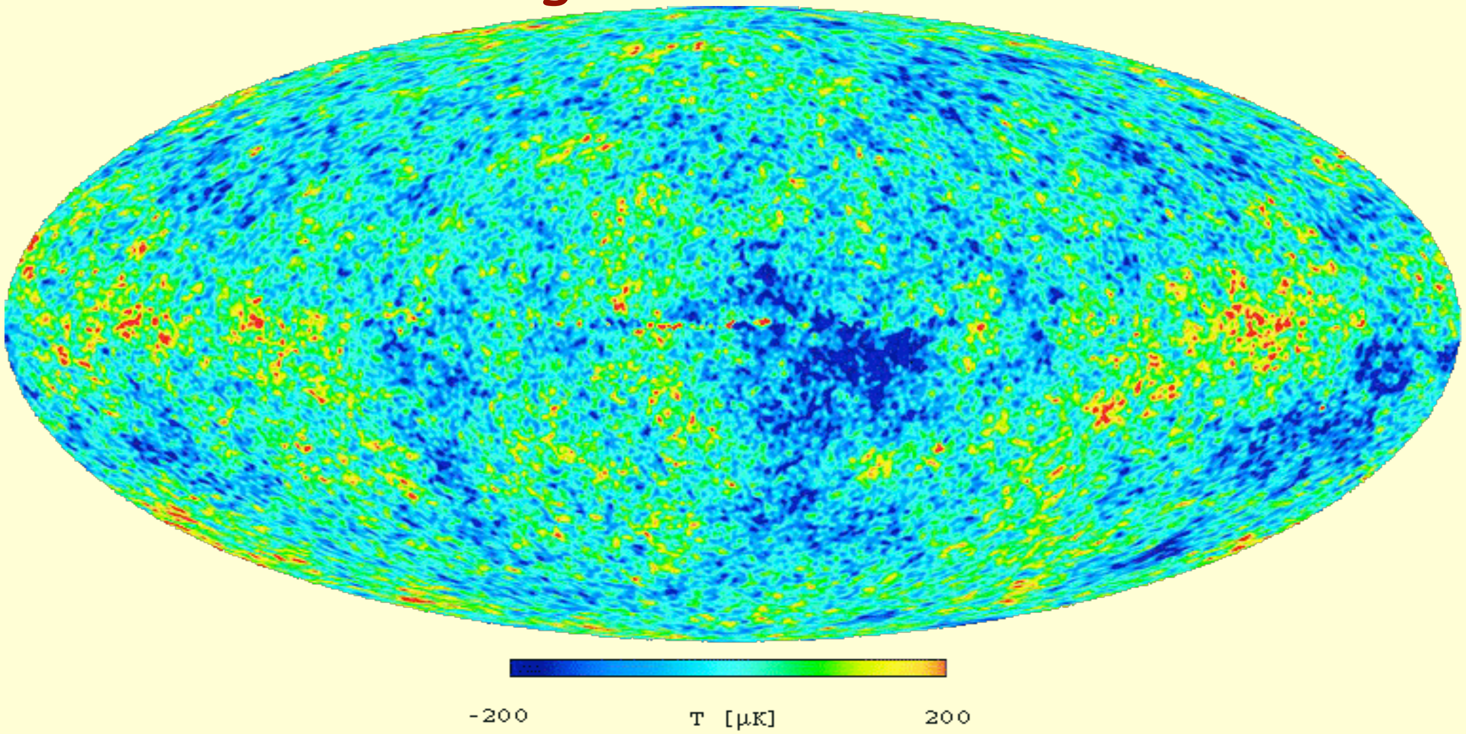
The Physics of the cosmic microwave background
Lecture 3

Cargèse, 8 aout, 2007



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The homogeneous and isotropic universe

cosmological parameters

$$
\begin{aligned}
H_0 &= \frac{\dot{a}}{a}(t_0) = h100\mathrm{km/sMpc} && \text{Hubble parameter} \\
\rho_c &= \frac{3}{8\pi G} H_0^2 && \text{critical density} \\
\Omega_m &= \frac{\rho_m(t_0)}{\rho_c} && \text{matter density parameter} \\
\Omega_b &= \frac{\rho_b(t_0)}{\rho_c} && \text{baryon density parameter} \\
\Omega_r &= \frac{\rho_r(t_0)}{\rho_c} && \text{radiation density parameter} \\
\Omega_k &= \frac{-k}{a_0^2 H_0^2} && \text{curvature parameter} \\
\Omega_\Lambda &= \frac{\Lambda}{3H_0^2} && \text{cosmological constant parameter}
\end{aligned}
$$

- reionisation

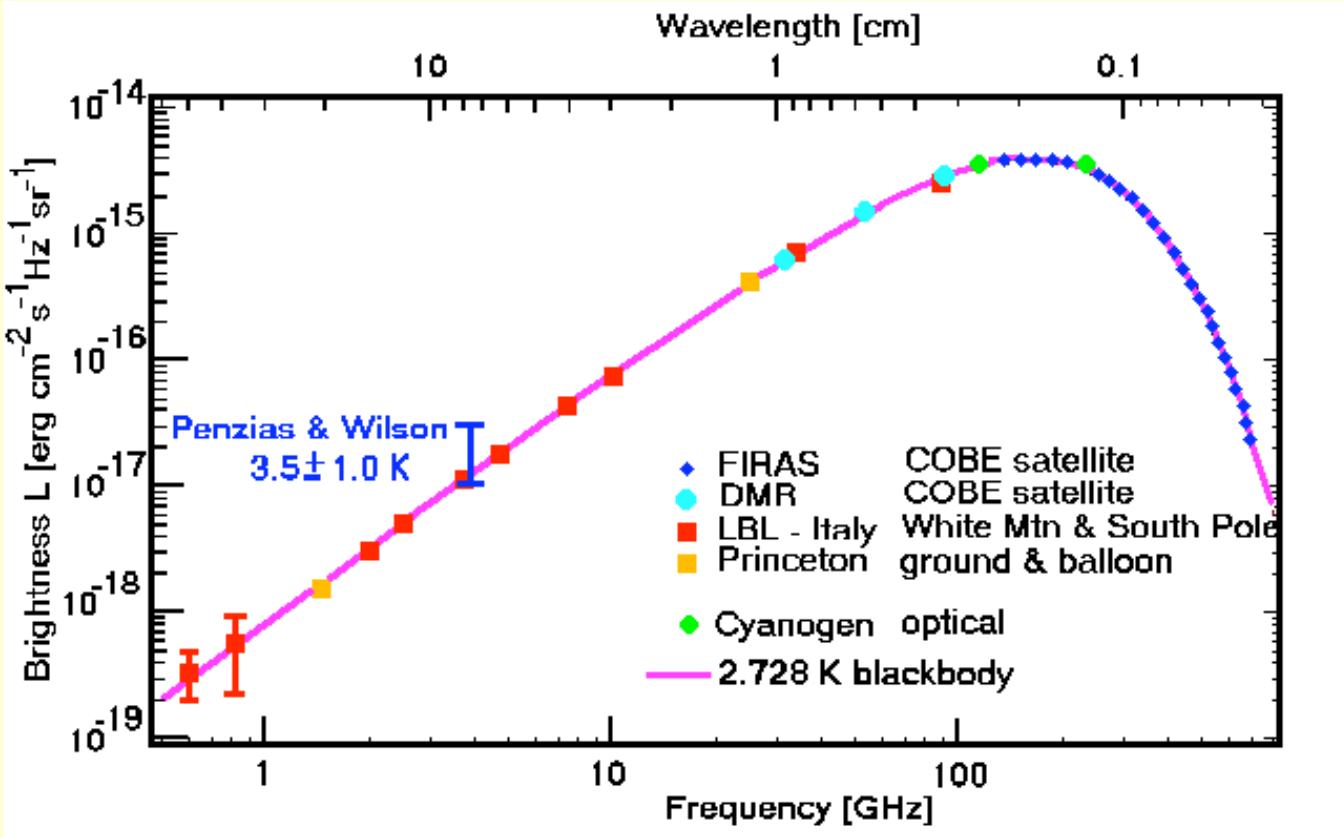
τ	optical depth to the last scattering surface
z_{rei}	redshift of reionisation

- Description of perturbations

A_s		amplitude of scalar perturbations
n_s		spectral index of scalar perturbations
R	$=$	A_T/A_s amplitude of tensor perturbations
n_T		spectral index of tensor perturbations
$(\sigma_8$		amplitude of perturbations at $8h^{-1}Mpc)$

The CMB

- After recombination (T ~ 3000K, t~3.8x10^5 years) the photons propagate freely, simply redshifted due to the expansion of the universe
- The spectrum of the CMB is a 'perfect' Planck spectrum:



|m| < 10^{-4}

y < 10^{-5}

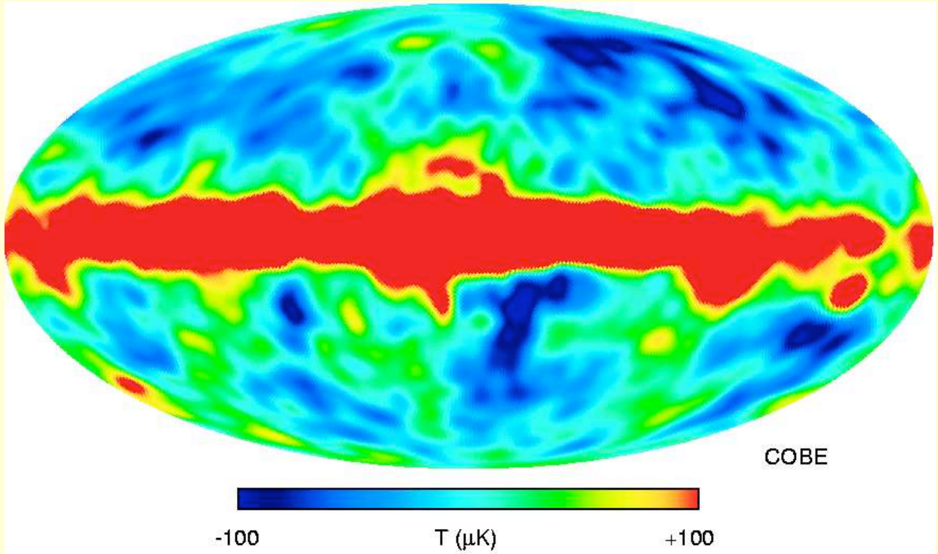
Y_{ff} < 2£ 10^{-5}

Þ ARCADE

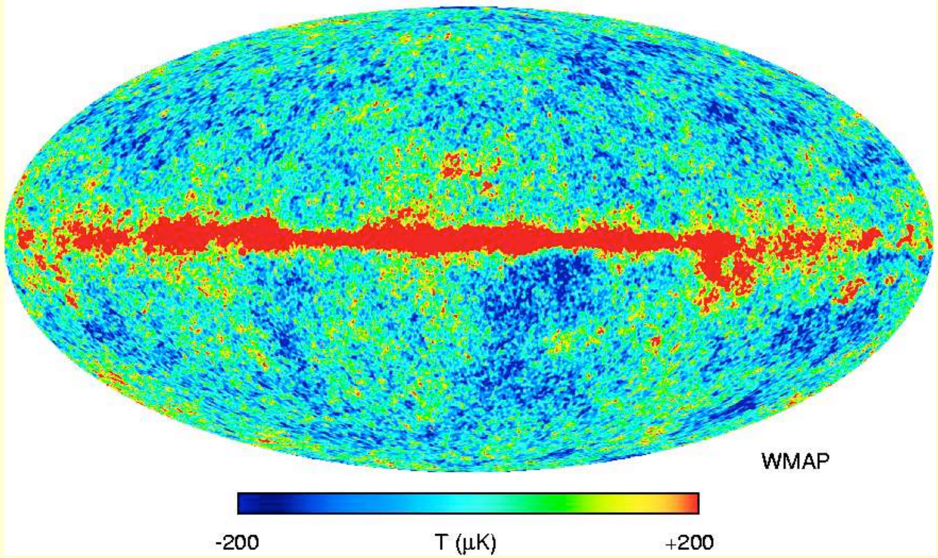
Þ DIMES

CMB anisotropies

COBE (1992)



