Lectures on Gauge-Higgs Unification in extra dimensions

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Thanks to my collaborators on this subject

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Outline

1. Extra dimensions, KK, gauge theories in XD
2. Orbifolds, SU(5) breaking via orbifolds
3. Interval vs. orbifold approach
4. Generics of gauge-Higgs unification
5. Higgs potential calculation
6. Fermion masses
7. A semi-realistic model
8. Bounds on the model from EWPO’s

1.Extra dimensions, KK decomposition

- If XD's exist, there has to be a reason why we have not seen them
  - Compactification
  - Localization of fields (will not use very much)
- Simplest example: scalar field on a circle

$$S = \int d^4x \int_0^{2\pi R} dy \partial_M \phi^* \partial^M \phi$$

- 5D EOM

$$(\partial_\mu \partial^\mu - \partial_y^2)\phi = 0$$

- But XD compact, ϕ has to periodic in y
- Fourier decomposition (=KK expansion)

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \phi_n(x) e^{iny/R}$$

- ϕ_n: KK modes. One 5D field=an infinite tower of 4D fields→KK expansion
- Dim of ϕ_n: 1 (like 4D). Dim. of ϕ:3/2 (like 5D)

$$\partial_\mu \partial^\mu \phi_n(x) + \frac{n^2}{R^2} \phi_n(x) = 0$$

- Different KK modes have different 4D masses, here $\mathbf{m_n^2=n^2/R^2}$
- Momentum along 5th dim~mass along 4D

- Can get 4D effective action by integrating explicitly over the y coordinate in KK exp. just collection of massive 4D fields

$$S_{eff} = \int d^4x \sum_{n=-\infty}^{\infty} \left(\partial_\mu \phi_n^* \partial^\mu \phi_n - m_n^2 |\phi_n|^2 \right)$$

- General KK expansion:
  - Take quadratic part of 5D action
  - Write fields as sum of ordinary 4D fields:

$$\phi = \sum_n \phi_n(x) f_n(y), \quad (\partial_\mu \partial^\mu + m_n^2)\phi_n = 0$$

  - **$f_n(y)$**: wave function of KK mode
  - Higher powers will give interactions of KK modes

Gauge theories in an extra dimension

$$S=\int d^5x(-\frac{1}{4}F^a_{MN}F^{MN\,a})=\int d^5x(-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a}-\frac{1}{2}F^a_{\mu 5}F^{\mu 5\,a})$$

$$F^a_{MN}=\partial_M A^a_N-\partial_N A^a_M+g_5 f^{abc}A^b_M A^c_N$$

- Gauge coupling $\mathbf{g_5}$ has dim. -1/2 (nonren.)
- Dim. of $\mathbf{A_M}$:3/2 like a 5D scalar
- From 4D point of view:

$$\mathbf{A_M \to A_\mu + A_5}$$

- $\mathbf{A_5}$ is like a scalar, that can be like an eaten GB (or a higgs). Eaten GB: mixing of $\mathbf{A_5}$ and $\mathbf{A_\mu}$

- The mixing term

$$-\int d^4x \int_0^{2\pi R} dy \frac{1}{2} F^a_{\mu 5} F^{\mu 5\ a}|_{quadratic} =$$
$$-\int d^4x \int_0^{2\pi R} dy \frac{1}{2} (\partial_\mu A^a_5 \partial^\mu A^{5\ a} + \partial_5 A^a_\mu \partial^5 A^{\mu\ a} - 2\partial_5 A^a_\mu \partial^\mu A^{5a})$$

Kinetic+mass term — Mixing term

- Assuming periodic BC for ALL fields (on circle appropriate) can integrate by parts

$$S_{mix} = -\int_0^{2\pi R} \partial^\mu A^a_\mu \partial_5 A^a_5$$

- As usual add gauge fixing term to cancel the mixings in R_ξ gauge

$$S_{GF} = -\int d^4x \int_0^{2\pi R} \frac{1}{2\xi}(\partial_\mu A^{\mu a} - \xi \partial_5 A_5^a)^2$$

- Chosen to reproduce normal gauge fixing piece for A_μ and to cancel mixing
- Action decoupled

Gauge bosons

$$\mathcal{L}_{A_\mu} = -\frac{1}{4}{F_{\mu\nu}^a}^2 + \frac{1}{2}\partial_5 A_\mu^a \partial_5 A^{\mu a} - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2$$

- In unitary gauge $\xi \to \infty$ a tower of massive gauge bosons $m_n^2 = n^2/R^2$

Scalars

$$\mathcal{L}_{A_5} = \frac{1}{2}\partial_\mu A_5^a \partial^\mu A_5^a - \frac{\xi}{2}(\partial_5 A_5^a)^2$$

- Tower of scalars with mass $\mathbf{m_n{}^2 = \xi\, n^2/R^2}$
- Unless **n=0** unphysical **($m \rightarrow \infty$)**
- A_5 provide longitudinal components of massive gauge fields. Only physical mode: massless zero mode

- Spectrum:
  1. Massive tower of GB's
  2. Massless GB+A_5 scalar

↑ This is what we want to eventually use for Higgs…

Fermions on a circle

- Somewhat tricky, 5D Dirac algebra contains γ_5
- Theory will NOT be chiral (only Dirac fermions)

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$$

- Action:

$$S = \int d^5x \left(\frac{i}{2} (\bar{\Psi}\, \Gamma^M \partial_M \Psi - \partial_M \bar{\Psi}\, \Gamma^M \Psi) - m \bar{\Psi}\Psi \right)$$

- In terms of components

$$S = \int d^5x \left(-i\bar{\chi}\bar{\sigma}^\mu \partial_\mu \chi - i\psi\sigma^\mu \partial_\mu \bar{\psi} + \frac{1}{2} (\psi \overleftrightarrow{\partial_5} \chi - \bar{\chi} \overleftrightarrow{\partial_5} \bar{\psi}) + m(\psi\chi + \bar{\chi}\bar{\psi}) \right)$$

- KK decomposition

$$\chi = \sum_n g_n(y)\, \chi_n(x),$$
$$\bar{\psi} = \sum_n f_n(y)\, \bar{\psi}_n(x)$$

- The KK modes are 4D Dirac fermions

$$-i\bar{\sigma}^\mu \partial_\mu \chi^{(n)} + m_n \bar{\psi}^{(n)} = 0$$
$$-i\sigma^\mu \partial_\mu \bar{\psi}^{(n)} + m_n \chi^{(n)} = 0$$

- Wave functions

$$g_n'' + (m_n^2 - m^2) g_n = 0$$
$$f_n'' + (m_n^2 - m^2) f_n = 0$$

- KK spectrum:

Tower of massive KK modes

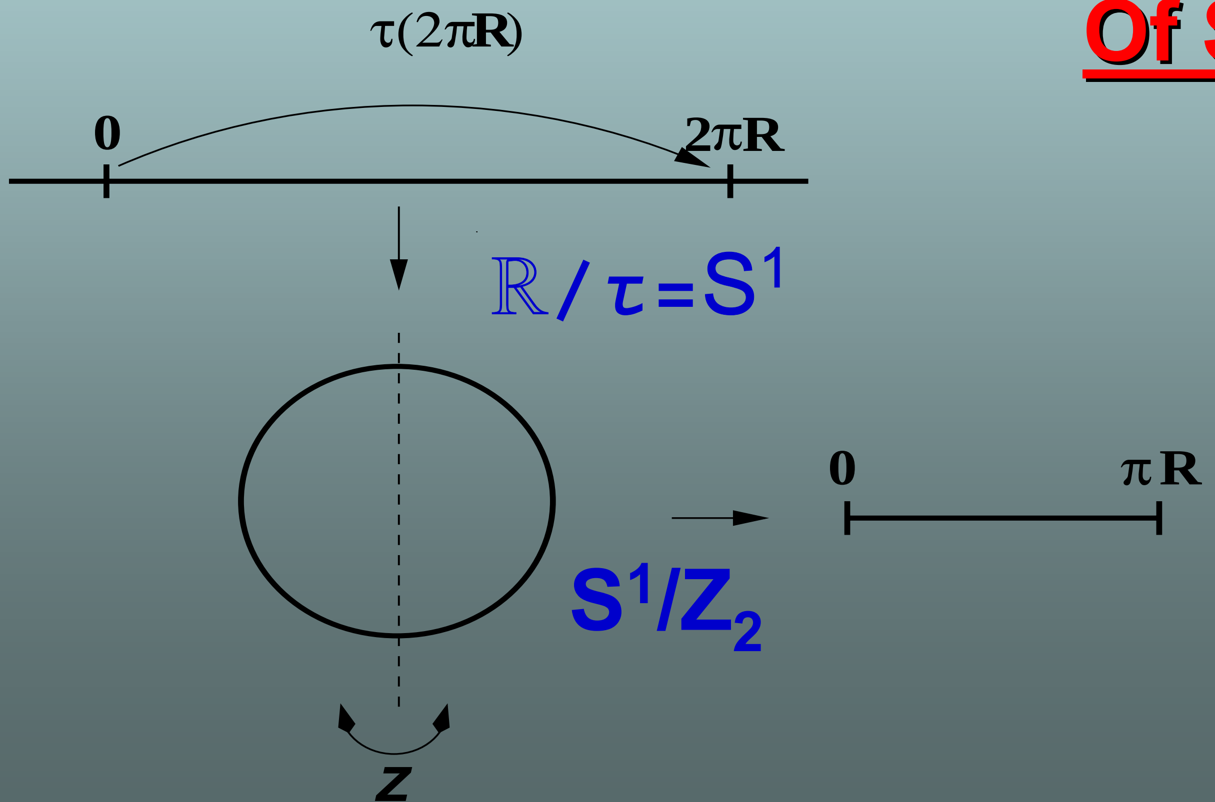
$$m_n^2=m^2+n^2/R^2$$

- On circle: no chiral zero mode even for **m=0**
- Clearly circle is too simple to
  - Reproduce SM
  - Give interesting possibilities for SSB
  - Give interesting zero mode spectra
- Look at next simplest possibility:
  - Orbifolds
  - Interval

2. Orbifolds

•Next simplest possibility: instead of circle comapctify on a line segment S^1/Z_2. Will look in two silghtly different approaches (orbifold vs. interval).

