Lecture II

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Global SUSY

SUSY algebra

- In addition to the momentum P_μ and Lorentz generators $M_{\mu\nu}$, add spinorial charges $Q_\alpha{}^i$, i=1, .., N

$$[P_\mu, Q^i_\alpha] = 0$$
$$[M^{\mu\nu}, Q^i_\alpha] = \frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^\beta Q^i_\beta$$
$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot\beta}\delta^{ij}$$
$$\{Q^i_\alpha, Q^j_\beta\} = \varepsilon_{\alpha\beta}C^{ij}$$

- $C^{ij}=-C^{ji}$ central charges, possible only if N>1

Massless particle

- Specialize to P_μ=E(1,0,0,1)
- helicity: h = ϵ_{0ijk} P^i $M^{jk}/2P^0$ = M^{12}

$$P^\mu = E(1,0,0,1)$$

$$\{Q^i_\alpha, \bar{Q}^j_{\dot\beta}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot\beta}\delta^{ij} = 2E\begin{pmatrix}2 & \\ & 0\end{pmatrix}\delta^{ij}$$

- creation/annihilation operators (C^{ij}=0)

$$b^i = Q^i_1/\sqrt{4E}, \qquad b^{i\dagger} = \bar{Q}^i_1/\sqrt{4E}$$

$$\{b^i, b^{j\dagger}\} = \delta^{ij}$$

$$[h, b^i] = -\frac{1}{2}b^i$$

$$[h, b^{i\dagger}] = -\frac{1}{2}b^{i\dagger}$$

N=1 multiplets

- Restrict $|h| \leq 1$ for renormalizability
- Multiplet structure: $\{|0\rangle, b^\dagger|0\rangle\}$
- chiral multiplet: pick $h|0\rangle=0$ or $-1/2$
  - $h=\{0, 1/2\}$ (anti-chiral) or $\{-1/2, 0\}$ (chiral)
  - i.e., a Weyl fermion and a complex scalar
- vector multiplet: pick $h|0\rangle=1/2|0\rangle$ or -1
  - $h=\{1/2, 1\}$ or $\{-1, -1/2\}$
  - i.e, a gauge field and a Weyl (Majorana) fermion
  - must be in adjoint (real) rep ⇒ non-chiral

N=2 multiplets

- Multiplet: $\{|0\rangle, b_1^\dagger|0\rangle, b_2^\dagger|0\rangle, b_1^\dagger b_2^\dagger|0\rangle\}$
- hypermultiplet: pick $h|0\rangle=-1/2$
  - $h=\{-1/2, 0, 0, 1/2\}$
  - under N=1, sum of chiral and anti-chiral
  - a Dirac fermion and two complex scalars
- vector multiplet: pick $h|0\rangle=0|0\rangle$
  - $h=\{0, 1/2, 1/2, 1\}$
  - under N=1, sum of chiral and vector
  - a gauge field and two Weyl (Majorana) fermions, and a real scalar
- Neither multiplet chiral: can't be used to supersymmetrize the Standard Model

N=1 superspace chiral superfield

- introduce Grassman-odd coordinates θ_α
- superspace: $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$

$$P_\mu = i\partial_\mu$$
$$M^{\mu\nu} = i(x^\mu\partial^\nu - x^\nu\partial^\mu)$$
$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar{\theta}^{\dot{\beta}}\sigma^\mu_{\dot{\beta}\alpha}\partial_\mu$$

- Covariant derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar{\theta}^{\dot{\beta}}\sigma^\mu_{\dot{\beta}\alpha}\partial_\mu$$
$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = 0$$
$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = -2i\partial_\mu(\sigma^\mu)_{\alpha\dot{\beta}}$$