WMAP (2003)



lightlike geodesics

From the surface of last scattering into our antennas the CMB photons travel along geodesics. By integrating the geodesic equation, we obtain the change of energy in a given direction n:

$E_f/E_i = (n{\cdot}u)_f/(n{\cdot}u)_i = [T_f/T_i](1+ DT_f/T_f - DT_i/T_i)$

This corresponds to a temperature variation. In first order perturbation theory one finds for scalar perturbations

$$\frac{\Delta T(\mathbf{n})}{T} = \left[\frac{1}{4}D_g^{(r)} + V_j^{(b)}n^j + \Psi + \Phi\right](\eta_{dec}, \mathbf{x}_{dec}) + \int_{\eta_{dec}}^{\eta_0} (\dot{\Psi} + \dot{\Phi})(\eta, \mathbf{x}(\eta))\, d\eta$$

acoustic oscillations

Doppler term

gravitat. potentiel (Sachs Wolfe)

integrated Sachs Wolfe ISW

The power spectrum of CMB anisotropies

DT(n) is a function on the sphere, we can expand it in spherical harmonics

$$\frac{\Delta T}{T}(\mathbf{x}_0, \mathbf{n}) = \sum_{\ell,m} a_{\ell m}(\mathbf{x}_0) Y_{\ell m}(\mathbf{n}) \qquad \langle a_{\ell m} \cdot a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$$

$$\langle \frac{\Delta T}{T}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n}') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\mathbf{n} \cdot \mathbf{n}')$$

consequence of statistical isotropy

observed mean

$$\frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2 \equiv C_\ell^{\text{obs}}$$

cosmic variance (if the a_{lm} 's are Gaussian)

$$\frac{\sqrt{\left\langle (C_\ell^{(obs)} - C_\ell)^2 \right\rangle}}{C_\ell} = \sqrt{\frac{2}{2\ell+1}}$$

Boltzmann eqn. I

Integrating the 1-particle distribution function of photons over energy, one arrives at the brightness,

$$\iota(t,\mathbf{x},\mathbf{n}) = \rho(t)\left[1+4\left(\frac{\Delta T}{T}(t,\mathbf{x},\mathbf{n})-\Phi(t,\mathbf{x})\right)\right]$$

Taking into acount elastic Thomson scattering before decoupling, one obtains a the following Boltzmann eqn. (in k-space, M ´ ΔT/T)

$$\dot{\mathcal{M}} + ik\mu\mathcal{M} = -ik\mu(\Psi+\Phi) + a\sigma_T n_e\left[-\mathcal{M}+\mathcal{M}_0+i\mu V_b+\frac{1}{2}P_2(\mu)\mathcal{M}_2\right]$$

with

$$\mathcal{M} = \sum_\ell (2\ell+1)(-i)^\ell \mathcal{M}_\ell(t,k)P_\ell(\mu)$$

we find the Boltzmann hierarchy

$$\dot{\mathcal{M}}_\ell + \frac{k(\ell+1)}{2\ell+1}\mathcal{M}_{\ell+1} - \frac{k\ell}{2\ell+1}\mathcal{M}_{\ell-1} + a\sigma_T n_e\mathcal{M}_\ell = \frac{k}{3}(\Psi+\Phi)\delta_{\ell 1}$$
$$+a\sigma_T n_e\left[\mathcal{M}_0\delta_{\ell 0}+\frac{1}{3}V_b\delta_{\ell 1}+\frac{1}{10}\mathcal{M}_2\delta_{\ell 2}\right]$$

Boltzmann eqn. II

Integral 'solution', $\kappa = s\, a\sigma_T\, n_e\, dt$

$$\mathcal{M}(t_0) = \int_{t_{\rm in}}^{t_0} dt e^{ik\mu(t-t_0)} e^{-\kappa}\left[-ik\mu(\Phi+\Psi) + \underbrace{a\sigma_T n_e e^{-\kappa}}_{g}\left(\mathcal{M}_0 + i\mu V_B + \frac{1}{2}P_2(\mu)\mathcal{M}_2\right)\right]$$

Via integrations by part we can move all μ dependence in the exponential,

$$\mathcal{M}(k,\mu) = \int_{t_{\rm in}}^{t_0} dt e^{ik\mu(t-t_0)} S(k,t)$$

$$S(k,t) = e^{-\kappa}(\dot\Phi + \dot\Psi) + g\left(\mathcal{M}_0 - \dot V_B/k - \frac{1}{4}\mathcal{M}_2 - \frac{3}{4k^2}\ddot{\mathcal{M}}_2\right) - \dot g\left(V_B/k + \frac{3}{4k^2}\dot{\mathcal{M}}_2\right) - \frac{3\ddot g}{4k^2}\mathcal{M}_2$$

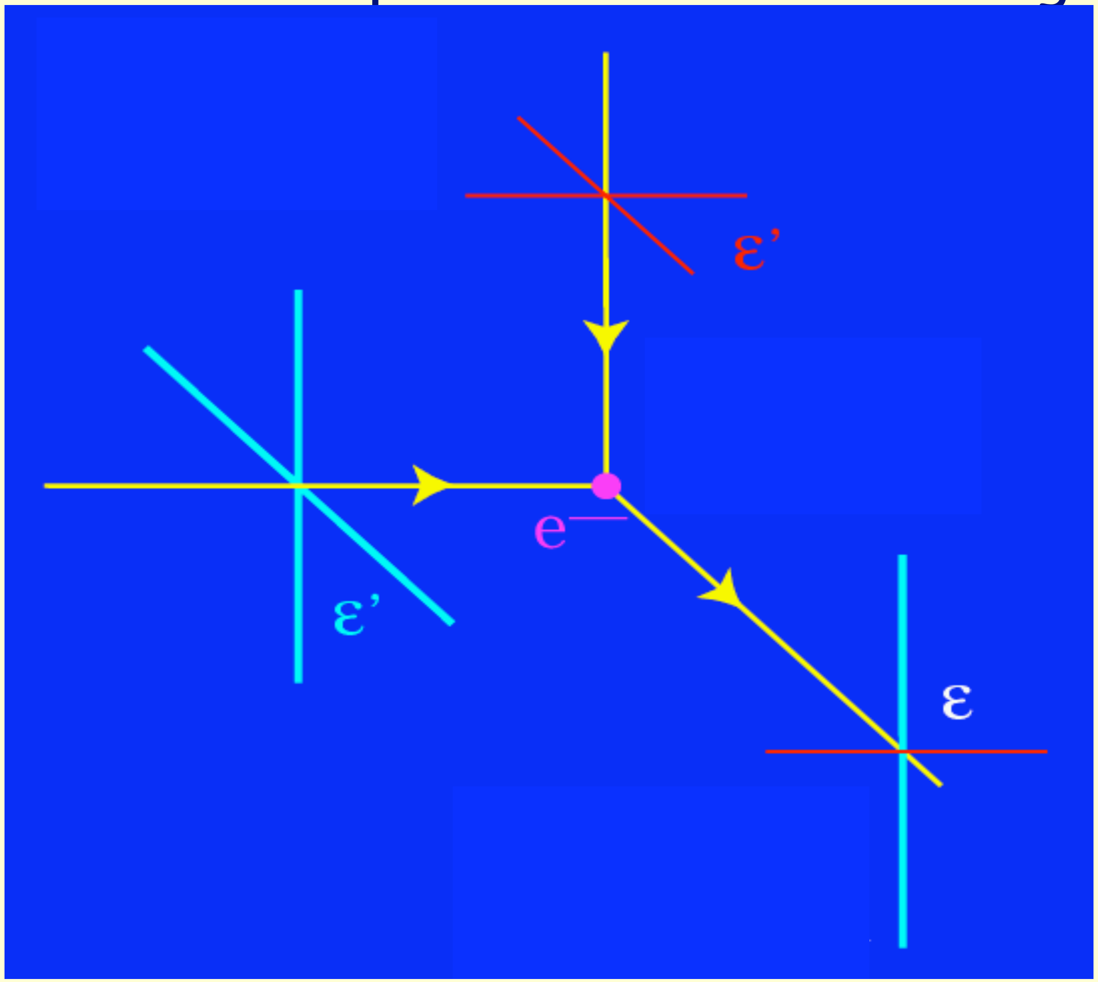
$$\mathcal{M}_\ell(k) = \int_{t_{\rm in}}^{t_0} dt j_\ell(k(t_0-t))S(k,t)$$

∞

$$C_\ell = (4\pi)^2\int dk k^2\langle|\mathcal{M}_\ell(k)|^2\rangle$$

Polarisation

- Thomson scattering depends on polarisation: a quadrupole anisotropy of the incoming wave generates linear polarisation of the outgoing wave.



Polarisation can be described by the Stokes parameters, but they depend on the choice of the coordinate system. The (complex) amplitude

$\varepsilon_i e^i$ of the 2-component electric field defines the spin 2 intensity $A_{ij} = \varepsilon_i^* \varepsilon_j(n)$ which can be written in terms of Pauli matrices as

$$A = \frac{1}{2}[I\sigma_0 + U\sigma_1 + V\sigma_2 + Q\sigma_3] = \frac{1}{2}[I\sigma_0 + V\sigma_2 + (Q+iU)\sigma_+ + (Q-iU)\sigma_-]$$

Q§ iU are the helicity § 2 eigenstates, which are expanded in spin 2 spherical harmonics. Their real and imaginary parts are called the 'electric' and 'magnetic' polarisations.

$$[Q(\mathbf{n}) \pm iU(\mathbf{n})]\sigma_\pm(\mathbf{n})_{ab} \;=\; \sum_{\ell m} a_{\ell m}^{(\pm)} [{}_{\pm 2}Y_{\ell m}(\mathbf{n})]_{ab}$$

$$a_{\ell m}^E = \frac{1}{2}\left(a_{\ell m}^{(+)} + a_{\ell m}^{(-)}\right) \;, \qquad a_{\ell m}^B = \frac{-i}{2}\left(a_{\ell m}^{(+)} - a_{\ell m}^{(-)}\right)$$

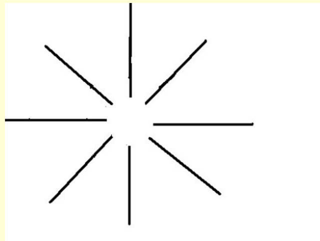
$$\langle a_{\ell m}^X a_{\ell' m'}^{*Y} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell^{XY}$$

(Seljak & Zaldarriaga, 97, Kamionkowski et al. '97, Hu & White '97)

Under parity operation $_{§2}Y_{l\,m}$!$(-1)^{l}{}_{..2}$ $Y_{l\,m}$ Hence E has the same parity as ΔT while B has parity $(-1)^{l+1}$. E describes gradient fields on the sphere (generated by scalar as well as tensor modes), while B describes the rotational component of the polarisation field (generated only by tensor or vector modes).

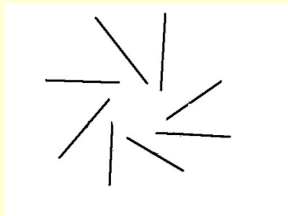
E-polarisation

(generated by scalar and tensor modes)



B-polarisation

(generated only by the tensor mode)



Due to their parity, T and B and E and B are not correlated while T and E are.

An additional effect on CMB fluctuations is Silk damping: on small scales, of the order of the size of the mean free path of CMB photons, fluctuations are damped due to free streaming: photons stream out of over-densities into under-densities.

To compute the effects of Silk damping and polarisation we have to solve the Boltzmann equation for M, E and B of the CMB radiation. This is usually done with the 'line of sight method' in standard, publicly available codes like CMBfast (Seljak & Zaldarriaga) , CAMBcode (Bridle & Lewis) or CMBeasy (Doran).

The physics of CMB fluctuations

- Large scales : The gravitational potential on the surface of last scattering, time dependence of the gravitational potential $\Psi \sim 10^{-5}$.