Geometric construction Of S^1/Z_2



Effects on the fields

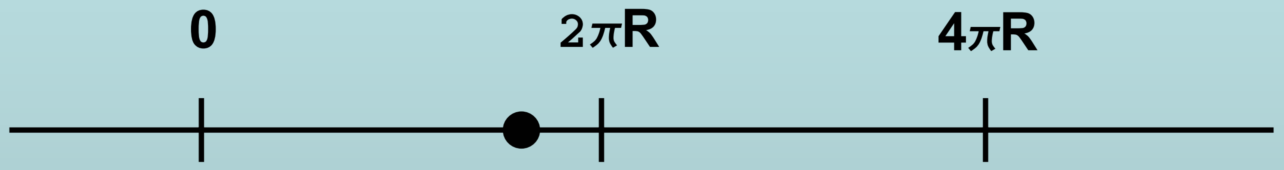
- τ, z have to be symmetries of action
- Fields have to agree UP TO a symmetry transformation **T,Z** (**T** is SS-twist)

$$\tau(2\pi R)\varphi(y) = T^{-1}\varphi(y + 2\pi R)$$
$$z\varphi(y) = Z\varphi(-y)$$

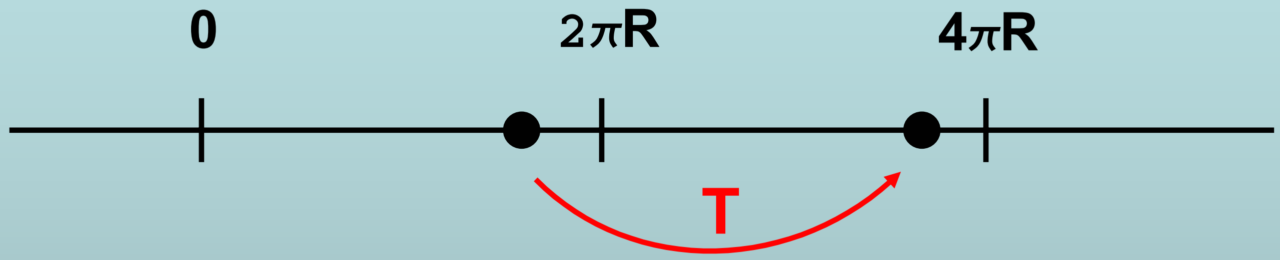
- Field identification will be

$$\varphi(y + 2\pi R) = T\varphi(y)$$
$$\varphi(-y) = Z\varphi(y)$$

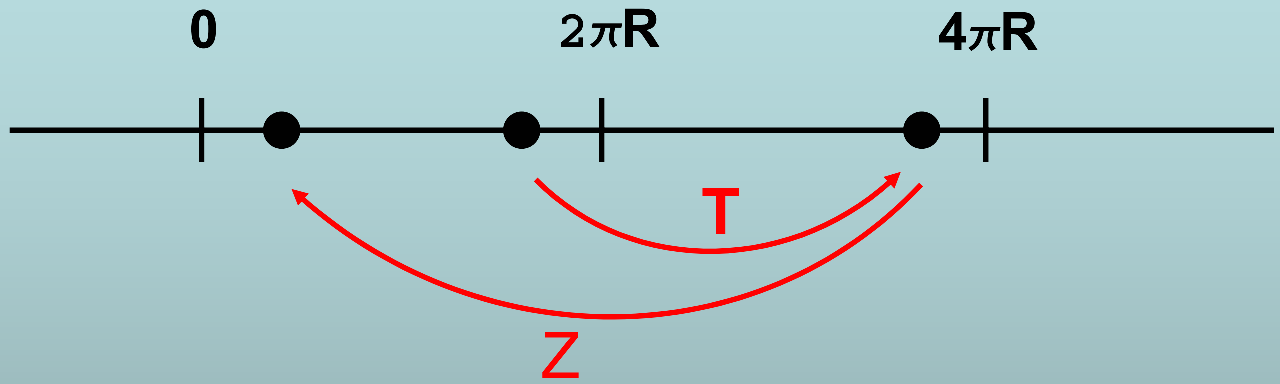
A consistency condition



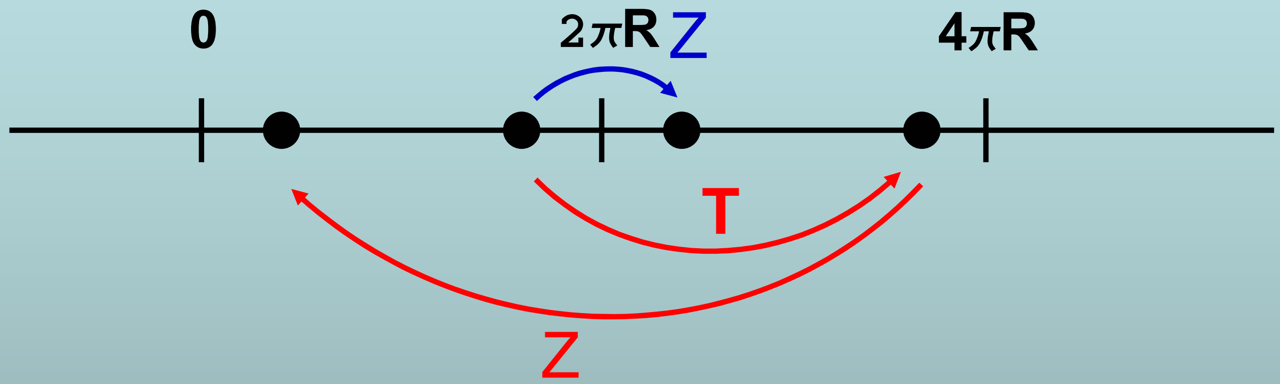
A consistency condition



A consistency condition



A consistency condition



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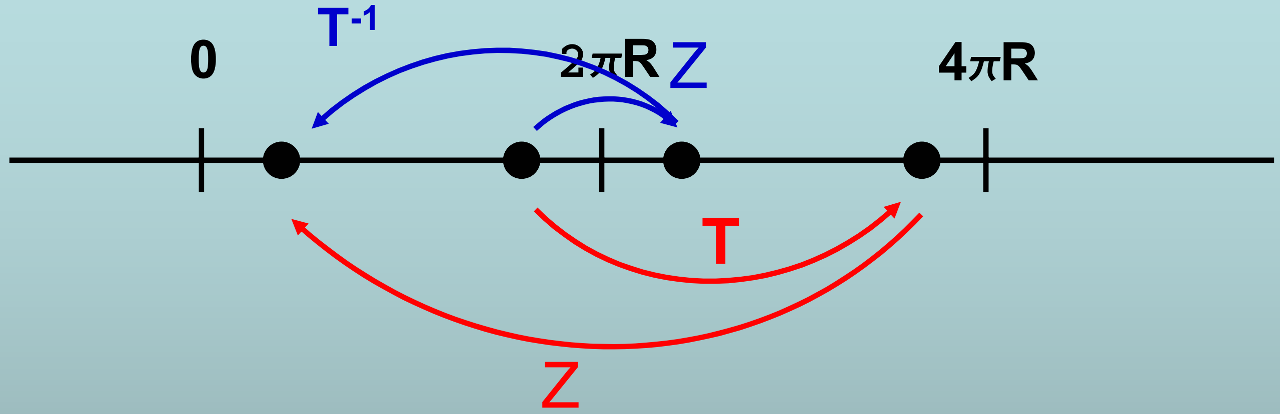
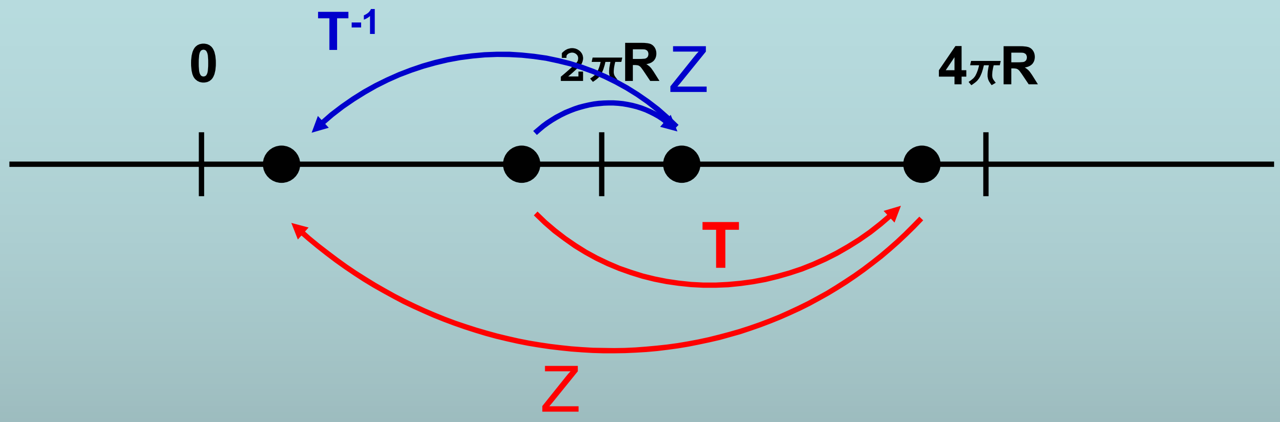


