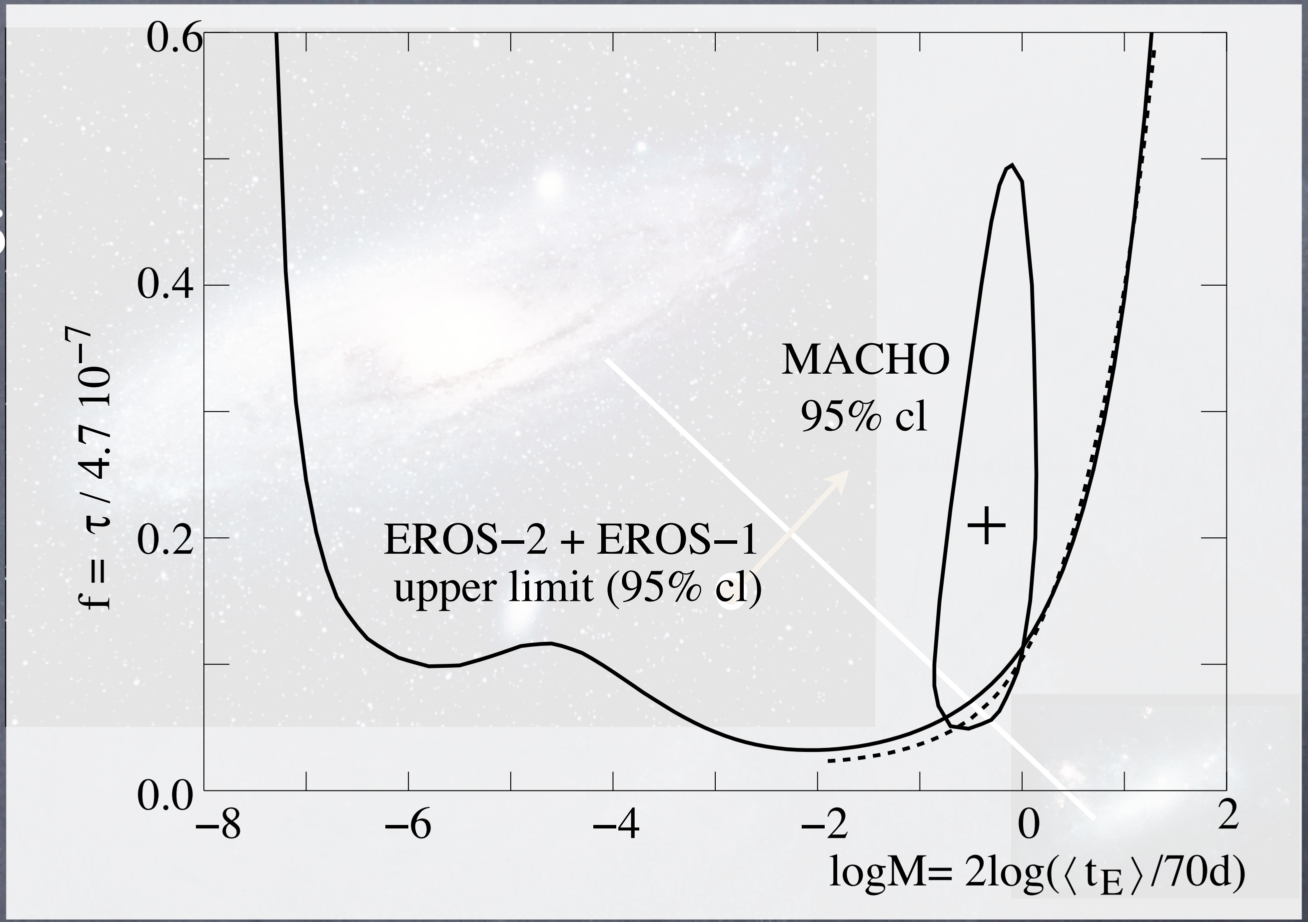
Energy Budget of the Universe

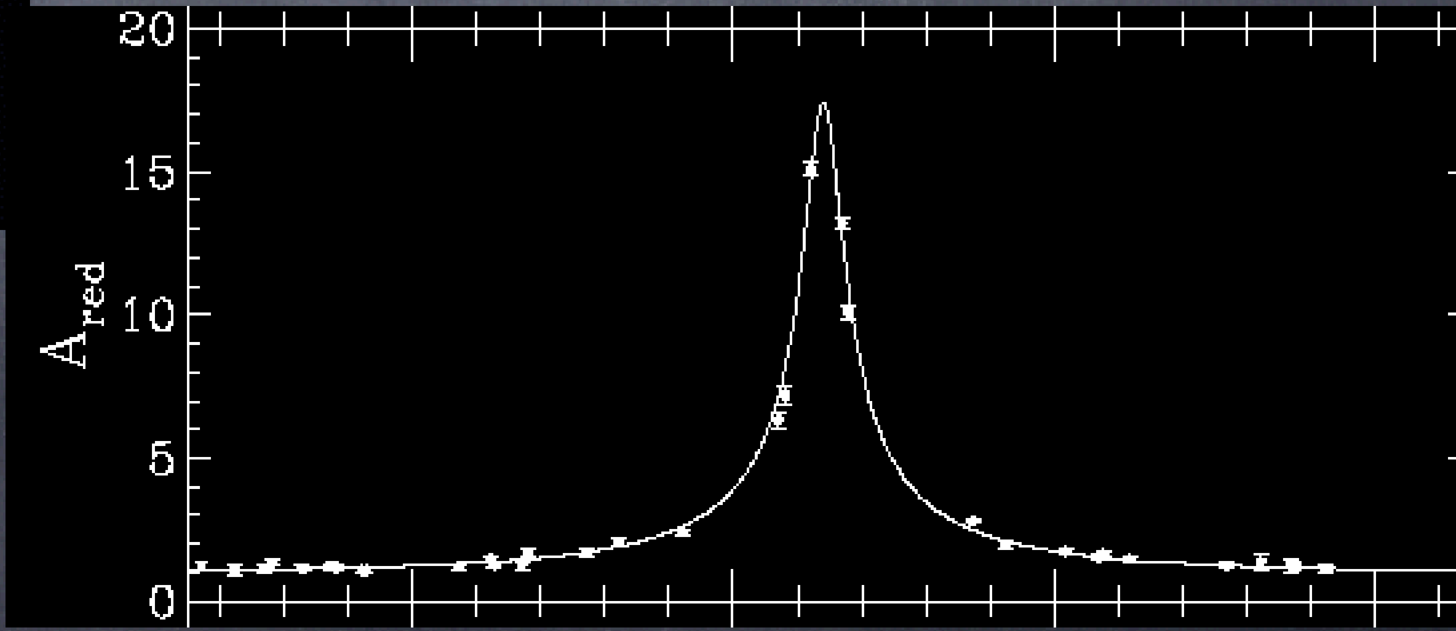
- Stars and galaxies are only ~0.5%
- Neutrinos are ~0.1–1.5%
- Rest of ordinary matter (electrons, protons & neutrons) are 4.4%
- Dark Matter 23%
- Dark Energy 73%
- Anti-Matter 0%
- Dark Field ~10^{62}%??



Dim Stars?

