Fundamental Concepts in high energy/Particle Physics

Lecture I : Introduction

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Plan

Lecture 1: intro to QFT: Relativity, kinematics, symmetries

Lecture 2: Towards Gauge theories

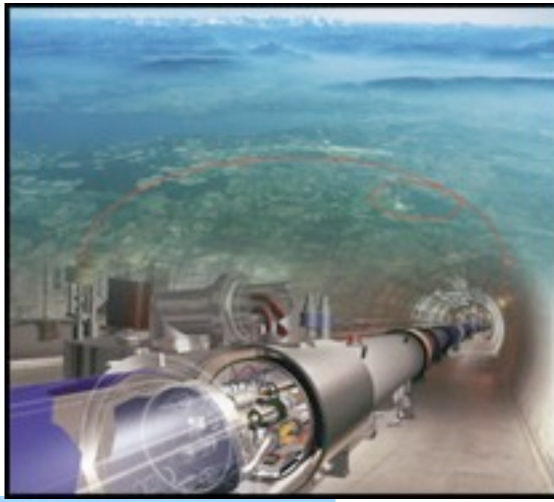
Lecture 3: Towards the Standard Model

Lecture 4: Towards beyond the Standard Model

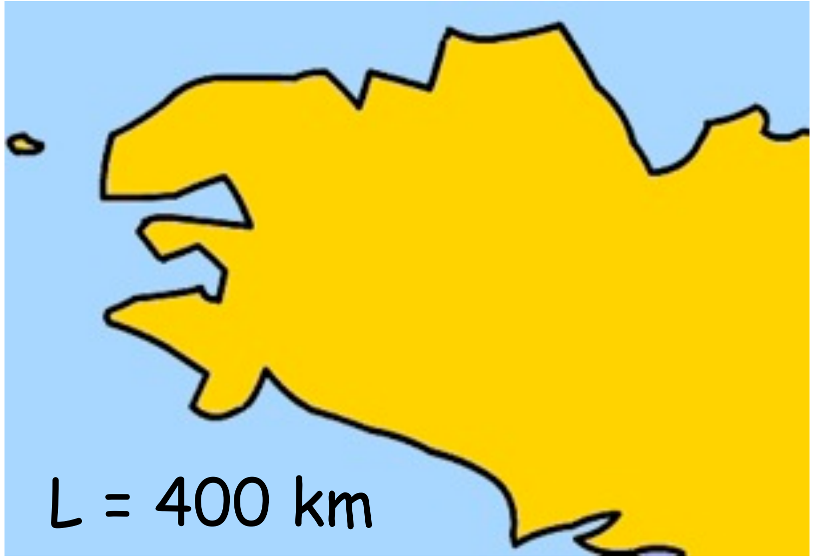
why high energies

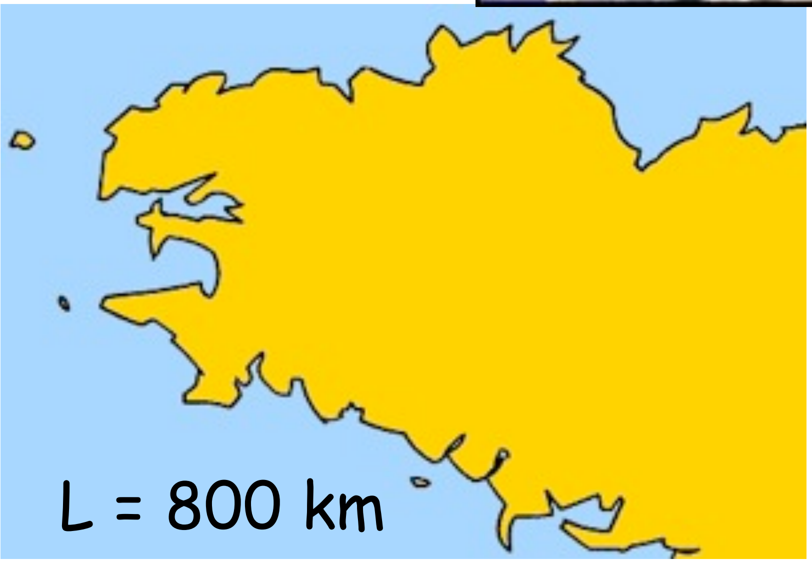
The Large Hadron Collider

➜ The LHC: the most gigantic microscope ever built



Going to higher energies ⇛ allows to study finer details





Particle Physics:
study of short distances

resolution limited by the de Broglie wavelength $\lambda = h/p$

→

High energy physics

high resolution necessitates large p

p>>m:
relativistic regime

What is a particle?