A consistency condition



$$ZT = T^{-1}Z$$

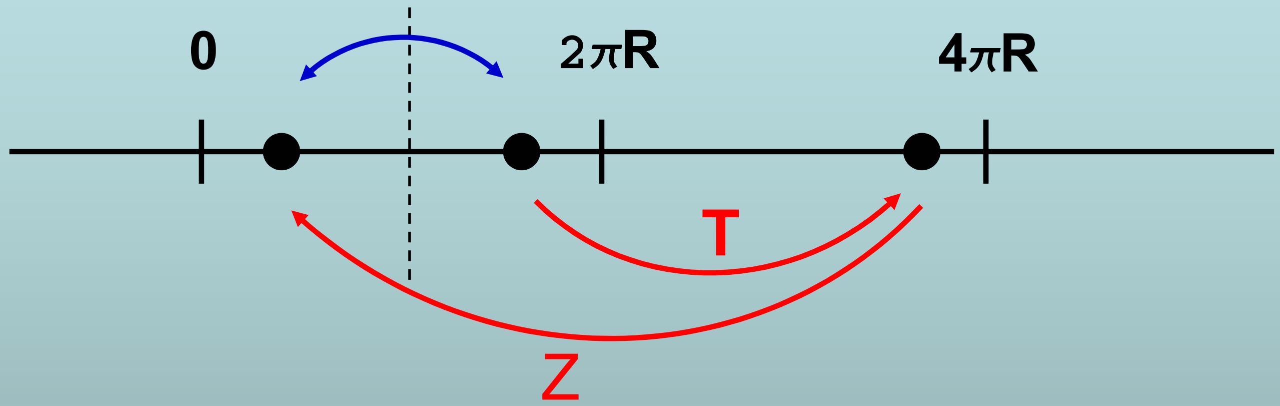
- **Z** is a projection $Z^2 = 1$

$$ZTZ = T^{-1}$$

- **ZT** is also a projection

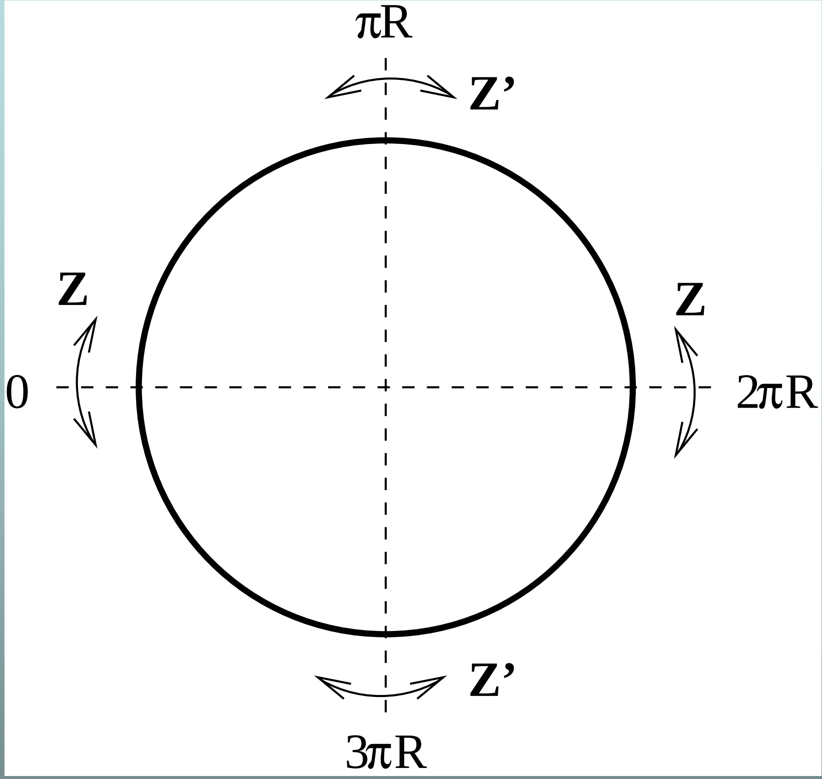
$$(ZT)^2 = ZTZT = T^{-1}T = 1$$

The effect of ZT



- **ZT** is a reflection around π**R**
- A generic $\mathbf{S^1/Z_2}$ is a combination of two (not necessarily commuting) parities **Z** and **ZT**

A simple way to picture the orbifold BC's



- Need to assign + or – parities to fields
- Parity assignments don't have to be in same basis (eg. there could be a Scherk-Schwarz twist if parities don't commute)

The orbifold BC's

- Assign parities under two Z_2's:
  - Scalars: $\varphi(-y)=P\varphi(y)$. $P=\pm 1$
  - Gauge fields: $A_\mu(-y)= PA_\mu(y)P^{-1}$
    $A_5(-y)=-PA_5(y)P^{-1}$
  - Fermions: $\chi(-y)=P\chi(y)$
    $\psi(-y)=-P\psi(y)$
- Reason:
  - A_5 opposite parity as A_μ (vector)
  - Term in fermion action: $\psi\partial_5\chi$

- The KK spectrum
  - Gauge bosons: If A_μ has zero mode, A_5 will NOT (and vice versa)
  - LH (χ) and RH (ψ) fermions have opposite BC's: if one has zero mode, the other doesn't → theory CHIRAL

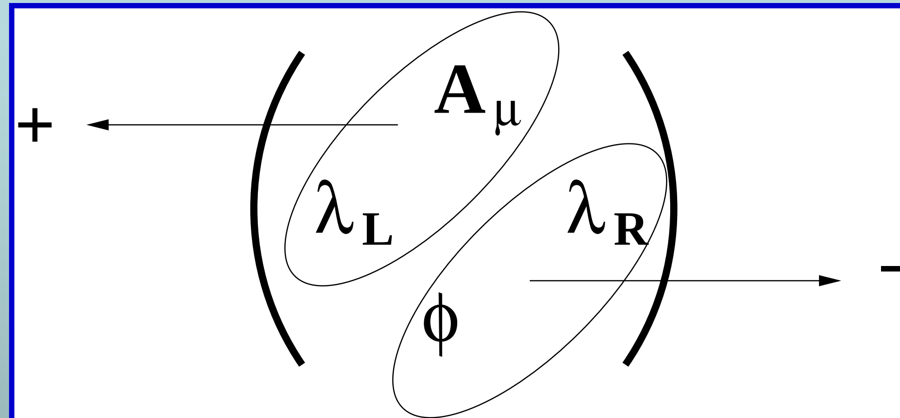
A simple example: GUT breaking via orbifolds

(Altarelli, Feruglio; Hall, Nomura)

- Assume we have SUSY SU(5) in an extra D
- 5D fermions non-chiral, smallest SUSY in 5D: 8 supercharges (like N=2 in 4D)
- Need to use orbifold BC's to
  - Break SUSY from N=2 to N=1
  - Break gauge SU(5)→SU(3)xSU(2)xU(1)

- BC to break SUSY (parities for VSF)

Z_2:



- Gauge breaking (on fundamental)

$$H = (5, \bar{5}), \qquad H' = (5', \bar{5}')$$

$$Z_2' : \begin{pmatrix} \\ 5 \\ \\ \end{pmatrix} \rightarrow \begin{pmatrix} - & & & & \\ & - & & & \\ & & - & & \\ & & & + & \\ & & & & + \end{pmatrix} \begin{pmatrix} \\ 5 \\ \\ \end{pmatrix}$$

•Action of Z_2' on adjoint:

$$Z_2' : \begin{pmatrix} 24 & \\ & \end{pmatrix} \rightarrow \left(\begin{array}{c|c} + & - \\ \hline - & + \end{array}\right)$$

•Decomposition of SU(5) adjoint

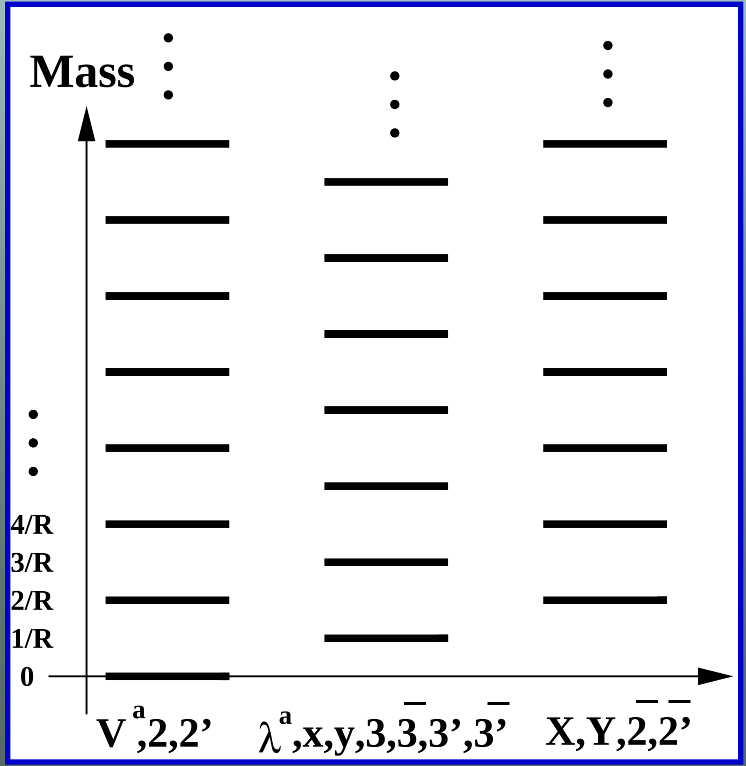
$$\left(\begin{array}{c|c} V^a_{SM} & X \\ \hline Y & V^a_{SM} \end{array}\right), \quad \left(\begin{array}{c|c} \lambda^a & x \\ \hline y & \lambda^a \end{array}\right)$$