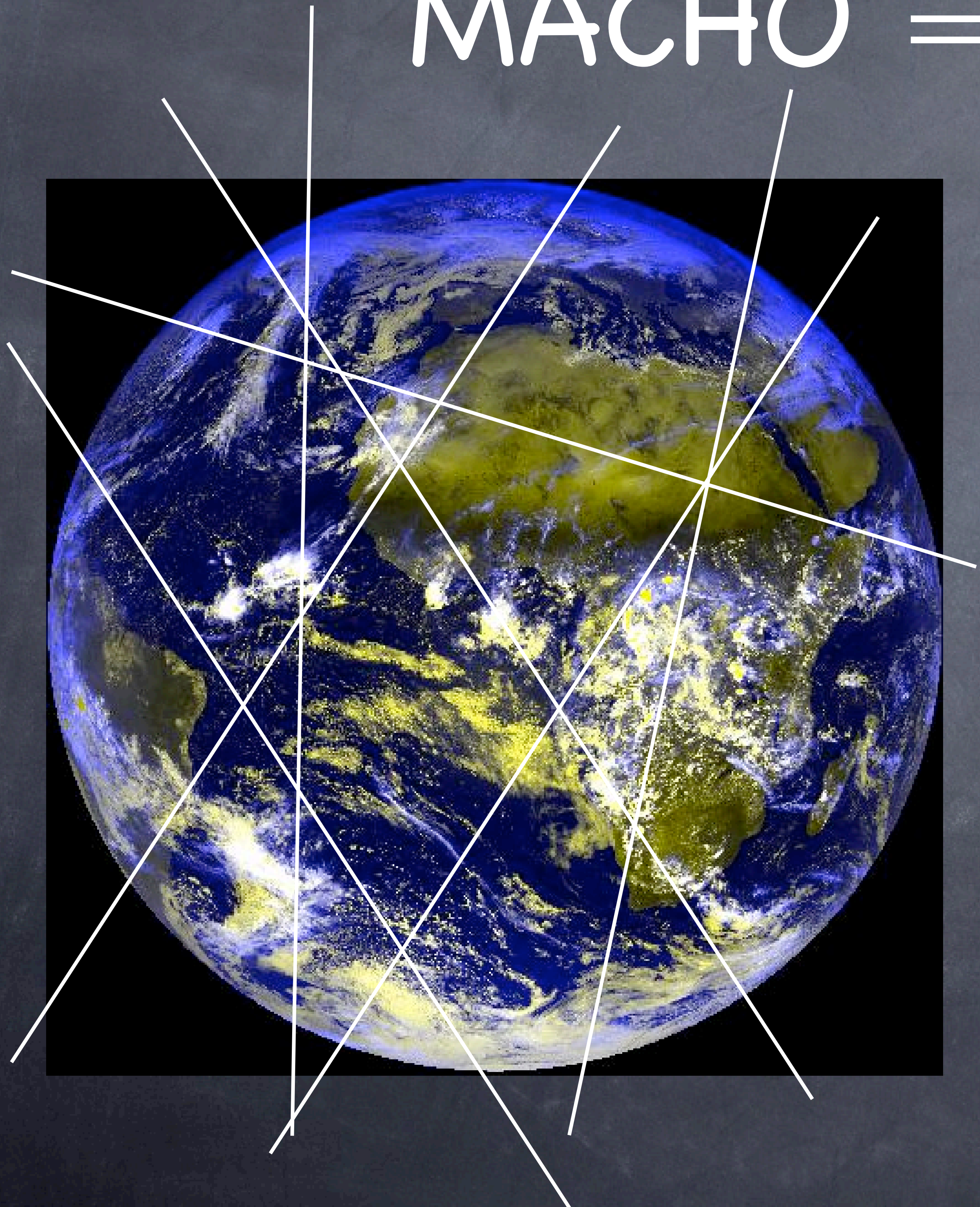
Search for MACHOs
(Massive Compact Halo Objects)





Not enough of them!

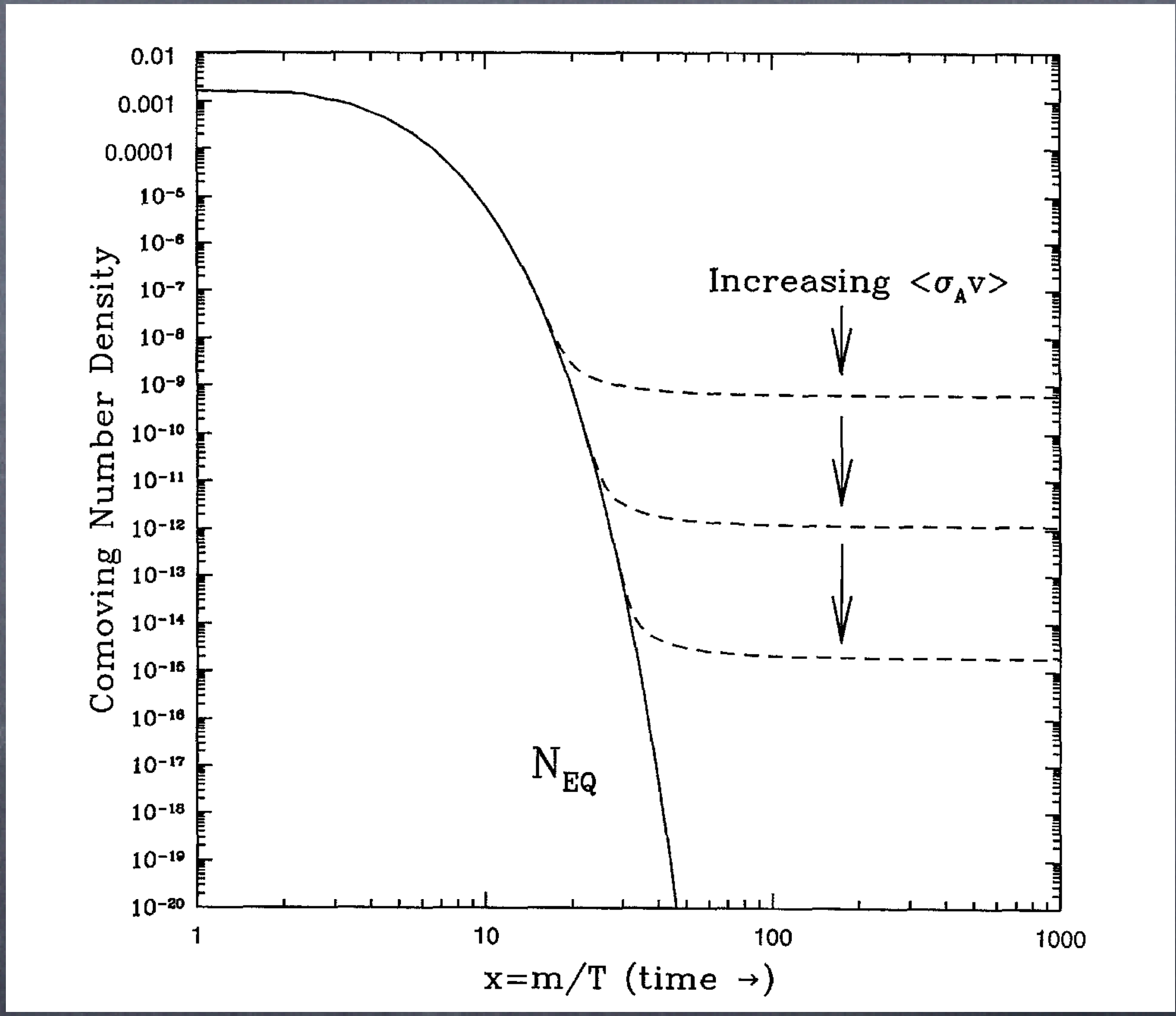
MACHO ⇒ WIMP



- It is probably WIMP (Weakly Interacting Massive Particle)
- Stable heavy particle produced in early Universe, left-over from near-complete annihilation

thermal relic

- thermal equilibrium when $T > m_\chi$
- Once $T < m_\chi$, no more χ created
- if stable, only way to lose them is annihilation
- but universe expands and χ get dilute
- at some point they can't find each other
- their number in comoving volume "frozen"



Freeze-out

- WIMP freezes out when the annihilation rate drops below the expansion rate
- Yield Y=n/s constant under expansion
- stronger annihilation ⇒ less abundance

$$H \approx g_*^{1/2} \frac{T^2}{M_{Pl}}$$

$$\Gamma_{\rm ann} \approx \langle \sigma_{\rm ann} v \rangle n$$

$$H(T_f) = \Gamma_{\rm ann}$$

$$n \approx g_*^{1/2} \frac{T_f^2}{M_{Pl} \langle \sigma_{\rm ann} v \rangle}$$

$$s \approx g_* T^3$$

$$Y = \frac{n}{s} \approx g_*^{-1/2} \frac{1}{M_{Pl} T_f \langle \sigma_{\rm ann} v \rangle}$$

$$\Omega_\chi = \frac{m_\chi Y s_0}{\rho_c}$$

$$\approx g_*^{-1/2} \frac{x_f}{M_{Pl}^3 \langle \sigma_{\rm ann} v \rangle} \frac{s_0}{H_0^2}$$

