Fundamental Concepts in Particle Physics

Lecture 3 :
Towards the Standard Model

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Abelian versus non-abelian gauge theories

The (Yang-Mills) action $\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$ is invariant under

$$\Psi(x) \rightarrow U(x)\Psi(x)$$

Abelian U(1) symmetry

$$U(x) = e^{iq\theta(x)}$$

$$A_\mu(x) \rightarrow A_\mu + \frac{i}{e}(\partial_\mu U)U^\dagger$$

Non-abelian SU(N)

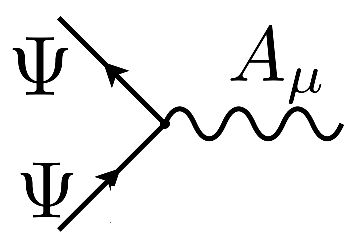
$$U(x) = e^{ig\theta^a(x)T^a}$$

T^a : N^2-1 generators (N×N matrices) acting on

$$A_\mu(x) = A_\mu^a T^a$$

$$A_\mu(x) \rightarrow U A_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

coupling constants



infinitesimal transformation $U(x) = 1 + ig\theta^a(x)T^a + \mathcal{O}(\theta^2)$

$$A_\mu^a(x) \longrightarrow A_\mu^a + \partial_\mu\theta^a - g f^{abc}\theta^b A_\mu^c$$

$$D_\mu\Psi = (\partial_\mu + iqA_\mu)$$

$$D_\mu\Psi = (\partial_\mu - igA_\mu^a T^a)$$

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi_R = \left(\partial_\mu + i\, g'\, Y\, B_\mu\right)\psi_R$$

Gauge Group $SU(2)_L$

acts on the two components of a doublet $\Psi_L=(u_L,d_L)$ or (ν_L, e_L)

$$\Psi_L \to e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a} \qquad T^a = \sigma^a/2$$

Pauli matrices

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1,\ldots,3$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = -i\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$D_\mu \psi_L = \left(\partial_\mu - i\, g\, W^a_\mu T^a\right)\psi_L$$

Gauge Group $SU(3)_c$

$q=(q_1,q_2,q_3)$ (the three color degrees of freedom)

$$q \to e^{-iT^a\alpha^a} q \qquad U = e^{-iT^a\alpha^a} \qquad \left[T^a, T^b\right] = i f^{abc} T^c$$

(3×3) Gell-Man matrices

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a = 1,\ldots,8$$

$$D_\mu q = \left(\partial_\mu - i\, g\, G^a_\mu T^a\right) q$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The gauge symmetries of the Standard Model

Gauge Group $U(1)_Y$ (abelian)

$$\psi' = e^{+iY\alpha_Y}\psi,$$

$$B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \alpha_Y$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu \psi_R = (\partial_\mu + i\, g'\, Y\, B_\mu)\, \psi_R$$

$$\mathcal{L}_{YM} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{2}F_{\mu\nu}F^{\mu\nu}$$

all Standard Model fermions carry U(1) charge

Gauge Group $SU(2)_L$

$$\Psi_L \to e^{-iT^a\alpha^a}\psi_L \qquad U = e^{-iT^a\alpha^a}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc}W^b_\mu W^c_\nu, \quad a = 1,\ldots,3$$

$$D_\mu \psi_L = (\partial_\mu - i\, g\, W^a_\mu T^a)\, \psi_L$$

Ψ_L=(u_L,d_L) or (ν_L,e_L)

only left-handed fermions charged under it -> chiral interactions

Gauge Group $SU(3)_c$

$$q \to e^{-iT^a\alpha^a} q \qquad U = e^{-iT^a\alpha^a}$$

$$G^a_\mu T^a \to U G^a_\mu T^a U^{-1} - \frac{i}{g}\partial_\mu U U^{-1}$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu, \quad a = 1,\ldots,8$$

$$D_\mu q = (\partial_\mu - i\, g\, G^a_\mu T^a)\, q$$

q=(q_1,q_2,q_3)