  $\theta > 1^{o}$

  $\ell < 100$

- Intermediate scales : Acoustic oscillations of the baryon/photon fluid before recombination.

  $6' < \theta < 1^{o}$

  $100 < \ell < 800$

- Small scales : Damping of fluctuations due to the imperfect coupling of photons and electrons during recombination (Silk damping).

  $\theta < 6'$

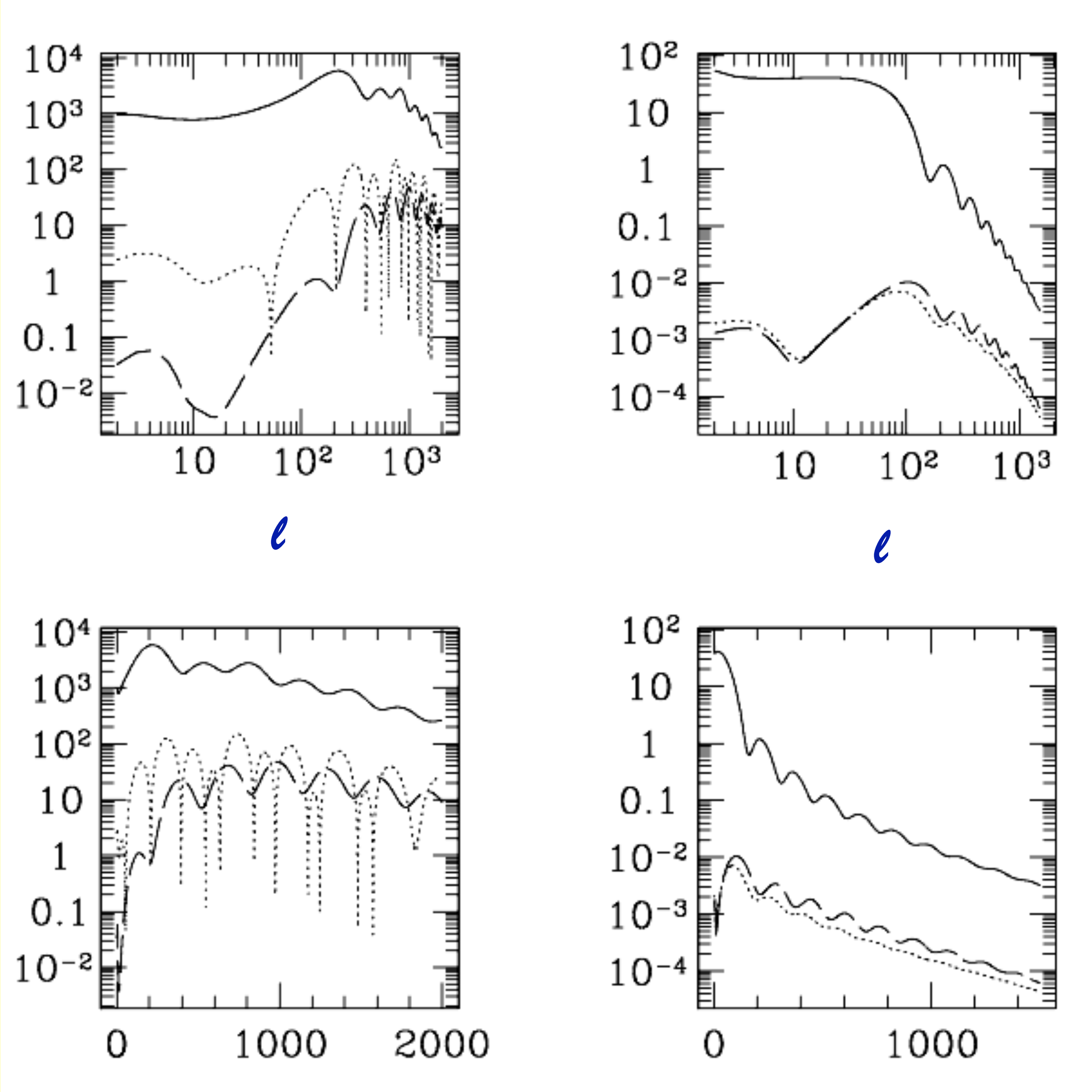
  $800 > \ell$

Power spectra

scalar

tensor

(in units of μK²)



Reionization

The absence of the so called Gunn-Peterson trough in quasar spectra tells us that the universe is reionised since, at least, z» 6.

Reionisation leads to a certain degree of re-scattering of CMB photons. This induces additional damping of anisotropies and additional polarisation on large scales (up to the horizon scale at reionisation). It enters the CMB spectrum mainly through one parameter, the optical depth t to the last scattering surface or the redshift of reionisation z_{re} .

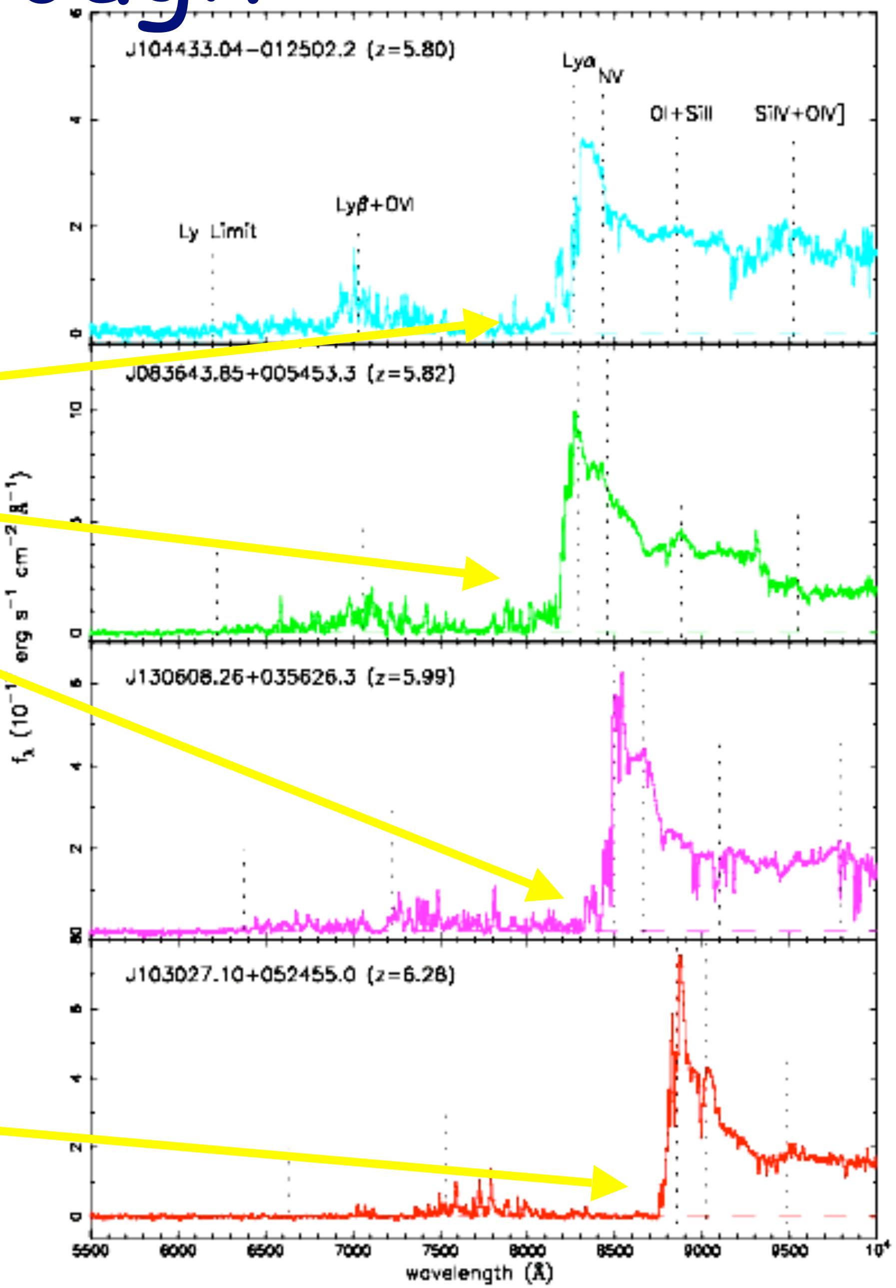
Gunn Peterson trough

In quasars with z<6.1 the photons with wavelength shorter that Ly-a are not absorbed.

normal emission

(Becker et al. 2001)