Order of magnitude

- "Known" Ω_χ=0.23 determines the WIMP annihilation cross section

$$\Omega_\chi \approx g_*^{-1/2} \frac{x_f}{M_{Pl}^3 \langle \sigma_{\rm ann} v \rangle} \frac{s_0}{H_0^2}$$

$$\langle \sigma_{\rm ann} v \rangle \approx \frac{1.12 \times 10^{-10} {\rm GeV}^{-2} x_f}{g_*^{1/2} \Omega_\chi h^2}$$

$$\sim 10^{-9} {\rm GeV}^{-2}$$

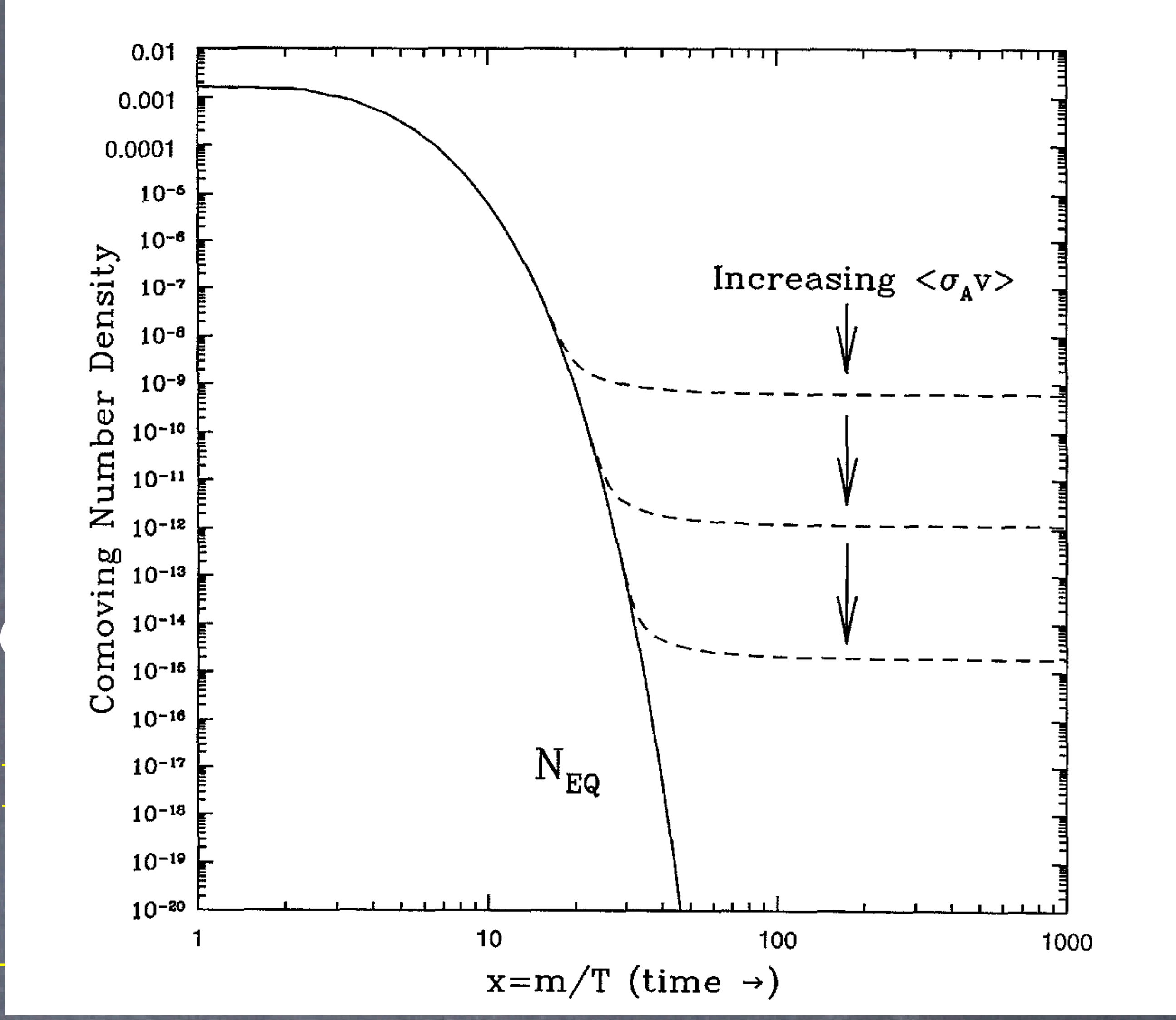
- simple estimate of the annihilation cross section

$$\langle \sigma_{\rm ann} v \rangle \approx \frac{\pi \alpha^2}{m_\chi^2}$$

- weak-scale mass!!!

$$m_\chi \approx 300 \ {\rm GeV}$$

therma



- Solve the Boltzmann e

$$\frac{dn_1}{dt} + 3Hn_1 \quad = \quad -\int \prod_{i=1}^{4} \frac{d^3p_i}{(2\pi)^3 2E_i} |\mathcal{M}($$

$$[f_1 f_2 (1 \pm f_3)(1 \pm f_4) -$$

- assume Maxwell distribution, $1=2=\chi$, $E_1=E_2=m_\chi$

$$\frac{dn}{dt} + 3Hn = -\langle \sigma_{\rm ann} v \rangle (n^2 - n_{eq}^2)$$

- Note momentum dependence may be important close to thresholds, resonances
- reproduce the estimate with

$$x_f \approx 24 + \ln \frac{m_\chi}{100\mathrm{GeV}} + \ln \frac{\langle \sigma_{\rm ann} v \rangle}{10^{-9}\mathrm{GeV}^{-2}} - \frac{1}{2} \ln \frac{g_*}{100}$$

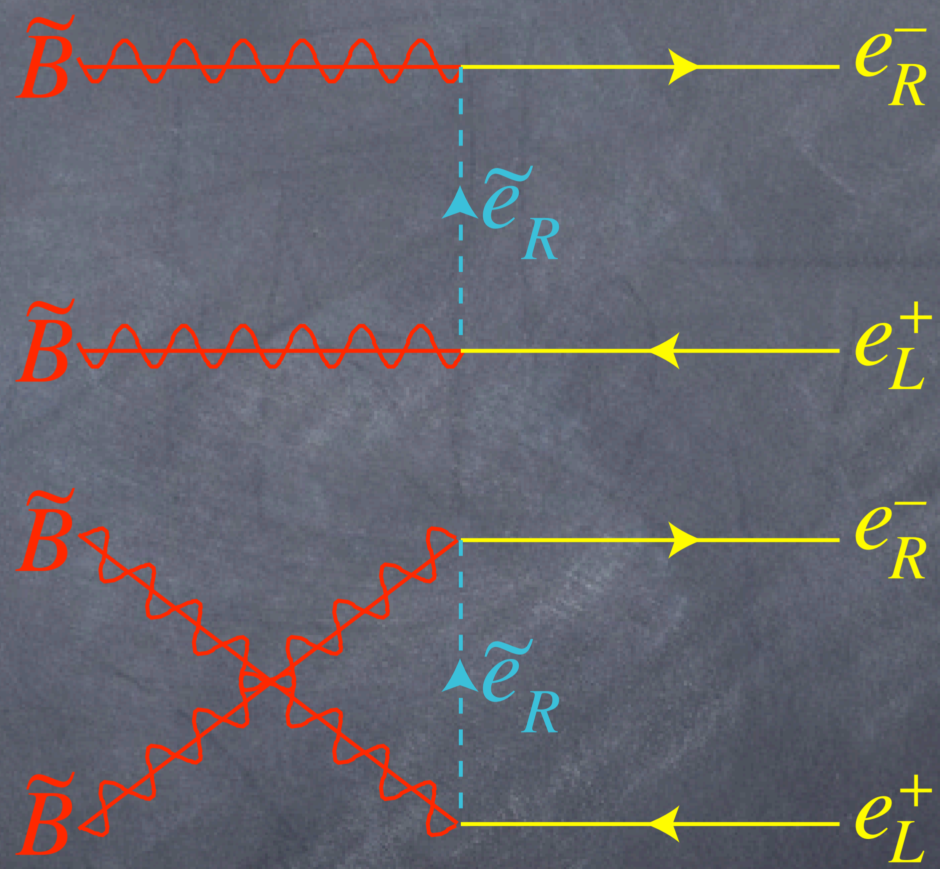
WIMP

- A stable particle at the weak scale with "EM-strength" coupling naturally gives the correct abundance
- This is where we expect new particles because of the hierarchy problem!
- Many candidates of this type: SUSY, little Higgs with T-parity, Universal Extra Dimensinos, etc
- If so, we may even create dark matter at accelerators

Example

- exchange of Majorana fermions with a relative minus sign

$$
\begin{aligned}
\mathcal{M}_{+-} &= 8g'^2 \frac{M_{\tilde{B}} p_{\tilde{B}}}{M_{\tilde{B}}^2 + m_{\tilde{e}_R}^2} \cos^2\frac{\theta}{2} \\
\mathcal{M}_{-+} &= 8g'^2 \frac{M_{\tilde{B}} p_{\tilde{B}}}{M_{\tilde{B}}^2 + m_{\tilde{e}_R}^2} \sin^2\frac{\theta}{2} \\
\mathcal{M}_{++} &= 0 \\
\mathcal{M}_{--} &= 0
\end{aligned}
$$