all quarks transform under it -> vector-like interactions

The lagrangian of the Standard Model

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

describe massless gauge bosons

$$\mathcal{L}_{\text{Fermion}} = \sum_{\text{quarks}} i\bar{q}\gamma^\mu D_\mu q + \sum_{\psi_L} i\overline{\psi_L}\gamma^\mu D_\mu \psi_L + \sum_{\psi_R} i\overline{\psi_R}\gamma^\mu D_\mu \psi_R$$

describe massless fermions and their interactions with gauge bosons

$$D_\mu \psi_R = \left[\partial_\mu + ig' Y B_\mu\right]\psi_R$$

($\sum_{\psi_L}$): only left-handed fermions

($\sum_{\psi_R}$): all fermions carrying a $U(1)_Y$ charge i.e. all Standard Model fermions

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\Phi)^\dagger D_\mu\Phi + \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2$$

→ gives mass to EW gauge bosons $\frac{1}{2}M_Z^2 Z_\mu Z^\mu + M_W^2 W^+_\mu W^{-\mu}$

$$D_\mu\Phi = \left[\partial_\mu - i\frac{g}{\sqrt{2}}\left(\tau^+ W^+_\mu + \tau^- W^-_\mu\right) - i\frac{g}{2}\tau_3 W^3_\mu + i\frac{g'}{2}B_\mu\right]\Phi$$

: covariant derivative of the Higgs

H charged under $SU(2) \times U(1)_Y$

$$\mathcal{L}_{\text{Yukawa}} = -Y_l\,\overline{L}\,\Phi\,\ell_R - Y_d\,\overline{Q}\,\Phi\,d_R - Y_u\,\overline{Q}\,\widetilde{\Phi}\,u_R + \text{h.c.}$$

→ gives mass to fermions

responsible for electroweak symmetry breaking!

$$SU(3) \times SU(2)_L \times U(1)_Y \longrightarrow SU(3) \times U(1)_{em}$$

8 massless gluons | **3 massive gauge bosons W^+ W^- Z_0** | **8 massless gluons** | **1 massless photon γ**

remaining unbroken symmetry

The W and Z bosons interact with the Higgs medium, the γ doesn't.

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$SU(3)_c$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g f^{abc} G^b_\mu G^c_\nu$$

$SU(2)_L$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu,$$

$U(1)_Y$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

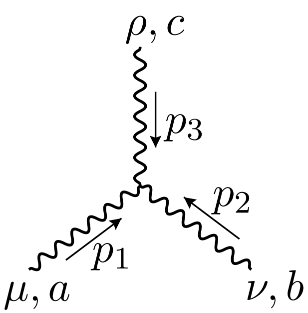
in mass eigen state basis

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}}$$

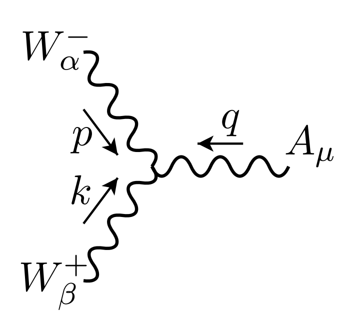
$$Z_\mu = W^3_\mu \cos\theta_W + B_\mu \sin\theta_W$$

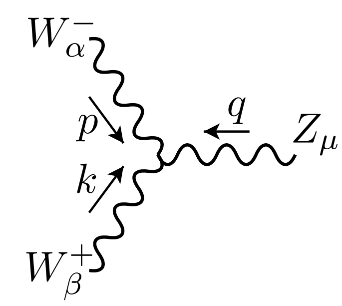
$$A_\mu = -W^3_\mu \sin\theta_W + B_\mu \cos\theta_W$$

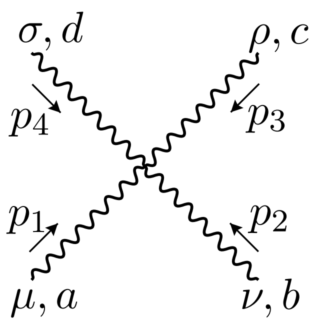
$$\cos\theta_W = g/\sqrt{g^2 + g'^2} \qquad \sin\theta_W = g'/\sqrt{g^2 + g'^2}$$



