Fundamental Concepts in Particle Physics

Lecture 4 : Towards Beyond the Standard Model

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Charges of Standard Model Fields

Field	SU(3)	$SU(2)_L$	T^3	$\frac{Y}{2}$	$Q = T^3 + \frac{Y}{2}$
g^a_μ (gluons)	8	1	0	0	0
(W^\pm_μ, W^0_μ)	1	3	$(\pm 1, 0)$	0	$(\pm 1, 0)$
B^0_μ	1	1	0	0	0
$Q_L = \left(\begin{array}{c} u_L \\ d_L \end{array}\right)$	3	2	$\left(\begin{array}{c}\frac{1}{2}\\-\frac{1}{2}\end{array}\right)$	$\frac{1}{6}$	$\left(\begin{array}{c}\frac{2}{3}\\-\frac{1}{3}\end{array}\right)$
u_R	3	1	0	$\frac{2}{3}$	$\frac{2}{3}$
d_R	3	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$
$E_L = \left(\begin{array}{c} \nu_L \\ e_L \end{array}\right)$	1	2	$\left(\begin{array}{c}\frac{1}{2}\\-\frac{1}{2}\end{array}\right)$	$-\frac{1}{2}$	$\left(\begin{array}{c}0\\-1\end{array}\right)$
e_R	1	1	0	-1	-1
$\Phi = \left(\begin{array}{c} \phi^+ \\ \phi^0 \end{array}\right)$	1	2	$\left(\begin{array}{c}\frac{1}{2}\\-\frac{1}{2}\end{array}\right)$	$\frac{1}{2}$	$\left(\begin{array}{c}1\\0\end{array}\right)$
$\Phi^c = \left(\begin{array}{c} \phi^0 \\ \phi^- \end{array}\right)$	1	2	$ \left(\begin{array}{c} \frac{1}{2} \\ -\frac{1}{2} \end{array}\right) $	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$

Left-handed versus right-handed spinors

Nature is symmetric under the group of Lorentz transformations, rotations, and translations which all together form the Poincaré group.

Particles are classified by spin: scalars, fermionic spinors, vector bosons. They correspond to irreducible representations of the Poincaré group

Spinors are of two types: the fundamental (left-handed) and the antifundamental (right-handed). The chirality of a spin 1/2 field refers to whether it is in the fundamental or the anti-fundamental and is therefore a label associated with a representation of the Lorentz group

Weyl spinors
$$\Psi_L:(rac{1}{2},0)$$
 $\Psi_R:(0,rac{1}{2})$

Dirac spinor

 $\Psi = \begin{bmatrix} \Psi_L \\ \Psi_{\Sigma} \end{bmatrix}$



helicity is a physical quantity: it is the projection of the spin onto the direction of motion



for a massless particle: chirality= helicity

SU(2)_L

(thus we call the fundamental spinors the left-handed spinors and the antifundamental spinors the right-handed spinors)

The Standard model is a chiral theory: the left-handed and right-handed spinors not only transform differently under the Lorentz group but also under the EW gauge group $SU(2)_{L}*U(1)$

The left-handed fields are denoted $Q=(u_L, d_L)$ and $L=(nu_L, e_L)$ while the right-handed fields are denoted $u_{,} d$ and $e_{,}$

Fermion masses

Lorentz invariant:

$$\boldsymbol{\psi}\boldsymbol{\psi} \Box \boldsymbol{\psi}_{L}^{*}\boldsymbol{\psi}_{R} \Box \boldsymbol{\psi}_{R}^{*}\boldsymbol{\psi}_{L}$$

-> Dirac mass

$$\mathfrak{L} = m e_L^* e_R \Box e_R^* e_L \Box$$

-> violates EW gauge symmetry

How do a left-handed and right-handed particle of different quantum numbers become a single massive particle?

The key is the gauge invariant Yukawa interaction:

$$\Delta \mathcal{L}_Y = -\phi(Q_i \lambda_{ij}^U u_j^c + L_i \lambda_{ij}^N N_j) - \phi^+(Q_i \lambda_{ij}^D d_j^c + L_i \lambda_{ij}^E e_j^c) + h.c.$$