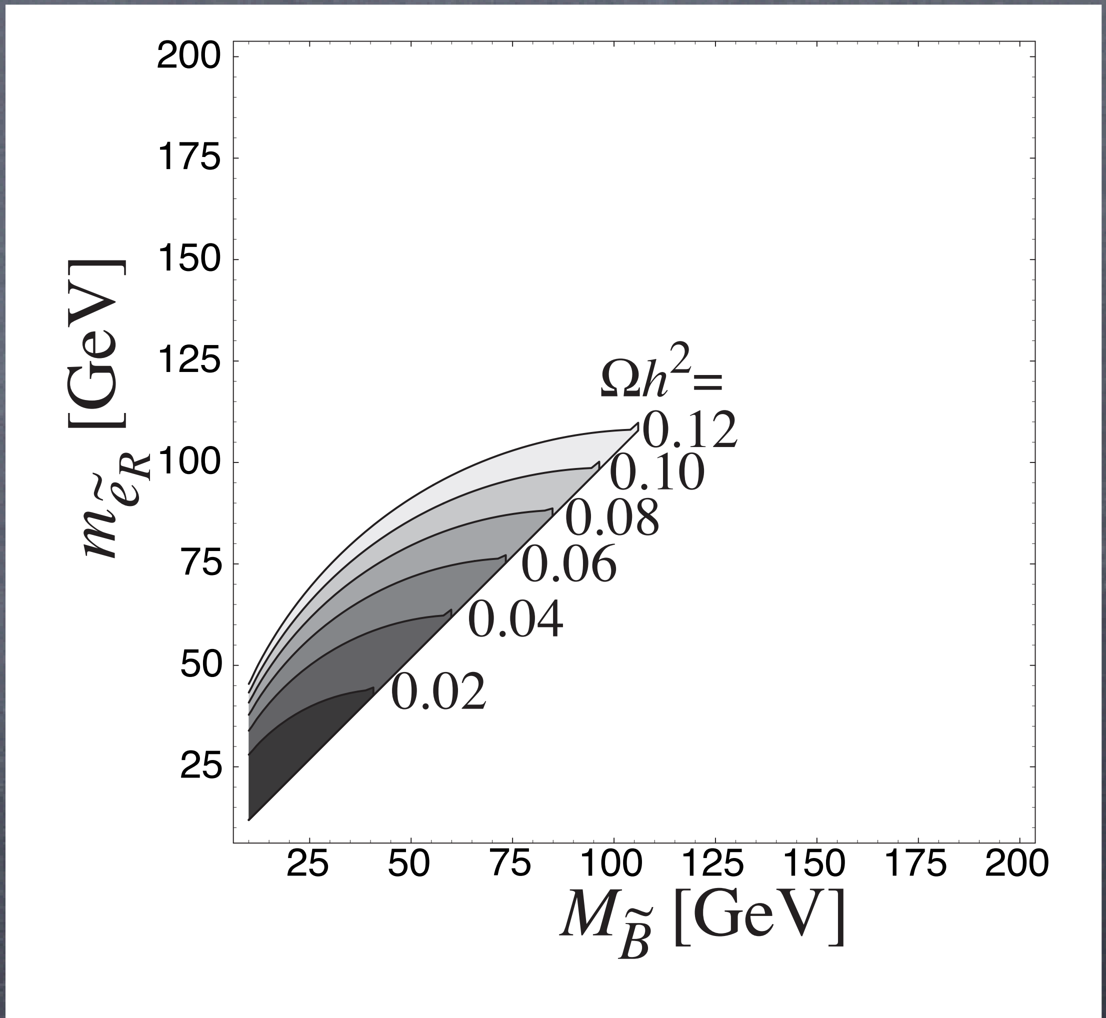


- P-wave annihilation
- Final state J=1
- L=0, S=1 not possible
- L=1, S=1 allowed

$$\sigma = \frac{4\pi\alpha^2 M_{\tilde{B}}^2 v_{\rm rel}}{3c_W^4 (M_{\tilde{B}}^2 + m_{\tilde{e}_R}^2)^2}$$

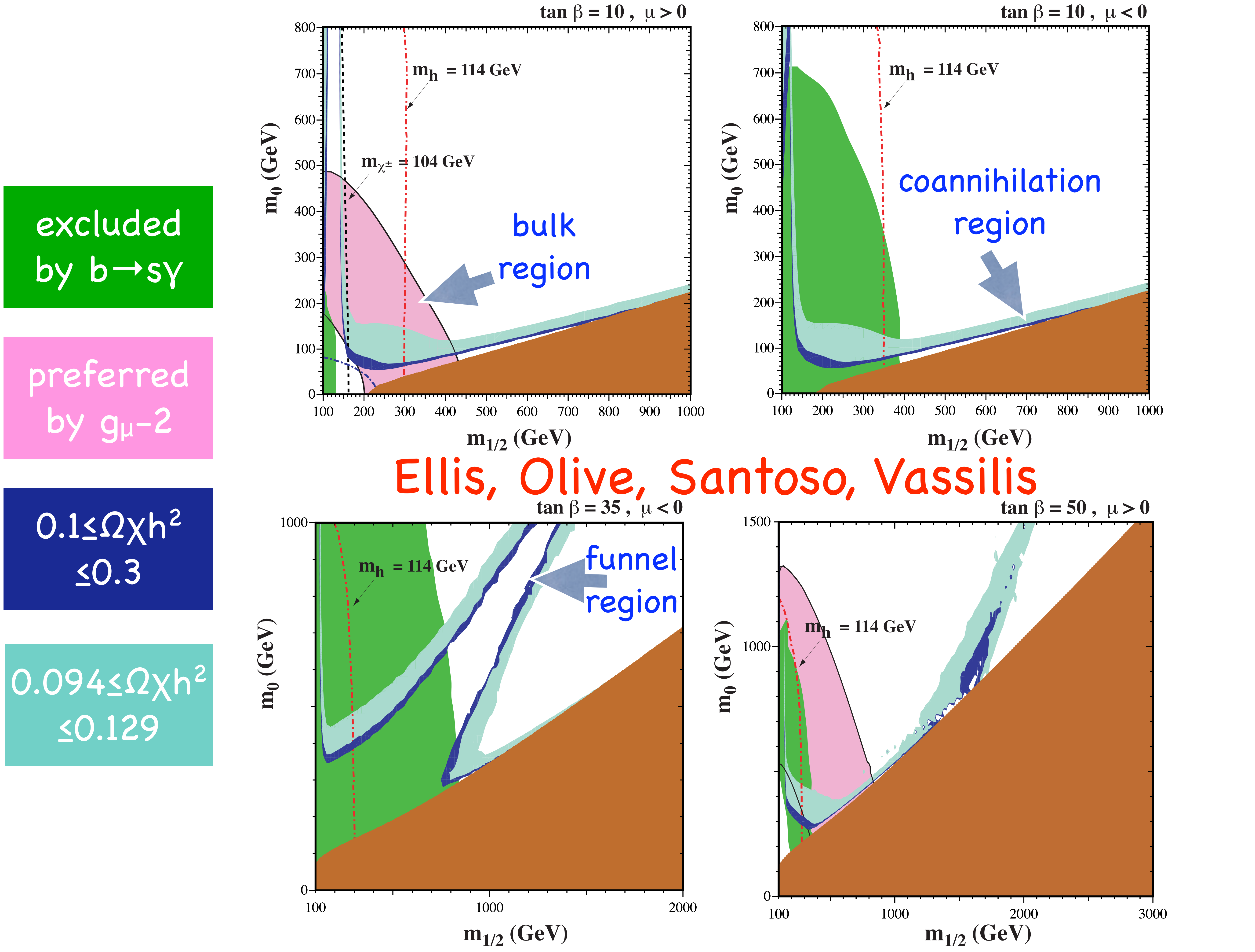
A little too much

- You get the right order of magnitude!
- But in detail, a little too much beyond the collider limits