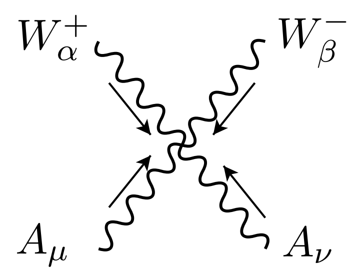
three gauge boson vertex

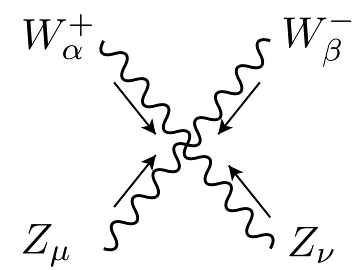


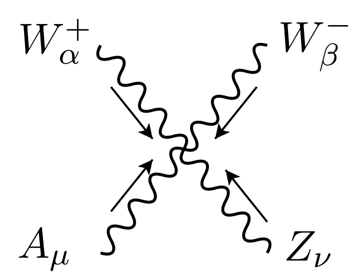


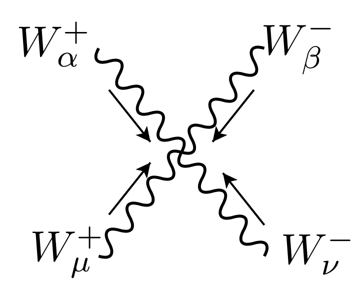


four gauge boson vertex





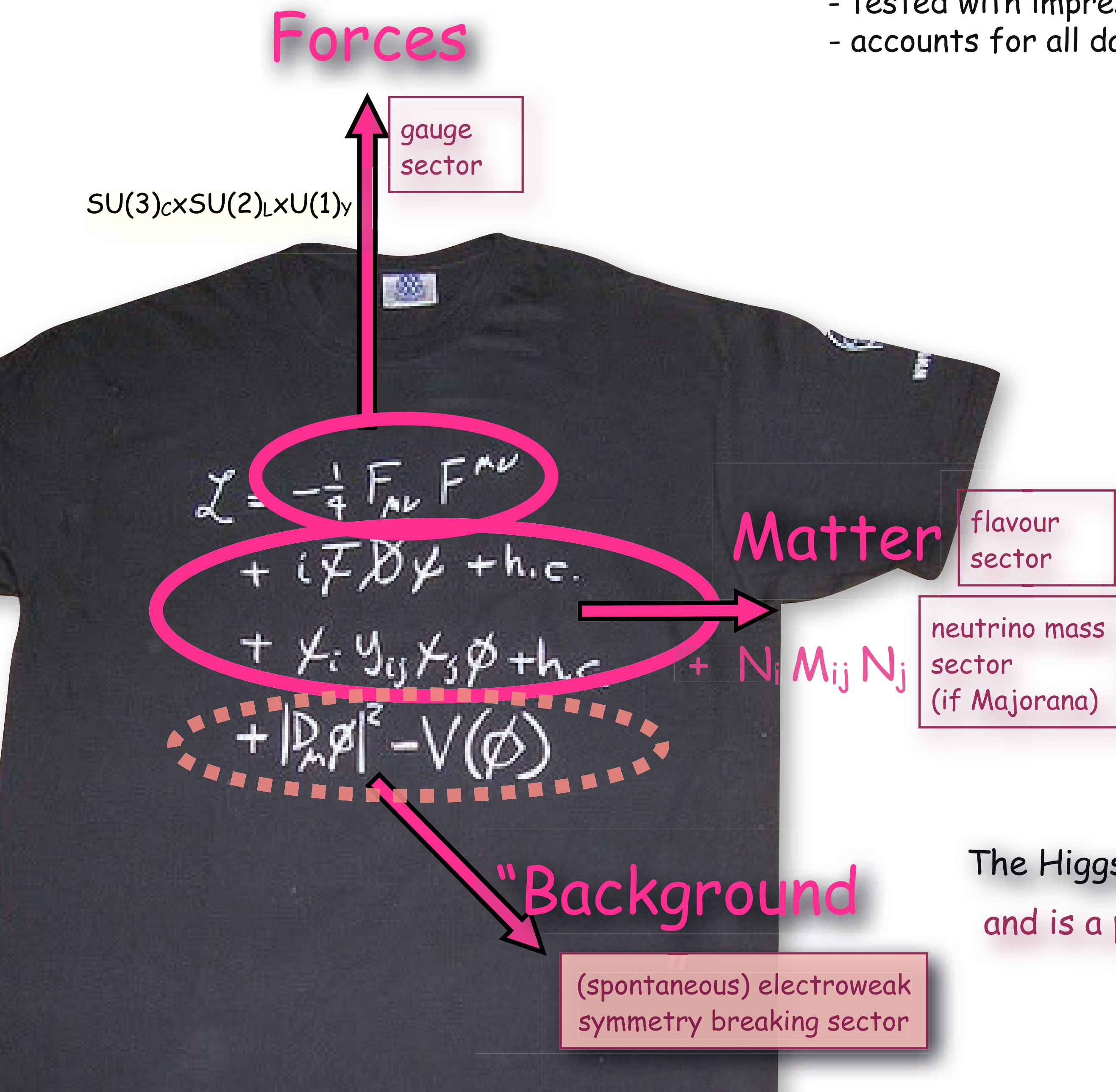


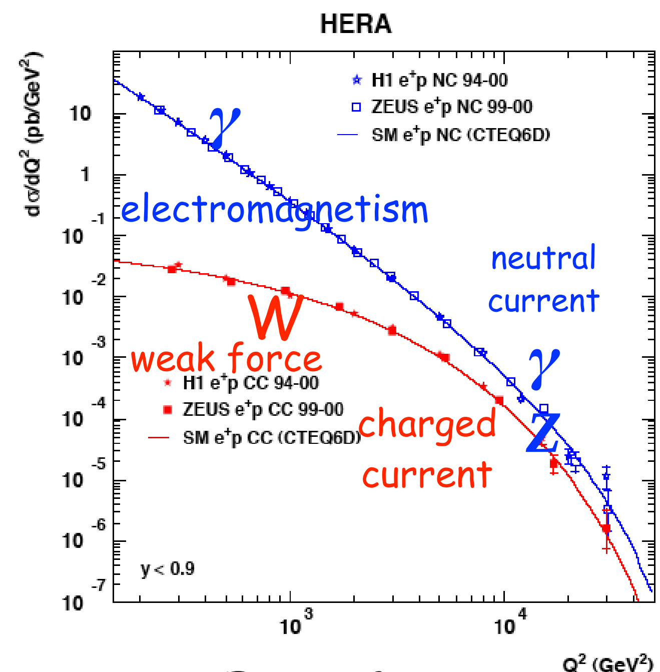


no such interactions for photon!