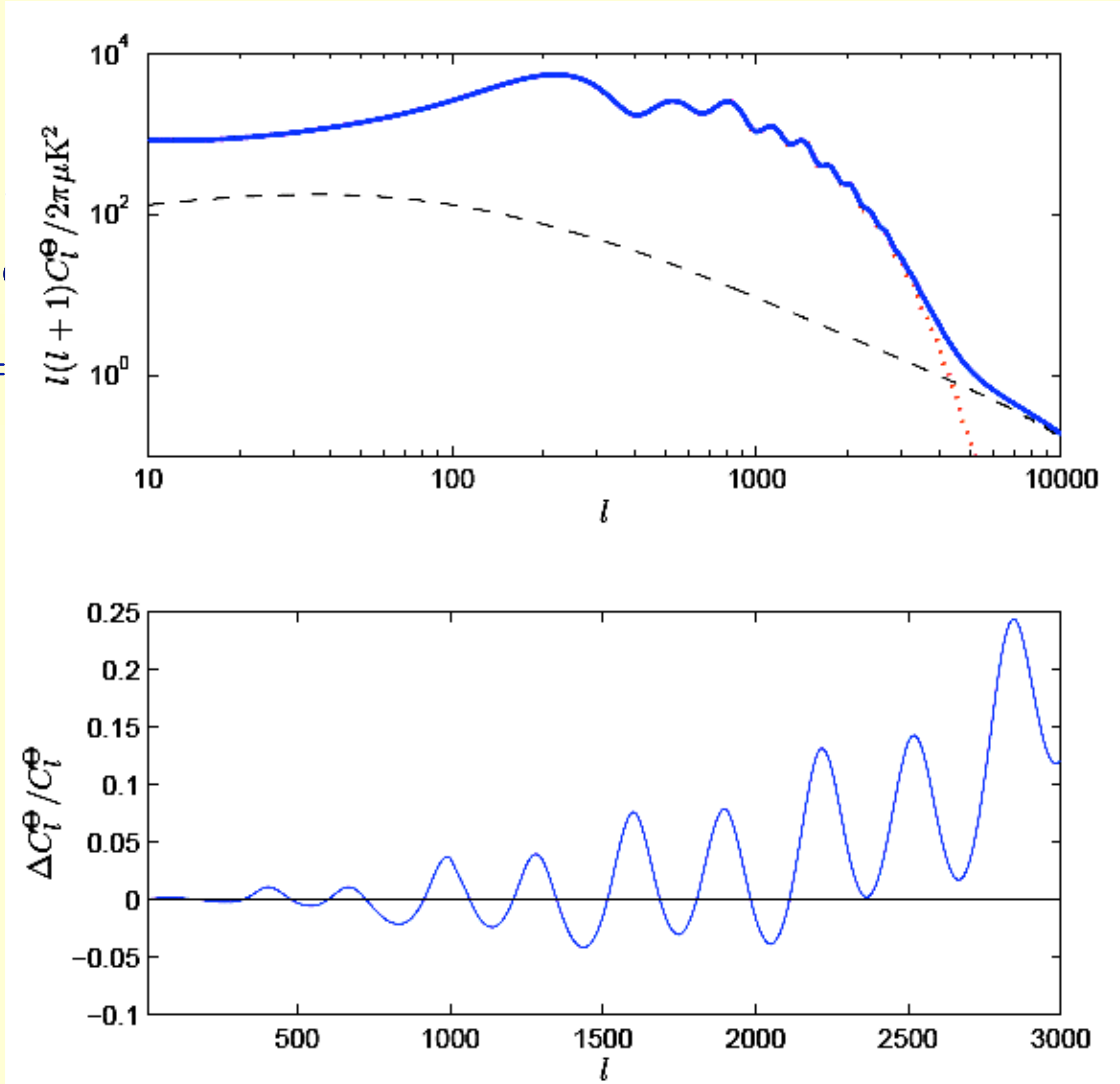
no emission



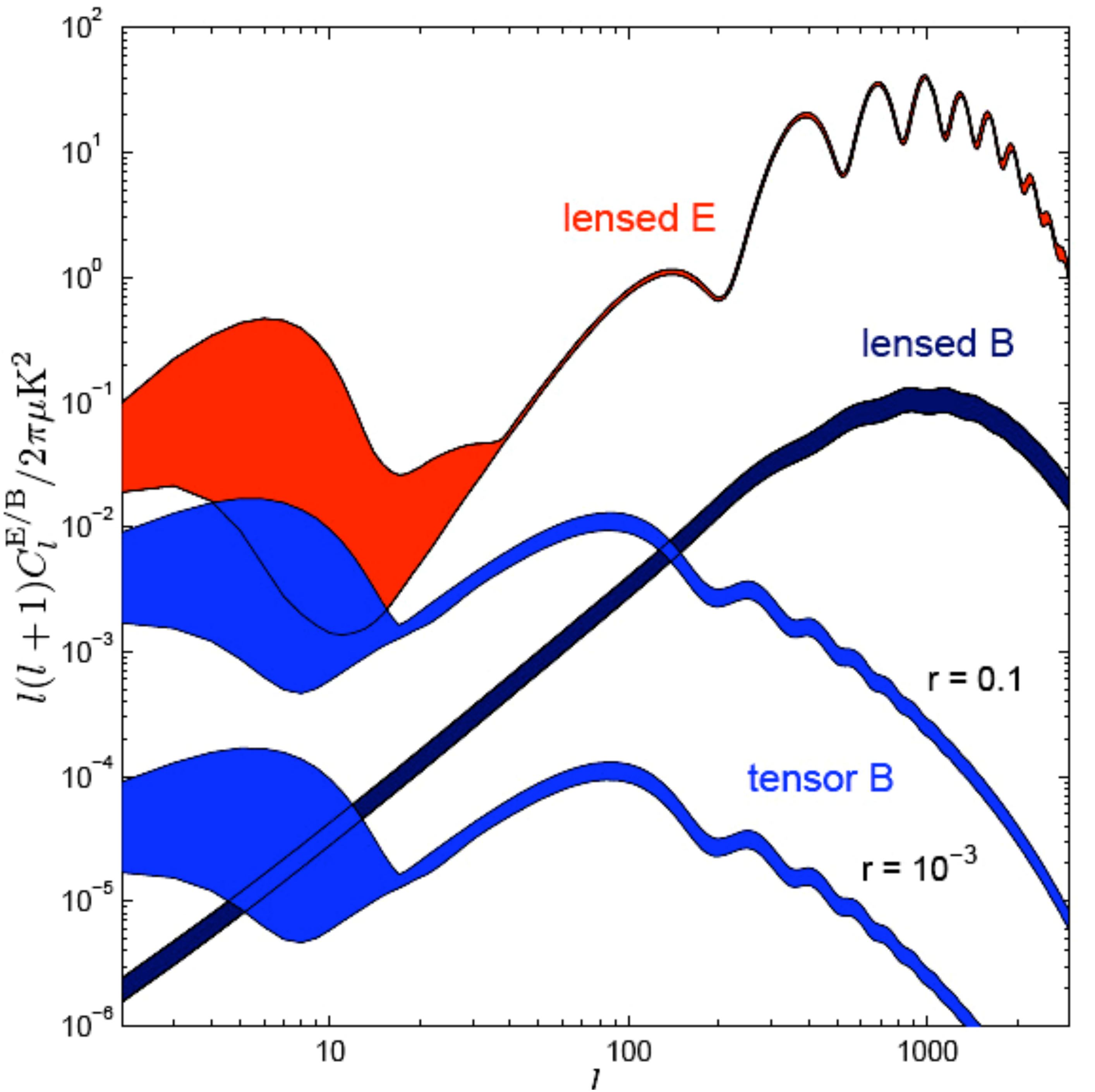
Due

phot

$\alpha =$

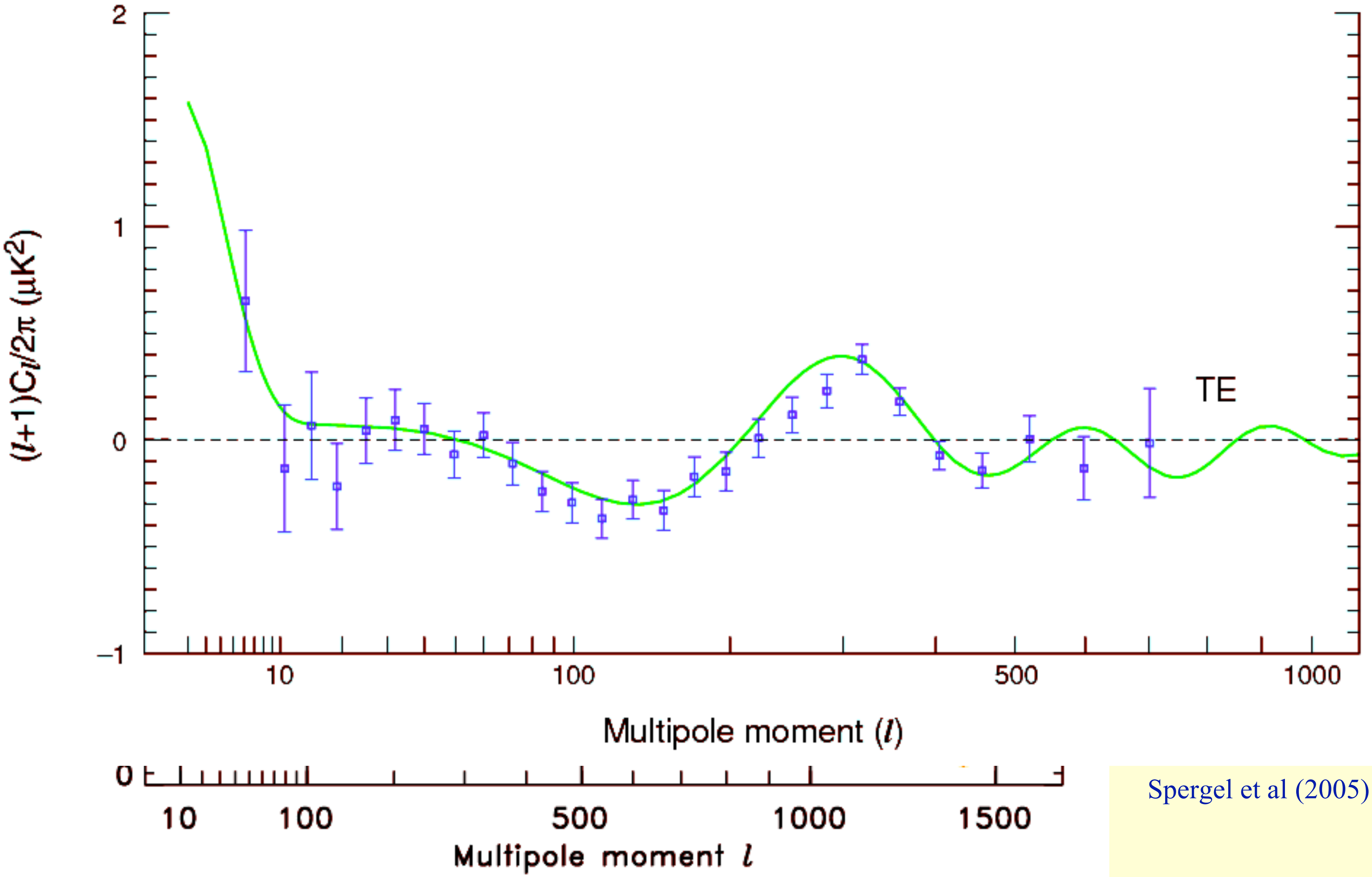


Challinor & Lewis '06



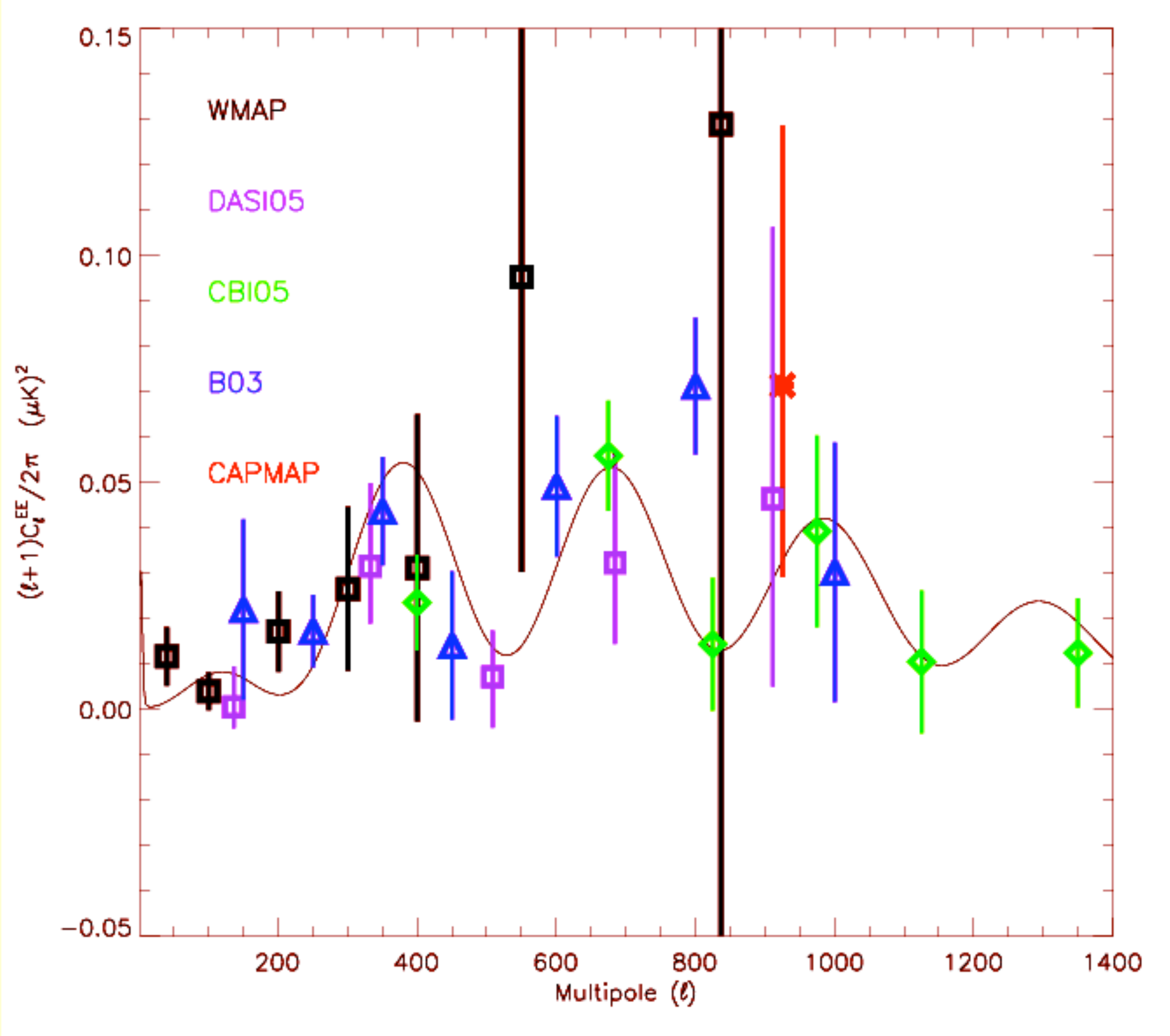
Challinor &
Lewis '06

WMAP data



Spergel et al (2005)