LSP

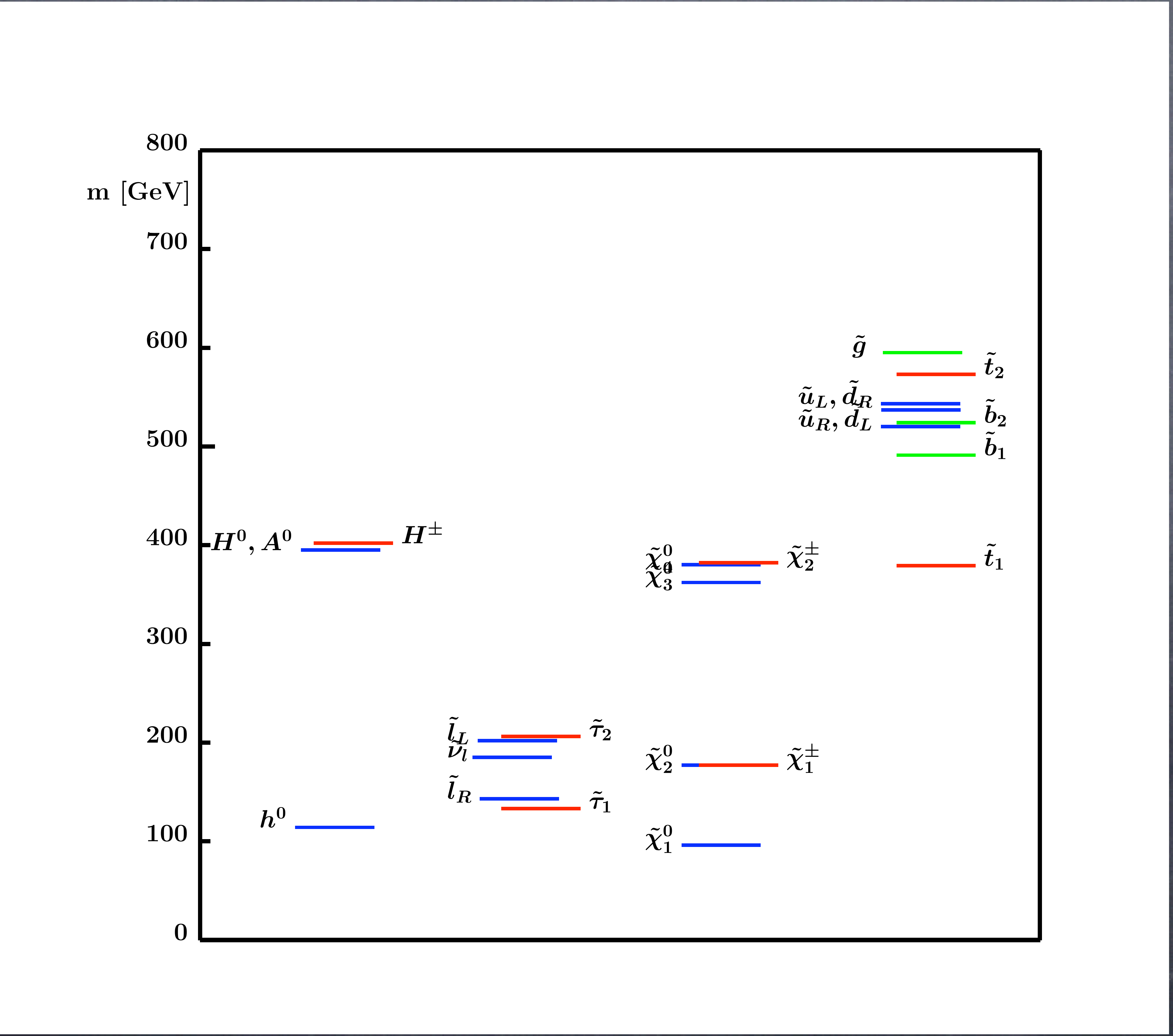
- The lightest Supersymmetric Particle is one of the best candidates for dark matter (assuming R-parity conservation)
- In the "Minimal Supergravity" or CMSSM, the LSP is bino-like
- Its annihilation cross section tends to be too small, abundance too large because it is P-wave suppressed $\tilde{B}\tilde{B} \to e^+e^-$
- Coannihilation region $\tilde{B}\tilde{\tau} \to \gamma\tau$
- Funnel region where annihilation goes through a Higgs resonance.



excluded by b→sγ
preferred by gμ-2
0.1≤Ωχh² ≤0.3
0.094≤Ωχh² ≤0.129
tan β = 10 , μ > 0
m_h = 114 GeV
m_χ± = 104 GeV
bulk region
m_0 (GeV)
m_1/2 (GeV)
tan β = 10 , μ < 0
m_h = 114 GeV
coannihilation region
Ellis, Olive, Santoso, Vassilis
tan β = 35 , μ < 0
m_h = 114 GeV
funnel region
tan β = 50 , μ > 0
m_h = 114 GeV