The Standard Model of Particle Physics

- one century to develop it
- tested with impressive precision
- accounts for all data in experimental particle physics





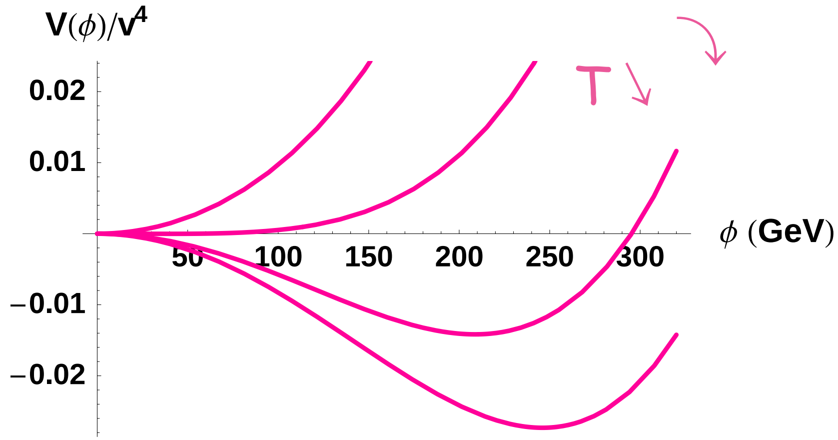
EW unification

The Higgs was the only remaining unobserved piece

and is a portal to new physics hidden sectors

The (adhoc) Higgs Mechanism (a model without dynamics)