WMAP and other polarisation data



From Page et al. 2006

Acoustic oscillations

Determine the angular distance to the last scattering surface, z_1

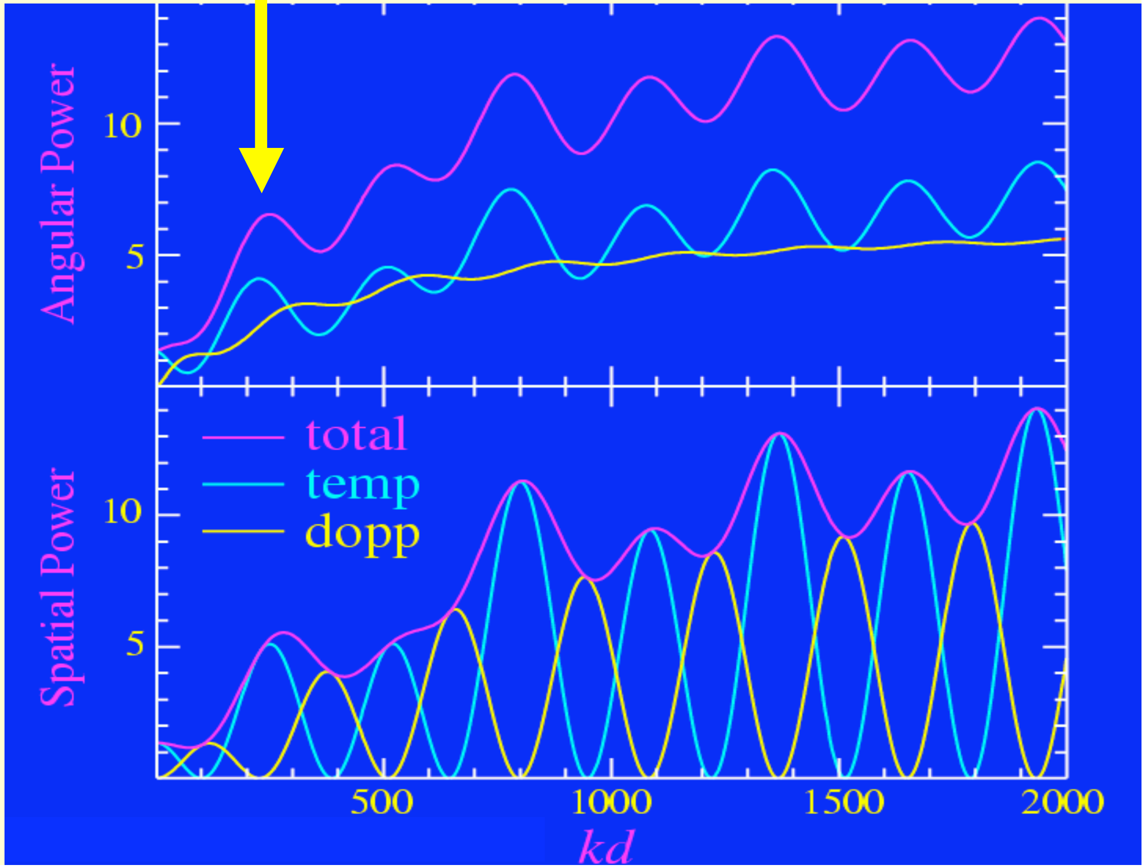
$$\eta_0 - \eta_1 = \frac{1}{H_0 a_0}\int_0^{z_1} \frac{dz}{[\Omega_{\rm rad}(z+1)^4 + \Omega_m(z+1)^3 + \Omega_\Lambda + \Omega_\kappa(z+1)^2]^{\frac{1}{2}}}$$

$$\eta_1 = \frac{1}{H_0 a_0}\int_{z_1}^{\infty} \frac{dz}{[\Omega_{\rm rad}(z+1)^4 + \Omega_m(z+1)^3 + \Omega_\Lambda + \Omega_\kappa(z+1)^2]^{\frac{1}{2}}}.$$

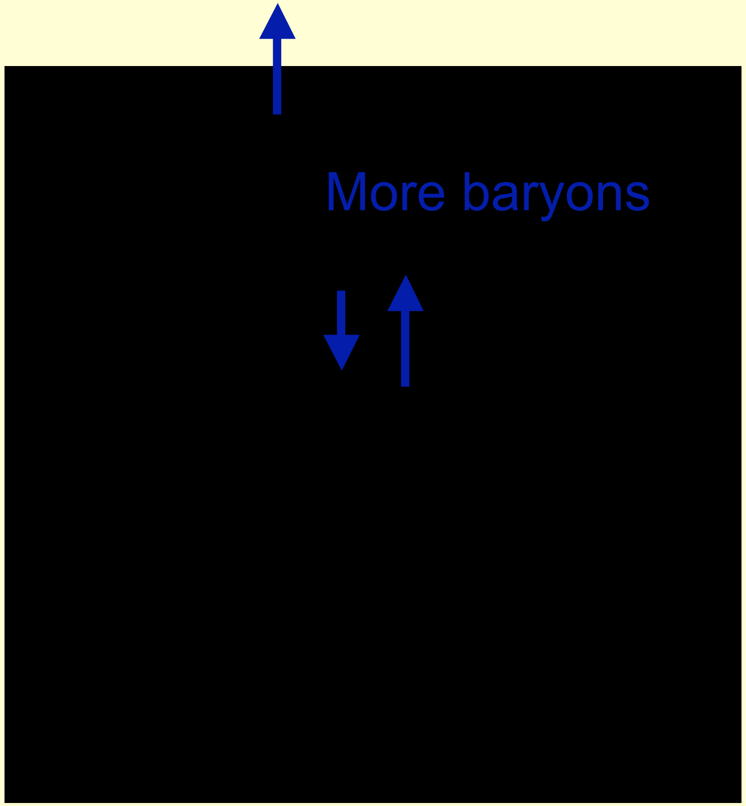
$$\vartheta_A = \frac{c_s \eta_1}{\chi(\eta_0 - \eta_1)}$$

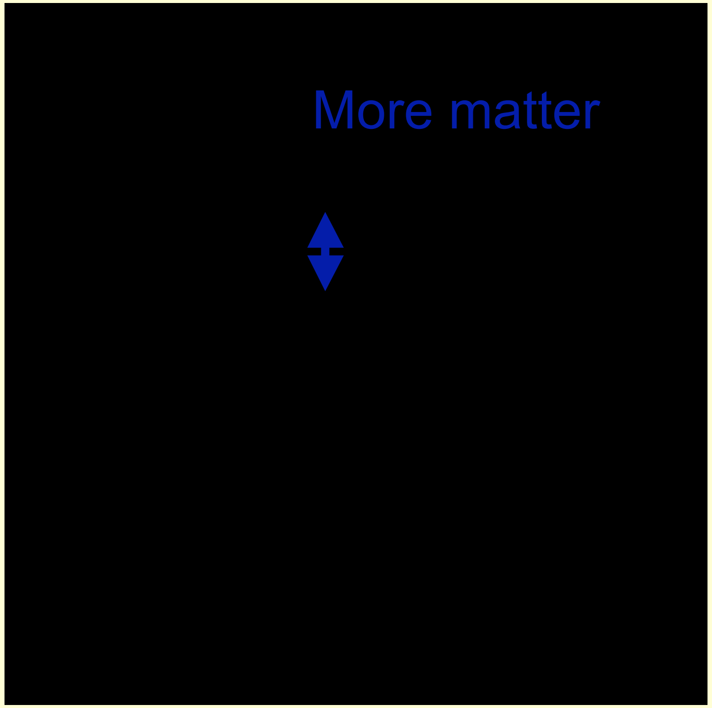
Is known with 1.7% accuracy from WMAP data

$c_s^2 = 1/3(1+3/4\rho_b/\rho_r)^{-1}$



Dependence on cosmological parameters

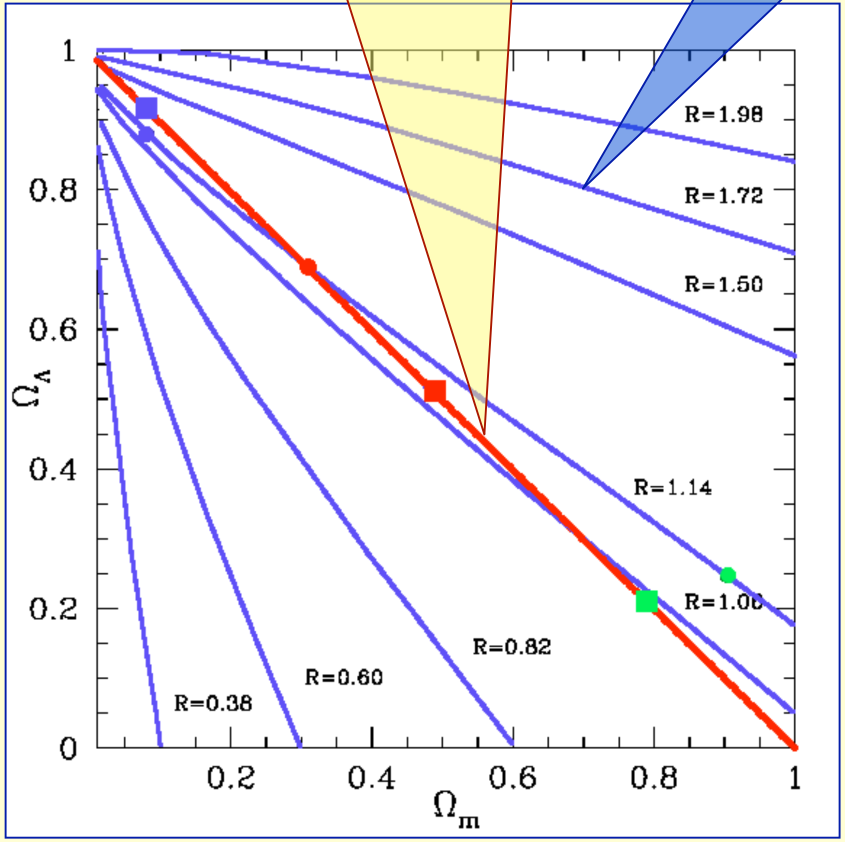




Geometrical degeneracy

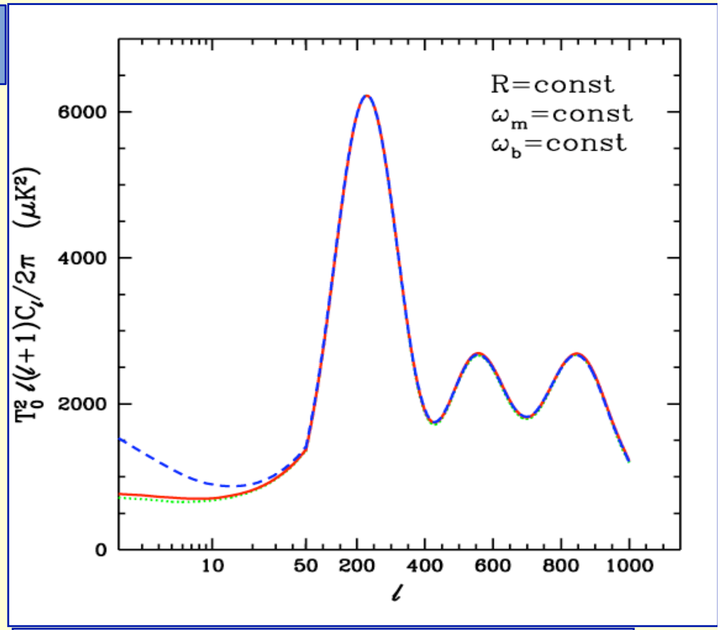
Flat Universe (ligne of constant curvature W_k=0)

degeneracy lines:

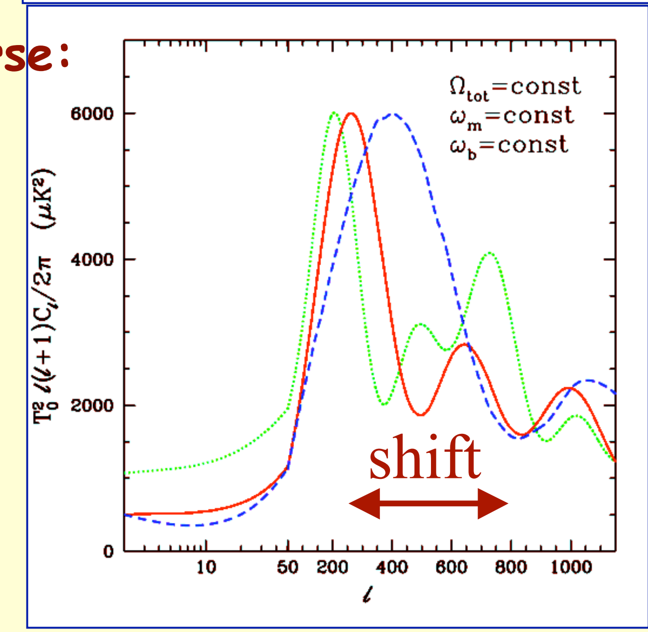


Degeneracy:

$\omega = \Omega h^2$

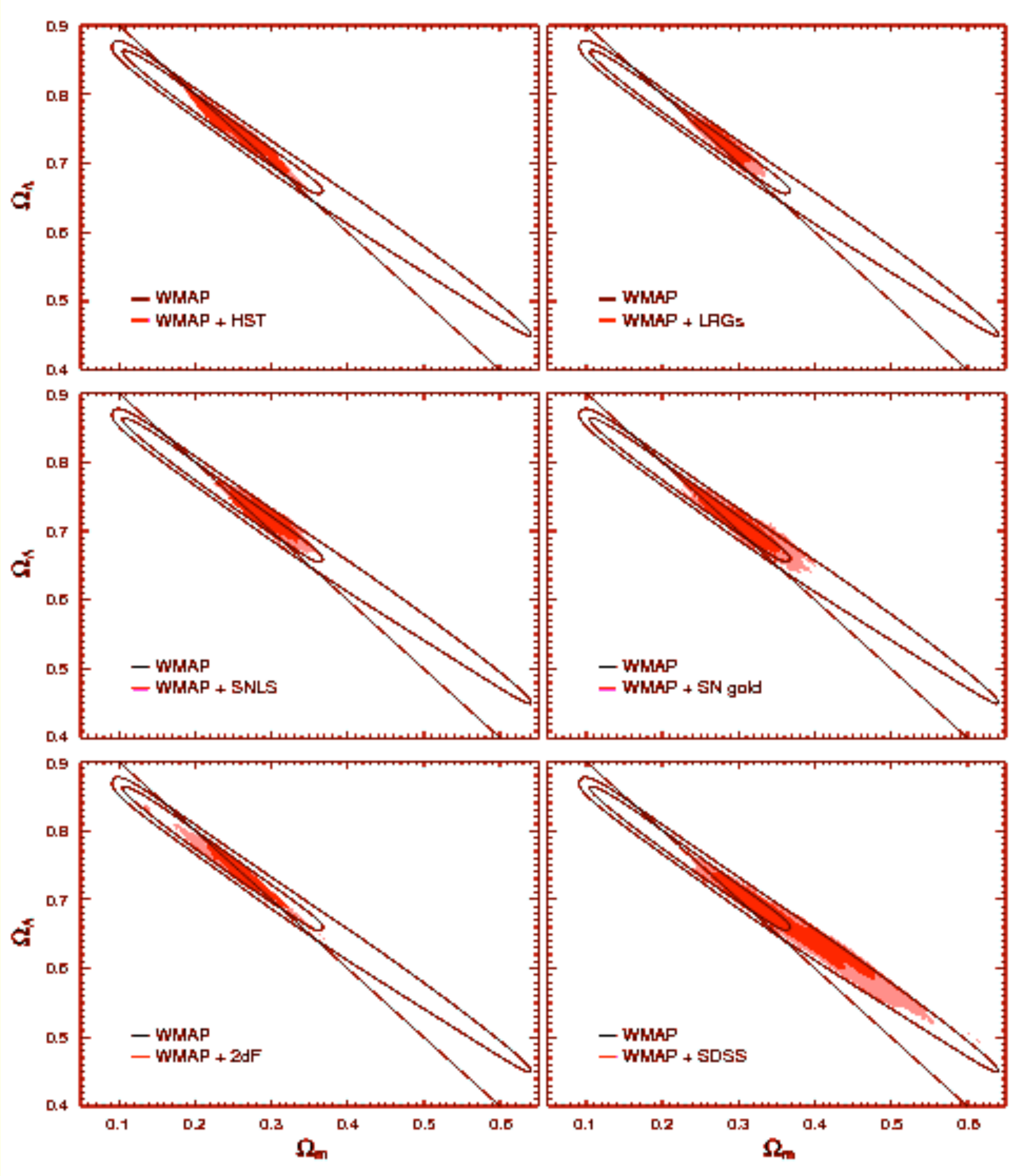


Flat Universe:



Shift parameter: $R = R(\Omega_\Lambda, \Omega_m)$

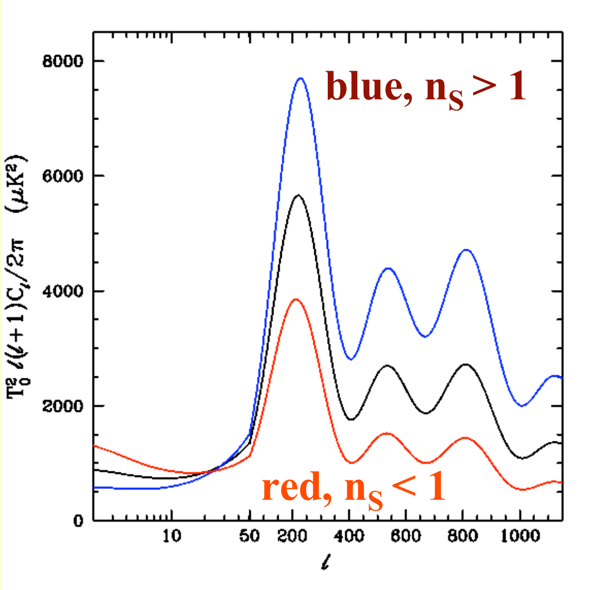
geometrical degeneracy II



Spergel et al. 2007

Primordial parameters

Scalar spectum:



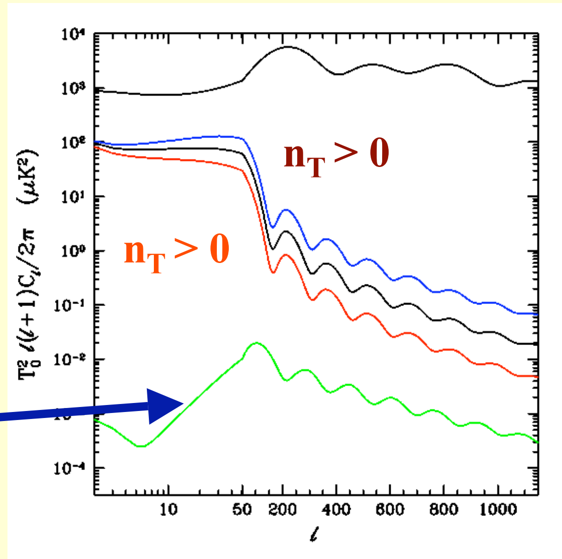
scalar spectral index n_S and amplitude A

$$\langle \Psi^2 \rangle = A k^{n_S - 1}$$

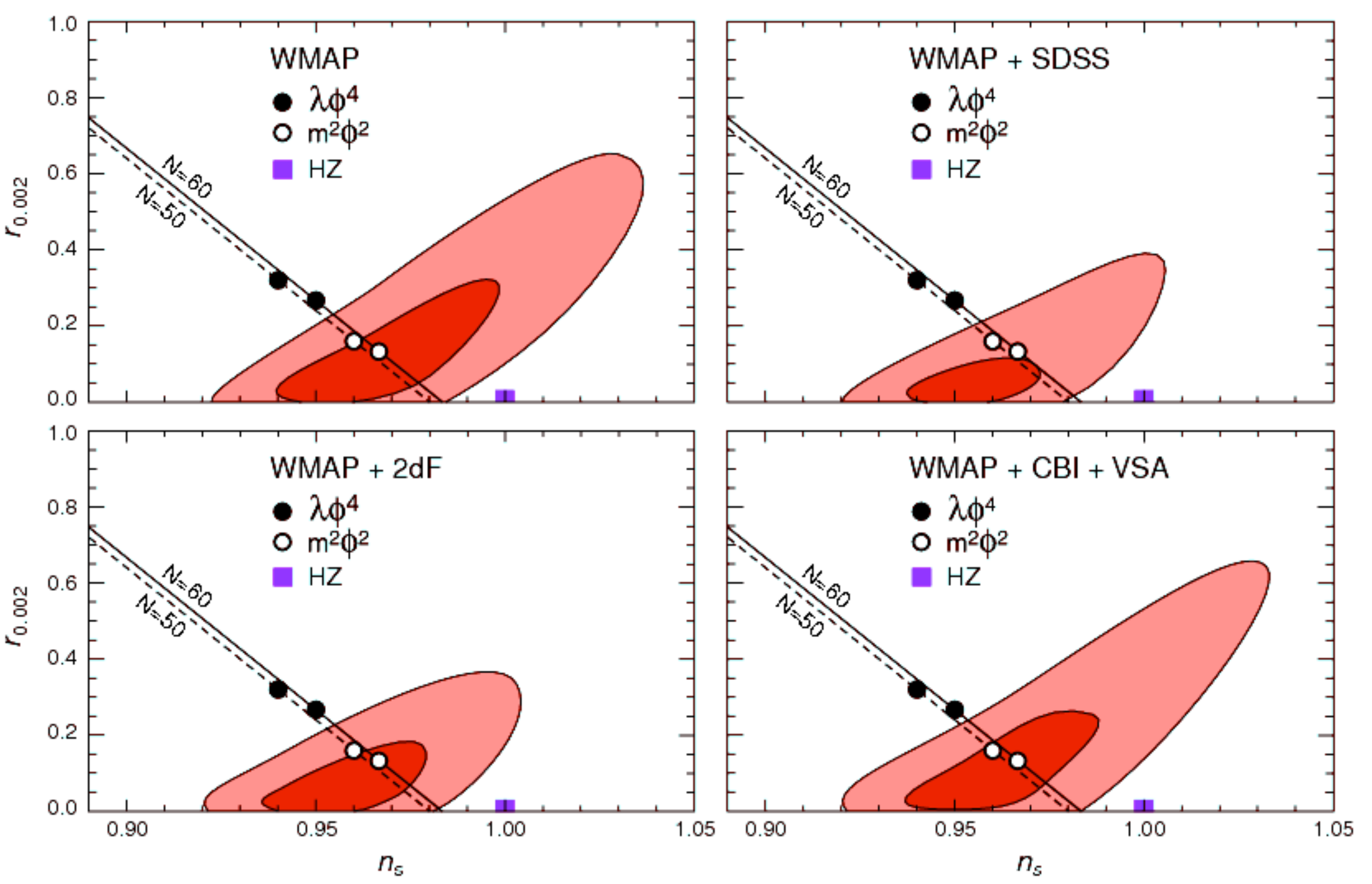
$n_S = 1$: scale invariant spectrum (Harrison-Zel'dovich)

Tensor spectum: (gravity waves)

The 'smoking gun' of inflation, has not yet been detected: B modes of the polarisation (Bpol, ...).