sample spectrum

$m_0 = 100,\ m_{1/2} = 250,\ A_0 = -100,\ \tan\beta = 10,\ \mu > 0$

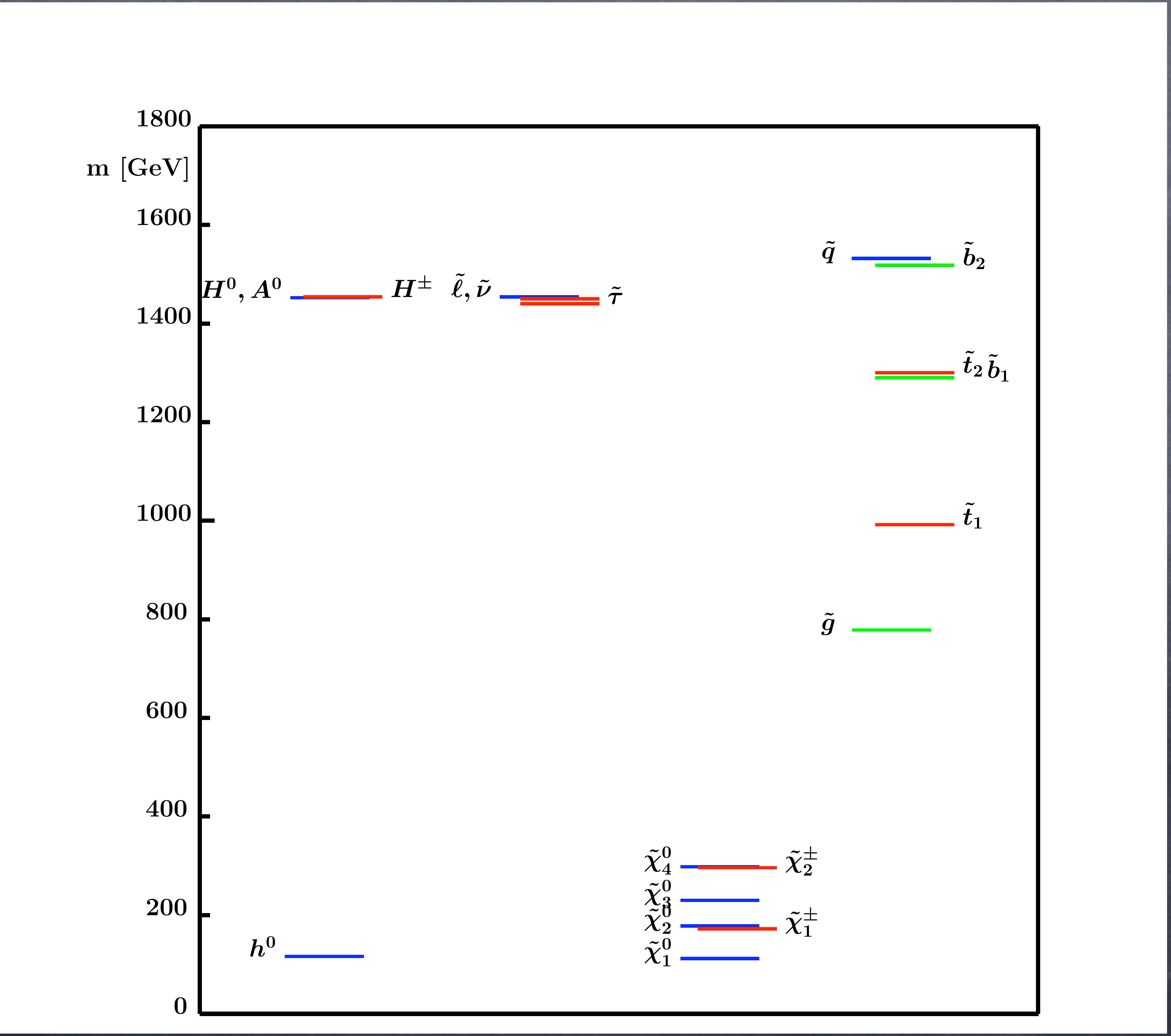


bulk region

SPS1a

sample spectrum

$m_0 = 1450,\ m_{1/2} = 300,\ A_0 = 0,\ \tan\beta = 10,\ \mu > 0$

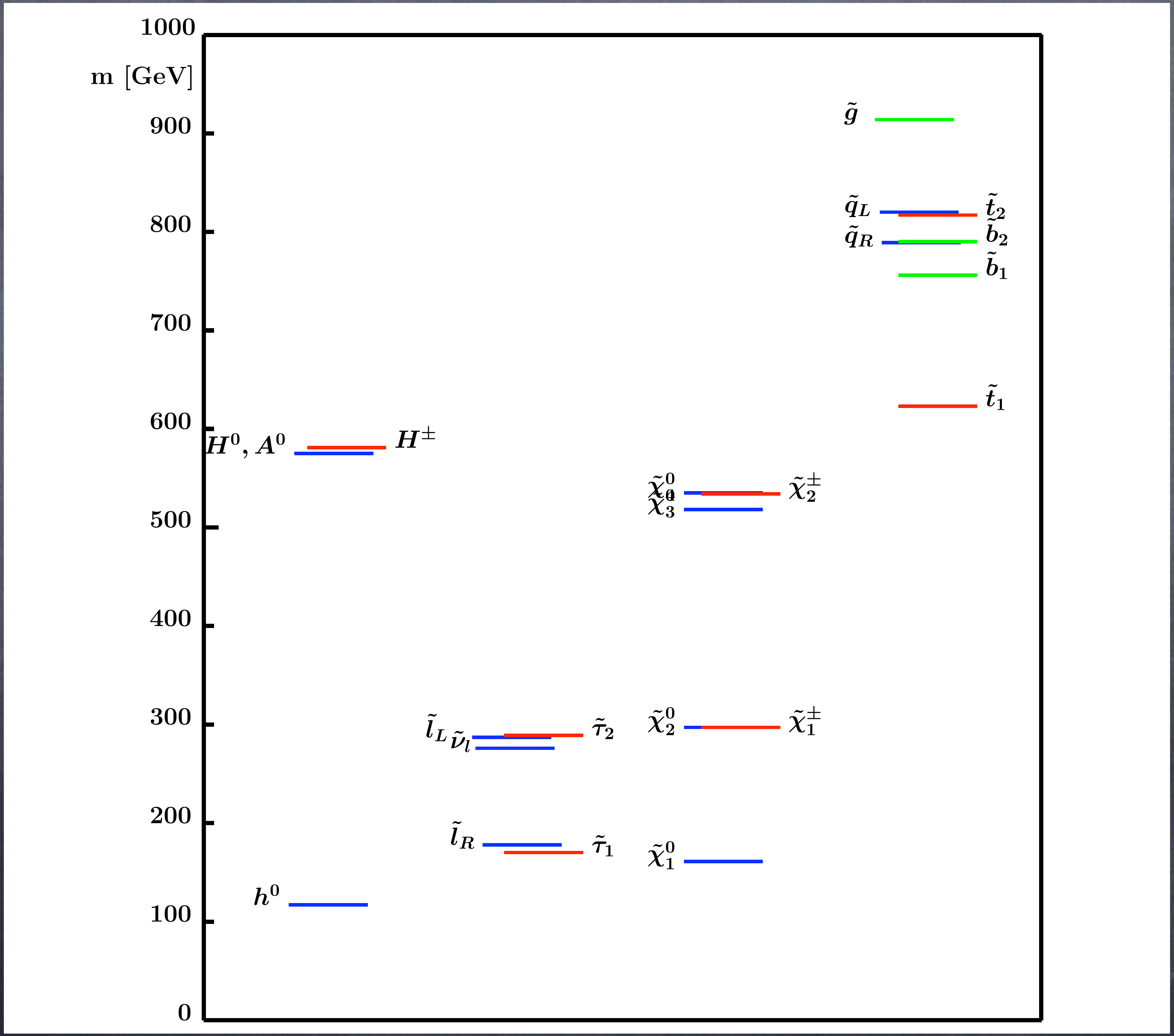


focus point region

SPS2

sample spectrum

$m_0 = 90,\ m_{1/2} = 400,\ A_0 = 0,\ \tan\beta = 10,\ \mu > 0$



coannihilation region

SPS3

Caveat

- The dark matter abundance is very sensitive to the superparticle spectrum, and hence on the mechanism of supersymmetry breaking
- "Minimal Supergravity" not well motivated theoretically, and its extension modifies the prediction
- Be careful about any strong claims!

Colliders

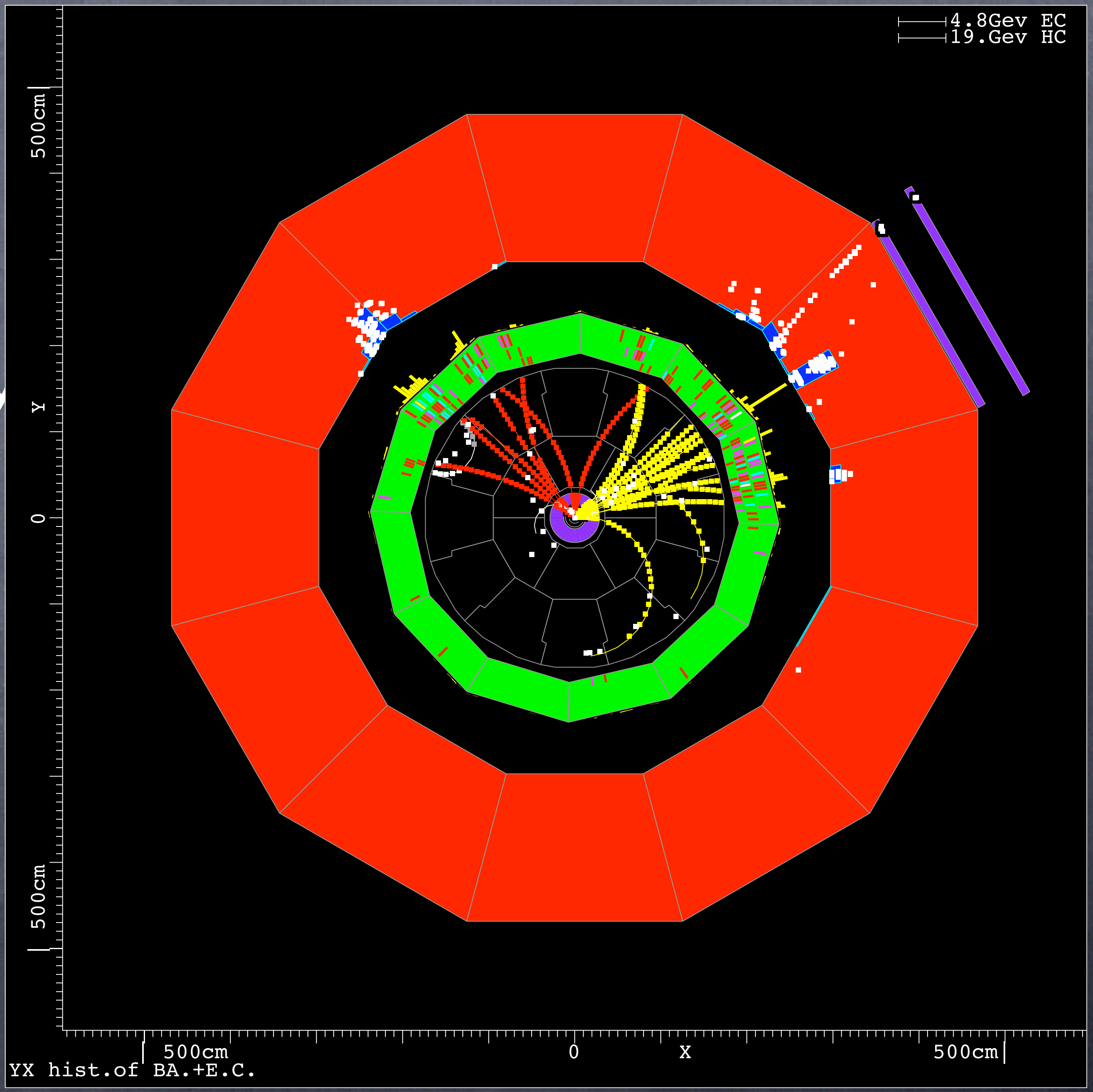
Producing Dark Matter in the laboratory

- Collision of high-energy particles mimic Big Bang
- We hope to create Dark Matter particles in the laboratory
- Look for events where energy and momenta are unbalanced

"missing energy" E_{miss}

- Something is escaping the detector
- electrically neutral, weakly interacting

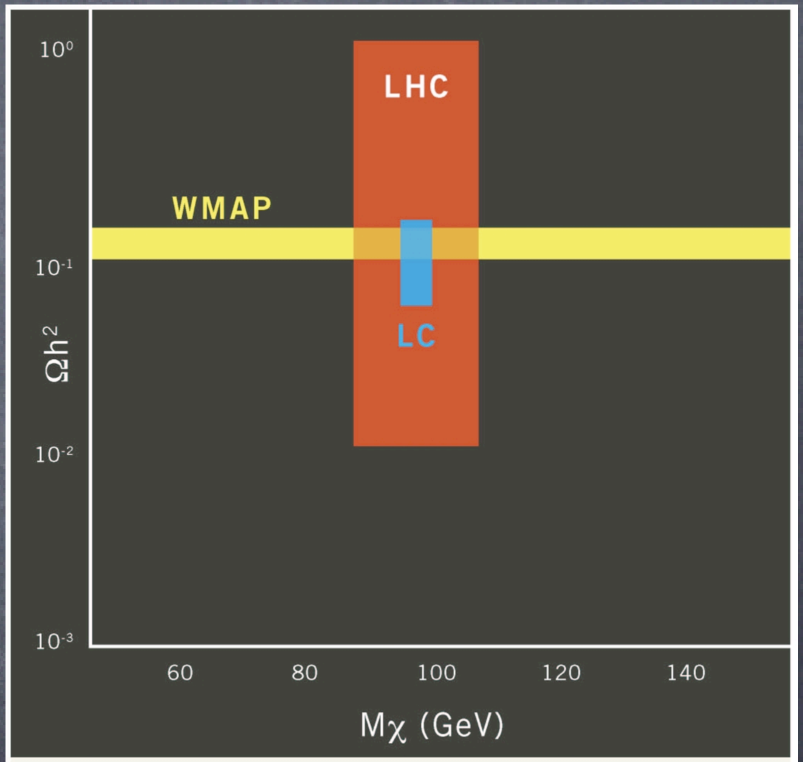
⇒Dark Matter!?