EW symmetry breaking is described by the condensation of a scalar field

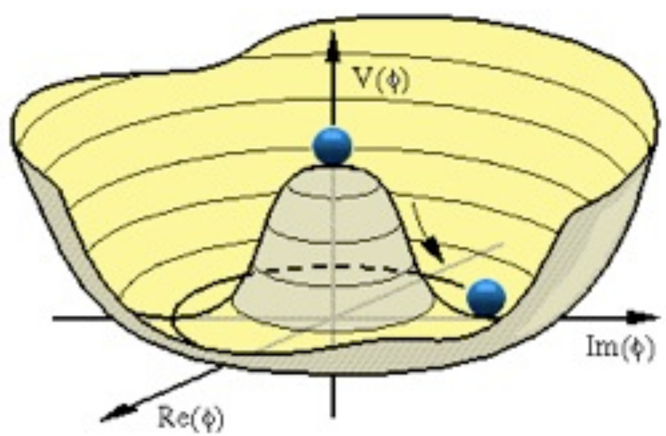


$$\Phi = \begin{bmatrix} \phi^+ \\ \frac{v + H + i\varphi_Z}{\sqrt{2}} \end{bmatrix}$$

Background value, Higgs medium

Higgs boson: excitation of the higgs medium

The Higgs selects a vacuum state by developing a non zero background value. When it does so, it gives mass to SM particles it couples to.



$$V(\Phi) = \frac{\mu^2}{2}\Phi^\dagger\Phi + \frac{\lambda}{4}\Phi^\dagger\Phi$$

Why is μ^2 negative ?

the puzzle:

We do not know what makes the Higgs condensate.
We ARRANGE the Higgs potential so that the Higgs condensates but this is just a parametrization that we are unable to explain dynamically.

Historically

Fermi Theory

(paper rejected by Nature: declared too speculative !)

$$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

$$\mathcal{L} = G_\mathcal{F}(\bar{n}p)(\bar{\nu}_e e)$$

exp: $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$$\mathcal{A} \propto G_\mathcal{F} E^2$$