- Can place a constraint $\bar{D}_{\dot{\alpha}}\phi(x,\theta,\bar{\theta}) = 0$
- Note $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}, \quad \bar{D}_{\dot{\alpha}}y^\mu = 0$
- solution: $\phi(x,\theta,\bar{\theta}) = \phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$

vector superfield

- introduce Grassman-odd coordinates θ_α
- real (vector) superfield:

$$V(x,\theta,\bar\theta) = C + \theta\chi + \bar\theta\bar\chi + \theta^2 M + \bar\theta^2 M^* + \theta\sigma^\mu\bar\theta A_\mu + \theta^2\bar\theta\bar\lambda + \bar\theta^2\theta\lambda + \theta^2\bar\theta^2 D$$

- gauge transformation: $V \to V + i\Lambda - i\bar\Lambda$

  Λ is a chiral superfield

  Can eliminate C, χ, M: Wess-Zumino gauge
- remaining dof: A_μ, λ, D
- Field strength chiral superfield:

$$W_\alpha = \bar D^2 D_\alpha V = \lambda_\alpha(y) + \theta^\beta \sigma^{\mu\nu}_{\beta\alpha} F_{\mu\nu} + \theta_\alpha D$$

Non-abelian gauge symmetry

- Generalization to non-abelian case

$$V = V^a T^a, \quad \Lambda = \Lambda^a T^a$$
$$e^V \to e^{-i\bar{\Lambda}} e^V e^{i\Lambda}$$
$$\phi \to e^{-i\Lambda}\phi$$
$$\phi^\dagger e^V \phi : \text{ invariant}$$

- Field strength chiral superfield

$$W_\alpha = \bar{D}^2 e^{-V} D_\alpha e^V$$
$$W_\alpha \to e^{-i\Lambda} W_\alpha e^{i\Lambda}$$

Kähler and superpotentials

- Two ways to construct invariants (up to total derivatives)
- full superspace integral of general superfield "Kähler potential"

$$\int d^4\theta \phi^* \phi = |\partial_\mu A|^2 + \bar{\psi} i \sigma^\mu \partial_\mu \psi + F^* F$$

- chiral superspace integral of chiral superfield "superpotential"

$$\int d^2\theta W(\phi) = \frac{\partial W}{\partial \phi^i} F^i + \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j$$

- Renormalizable theory is fixed by W

gauge theory (Wess-Zumino gauge)

matter kinetic term

$$\int d^4\theta \phi^\dagger e^V \phi = |D_\mu A|^2 + \bar{\psi} i \not{D} \psi + F^\dagger F + D^a A^\dagger T^a A + (\sqrt{2} A^\dagger \lambda^a T^a \psi + h.c.)$$

gauge kinetic term

$$\int d^2\theta \frac{1}{g^2} W^a_\alpha W^{a\alpha} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{g^2} \bar{\lambda}^a i \not{D} \lambda^a + \frac{1}{2g^2} D^a D^a$$

superpotential

$$\int d^2\theta W(\phi) = \frac{\partial W}{\partial \phi^i} F^i + \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j$$