Once the Higgs condenses, these couplings become the mass of the SM fermions

The CKM matrix

Both up and down-type quarks acquire mass matrices

$$M_u = Y_u v, \qquad M_d = Y_d v.$$

Need to be diagonalized with four independent unitary rotations:

$$M_{u}^{\dagger}M_{u} = V_{u_{R}}D_{u}^{2}V_{u_{R}}^{\dagger}, \qquad M_{u}M_{u}^{\dagger} = V_{u_{L}}D_{u}^{2}V_{u_{L}}^{\dagger}, M_{d}^{\dagger}M_{d} = V_{d_{R}}D_{d}^{2}V_{d_{R}}^{\dagger}, \qquad M_{d}M_{d}^{\dagger} = V_{d_{L}}D_{d}^{2}V_{d_{L}}^{\dagger}.$$

mass eigen values:
$$D_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad D_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

u_L and d_L live in a doublet. There is no basis in which both components are in a mass eigenstate.

$$Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} = \begin{pmatrix} (V_{u_L})_{ij} u_{Lj}^m \\ (V_{d_L})_{ij} d_{Lj}^m \end{pmatrix}$$

-> Cabbibo-Kobayashi-Maskawa mixing matrix of the Standard Model $V_{CKM} = V_{u_L}^{\dagger} V_{d_L}$

The hierarchy problem

As soon as we introduce a fundamental scalar field in the theory (the Higgs), this generates a puzzle: the so-called "hierarchy problem".

= the fact that the Higgs self-energy receives radiative contributions that are quadratically divergent.



There are examples in physics where unexpected and precise parameter cancellations were actually the signal of the existence of new particles.

For instance, the electron self energy has a power divergence that can be cured only by the introduction of the positron.

Similarly, the extreme sensitivity of the Higgs self energy with respect to physics at high momentum can be naturally reduced by introducing new symmetries and new degrees of freedom, such as supersymmetry, or extra spacial dimensions or additional global symmetries.



new states \approx softer UV behavior, small correction to the mass

The hierarchy problem: What is keeping the Higgs boson light?

we need new degrees of freedom to cancel Λ^2 divergences and ensure the stability of the weak scale

a problem that arises for any elementary SCALAR particle

does not arise for fermions (protected by chiral symmetry) or gauge bosons (protected by the gauge symmetry)

What is cancelling the divergent diagrams? $\Rightarrow \delta M_{H}^2 \propto \Lambda^2$

A light Higgs calls for New Physics at the TeV scale

the "hierarchy problem": the main motivation for building the LHC

Addressing the hierarchy problem $\left(\left. \delta m_h^2 \right|_{1-loop} \sim - \frac{y_t^2}{8\pi^2} \Lambda_{UV}^2 \right)$ with a new symmetry new global symmetry Extra dimensions Supersymmetry fermion vector scalar $H \to H + \theta$ $\Psi \rightarrow e^{i\theta\gamma_5} \Psi$ $A_{\mu} \to A_{\mu} + \partial \theta$ Ψ massless: A_{μ} massless: H massless: protected by a protected by protected by global symmetry chiral symmetry gauge invariance Ψ <---> H In 5 dimensions: $H=A_5$

(The Minimal Supersymmetric Standard Model (MSSM)

Supersymmetry can solve the "big" hierarchy and naturalness is preserved up to very high scales if superparticle masses are at the weak scale

$$\Delta(m_{h^0}^2) = \stackrel{h^0}{-} - \stackrel{t}{-} \stackrel{h^0}{-} - \stackrel{t}{-} \stackrel{h^0}{-} \stackrel{t}{-} \stackrel{t$$

$$\delta m_H^2 \sim -\frac{3 h_t^2}{8 \pi^2} m_{\widetilde{t}}^2 \log \frac{\Lambda^2}{m_{\widetilde{t}}^2}$$

An elegant solution to the hierarchy pb: Supersymmetry

2 categories of particles: Fermions

matter particles

fermions repel each other





[Enrico Fermi 1901-1954]

Bosons

force carriers

bosons can pile up





[Satyendra Bose 1894-1974]



String Theory

unnification of the Standard Model forces with gravity



Extra Dimensions

String theories are (well) defined only in spacetime with 10 or 11 dimensions These extra dimensions are assumed to be curled up







Highly non-trivial cancellation and suggestive connection between quarks and leptons

The Standard Model as a remnant of a Grand Unified Theory ?

There are gauge groups for which the anomalies automatically cancel, e.g. SO(10)

Good reason for unification II : Charge quantization $Q_e = T_3 + Y$

How come is the electric charge quantized?

- Eigen values of the generators of the abelian U(1) are continuous e.g. in the symmetry of translational invariance of time, there is no restriction in the (energy) eigen values.
- Eigen values of the generators of a simple non-abelian group are discrete

e.g. in SO(3) rotations, the eigen values of the third component of angular momentum can take only integers or 1/2 integers values. In SU(5), since the electric charge is one of the generators, its eigen values are discrete and hence quantized.

simple unification group -> charge quantization

 $SU(3)_c x SU(2)_L x U(1)_Y \subset SU(5)$

SM matter content fits nicely into SU(5) relation between color SU(3) and electric charge.