A small, quantum and fast-moving object

Quantum Mechanics

- duality wave-particle
- Heisenberg inequalities

energy non-conservation
on time intervals Δt energy fluctuations ΔE

$$\Delta t \times \Delta E \sim \hbar$$

Special Relativity

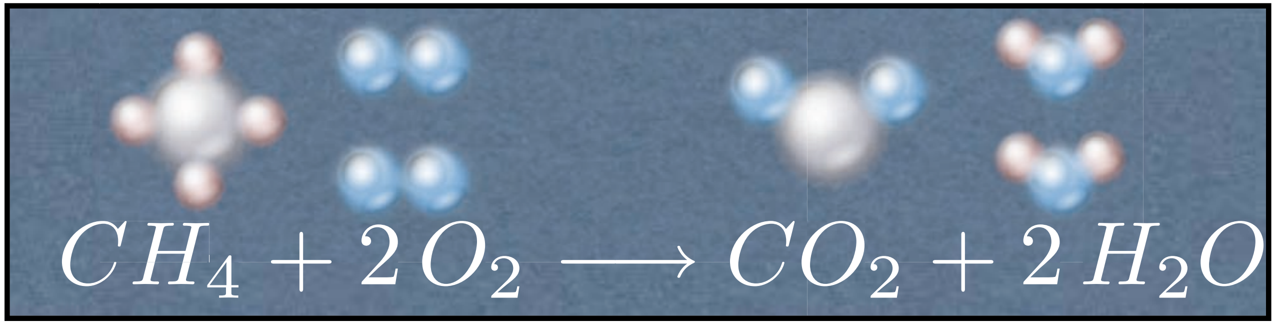
- space-time
- energy = mass

particle number non constant

Creation of matter from energy

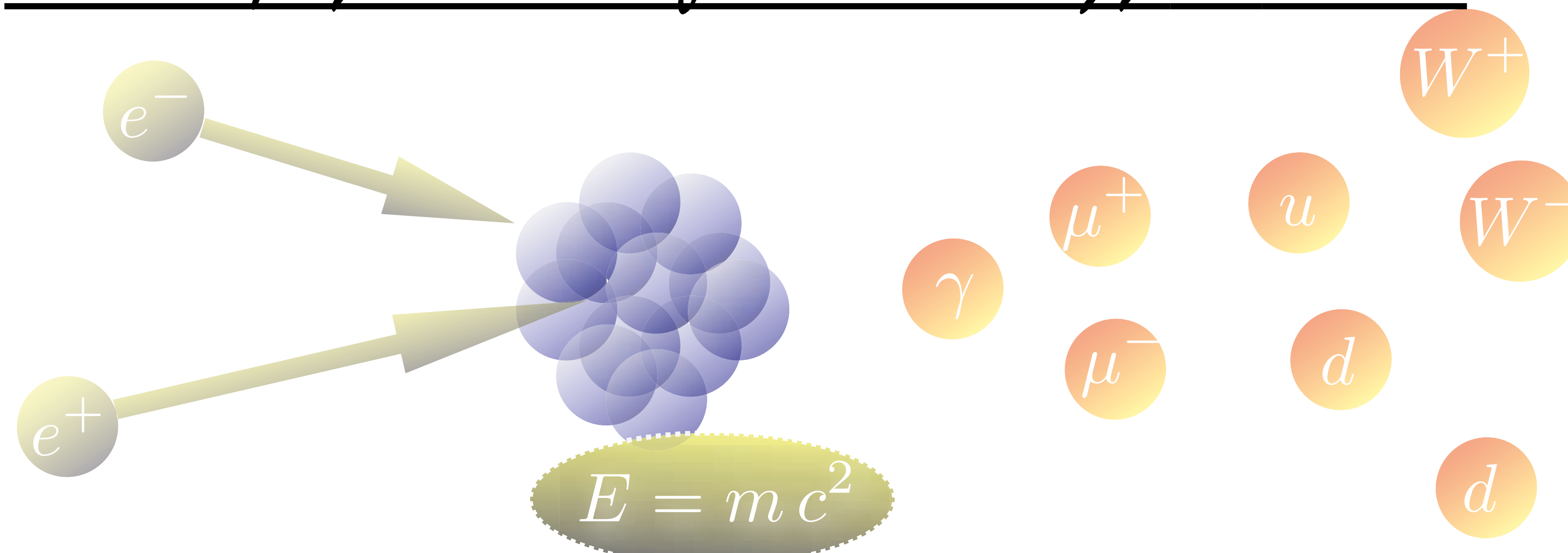
- ***Chemistry : rearrangement of matter***

the different constituents of matter reorganize themselves



$$CH_4 + 2\,O_2 \longrightarrow CO_2 + 2\,H_2O$$

- ***Particle physics : transformation energy ↔ matter***



Equivalence between mass and energy (Einstein's idea) plays a very fundamental role in particle physics

Natural units in high energy physics

The fundamental units have dimension of length (L), mass (M) and time (T).
All other units are derived from these.

The two universal constants in SI units

$\hbar = 1.055\times10^{-34}$ J s = 1.055×10^{-34} kg m^2/s and $c = 3\ 10^8$ m/s

In particle physics we work with units $\hbar = c = 1$

Thus, velocity of particle is measured in units of the speed of light, very natural in particle physics where $0 \leq v < 1$ for massive particles and $v = 1$ for massless particles

In the c=1 unit:
[velocity]=pure number
[energy]=[momentum]=[mass]

notation: dimension of quantity P is [P]

Natural units in high energy physics

$\hbar$ has dimension of [Energy]×[time].