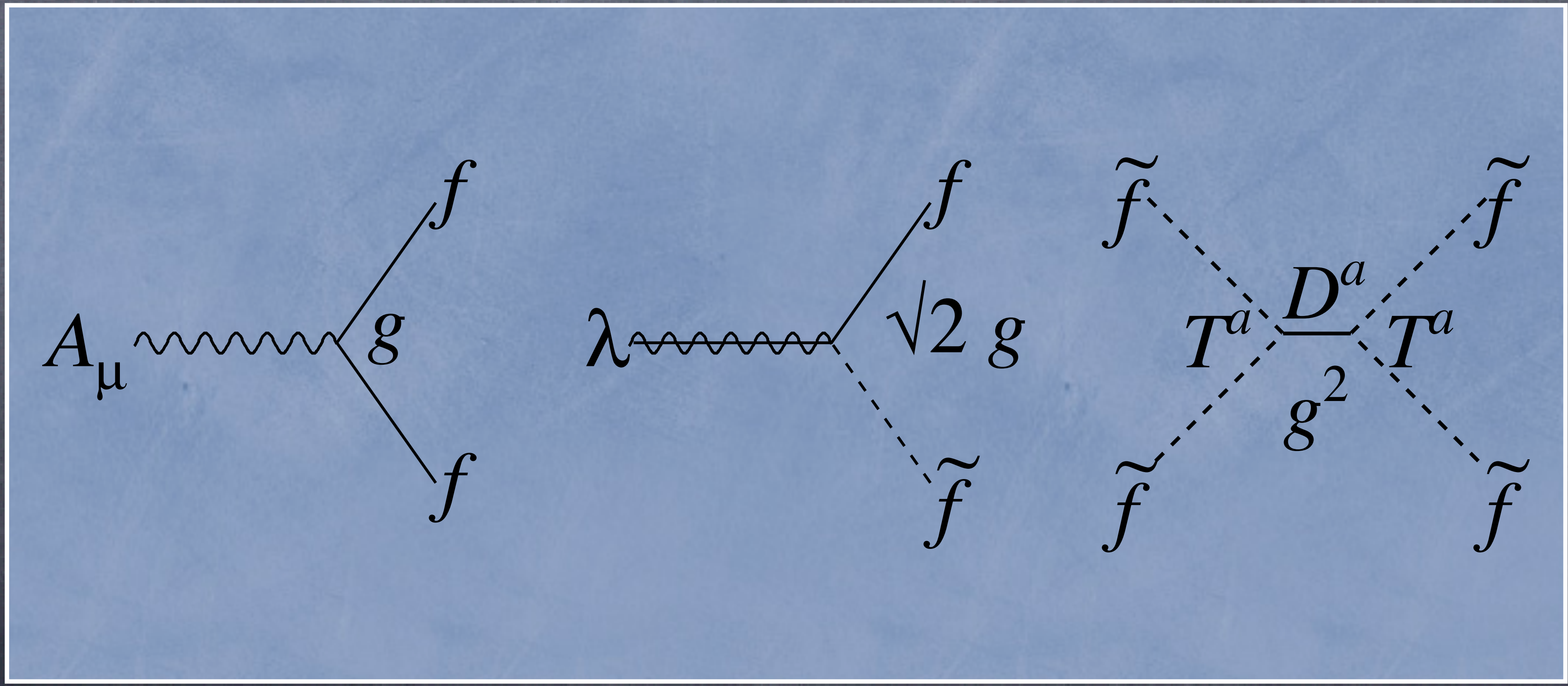
Solve for auxiliary fields

$$D^a = g^2 A^\dagger T^a A, \quad F^{i*} = \frac{\partial W}{\partial \phi^i}$$

$$V = \frac{1}{2g^2} D^a D^a + |F^i|^2 = \frac{g^2}{2} (A^\dagger T^a A)^2 + \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