•Decomposition of bulk Higgses:

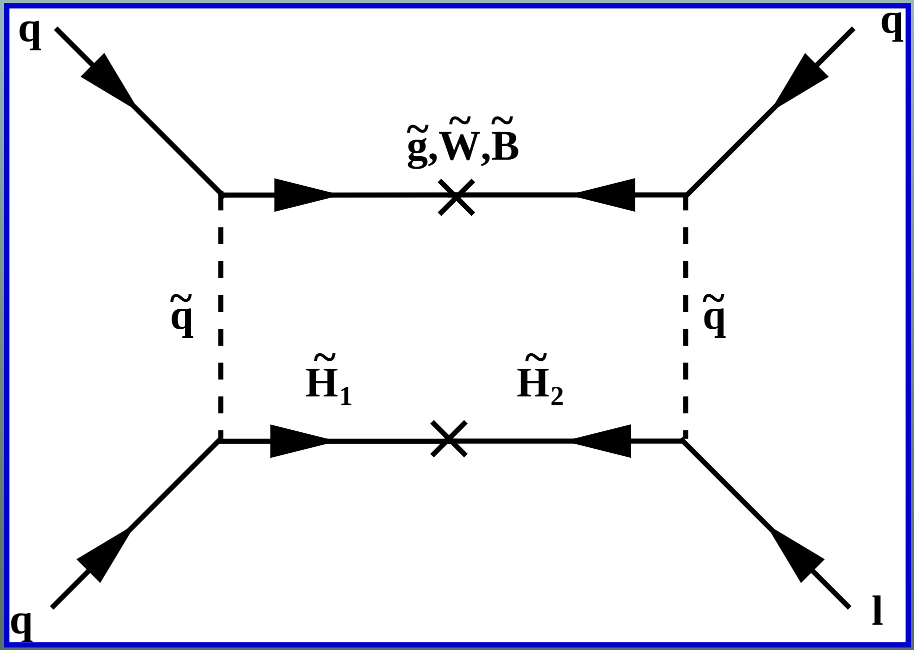
$$H = (3+2, \bar{3}+\bar{2}), \qquad H' = (3'+2', \bar{3}'+\bar{2}')$$

- The KK decomposition will be

(Z_2, Z_2')	mode	KK mass	wave function	
$(+,+)$	$V^a_{SM}, 2, 2'$	$\frac{2n}{R}$	$\cos\frac{2ny}{R}$	
$(+,-)$	$\lambda^a, 3, \bar{3}'$	$\frac{2n+1}{R}$	$\sin\frac{(2n+1)y}{R}$	$, \quad n = 0, 1, 2, \ldots$
$(-,+)$	$x, y, \bar{3}, 3'$	$\frac{2n+1}{R}$	$\cos\frac{(2n+1)y}{R}$	
$(-,-)$	$X, Y, \bar{2}, \bar{2}'$	$\frac{2n+2}{R}$	$\sin\frac{2ny}{R}$	



- Important difference from 4D theory: **doublet-triplet** splitting automatically solved
- In 4D hard to understand why $m_3>m_2$
- Here parity assignments solve it
- No dim 5 proton decay since no **3 $\bar{\mathbf{3}}$** mass



3. Interval vs. orbifold approach

- Could just start with field theory in line segment [0,πR] and specify some BC's
- For example scalar field

$$S_{bulk} = \int d^4x \int_0^{\pi R} \left(\frac{1}{2}\partial^M \phi \partial_M \phi - V(\phi)\right) dy$$

- Variation of action (after integrating by parts)

$$\delta S = \int d^4x \int_0^{\pi R} \left[-\partial_M \partial^M \phi - \frac{\partial V}{\partial \phi}\right] \delta\phi - \left[\int d^4x \partial_y \phi \delta\phi\right]_0^{\pi R}$$

- Require δS_{bulk} and δS_{bound} vanish separately for arbitrary $\delta\Phi$. Fixes BC!

$$\partial_y \phi|_{y=0,\pi R} = 0$$

- Neumann (flat) BC is natural
- How to interpret Dirichlet BC? Add a large localized mass on boundary!

$$S = S_{bulk} - \int d^4x \tfrac{1}{2} M_1^2 \phi^2|_{y=0} - \int d^4x \tfrac{1}{2} M_2^2 \phi^2|_{y=\pi R}$$

- Boundary variation will be

$$\delta S_{bound} = -\int \delta\phi(\partial\phi + M_2^2\phi)|_{y=\pi R} + \int d^4x \delta\phi(\partial_y\phi - M_1^2\phi)|_{y=0}$$

- The BC will be:

$$\begin{aligned} \partial_y\phi + M_2^2\phi &= 0 \quad \text{at} \quad y = \pi R, \\ \partial_y\phi - M_1^2\phi &= 0 \quad \text{at} \quad y = 0 \end{aligned}$$

•Interpretation of Dirichlet BC: limit of **M**$\rightarrow\infty$ limit of localized mass term.

•Can repeat procedure for gauge fields on interval. General BC with localized scalar VEV on boundaries:

•Gauge fields with $\mathbf{v_{1,2}}$ boundary scalar VEV

$$\partial_y A_\mu \mp v_{1,2}^2 A_\mu = 0$$

•$\mathbf{A_5}$ BC:

$$\partial_y A_5 - \frac{\xi_1}{\xi}\frac{m^2/\xi_1}{m^2/\xi_1 - v_1^2} A_5|_{y=0} = 0$$

$$\partial_y A_5 + \frac{\xi_2}{\xi}\frac{m^2/\xi_2}{m^2/\xi_2 - v_2^2} A_5|_{y=\pi R} = 0$$

•$\boldsymbol{\xi}$: bulk gf., $\boldsymbol{\xi_{1,2}}$: boundary gf. terms

- Meaning: if v=0 $\partial_y A_\mu| = 0, \quad A_5| = 0$

- If v$\rightarrow\infty$ $A_\mu| = 0, \quad \partial_y A_5| = 0$

- Just like in the case of orbifolds. Except: symmetry used for orbifolding form a Z_2 subgroup $\subset$ Cartan subalgebra.
- Can NOT reduce rank with a single orbifolding
- Can reduce rank with localized scalar
- Orbifold BC's subset of possible BC's on interval
- For example can break EWS via BC's using the interval approach$\rightarrow$HIGGSLESS EWSB (will not discuss here)

•Is there any advantage for using ONLY orbifolds?

Yes! Localized VEVs vs. flat VEVs:

•**<u>Orbifolds</u>**: wave functions all orthogonal. If Higgs VEV of A_5: also flat, does NOT mix KK modes. No tree-level corrections to EWPO

•**<u>Localized Higgs:</u>** will mix KK modes, induce corrections to EWPO (**$\Delta\rho$, Zbb**,…)

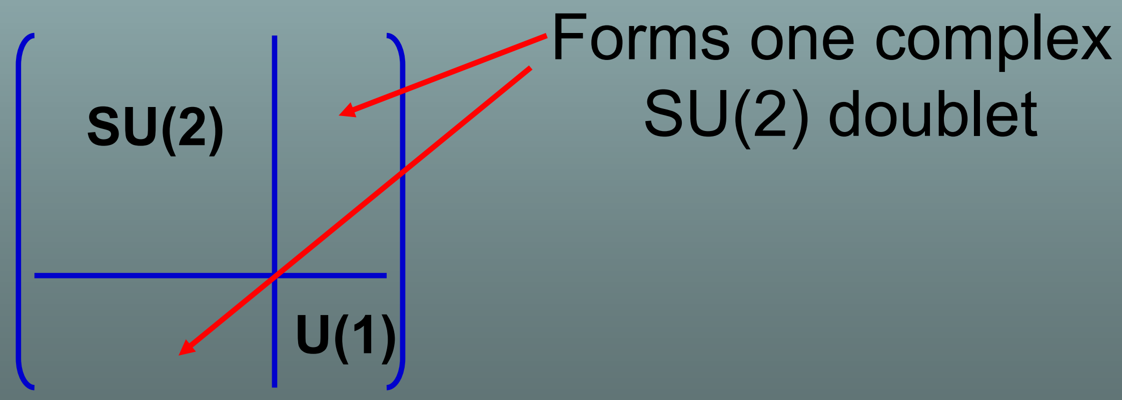
•Flat Higgs VEV in flat orbifold theory~like T-parity in little Higgs (protects EWPO)…

4. Basics of gauge-Higgs unification (GHU)

- Idea: A_5 4D scalar could be Higgs. How to find a setup where A_5 is a doublet of SU(2)xU(1) with correct hypercharge?
- Ideally, use flat space, and NO induced Scalars, just orbifold BCs

History: 1979 Manton, use 6D with monopole in sphere
1983 Hosotani, "Wilson line" breaking: "Hosotani mechanism"
1998 Hatanaka, Inami, Lim: revive idea, no concrete model
2001 Antoniadis, Benakli, Quiros: basic model, Higgs potential calculation
2002 C.C., Grojean, Murayama; von Gersdorrf, Irges, Quiros: 6D problems, basics of flavor construction
2003 Scrucca, Serone, Silvestrini (+Wulzer): basic 5D model introduced and alayzed
2005 Cacciapaglia, C.C., Park; Panico, Serone, Wulzer: close to realistic model

- A_5 is in adjoint of gauge group, but Higgs is doublet: need to enlarge gauge group.
- If we want to use simplest orbifold (does not reduce rank): extended gauge group would be rank 2
- Simplest rank 2 group SU(3)