- no continuous limit
- inconsistent above 300 GeV

Gauge theory

microscopic theory

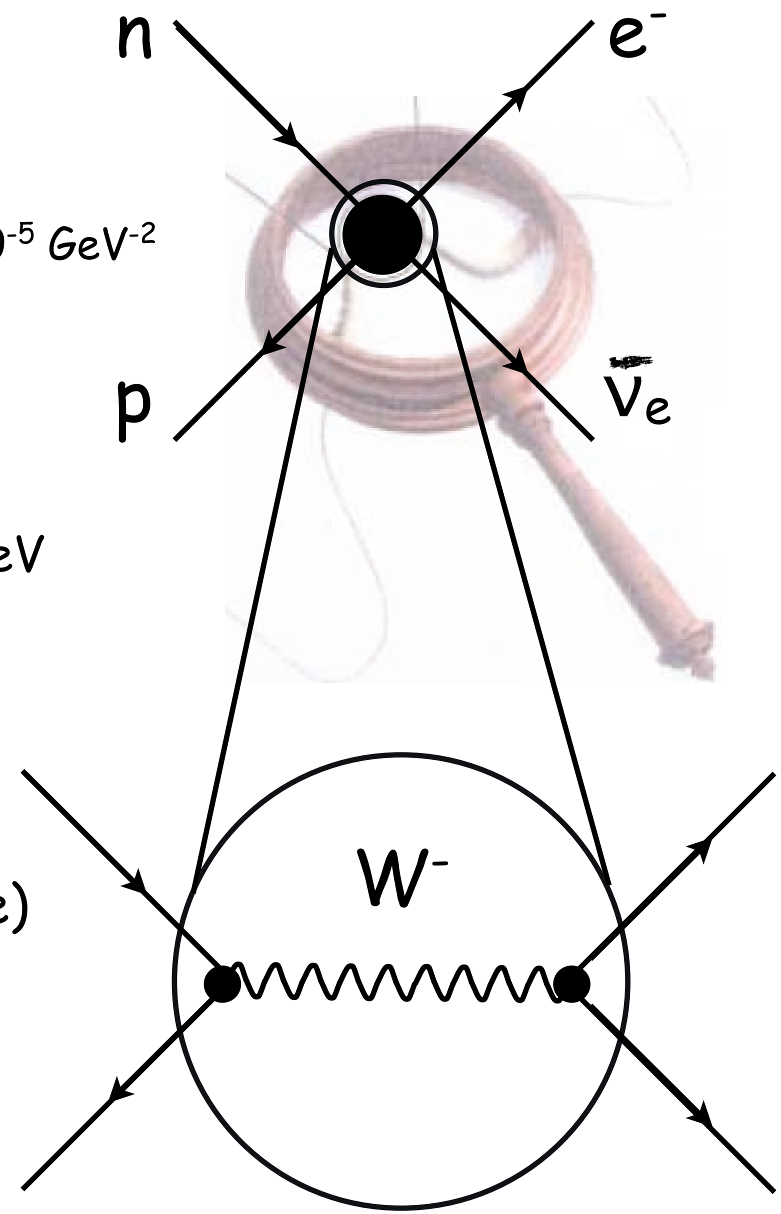
(exchange of a massive spin 1 particle)

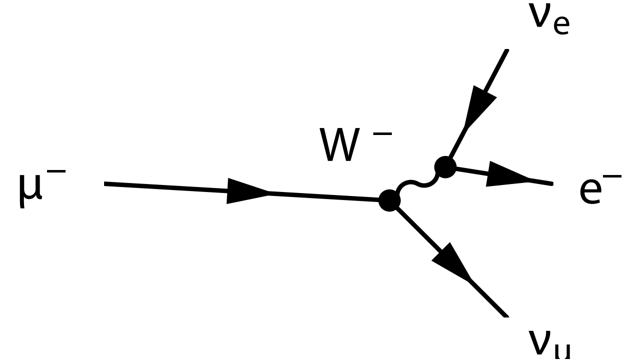
$$G_\mathcal{F} = \frac{\sqrt{2}g^2}{8m_W^2}$$

exp: m_W=80.4 GeV

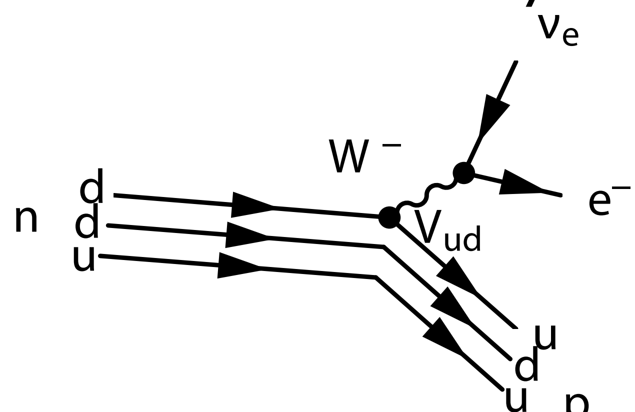
➔ g ≈0.6, ie, same order as e=0.3

unification EM & weak interactions





muon decay



beta decay

- We have quantized free fields

- We have introduced interactions

(particle creation and annihilation can only take place in theory with interactions)

We now would like to compute probability of processes like for instance a two-body decay a->c+d or a two-body reaction a+b->c+d

"S-matrix approach"-> calculate probability of transition between two asymptotic states