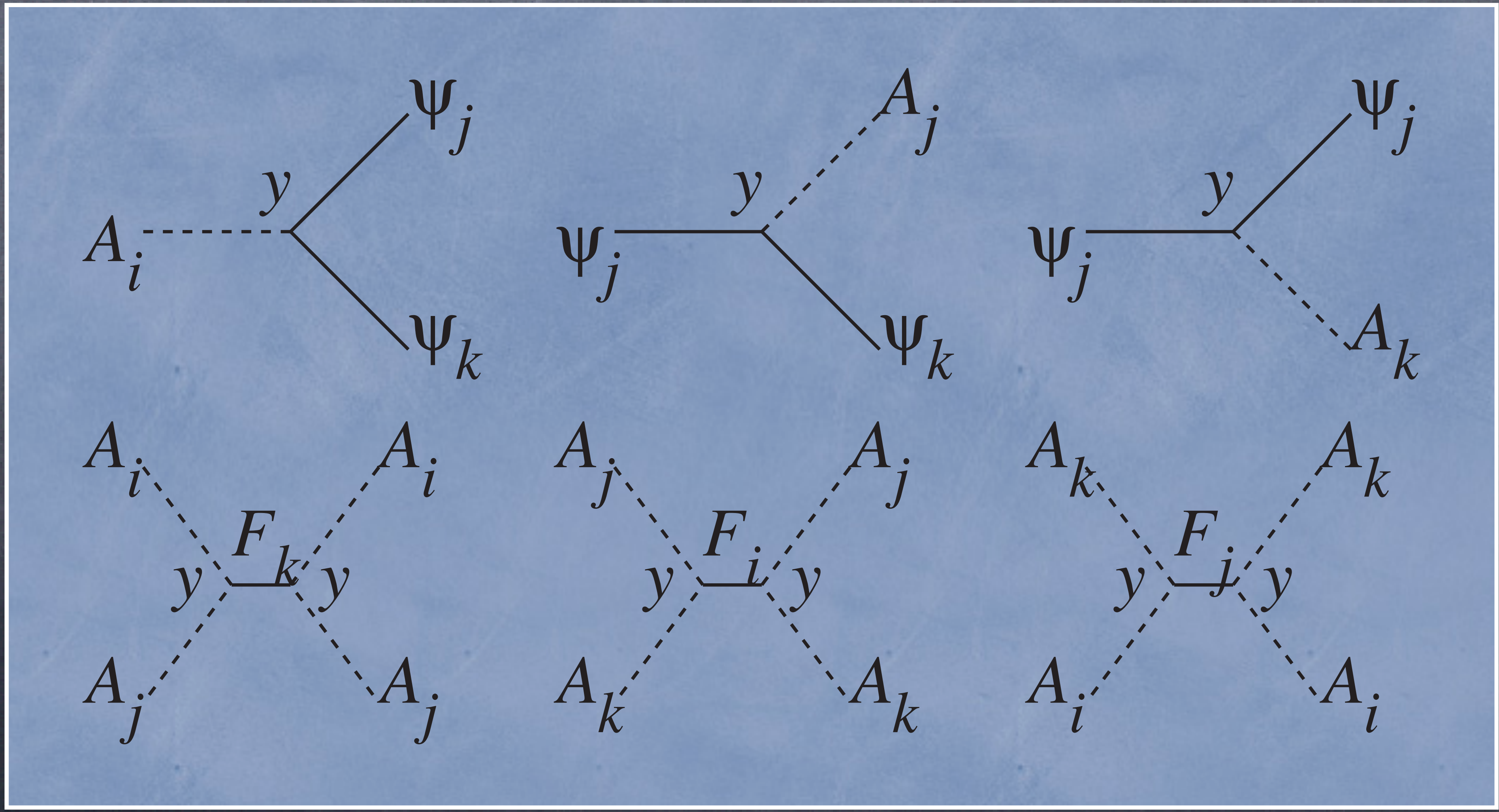
Feynman rules

- Single gauge coupling constant gives all of these Feynman vertices



Feynman rules

- Single Yukawa coupling constant gives all of these Feynman vertices



Fayet-Illiopoulos D-term

- Only for U(1) gauge factors, there is another possible term
$$\int d^4\theta V \to \int d^4\theta(V + i\Lambda - i\bar{\Lambda}) = \int d^4\theta V + \text{surface terms}$$
- constant term: ξD
- changes the D-term potential: $\frac{g^2}{2}\left(A^{i\dagger}Q_i A^i - \frac{\xi}{g^2}\right)^2$
- not consistent with supergravity by itself unless $U(1)_R$ is gauged or Green-Schwarz mechanism is employed
- I will not discuss it any further

Renormalization

- Start with the Wilsonian action at scale μ

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\frac{1}{g_0^2} W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- non-renormalization theorem, holomorphy, and transivity says at scale $\mu' = \mu e^{-t}$,

$$\int d^4\theta \sum_i Z_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t \right) W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

$$b_0 = 3C_A - \Sigma_i T_F^i$$

- to identify coupling constants, need to rescale fields to canonical normalization
- However, rescaling fields yield anomalous Jacobians

Renormalization

- Konishi anomaly $\int \mathcal{D}\phi_i = \int \mathcal{D}(e^{\sigma}\phi_i)e^{-\int d^2\theta T_F^i \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- rescaling anomaly $\int \mathcal{D}V = \int \mathcal{D}(e^{\sigma}V)e^{+\int d^2\theta C_A \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$
- first rescale matter fields