Concordance model of Dark Matter?

- cosmological measurement of dark matter
  - $\Rightarrow$ abundance $\propto$ (annihilation cross section)$^{-1}$
- detection experiments
  - $\Rightarrow$ scattering cross section
- production at colliders
  - $\Rightarrow$ mass, couplings
  - $\Rightarrow$ can calculate cross sections
- Will know what Dark Matter is
- Will understand universe back to $t \sim 10^{-10}$sec

just like BBN!



STAU COANIHILLATION

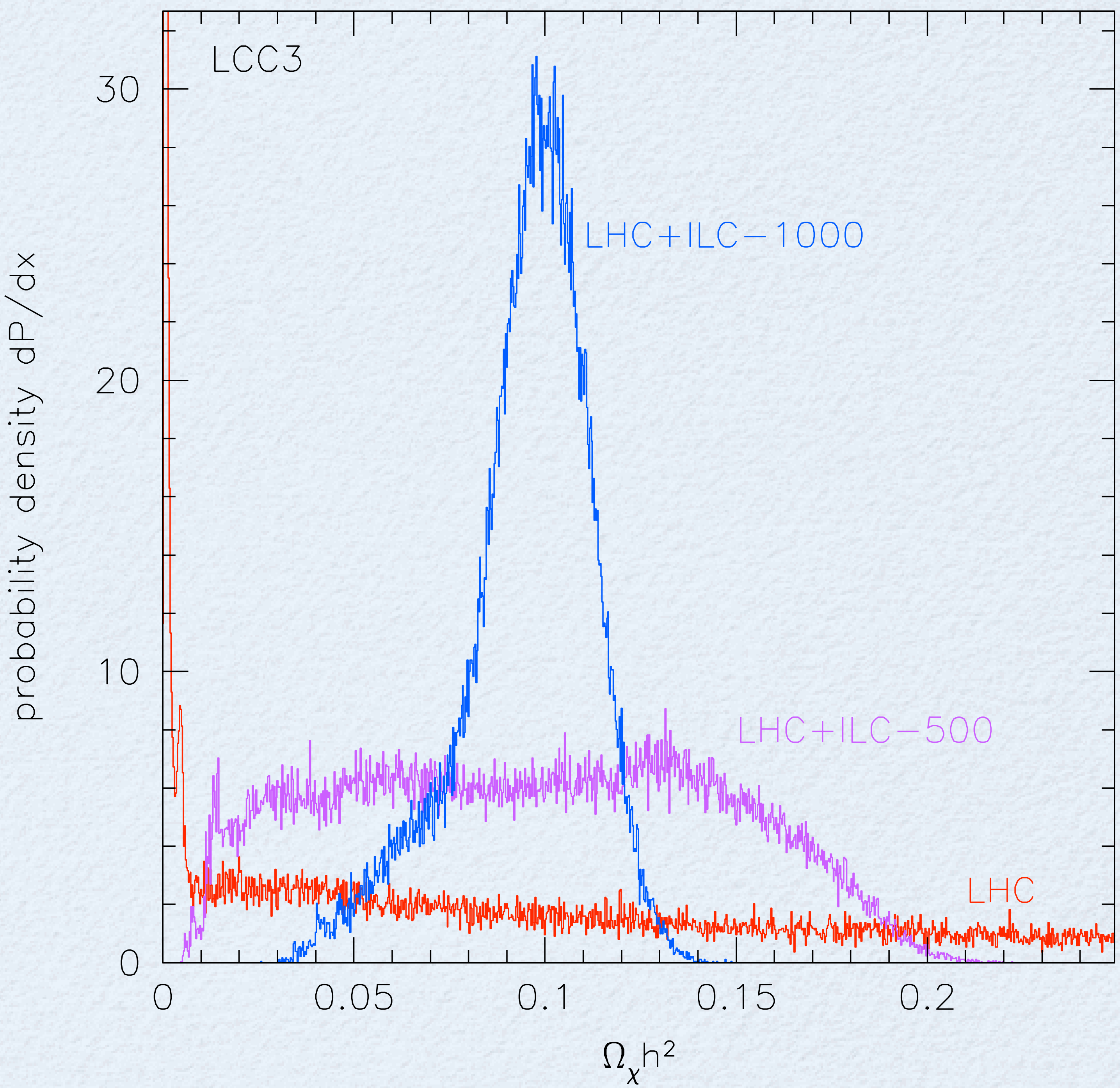
LHC data are not sensitve to mass difference betewnn LSP and stau

ILC@1TeV give important imformation

$\delta\Omega$

167% (LHC@300fb^-1)

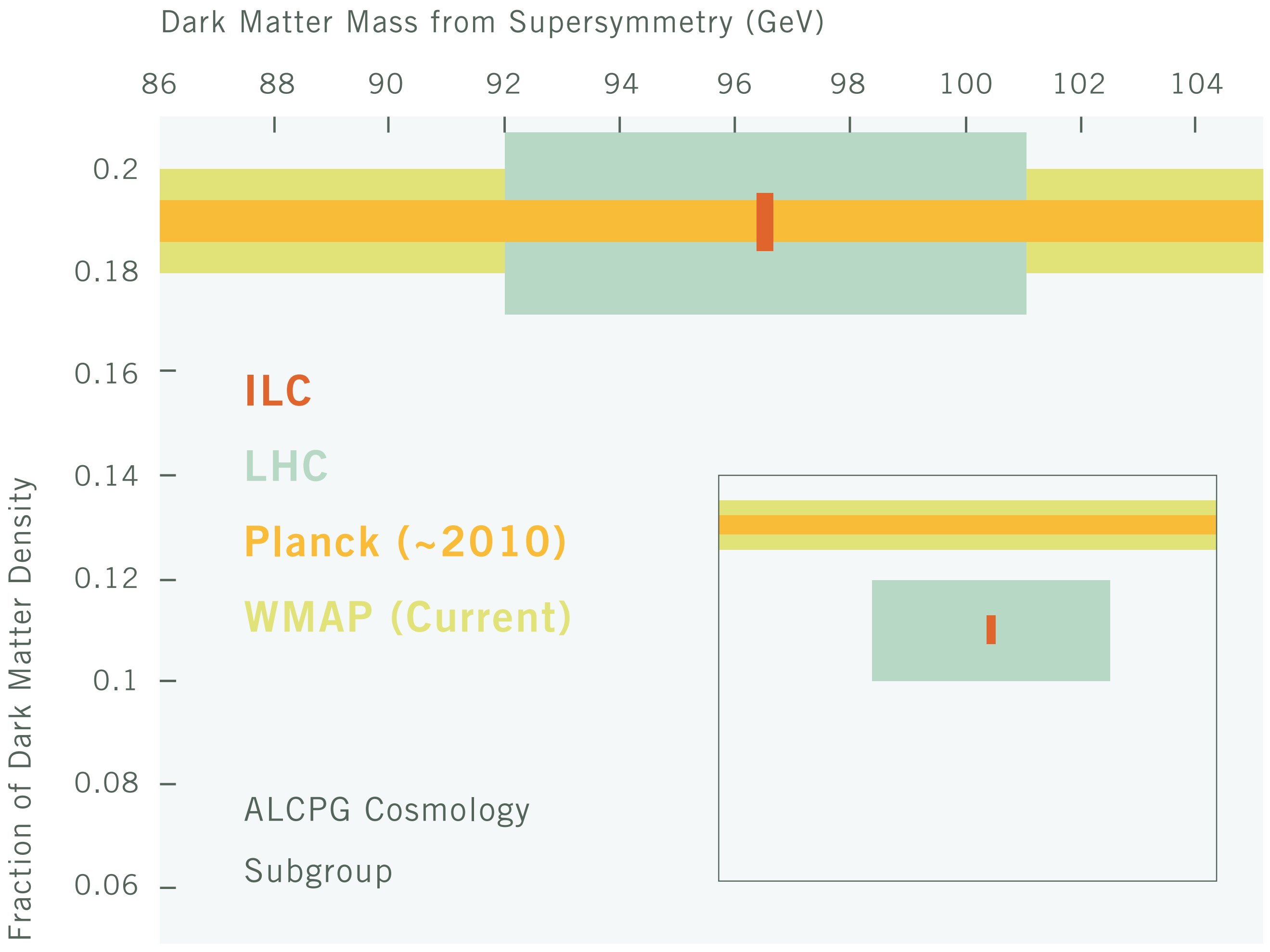
18% (ILC@500fb^-1)