The S-matrix

We consider a state |a>(t) which at an initial time t_i is labelled |a>.
Similarly we consider a state |b>(t) which at a final time t_f is labelled |b>

At t_f the state |a>(t) as evolved as $e^{-iH(t_f - t_i)}|a>$

where H is the hamiltonian of the theory

The amplitude for the process in which the initial state |a> evolves into the final state |b> is given by

$$\mathcal{M} = < b|e^{-iH(t_f - t_i)}|a >$$

the final state is a set of well-separated particles long after the interaction

evolution operator "S-matrix"

the initial state is either a one-particle (decay) or two well-separated particles (scattering), long before interaction happens

|a> and |b> are both described by free fields

The probability of the process is given by $|\mathcal{M}|^2$

and that can be linked to a transition rate per volume unit as measured by an experiment

Link to observables

- cross section: reaction rate per target particle per unit incident flux

$$\frac{[1/\text{time}]}{[1/(\text{time length}^2)]}$$

--> has units of a surface

measured in multiples of 1 barn= $10^{-24}\ \mathrm{cm}^2$

typical relevant LHC cross sections ~ in pb

1 picobarn= 1 pb= $10^{-36}\mathrm{cm}^2$

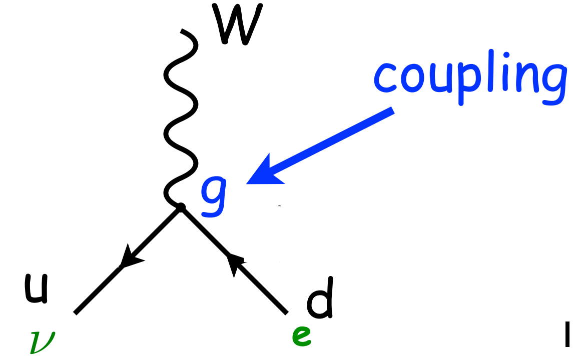
- Decay width (inverse of lifetime of a particle) =transition rate

has dimension [1/time]

Example: decay width of EW gauge bosons

$$\Gamma \propto |\mathcal{M}|^2$$

scales as the square of the coupling constant

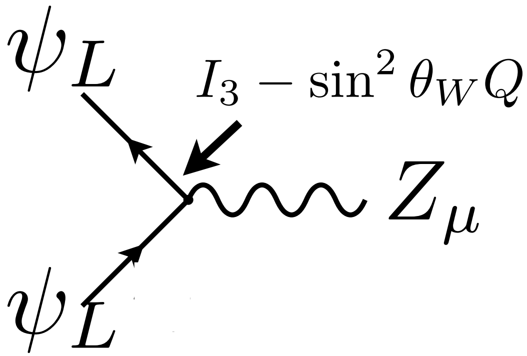


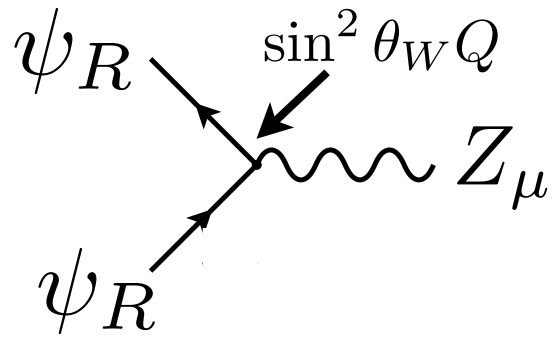
Z couplings to fermions

The coupling of Z to any fermion is proportional to $I_3 - \sin^2\theta_W Q$

where $I_3 = \pm\frac{1}{2}$ is z-component of weak isospin and Q is electric charge