Necessary BC's

- The necessary projection (at both endpoints):

$$P=\left(\begin{array}{cc|c} 1 & & \\ & 1 & \\ \hline & & -1 \end{array}\right)$$

- Action on A_μ

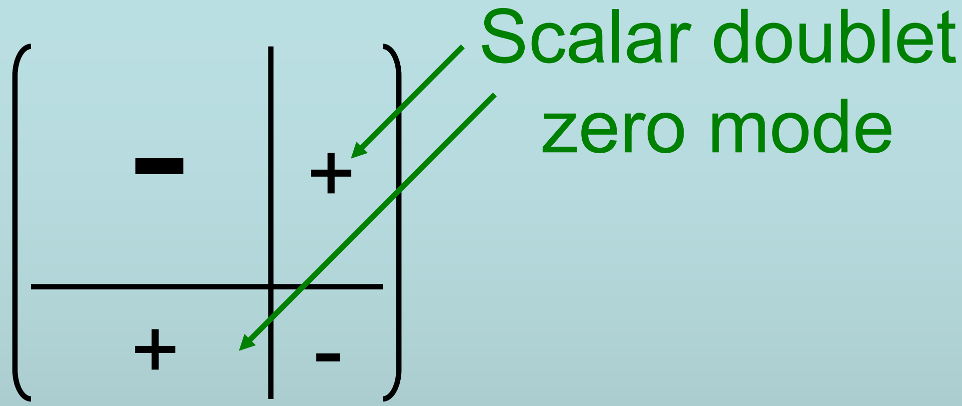
$$\left(\begin{array}{c|c} + & - \\ \hline - & + \end{array}\right)$$

SU(2)xU(1) gauge zero modes

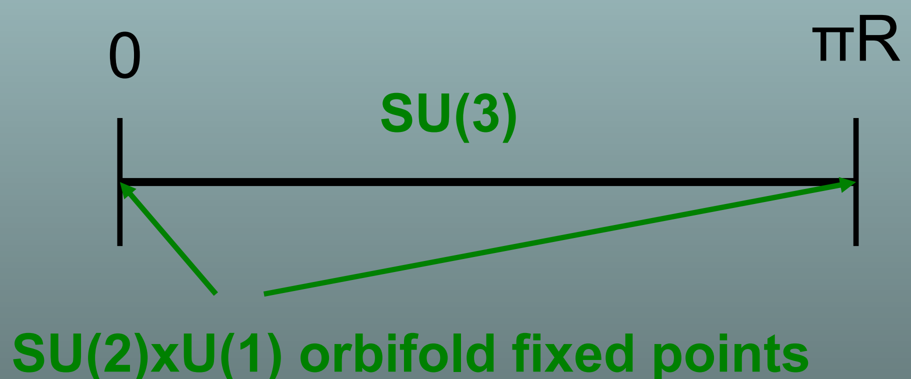
$PA_\mu(-y)P^{-1}$

•Action on A_5:

$-PA_5(-y)P^{-1}$



•Picture:



•For (+,+) fields: **cos(ny/R), $m_n^2=n^2/R^2$**

•For (-,-) fields: **sin(ny/R), $m_n^2=n^2/R^2$**

- Why is this interesting? 5D gauge invariance:

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \epsilon(x,y) + i[\epsilon(x,y), A_\mu] \\ A_5 &\rightarrow A_5 + \partial_5 \epsilon(x,y) + i[\epsilon(x,y), A_5] \end{aligned}$$

- ε: gauge transformation param., has its own KK expansion (same as A_μ). For broken dir. **ε(0,πR)=0**, BUT **∂_5ε≠0.**
- **Shift symmetry** protects **A_5** from mass even at fixed points where gauge symmetry broken
- Shift symmetry analog of broken global sym. in little Higgs models protecting Higgs.

•Shift symmetry forbids tree-level potential Also **local** radiative potential for Higgs forbidden (formulation as SS theory)

•**Non-local** loop effects could still give a **finite** Higgs potential (loop has to stretch from one fixed point to other – does not shrink to zero – result must be finite…)

•Gauge-Higgs unification protects Higgs from divergences due to higher dim. gauge invar.

•Higgs potential only generated through finite loop effects

5. The calculation of the Higgs potential

- Need Coleman-Weinberg potential for Higgs
- Assume simplest SU(3) model for now
- Higgs VEV normalization:

$$A_5 = \frac{1}{\sqrt{2}} \begin{pmatrix} - & H_5 \\ H_5^\dagger & - \end{pmatrix}$$

$$\langle H_5 \rangle = \sqrt{2} \begin{pmatrix} 0 \\ \alpha/R \end{pmatrix}$$

- **α**: VEV in units of radius. For realistic model needs to be **≪1** (to separate KK modes from) SM particles

- For Coleman-Weinberg need α-dependent Mass spectrum. For example gauge KK:

cos(ny/R) sin(ny/R)

$$\left(\begin{array}{cc|c} \frac{1}{\sqrt{2}}A^3+\frac{1}{\sqrt{6}}A^8 & W^+ & \tilde{W}^1 \\ W^- & -\frac{1}{\sqrt{2}}A^3+\frac{1}{\sqrt{6}}A^8 & \tilde{W}^2 \\ \hline \tilde{W}^{1*} & \tilde{W}^{2*} & -\frac{2}{\sqrt{6}}A^8 \end{array}\right)$$

- Mass terms come from: $-\int_0^{\pi R} \frac{1}{2}\mathrm{Tr}\; F_{5\mu}^2$

0 in unitary gauge

$$-\frac{1}{2}\int_0^{\pi R}\mathrm{Tr}\left(\partial_5 A_\mu - \partial_\mu A_5 + g_5[\langle A_5\rangle, A_\mu]\right)^2$$

- The mass matrix mixes various components In the 8x8 basis A_1-A_8 the mixing matrix is:

- TeXForm on the Mathematica output:

$$\frac{1}{R^2}\begin{pmatrix} 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & 4\alpha n & 0 & 0 & 0 \\ 0 & 2\left(\alpha^2+n^2\right) & 0 & -4\alpha n & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & -4\alpha n & -2\sqrt{3}\alpha^2 \\ 0 & -4\alpha n & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 & 0 \\ 4\alpha n & 0 & 0 & 0 & 2\left(\alpha^2+n^2\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2n^2 & 0 & 0 \\ 0 & 0 & -4\alpha n & 0 & 0 & 0 & 2\left(4\alpha^2+n^2\right) & 4\sqrt{3}\alpha n \\ 0 & 0 & -2\sqrt{3}\alpha^2 & 0 & 0 & 0 & 4\sqrt{3}\alpha n & 2\left(3\alpha^2+n^2\right) \end{pmatrix}$$

- Eigenvalues: **n^2/R^2** x2 $\leftarrow\gamma$

  $(n\pm\alpha)^2/R^2$ x2 $\leftarrow W_\pm$

  $(n\pm2\alpha)^2/R^2$ x1 $\leftarrow Z$

- Implies most problematic part of model:

$$M_Z^2/M_W^2=2$$

- Obviously due to wrong U(1) quantum number of Higgs

- Unbroken U(1) after orbifolding: T_8
- Higgs quantum number:

Usual normalization:

- $g \rightarrow 1/2$ diag (1,-1), etc
- $g' \rightarrow$ Higgs quantum number 1/2

- Here: for Tr $T_a T_b = 1/2$: $T_8 = 1/(2\sqrt{3})$ diag(1,1,-2)
- Higgs quantum number $\sqrt{3}/2$. Rescale U(1):
- **$\sqrt{3}/2 g = g'/2$**

$\sin^2\theta_W = g'^2/(g^2+g'^2) = 3/(1+3) = 3/4$

- Wrong U(1) normalization, need another U(1)

The Coleman-Weinberg potential

(Antoniadis, Benakli, Quiros)

$$V_{CW}(\phi) = \frac{1}{2}\sum_I (-1)^{F_I} \int \frac{d^4p}{(2\pi)^4} \log(p^2 + M_I^2(\phi))$$

- Can be rewritten in the form

$$V_{CW} = -\frac{1}{32\pi^2}\sum_I (-1)^{F_I} \int_0^\infty dl l e^{-\frac{M_I^2(\phi)}{l}}$$

- General form of KK mass spectrum (ABQ)

$$M_{\vec{m}}^2 = \mu^2 + \sum_{i=1}^d \frac{(m_i + a_i(\phi))^2}{R_i^2}$$

↓

$$V_{CW} = -\sum_I \frac{(-1)^{F_I}}{32\pi^2} \sum_{\vec{m}} \int_0^\infty dl l e^{-\frac{\mu^2}{l}} e^{-\sum_i \frac{(m_i + a_i)^2}{R_i^2 l}}$$

- Using a Poisson resummation

$$\frac{1}{2\pi R}\sum_m F(m/R) = \sum_n \tilde{F}(2\pi n R)$$

$$\sum_{\vec{m}} e^{-\sum_i \frac{(m_i+a_i)^2}{r_i^2}} = \pi^{\frac{d}{2}}\prod_{i=1}^{d} R_i \sum_{\vec{n}} e^{2\pi i \sum_j n_j a_j} e^{-\pi^2 \sum_j n_j^2 r_j^2}$$

$$V_{CW}(\phi) = -\sum_I \frac{(-1)^{F_I}}{32\pi^2}\pi^{\frac{d}{2}}(\prod R_i)\sum_{\vec{n}} e^{2\pi i \sum_j n_j a_j} \int_0^\infty dl\, l^{1+\frac{d}{2}} e^{-\frac{\mu^2}{l}} e^{-\pi^2(\sum_j n_j^2 R_j^2) l}$$

- For example, if μ=0 (no bulk mass)

$$V_{CW} = -\sum_I \frac{(-1)^{F_I}}{32\pi^2}(\prod R_i)\pi^{\frac{d}{2}}\Gamma(2+\tfrac{d}{2})\sum_{\vec{n}\neq 0}\frac{e^{2\pi i \vec{n}\cdot\vec{a}}}{(\pi^2\sum_j n_j^2 R_j^2)^{2+\frac{d}{2}}}$$

- As expected potential finite (dropped a divergent constant piece…)

- Expression for potential in general case in 5D:

$$V_{eff}(\beta) = \frac{\mp 1}{32\pi^2} \frac{1}{(\pi R)^4} \mathcal{F}(\beta)$$

β=k α

- Where for no bulk mass term **$m_n^2 = (n+\beta)^2/R^2$**

$$\mathcal{F}(\beta) = \frac{3}{2} \sum_{n=1}^{\infty} \frac{\cos(2\pi\beta n)}{n^5}$$

- With bulk mass term **$m_n^2 = M^2 + (n+\beta)^2/R^2$**

$$\mathcal{F}_\kappa(\beta) = \frac{3}{2} \sum_{n=1}^{\infty} \frac{e^{-\kappa n} \cos(2\pi\beta n)}{n^3} \left(\frac{\kappa^2}{3} + \frac{\kappa}{n} + \frac{1}{n^2} \right)$$

- Where **κ=2πMR**. For large κ exponentially suppressed.