Quarks carry 1/3 of the lepton charge because they have 3 colors. The SU(5) theory provides a rationale basis for understanding particle charges and the weak hypercharge assignment in the SM Evolution of coupling constants



the forces depend on distances

the charges depend on distances

The electromagnetic coupling decreases at large distances. Charge screening (vacuum polarization) due to virtual fermion-antifermion pairs

The vacuum behaves as a polarized dielectric medium



virtual particles



An opposite effect for the strong coupling

because of the non-abelian nature of the underlying SU(3) gauge symmetry: the gauge boson self-interactions generate an anti-screening effect through gauge boson loops. This effect is larger than the one from fermion loops --> the strong coupling decreases at short distances

 $\frac{\partial \alpha_s}{\partial \log E} = \beta(\alpha_s) = \frac{\alpha_s^2}{\pi} \left(-\frac{11N_c}{6} + \frac{N_f}{3} \right) \qquad \alpha_s \mathcal{T} \quad \text{when } d\mathcal{T}$ $\alpha_s = g_s^2 / 4\pi \qquad \text{property of 'asymptotic freedom'}$

quarks behave as free particles when the energy becomes very large

Evolution of gauge couplings

The evolution of gauge couplings is controlled by the renormalization group equations $\frac{d\alpha(\mu)}{d\log\mu} \equiv \beta(\alpha(\mu))$

At one loop:

$$\beta(\alpha) \equiv \frac{d\alpha(\mu)}{d\log\mu} = \frac{-b}{2\pi} \ \alpha^2 + \mathcal{O}(\alpha^3)$$

So couplings vary logarithmically as a function of the energy scale:

$$\frac{1}{\alpha_{i}(\mu)} = \frac{1}{\alpha_{i}(\mu_{0})} + \frac{b_{i}}{2\pi} \log \frac{\mu}{\mu_{0}}$$
$$\alpha_{i} = g_{i}^{2}/4\pi \qquad i = SU(3), SU(2), U(1)$$

$$D_i$$
 : defined by the particle content

we observe different couplings but it looks like a low energy artefact



Good reason for unification

$$\mathbf{O}$$
 SU(3)_cxSU(2)_LxU(1)_y \subset SU(5)

SM matter content fits nicely into SU(5)

SU(5) adjoint rep.

$$\left(\begin{array}{c|c} SU(2) \\ \hline \\ SU(3) \end{array}\right)$$

$$\nu_e)_L^T, \qquad T^{12} = \sqrt{\frac{3}{5}} \begin{pmatrix} 1/2 & | & & \\ & 1/2 & & \\ & & -1/3 & \\ & & & -1/3 \end{pmatrix} = \sqrt{\frac{3}{5}} Y$$

additional U(1) factor that
commutes with SU(3)*SU(2)
$$Tr(T^{a}T^{b}) = \frac{1}{2}\delta^{ab}$$
$$g_{5}T^{12} = g'Y \qquad g_{5}\sqrt{\frac{3}{5}} = g' \qquad g_{5} = g = g_{s}$$
$$\overbrace{\sin^{2}\theta_{W} = \frac{3}{8} \text{ @ M_{GUT}}}^{\text{Subsect}} 21$$

$$Q_{L} = \begin{pmatrix} u_{L} \\ d_{L} \end{pmatrix} = (3,2)_{1/6}, \quad u_{R}^{c} = (\bar{3},1)_{-2/3}, \quad d_{R}^{c} = (\bar{3},1)_{1/3},$$

$$L = \begin{pmatrix} \nu_{L} \\ e_{L} \end{pmatrix} = (1,2)_{-1/2}, \quad e_{R}^{c} = (1,1)_{1}$$

$$\bar{5} = (1,2)_{-\frac{1}{2}}\sqrt{\frac{3}{5}} + (\bar{3},1)_{\frac{1}{3}}\sqrt{\frac{3}{5}},$$

$$\bar{5} = L + d_{R}^{c}$$

$$10 = (5 \times 5)_{A} = (\bar{3},1)_{-\frac{2}{3}}\sqrt{\frac{3}{5}} + (3,2)_{\frac{1}{6}}\sqrt{\frac{3}{5}} + (1,1)_{\sqrt{\frac{3}{5}}},$$