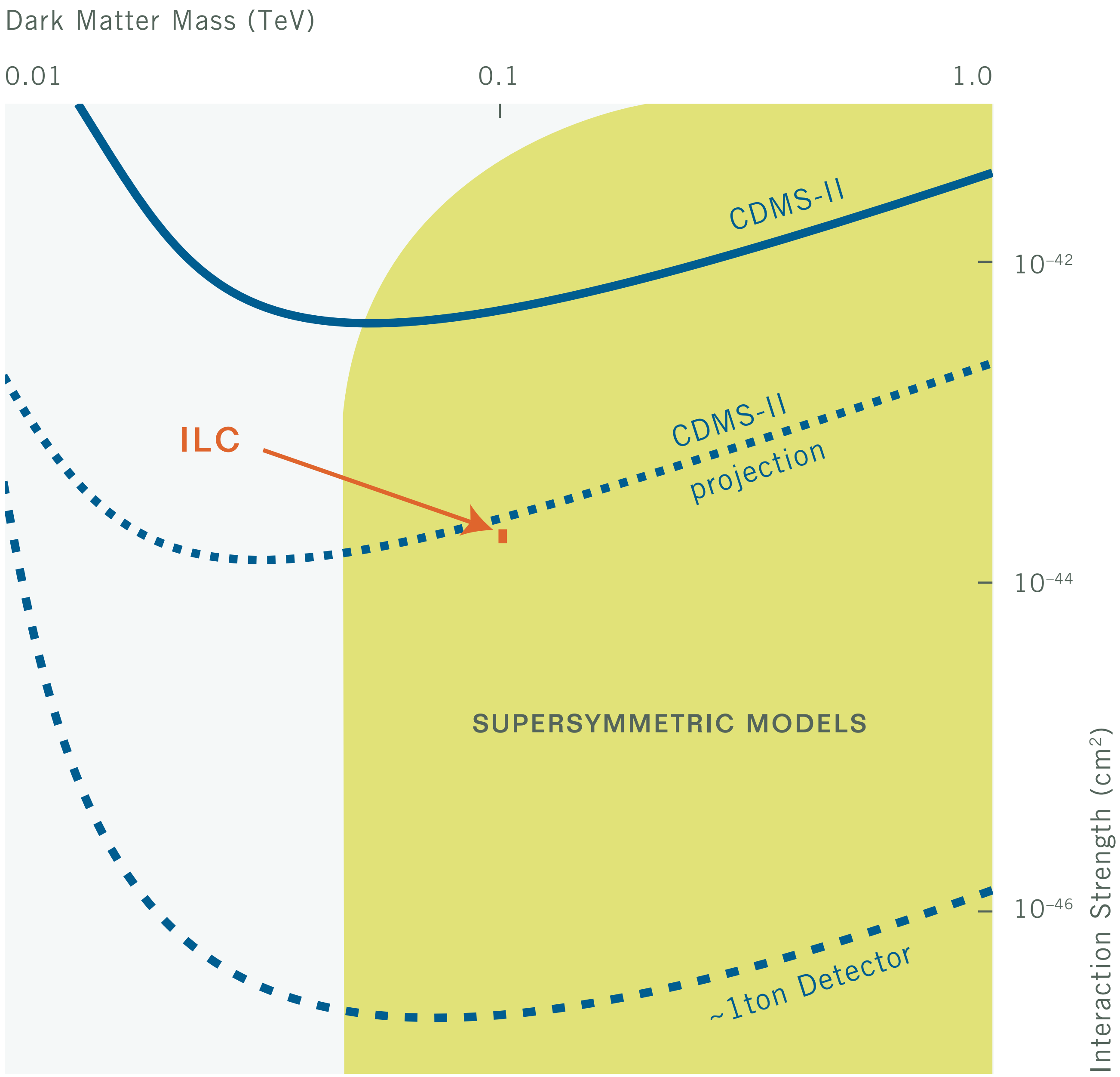
Shimizu, taken from E. Baltz et al

Dark Matter

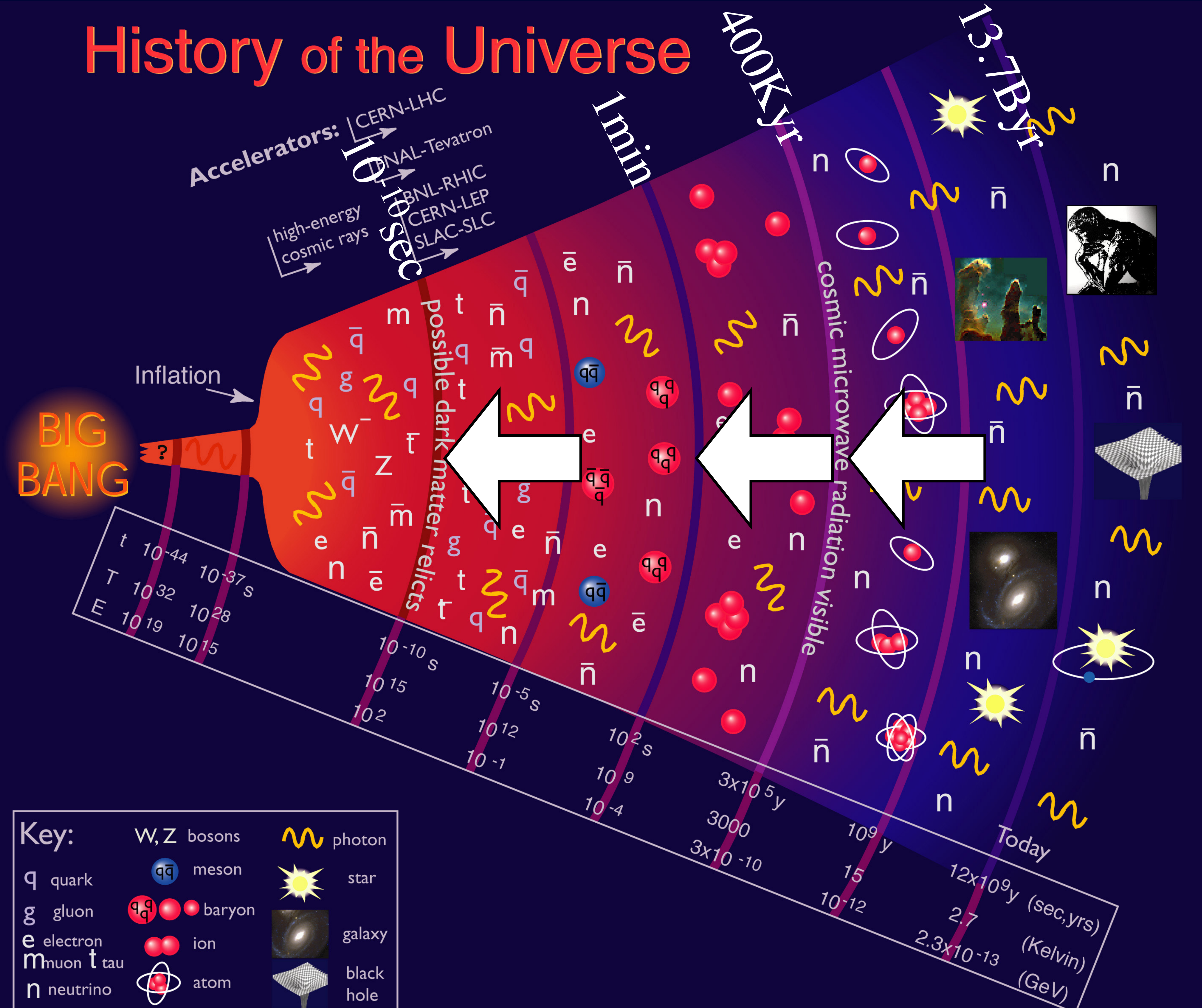
abundance



direct cross section



History of the Universe



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Conclusions

- Supersymmetry is still the best solution to the hierarchy problem
- To be present at TeV, it needs to be flavor-blind, quite possible in many models
- It provides a very good candidate for dark matter
- It may help us understand the dark energy
- It connects string theory, unification, collider physics, cosmology, astrophysics in a remarkable way
- We may see if it is true in the next year!