Comments

- n=1 term most important in series **$\pm\cos 2\pi\beta$**
- For fermions min. for **$\beta=\frac{1}{2}$**
- For bosons min. for **$\beta=0$**
- For twisted fermions (will see later) spectrum

$$m_n^2 = M^2 + (n+\tfrac{1}{2}+\beta)^2/R^2$$

- Effect in potential **$\beta\rightarrow\beta+\frac{1}{2}$**

Summary:

Can calculate finite Higgs potential for arbitrary bulk fields. Need to know, what bulk fields…

6.The fermion fields & flavor structure

- Apparent problem: since Higgs=$\mathbf{A_5}$, Yukawa coupling=gauge coupling. How to get fermion mass hierarchy?

1. Use Arkani-Hamed **Schmaltz** idea of localizing fermions at different parts of 5D
2. Use bulk fermions mixed with localized fermions at the fixed points (an X-D version of Frogatt-Nielsen)

- Will use second approach

- Every SM field→Dirac fermion in 5D **Ψ**
- Arrange BC's such, that only one zero mode
- In order to avoid masses of order **M_W** add a second bulk field with same quantum # but opposite parity assignments **Ψ'**
- Two fields will marry up with bulk mass

$$M\Psi\bar{\Psi}'$$

- At this point no chiral zero modes. We add them as fields localized at the fixed points and mix them with the bulk fields

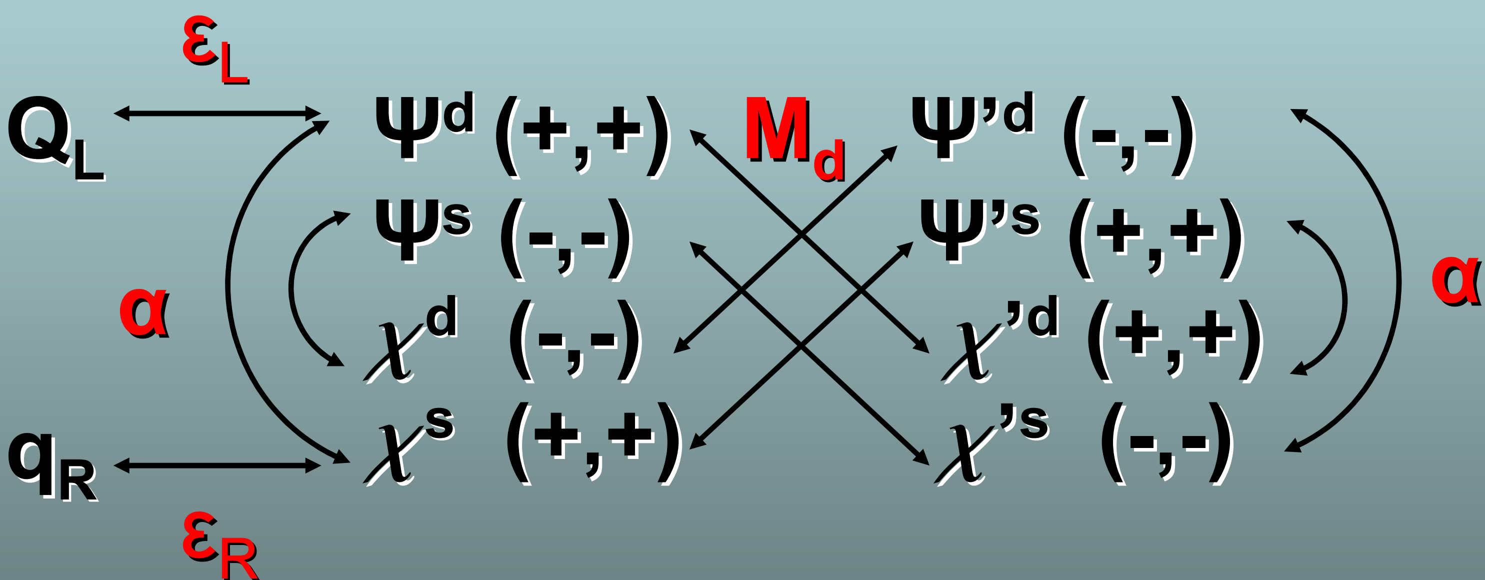
$$\mathcal{L}_{loc} = \left[-i\bar{Q}_L\bar{\sigma}^\mu\partial_\mu Q_L + \frac{\epsilon_L}{\sqrt{\pi R}}\psi^d Q_L + h.c.\right]\delta(y-y_L) + \left[-iq_R\sigma^\mu\partial_\mu\bar{q}_R + \frac{\epsilon_R}{\sqrt{\pi R}}q_R\chi^s + h.c.\right]\delta(y-y_R)\,,$$

- Here Ψ^d is the doublet and χ^s is the singlet in the bulk field. Depending on choices of parity there is always a unique choice of which to add
- y_L and y_R could be either fixed points
- 4 distinct possibilities (same fp or opposite, fermions twisted or not…)

- In the $\alpha=0$ limit still a zero mode (odd number of chiral fermions), so light modes $m_0 \propto \alpha$
- Mass spectrum will depend on $\alpha, \varepsilon_L, \varepsilon_R, M$

Example: down quark

- Use bulk triplets **3** and no twisting



- Need to write down coupled bulk equations
- Can diagonalize bulk equations
- BC's will provide equation for KK masses

•Equation for spectrum for **3** with untwisted fermions:

$$\mathcal{Y}_3(w) = (\cos w - \cos(2\pi\alpha))^2 + 2\frac{\epsilon_L^2 + \epsilon_R^2}{w} \sin w\,(\cos w - \cos(2\pi\alpha)) + $$
$$ - \frac{4\epsilon_L^2\epsilon_R^2}{w^2} \cdot \begin{cases} (\cos w + 1)\left(\cos w - 1 + 2\frac{w^2}{w^2+\kappa^2}\sin^2(\pi\alpha)\right) & \text{different branes}, \\ \frac{1}{2}\left(\cos 2w - 1 + 2\frac{w^2}{w^2+\kappa^2}\sin^2(2\pi\alpha)\right) & \text{same brane}. \end{cases}$$

- **$y_3(w)=0$** determines mass eigenmodes **m**
- **$\kappa=2\pi RM$**, **$w^2=(2\pi Rm)^2-\kappa^2$**

•Similar equation for the twisted case

$$\tilde{\mathcal{Y}}_3(w) = (\cos w + \cos(2\pi\alpha))^2 + 2\frac{\epsilon_L^2 + \epsilon_R^2}{w} \sin w\,(\cos w + \cos(2\pi\alpha)) + $$
$$ - \frac{4\epsilon_L^2\epsilon_R^2}{w^2} \cdot \begin{cases} (\cos w - 1)\left(\cos w + 1 - 2\frac{\kappa^2}{w^2+\kappa^2}\sin^2(\pi\alpha)\right) & \text{different branes}, \\ \frac{1}{2}\left(\cos 2w - 1 + 2\frac{w^2}{w^2+\kappa^2}\sin^2(2\pi\alpha)\right) & \text{same brane}. \end{cases}$$

- Simple limits:
  - No boundary mixings **($\epsilon_{L,R} \to 0$)**

$$m_n^2 = M^2 + \begin{cases} \frac{(n+\alpha)^2}{R^2} & \text{untwisted} \\ \frac{(n+1/2+\alpha)^2}{R^2} & \text{twisted} \end{cases}$$

  - Small Higgs VEV **($\alpha \ll 1$)**, large bulk mass **($\kappa \gg 1$)**

$$\text{diff. branes} \rightarrow \frac{4\epsilon_L \epsilon_R}{\sqrt{(2\epsilon_L^2+1)(2\epsilon_R^2+1)}} \frac{\kappa}{2} e^{-\kappa/2}$$

$$\text{same brane} \rightarrow \frac{4\epsilon_L \epsilon_R}{\sqrt{(2\epsilon_L^2+1)(2\epsilon_R^2+1)}} \kappa\, e^{-\kappa}$$