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i \right) W_\alpha W^\alpha + Z_i^{-1/2} Z_j^{-1/2} Z_k^{-1/2} \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- then rescale the gauge field $V \to g_c V$

$$\frac{1}{g_c^2} = \frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i - C_A \frac{1}{8\pi^2} \ln g_c^2$$

$$\int d^2\theta \frac{1}{g_c^2} (\bar{D}^2 e^{-g_c V} D_\alpha e^{g_c V})^2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \cdots$$

The Minimal Supersymmetric Standard Model (MSSM)

supersymmetrize it

- All quarks and leptons are Weyl fermions
- chiral superfields have left-handed Weyl fermions
- Use charge conjugation to make them all left-handed: Q, L, u^c, d^c, e^c
- Promote them to chiral superfields, namely add their scalar partners
- Naming convention: add "s" as a prefix, which stands for supersymmetry or scalar

  terrible convention!
- e.g.: selectron, smuon, stop, sup, sstrange

supersymmetrize it

- All gauge fields are promoted to vector multiplets
- namely add massless Weyl=Majorana fermions "gauginos"
- Naming convention: add "ino" as a suffix, which doesn't mean "small" in any sense
  terrible convention!
- e.g.: gluino, wino, photino, zino, bino

supersymmetrize it

- Minimal Standard Model has only one Higgs doublet
- It gives mass to both up- and down-type fields $\mathcal{L}_{Yukawa} = Y_u^{ij}\bar{Q}_i u_j \tilde{H} + Y_d^{ij}\bar{Q}_i d_j H + Y_l^{ij}\bar{L}_i e_j H, \quad \tilde{H} = i\sigma_2 H^*$
- Promote it to a chiral superfield, namely add a "Higgsino"
- But having only one higgsino makes the SU(2) xU(1) anomalous
- Also complex conjugation not allowed in a superpotential
- solution: introduce two Higgs doublets

  $H_u(1,2,+1/2)$, $H_d(1,2,-1/2)$

The MSSM

- $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory

	Q	d^c	u^c	L	e^c	H_u	H_d	g	B	W
$SU(3)_C$	3	3^*	3^*	1	1	1	1	8	1	1
$SU(2)_L$	2	1	1	2	1	2	1	1	1	3
$U(1)_Y$	+1/6	+1/3	−2/3	−1/2	−1	+1/2	−1/2	0	0	0
mltplt	χ	χ	χ	χ	χ	χ	χ	V	V	V
flavor	3	3	3	3	3	1	1	1	1	1
Z_2	−	−	−	−	−	+	+	+	+	+

The superpotential

- The terms we want

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

- The terms we don't want (violates B or L)

$$W_{\not{R}_p} = \lambda_{ijk} u_i^c d_j^c d_k^c + \lambda'_{ijk} Q_i d_j^c L_k + \lambda''_{ijk} L_i L_j e_k^c + \mu_i L_i H_u$$

- Impose Z_2 symmetry ("matter parity") that all matter chiral superfields are odd, Higgs even
- combined with 2π rotation of space $(-1)^{2S}$ (is equivalent to $\theta \to -\theta$), it gives R-parity

The Higgs potential

Without supersymmetry breaking effects, the superpotential

$$W_{MSSM} = Y_u^{ij} Q_i u_j^c H_u + Y_d^{ij} Q_i d_j^c H_d + Y_l^{ij} L_i e_j^c H_d + \mu H_u H_d$$

gives the potential for the Higgs field

$$V = \mu^2 (H_u^\dagger H_u + H_d^\dagger H_d) + \frac{g^2}{8} (H_u^\dagger \vec{\tau} H_u + H_d^\dagger \vec{\tau} H_d)^2 + \frac{g'^2}{8} (H_u^\dagger H_u - H_d^\dagger H_d)^2$$

which has only one ground state

$$\langle H_u \rangle = \langle H_d \rangle = 0$$

Namely the electroweak SU(2)xU(1) is unbroken unless supersymmetry is broken