$$10 = u_{R}^{c} + Q_{L} + e_{R}^{c}$$

$$w_{ab} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u^{c1} & -u^{c2} \\ -u^{c1} & 0 & u^{c1} \\ u^{c2} & -u^{c1} & 0 \\ \end{pmatrix},$$

$$b_{b} = \frac{1}{\sqrt{2}} \begin{bmatrix} -u^{-1} & 0 & u^{-1} & -u_{2} & -u_{2} \\ u^{-2} & -u^{-1} & 0 & -u_{3} & -u_{3} \\ \hline u_{1} & u_{2} & u_{3} & 0 & -e^{+} \\ d_{1} & d_{2} & d_{3} & e^{+} & 0 \end{bmatrix}$$

Standard Model beta functions

$$b = \frac{11}{3}T_{2}(\text{spin-1}) - \frac{2}{3}T_{2}(\text{chiral spin-1/2}) - \frac{1}{3}T_{2}(\text{complex spin-0})$$

$$Tr(T^{a}(R)T^{b}(R)) = T_{2}(R)\delta^{ab} \quad T_{2}(\text{find}) = \frac{1}{2} \quad T_{2}(\text{adj}) = N$$
universal contribution coming from complete SU(5) representations (4N_F/3 in SM in 4N_F/3 *3/2 in susy)
bosons
So in the SM:

$$b_{3} = \frac{11}{3} \times N_{c} - \frac{2}{3} \times N_{f} \left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1 + \frac{1}{2} \times 1\right) = 7$$

$$b_{2} = \frac{11}{3} \times 2 - \left(\frac{2}{3} \times N_{f} \left(\frac{1}{2} \times 3 + \frac{1}{2} \times 1\right)\right) - \frac{1}{3} \times \frac{1}{2} = \frac{19}{6}$$

$$b_{Y} = -\frac{\frac{12}{3} \times N_{f} \left((\frac{1}{6})^{2} \times 2 \times N_{c} + (\frac{-2}{3})^{2} \times N_{c} + (\frac{1}{3})^{2} \times N_{c} + (\frac{-1}{2})^{2} \times 2 + (1)^{2}\right)}{-\frac{1}{3}(\frac{1}{2})^{2} \times 2 = -\frac{41}{6}} \rightarrow b_{1} = b_{Y} \times \frac{3}{5} = -\frac{41}{10}$$

Only the Higgs and the SM gauge bosons can affect the relative running

In the MSSM, extra contributions from the higgsinos and gauginos

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Comparison

1-loop evolution of gauge couplings



Proton decay

Baryon number is violated via the exchange of GUT gauge bosons with GUT scale mass resulting in effective interactions suppressed by $1/M_{GUT}^2$

The dominant decay mode is $\ p
ightarrow e^+ \pi_0$

The proton lifetime is given by:

$$\overline{\tau(p \to \pi_0 e^+)} \approx \left(\frac{M_{GUT}}{10^{16}}\right)^4 \left(\frac{1/35}{(\alpha_{GUT})}\right)^2 \times 4.4 \times 10^{34} \text{ yr}$$

Experimental constraints lead to: $\tau_p > 5.3 \times 10^{33} \text{ yr}$

i.e
$$M_{GUT} > \left(\frac{\alpha_{GUT}}{1/35}\right)^{1/2} \times 6 \times 10^{15} \text{ GeV}$$





The Dark Matter of the universe

Some invisible transparent matter (that does not interact with photons) which presence is deduced through its gravitational effects



15% baryonic matter (1% in stars, 14% in gas)

85% dark unknown matter

 $\Omega_{\rm DM} = \frac{\text{energy density of the universe stored in dark matter}}{\text{total energy density of the universe}} \sim 25 \%$

 $\Omega_{dark energy} \sim 70 \%$ $\Omega_{SM} = \sim 5 \%$

Dark matter can't be explained by the Standard Model

Matter Forces mediators force III II

baryonic massless

charged/unstable

3 families of matter

contribution to the energy budget of the universe

Particle	Ω	type
Baryons	4 - $5~%$	cold
Neutrinos	$< 2 \ \%$	hot
Dark matter	20 - $26~%$	cold



leptons quarks

The relic abundance of a stable particle follows from the generic thermal freeze-out mechanism in the expanding universe



 $\sigma_{anni} \approx 1 \text{ pb}$ leads to the correct dark matter abundance.

Producing Dark Matter at LHC = "Missing Energy" events













Search for neutrinos in the South Pole



Search for antiprotons in space

In the Mediterranean



Antarès





Search for dark matter photons on Earth





and in space



Fermi