Exponentially suppressed by bulk mass…

•Small bulk mass **($\kappa \ll 1$):** if untwisted there will be a mode with **$m=M_W$.** Reason: bulk mass couples two fermions, and only one mixes with localized fields. Other light:

$$\text{diff. branes} \rightarrow m_q \pi R = \frac{\epsilon_L \epsilon_R}{\sqrt{(1+\epsilon_L^2)(1+\epsilon_R^2) - \cos^2 \pi\alpha}}$$

$$\text{same brane} \rightarrow m_q \pi R = \frac{\epsilon_L \epsilon_R \cos \pi\alpha}{\sqrt{(1+\epsilon_L^2)(1+\epsilon_R^2) - \cos^2 \pi\alpha}}$$

Lessons:

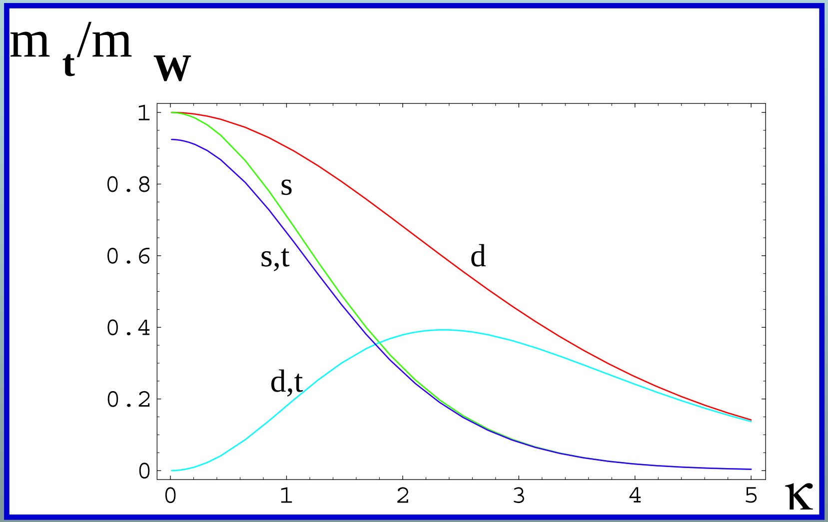
•Many ways to get suppression: large bulk mass, small boundary mixing

•Hard to get a large mass. Upper limit M_W.

•Upper limit achievable for vanishing bulk mass

•Upper limit can be relaxed for bigger reps due to non-trivial group-theory factors (Dynkin)

•Example: behavior of lowest eigenmode for different (d) or same (s) brane, untwisted or twisted (t) bulk fermions:



•Final remark: for large mixings spectrum can be deformed a lot. Need to modify formula for Higgs potential! Using result of Goldberger& Rothstein:

- Mass given by **y(m)=0,** contribution to CW:

$$V_{eff} = \frac{1}{2}\int_0^\infty \frac{d^4p}{(2\pi)^4} \ln \mathcal{Y}(ip)$$

- In our case

$$\mathcal{F}_\epsilon(\kappa,\alpha) = \frac{1}{8}\int_\kappa^\infty d\zeta\zeta(\zeta^2-\kappa^2)\ln\frac{\mathcal{Y}(i\zeta)}{K(\zeta)}$$

- Function **K** to regulate divergent constant
- Contribution of various bulk fields to CW:

bulk field	multiplicity	
gauge (adj.)	-3	$2\mathcal{F}(\alpha)+\mathcal{F}(2\alpha)$
down (3)	3×8	$\mathcal{F}_{\kappa_d}(\alpha)$
up (6)	3×8	$\mathcal{F}_{\kappa_u}(\alpha)+\mathcal{F}_{\kappa_u}(2\alpha)$
lepton (10)	8	$2\mathcal{F}_{\kappa_l}(\alpha)+\mathcal{F}_{\kappa_l}(2\alpha)+\mathcal{F}_{\kappa_l}(3\alpha)$

7.A semi-realistic model

- To fix $\sin^2\theta_W$ we add an additional $U(1)_X$
- Gauge group $SU(3)xU(1)_X$ broken by orbifold to $SU(2)_L xU(1)_8 xU(1)_X$, and $U(1)_8 xU(1)_X \rightarrow U(1)_Y$ on the fixed point (localized Higgs or anomaly)
- This last breaking distorts wave functions, we'll have to pay the price for that…

Two main problems:

(Scrucca, Serone, Silvestrini)

- Higgs mass too small (& KK modes light)
- Top mass too small

- Reason: if assume (well motivated)
    - all mixings of same order
    - fermion hierarchy only from bulk masses
- Most bulk masses very large, contribution to CW very suppressed. Basically top dominates radiative potential, and minimum of top+gauge contribution gives

$$\alpha \sim 0.3, \quad m_h \sim 0.2 - 0.3 m_W$$
$$1/R \sim 3 - 5 m_W \sim 250 - 400\text{GeV}$$
$$m_t \leq m_W$$

- This is obviously bad

•**Fix Higgs mass and VEV**: assume that some light fermions light **due to small mixing** rather than due to large bulk mass

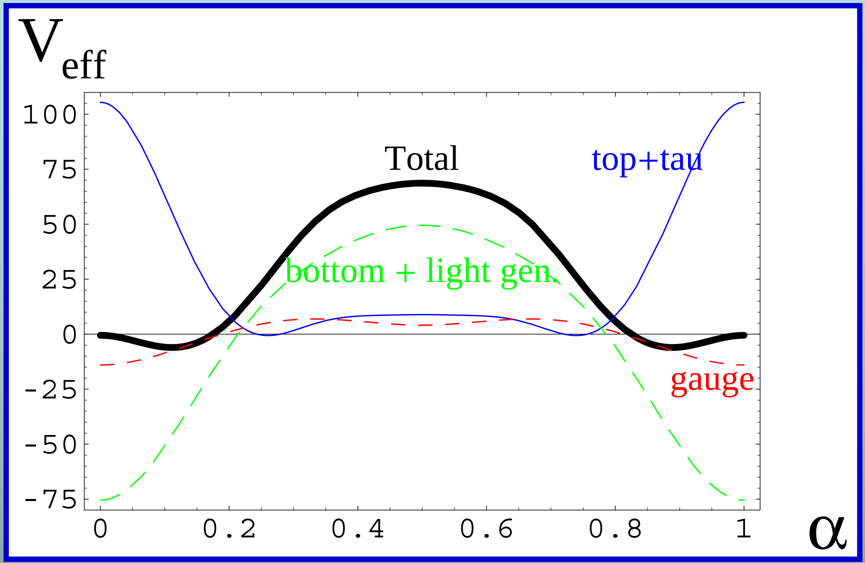
•These bulk fermions will also contribute

•Take different representations and twist some of fermions→get a much more versatile Higgs potential

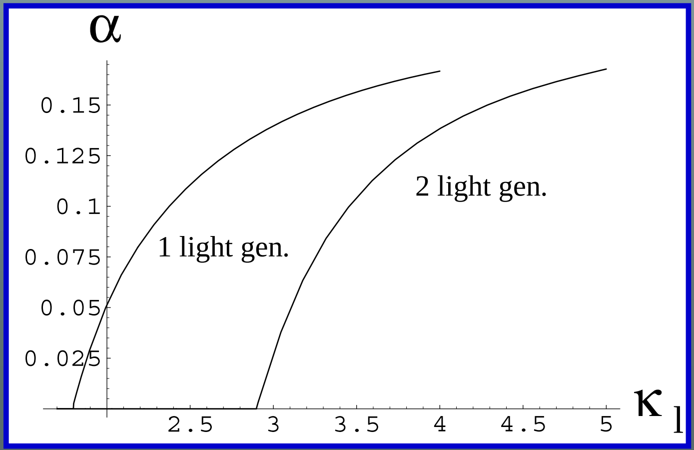
A successful example

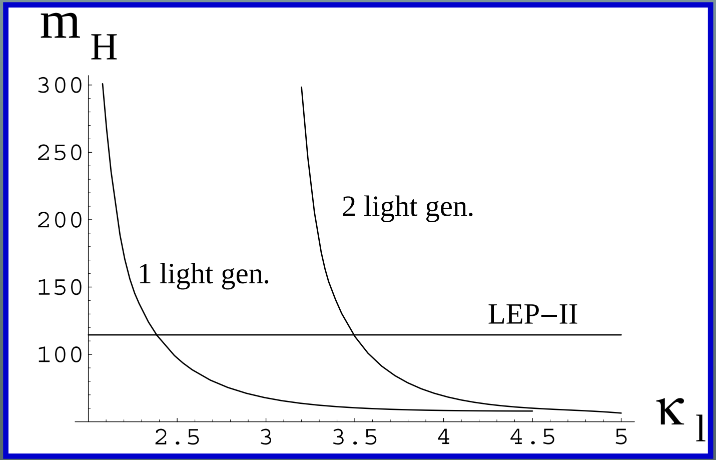
- Top: rep. $\bar{\mathbf{6}}$, large mixing $\epsilon_{L,R}\sim 3$, $\kappa_t \sim 1$
- Bottom: twisted **3**, $\kappa_b = 0$
- Tau: **10**, $\kappa_\tau = 1$
- Light gens: twisted $\mathbf{3}+\bar{\mathbf{6}}+\mathbf{10}$, common κ_l

• The Higgs potential:



• VEV and Higgs mass





•**Fix top mass:** upper bound on fermion mass actually depends on representation

$$m_t \leq k m_W$$

•**k^2:** number of indices of rep. top is embedded

•For **m_t=2m_W** need a 4-index irrep…

•Simplest possibility **$\overline{15}$** dim rep:

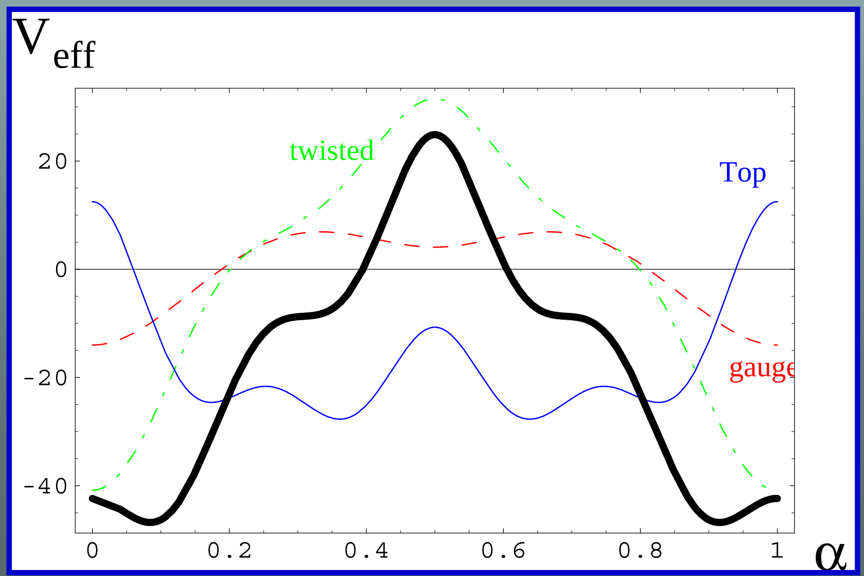
$$(\overline{15})_{-2/3} \rightarrow (\mathbf{1}, 2/3) + (\mathbf{2}, 1/6) + (\mathbf{3}, -1/3) + (\mathbf{4}, -5/6) + (\mathbf{5}, -4/3)$$

•To get biggest top mass (**2m_W**) need top to be a bulk zero mode. So we only add a single **$\overline{15}$** with usual orbifold projections. Remove ad'l zero modes via mixing with localized fields

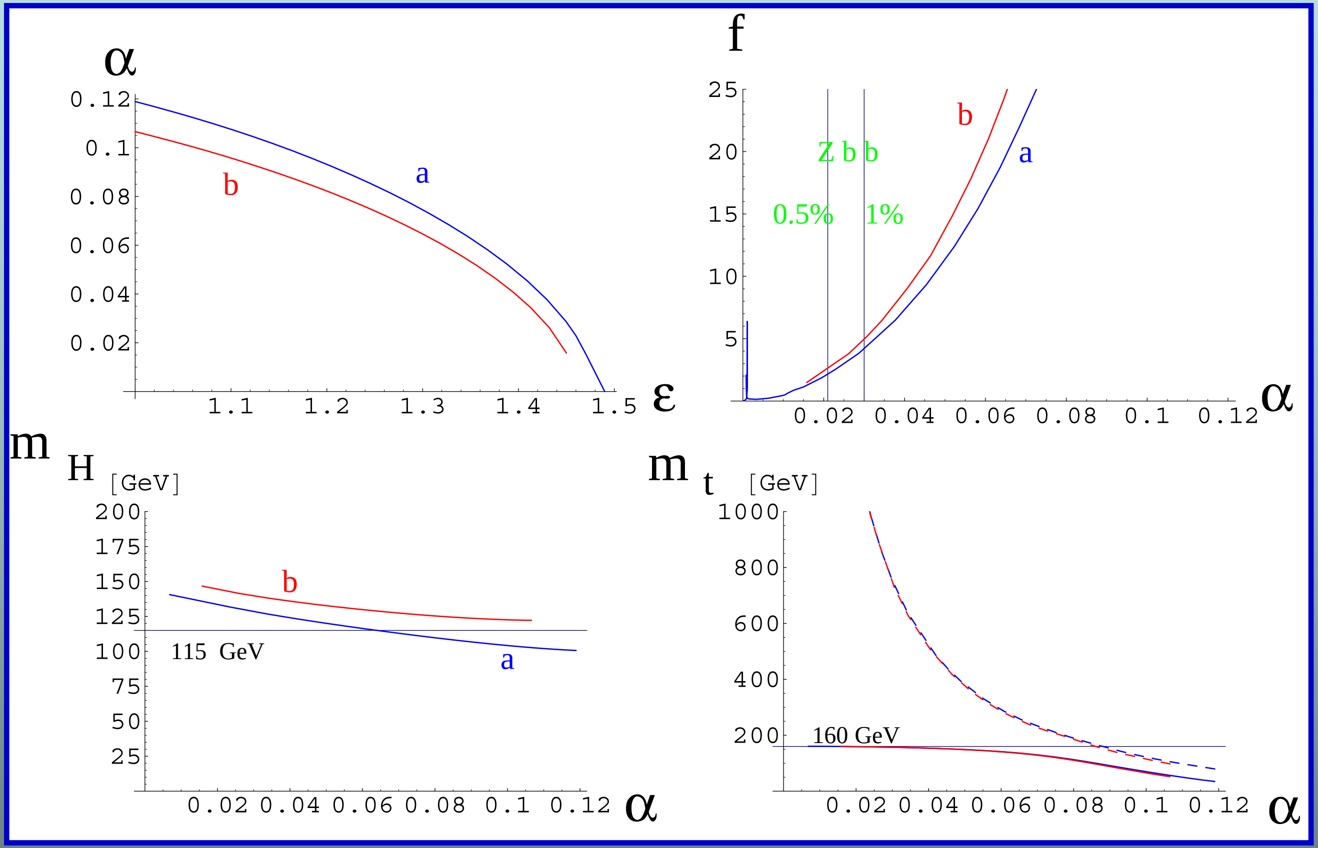
- For EWSB third generation enough (twisted fermions for **b**,τ). Possible reps (choose them as small as possible to not lower cutoff further)

	bottom	tau
model a	$(\mathbf{3},\mathbf{3})_0$	$(\mathbf{1},\mathbf{10})_0$
model b	$(\mathbf{3},\mathbf{6})_{1/3}$	$(\mathbf{1},\mathbf{3})_{-2/3}$

- The Higgs potential



•Results for α, $\mathbf{m_H}$, $\mathbf{m_{top}}$ and fine tuning (**f**)



•Fine tuning defined via the usual log derivative

$$f = \frac{d \log \alpha(\epsilon)}{d \log \epsilon}$$

•Some particular model points:

α	$1/R$	f	m_H	m_t	m_t'
0.08	1 TeV	31% 42%	110 125 GeV	113 110 GeV	189 186 GeV
0.05	1.6 TeV	11% 14%	120 133 GeV	149 149 GeV	381 375 GeV
0.04	2 TeV	7% 9%	124 136 GeV	154 154 GeV	519 514 Gev
0.03	2.7 TeV	4% 5%	128 140 GeV	157 157 GeV	753 746 Gev
0.02	4 TeV	2% 2%	134 144 GeV	159 159 GeV	1224 1213 Gev

•Introducing a large representation dangerous for lowering cutoff scale: only a **few x 1/R**.

•Would need to check stability of results under loop corrections (two loop Higgs potential?)

8. Bounds on the model from EWPT

- **W,Z,<H>** flat: no mixing induced among KK modes, no correction to EWPO from these at tree level, and loop should be small
- Only possible source: exotic zero modes that mix with SM fields and pick up mass via **boundary** terms (otherwise orthogonality OK)
- **Two** such sources:
  - Fermion zero modes needed to generate fermion masses: **$Zb\bar{b}$** affected
  - Additional **$U(1)_X$** to fix **$\sin^2\theta_W$**: will affect **$\Delta\rho$**

$Zb\bar{b}$ from mixing with heavy quarks

- Light fermions mixing negligible. Only 3rd gen. problematic. Lowest order Yukawa by gauge inv.

$$\mathcal{Y}_{-1/3}\, Q_L H^\dagger \bar{\mathbf{3}}_{-1/3} + \mathcal{Y}_{2/3}\, Q_L H \bar{\mathbf{3}}_{2/3}$$

- General expression for corr. of Z-vertex:

$$\Delta = \frac{\delta g}{g} = \frac{1}{1-\frac{2}{3}\sin^2\theta_W}\left(\mathcal{Y}^2_{2/3} - \mathcal{Y}^2_{-1/3}\right)\left(\frac{m_W}{m_3}\right)^2$$

- For the **15** rep $\mathbf{y_{-1/3}=\sqrt{3}}$, $\mathbf{m_3^2=3/(R^2\pi^2)}$

$$\Delta \simeq -11\alpha^2$$

- Bound from LEP: $\boldsymbol{\alpha<0.021}$, **1/R>3.9 TeV**

Effects of additional $U(1)_X$

$$X_\mu = \frac{1}{\sqrt{3g^2+g_x^2}}\left(\sqrt{3}gA_\mu^8 - g_x A_\mu^x\right)$$

- X_μ gets a localized mass. After EWSB mixing with Z induced, correction to T:

$$T = \frac{4\pi}{e^2}\Delta\rho = \frac{4\pi}{e^2}\frac{\pi^2}{3}\frac{3-4\sin^2\theta_W}{\cos^2\theta_W}\alpha^2 \approx 1.2\cdot 10^3\,\alpha^2$$

- Strongest bound on model **1/R>5 TeV**, $\boldsymbol{\alpha}$**<0.018**
- Also contibutes to $\boldsymbol{\delta}$**g/g** bound **1/R>4.5 TeV**

Summary

- In extra dim's a possible solution to hierarchy problem is via gauge-Higgs unification
- Need to extand gauge group and orbifold it to **SU(2)xU(1)**
- Simplest (and most realistic) example in 5D **SU(3)xU(1)$_X$**
- Generically hard to get a large separation of Higgs VEV and KK modes, and heavy Higgs, top
- Can use many bulk fermions to generate a sufficiently generic Higgs pot.
- Top mass fixed via large bulk representation
- Constraints from **Zbb**, $\Delta\rho$: little hierarchy …