Primordial parameters



Spergel et al. 2007

Measured cosmological parameters

(With CMB + flatness or CMB + Hubble)

Parameter	First Year Mean	WMAPext Mean	Three Year Mean	First Year ML	WMAPext ML	Three Year ML
$100\Omega_b h^2$	$2.38^{+0.13}_{-0.12}$	$2.32^{+0.12}_{-0.11}$	2.23 ± 0.08	2.30	2.21	2.23
$\Omega_m h^2$	$0.144^{+0.016}_{-0.016}$	$0.134^{+0.006}_{-0.006}$	0.126 ± 0.009	0.145	[illegible]	28
H_0	72^{+5}_{-5}	73^{+3}_{-3}	74^{+3}_{-3}	68	[illegible]	3
τ	$0.17^{+0.08}_{-0.07}$	$0.15^{+0.07}_{-0.07}$	0.093 ± 0.029	0.10	0.10	0.092
n_s	$0.99^{+0.04}_{-0.04}$	$0.98^{+0.03}_{-0.03}$	0.961 ± 0.017	0.97	0.96	0.958
Ω_m	$0.29^{+0.07}_{-0.07}$	$0.25^{+0.03}_{-0.03}$	0.234 ± 0.035	0.32	0.27	0.24
σ_8	$0.92^{+0.1}_{-0.1}$	$0.84^{+0.06}_{-0.06}$	0.76 ± 0.05	0.88	0.82	0.77

$\Omega_\Lambda = 0.75 \pm 0.07$

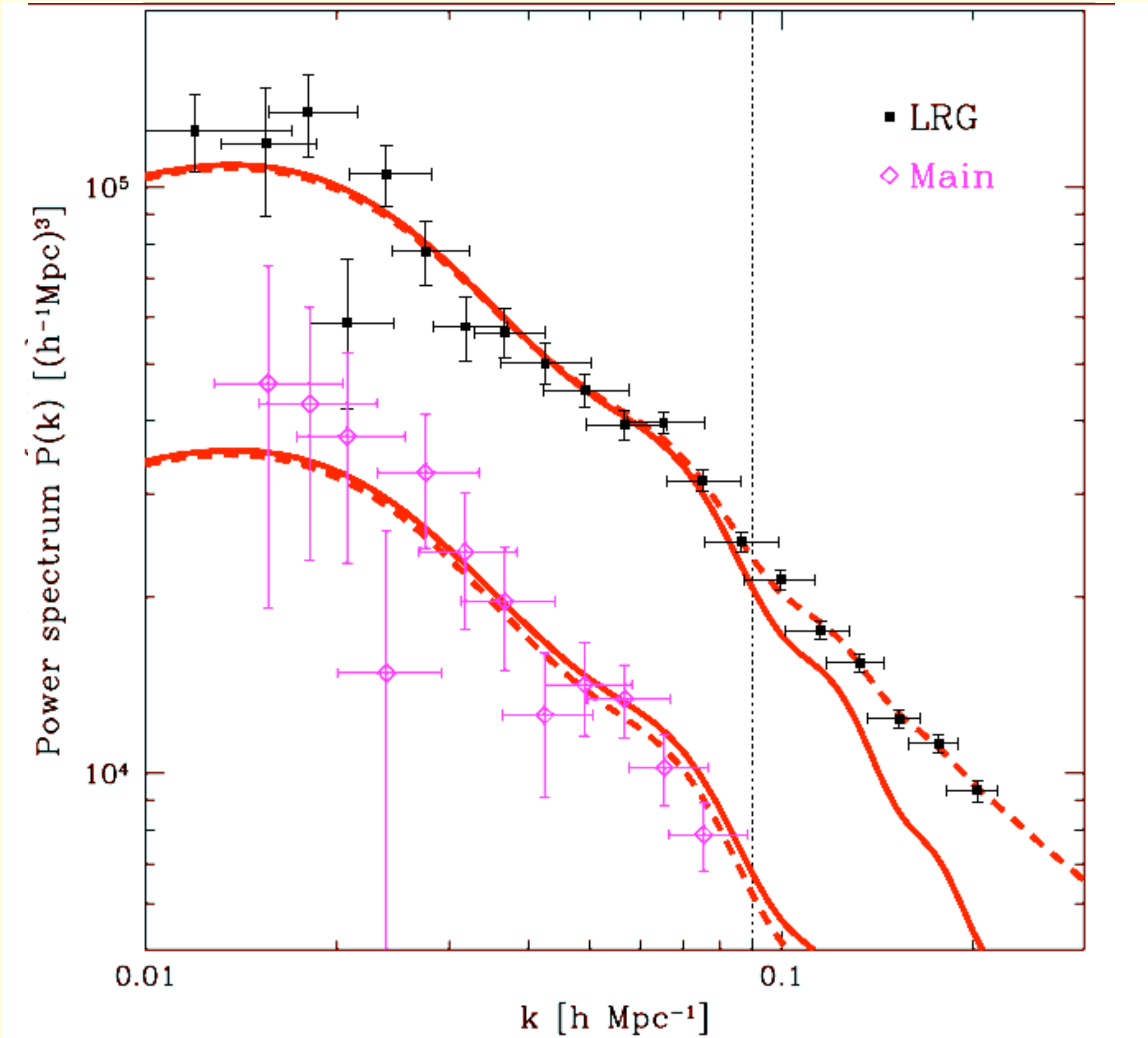
(Spergel et al. 2007)

Attention: **FLATNESS** imposed!!!

On the other hand: $\Omega_{tot} = 1.02 \pm 0.02$ with the HST prior on *h*...

Galaxy distribution (LSS)

Tegmark et al. 2006



Sloan LRG combined with WMAP 3

(Tegmark et al. 2006)

e 2: Cosmological parameters measured from WMAP and SDSS LRG data with the Occam's razor approach d
marginalized over all other parameters in the vanilla set (ω_b, ω_c, Ω_Λ, A_s, n_s, τ, b, (

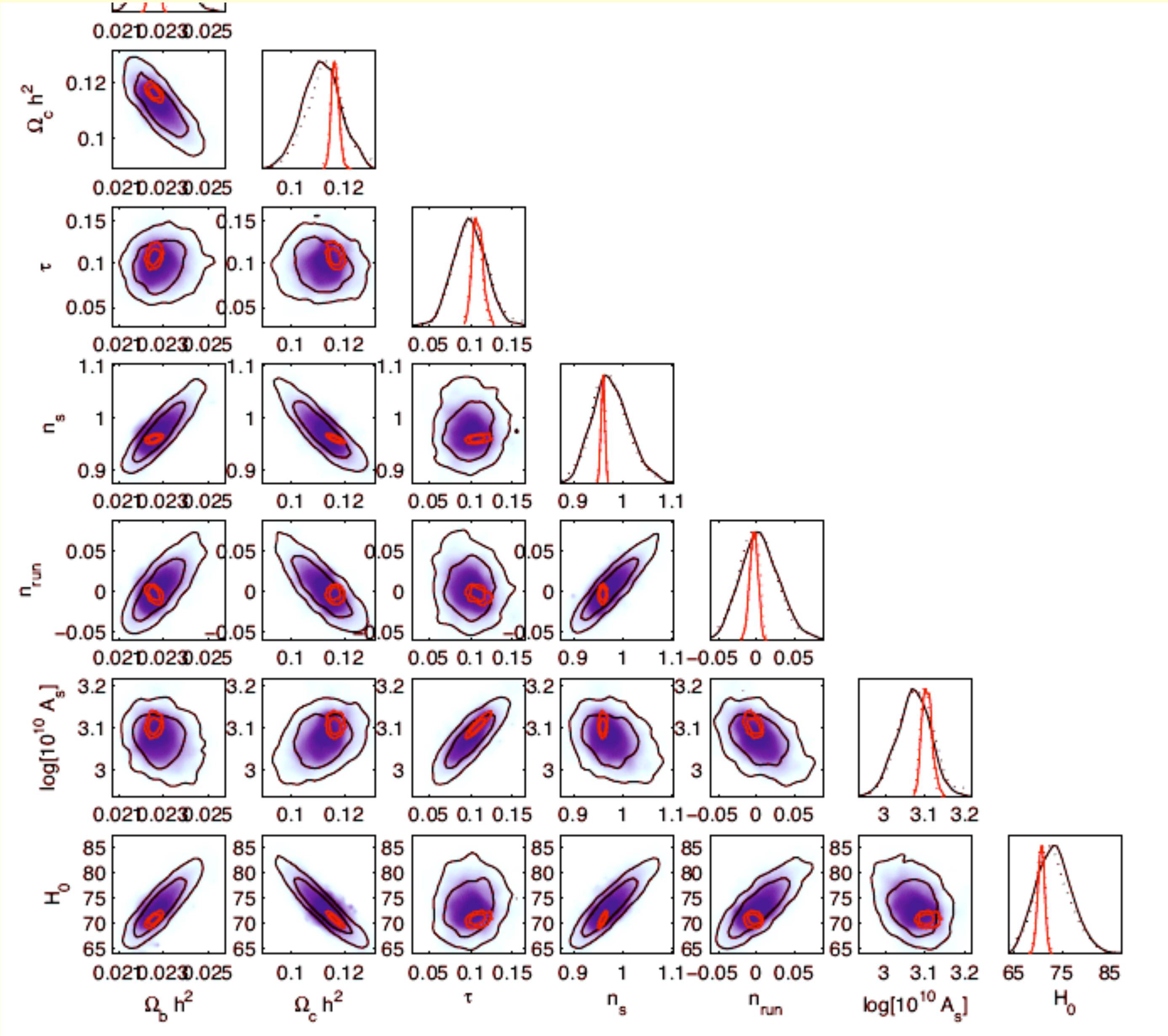
Parameter	Value	Meaning	Definition
Matter budget parameters:			
Ω_{tot}	$1.003^{+0.010}_{-0.009}$	Total density/critical density	$\Omega_{tot} = \Omega_m + \Omega_\Lambda = 1 -$ (
Ω_Λ	$0.761^{+0.017}_{-0.018}$	Dark energy density parameter	$\Omega_\Lambda \approx h^{-2}\rho_\Lambda(1.88 \times 10^{-}$
ω_b	$0.0222^{+0.0007}_{-0.0007}$	Baryon density	$\omega_b = \Omega_b h^2 \approx \rho_b/(1.88 \times$
ω_c	$0.1050^{+0.0041}_{-0.0040}$	Cold dark matter density	$\omega_c = \Omega_c h^2 \approx \rho_c/(1.88 \times$
ω_ν	< 0.010 (95%)	Massive neutrino density	$\omega_\nu = \Omega_\nu h^2 \approx \rho_\nu/(1.88$ >
w	$-0.941^{+0.087}_{-0.101}$	Dark energy equation of state	p_Λ/ρ_Λ (approximated as
Seed fluctuation parameters:			
A_s	$0.690^{+0.045}_{-0.044}$	Scalar fluctuation amplitude	Primordial scalar power s
r	< 0.30 (95%)	Tensor-to-scalar ratio	Tensor-to-scalar power ra
n_s	$0.953^{+0.016}_{-0.016}$	Scalar spectral index	Primordial spectral index
$n_t + 1$	$0.9861^{+0.0096}_{-0.0142}$	Tensor spectral index	$n_t = -r/8$ assumed
α	$-0.040^{+0.027}_{-0.027}$	Running of spectral index	$\alpha = dn_s/d\ln k$ (approxin
Nuisance parameters:			
τ	$0.087^{+0.028}_{-0.030}$	Reionization optical depth	
b	$1.896^{+0.074}_{-0.069}$	Galaxy bias factor	$b = [P_{galaxy}(k)/P(k)]^{1/}$
Q_{nl}	$30.3^{+4.4}_{-4.1}$	Nonlinear correction parameter [29]	$P_g(k) = P_{dewiggled}(k)b^2$