$\sin^2\theta_W = 0.231$





for the quarks:

u_L	$I_3 = +1/2$	$Q = +2/3$
u_R	$I_3 = 0$	$Q = +2/3$
d_L	$I_3 = -1/2$	$Q = -1/3$
d_R	$I_3 = 0$	$Q = -1/3$

and similarly for c,s, and b (t is too heavy for the Z to decay into it)

for the leptons:

e_L	$I_3 = -1/2$	$Q = -1$
e_R	$I_3 = 0$	$Q = -1$
ν_{e_L}	$I_3 = +1/2$	$Q = 0$

and similarly for $\nu, \tau, \nu_\mu, \nu_\tau$

Branching fractions for Z decay

for the quarks:

u_L $I_3 = +1/2$ $Q = +2/3$

u_R $I_3 = 0$ $Q = +2/3$

d_L $I_3 = -1/2$ $Q = -1/3$

d_R $I_3 = 0$ $Q = -1/3$

and similarly for c,s, and b (t is too heavy for the Z to decay into it)

for the leptons:

e_L $I_3 = -1/2$ $Q = -1$

e_R $I_3 = 0$ $Q = -1$

ν_{e_L} $I_3 = +1/2$ $Q = 0$

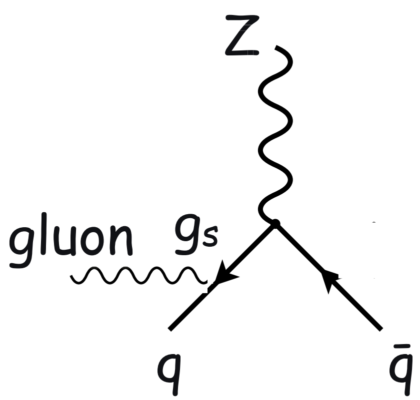
and similarly for $\nu, \tau, \nu_\mu, \nu_\tau$

The decay rate is proportional to the square of the coupling constant $I_3 - \sin^2\theta_W Q$

Also, for quarks, there is an additional factor $(1 + \frac{\alpha_s}{2\pi})$ where $\alpha_s = g_s^2/4\pi = 0.118$ due to the additional gluon emission

$$B(Z \to e^+e^-) = B(Z \to e_L^+e_L^-) + B(Z \to e_R^+e_R^-)$$

$$B(Z \to e_L^+e_L^-) = \frac{\Gamma(Z \to e_L^+e_L^-)}{\sum_{all\ particles}\Gamma(Z \to particle, antiparticle)}$$



$$B(Z \to \nu\bar{\nu}) = B(Z \to \nu_e\bar{\nu}_e) + B(Z \to \nu_\mu\bar{\nu}_\mu + B(Z \to \nu_\tau\bar{\nu}_\tau)$$
$$= 3B(Z \to \nu_e\bar{\nu}_e) = 20\%$$

$$B(Z \to e^+e^-) = B(Z \to \mu^+\mu^-) = B(Z \to \tau^+\tau^-) = 3.33\%$$
$$B(Z \to all\ hadrons) = 3 \times [B(Z \to u\bar{u}) + B(Z \to d\bar{d}) + B(Z \to s\bar{s})$$
$$+B(Z \to c\bar{c}) + B(Z \to b\bar{b})] = 69.9\%$$

Branching fractions for W decay

$$W^- \to e^-\bar{\nu}_e, \mu^-\bar{\nu}_\mu, \tau^-\bar{\nu}_\tau, d'\bar{u}, s'\bar{c}.$$

$$BR(W^- \to e^-\bar{\nu}_e) = BR(W^- \to \mu^-\bar{\nu}_\mu) = BR(W^- \to \tau^-\bar{\nu}_\tau)$$
$$= \frac{1}{3 + 6(1 + \alpha_s/\pi)} = 0.108,$$

$$BR(W^- \to \text{hadrons}) = \frac{6(1 + \alpha_s/\pi)}{3 + 6(1 + \alpha_s/\pi)} = 0.675.$$

The Standard Model

	Q	d	u	L	e	B	W	g	H
$SU(3)_C$	3	3	3	1	1	1	1	8	1
$SU(2)_L$	2	1	1	2	1	1	3	1	2
$U(1)_Y$	+1/6	-1/3	+2/3	-1/2	+1	0	0	0	-1/2
spin	-1/2	+1/2	+1/2	-1/2	+1/2	1	1	1	0
flavor	3	3	3	3	3	1	1	1	1