Sloan LRG combined with WMAP 3

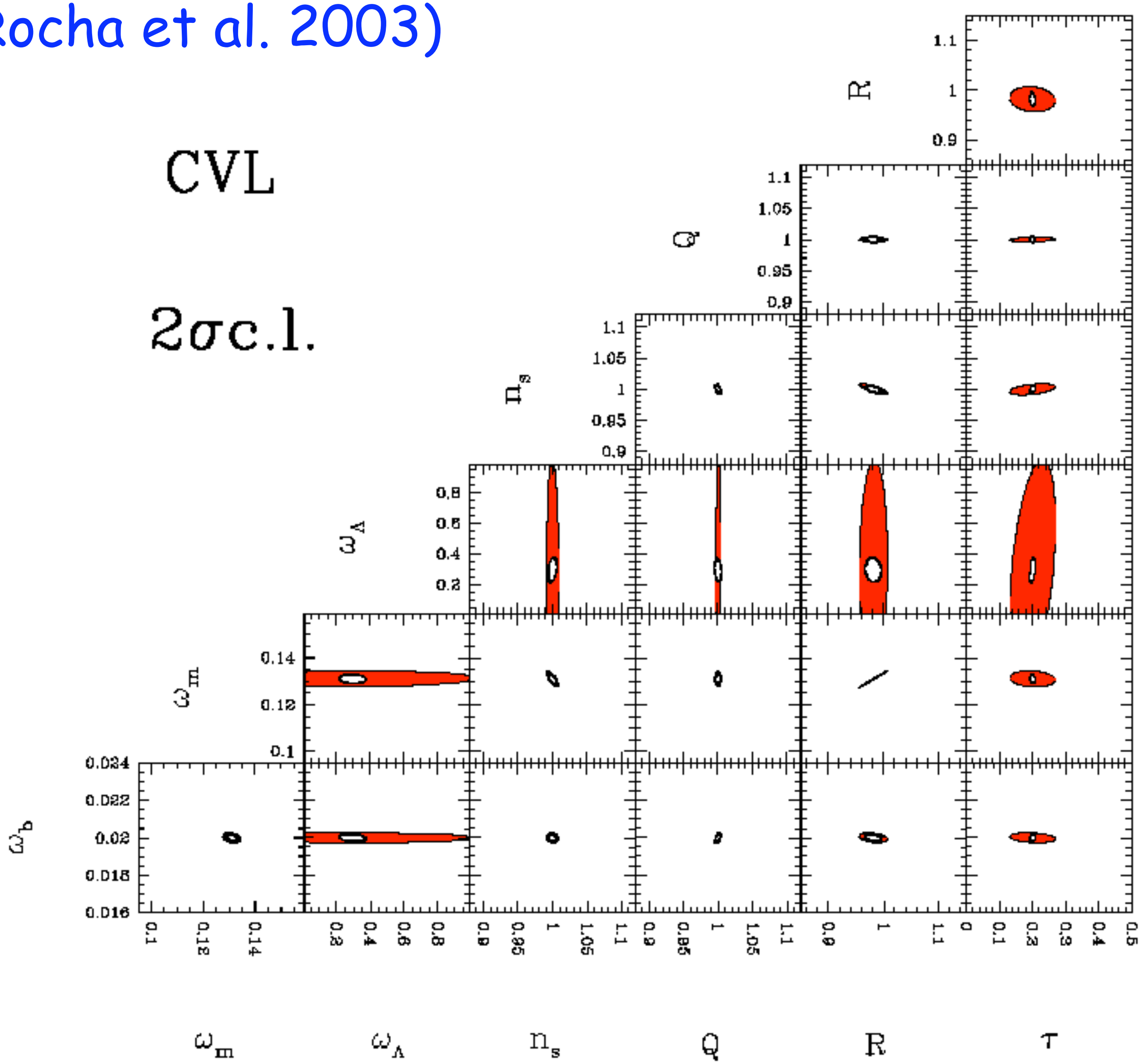
Other popular parameters (determined by those above):			
h	$0.730^{+0.019}_{-0.019}$	Hubble parameter	$h = \sqrt{(\omega_b + \omega_c + \omega_\nu)/(\Omega_{tot} - \Omega_\Lambda)}$
Ω_m	$0.239^{+0.018}_{-0.017}$	Matter density/critical density	$\Omega_m = \Omega_{tot} - \Omega_\Lambda$
Ω_b	$0.0416^{+0.0019}_{-0.0018}$	Baryon density/critical density	$\Omega_b = \omega_b/h^2$
Ω_c	$0.197^{+0.016}_{-0.015}$	CDM density/critical density	$\Omega_c = \omega_c/h^2$
Ω_ν	< 0.024 (95%)	Neutrino density/critical density	$\Omega_\nu = \omega_\nu/h^2$
Ω_k	$-0.0030^{+0.0095}_{-0.0102}$	Spatial curvature	$\Omega_k = 1 - \Omega_{tot}$
ω_m	$0.1272^{+0.0044}_{-0.0043}$	Matter density	$\omega_m = \omega_b + \omega_c + \omega_\nu = \Omega_m h^2$
f_ν	< 0.090 (95%)	Dark matter neutrino fraction	$f_\nu = \rho_\nu/\rho_d$
A_t	< 0.21 (95%)	Tensor fluctuation amplitude	$A_t = rA_s$
M_ν	< 0.94 (95%) eV	Sum of neutrino masses	$M_\nu \approx (94.4\ \mathrm{eV}) \times \omega_\nu$ [105]
$A_{.002}$	$0.801^{+0.042}_{-0.043}$	WMAP3 normalization parameter	A_s scaled to $k = 0.002$/Mpc: $A_{.002} = 25^{1-n_s}$
$r_{.002}$	< 0.33 (95%)	Tensor-to-scalar ratio (WMAP3)	Tensor-to-scalar power ratio at $k = 0.002$/Mpc
σ_8	$0.756^{+0.035}_{-0.035}$	Density fluctuation amplitude	$\sigma_8 = \{4\pi \int_0^\infty [\frac{3}{x^3}(\sin x - x\cos x)]^2 P(k) \frac{k^2 dk}{(2\pi)^3}$
$\sigma_8 \Omega_m^{0.6}$	$0.320^{+0.024}_{-0.023}$	Velocity fluctuation amplitude	
Cosmic history parameters:			
z_{eq}	3057^{+105}_{-102}	Matter-radiation Equality redshift	$z_{eq} \approx 24074\omega_m - 1$
z_{rec}	$1090.25^{+0.93}_{-0.91}$	Recombination redshift	$z_{rec}(\omega_m, \omega_b)$ given by eq. (18) of [106]
z_{ion}	$11.1^{+2.2}_{-2.7}$	Reionization redshift (abrupt)	$z_{ion} \approx 92(0.03h\tau/\omega_b)^{2/3}\Omega_m^{1/3}$ (assuming abru
z_{acc}	$0.855^{+0.059}_{-0.059}$	Acceleration redshift	$z_{acc} = [(-3w - 1)\Omega_\Lambda/\Omega_m]^{-1/3w} - 1$ if $w <$
t_{eq}	$0.0634^{+0.0045}_{-0.0041}$ Myr	Matter-radiation Equality time	$t_{eq} \approx (9.785\ \mathrm{Gyr}) \times h^{-1} \int_{z_{eq}}^\infty [H_0/H(z)(1+z)]$
t_{rec}	$0.3856^{+0.0040}_{-0.0040}$ Myr	Recombination time	$t_{req} \approx (9.785\ \mathrm{Gyr}) \times h^{-1} \int_{z_{rec}}^\infty [H_0/H(z)(1+z$
t_{ion}	$0.43^{+0.20}_{-0.10}$ Gyr	Reionization time	$t_{ion} \approx (9.785\ \mathrm{Gyr}) \times h^{-1} \int_{z_{ion}}^\infty [H_0/H(z)(1+z$
t_{acc}	$6.74^{+0.25}_{-0.24}$ Gyr	Acceleration time	$t_{acc} \approx (9.785\ \mathrm{Gyr}) \times h^{-1} \int_{z_{acc}}^\infty [H_0/H(z)(1+z$
t_{now}	$13.76^{+0.15}_{-0.15}$ Gyr	Age of Universe now	$t_{now} \approx (9.785\ \mathrm{Gyr}) \times h^{-1} \int_0^\infty [H_0/H(z)(1+z)]$

Forecast2: Planck 1 year data vs. WMAP 4 year

(Planck consortium 2006)

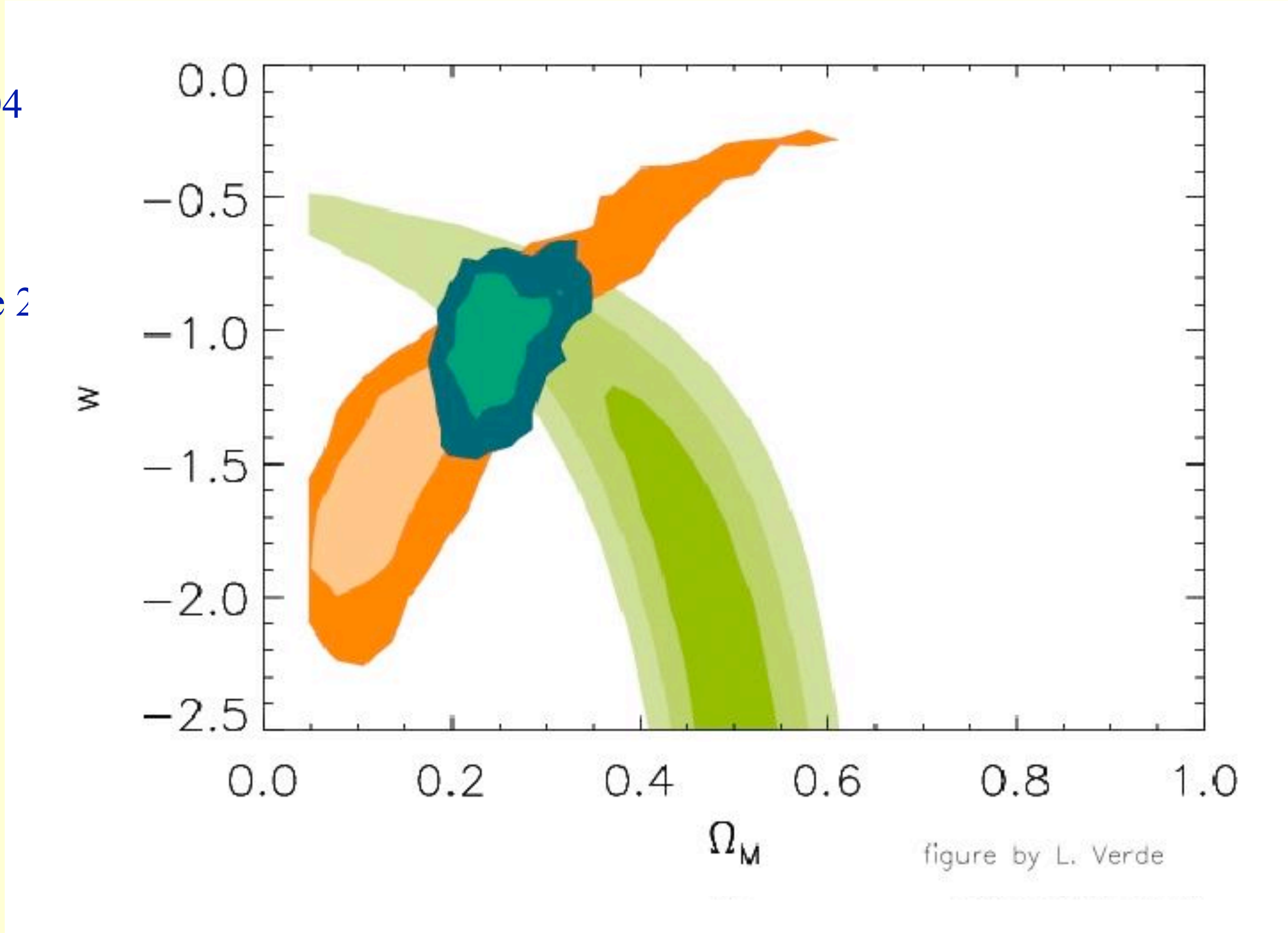


Forecast3: Cosmic variance limited data (Rocha et al. 2003)



Evidence for a cosmological constant

Sn1a, Riess et al. 2004
(green)
CMB + Hubble
(orange)
Bi-spectrum β, Verde 2
(blue)



Conclusions

- The CMB with its small perturbations has h… enormously in determining properties & parameters … universe and it will continue to do so.
- We know the cosmological parame… …h impressive precision which will still impro… …derably during the next years.
- We don't understand at … …izarre 'mix' of cosmic components: W… …22, $W_m h^2 \sim 0.13$, $W_L \sim 0.73$
- The simplest m… …nflation (a nearly scale invariant spectrum of … perturbations, vanishing curvature) is a good fit t… …ata.
- Wh… …rk matter?
- …t is dark energy?
- What is the inflaton?

! We have not run out of problems in cosmology!