$\hbar$/mc~length (from uncertainty principle $\Delta p \Delta x \geq \hbar/2$)
or de Broglie's formula $\lambda = h/p$

$\hbar = 1$ ---> [length]=[mass]$^{-1}$

Thus all physical quantities can be expressed as powers of mass or of length.

e.g. energy density, $E/L^3 \sim M^4$

$$\alpha = \frac{e^2}{4\pi\hbar c}$$ pure number

We specify one more unit taken as that of the energy, the GeV.

mass unit: $M\ c^2/c^2 = 1$ GeV

length unit: $\hbar c/M\ c^2 = 1\ \mathrm{GeV}^{-1} = 0.1975$ fm

time unit: $\hbar c/M\ c^3 = 1\ \mathrm{GeV}^{-1} = 6.59\ 10^{-25}$s.

Natural units in high energy physics

1 eV = 1.6 10^{-19} J

--> $\hbar$ c = 1.055×10^{-34} J s × 3 10^{8} m/s= 1.978×10^{-7} eV m

Using 1 fm = 10^{-15} m and 1 MeV = 10^{6} eV: $\hbar$ c = 197.8 MeV fm

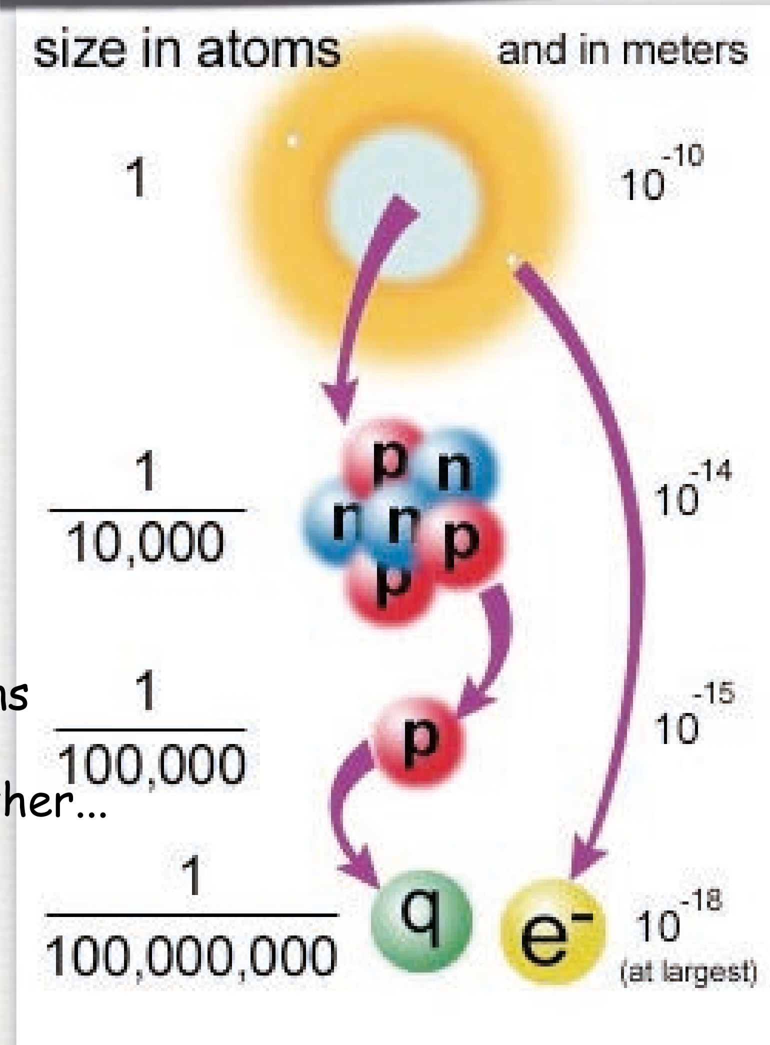
So in natural units: $$1\ \mathrm{fm} \approx 1/(200\ \mathrm{MeV})$$

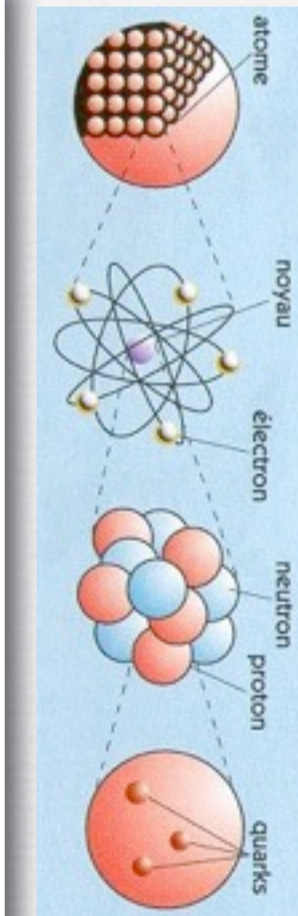
also, c=1 --> 1 fm ~ 3×10^{-24} s^{-1} --> GeV^{-1} ~ 6 × 10^{-25} s^{-1}

$\hbar$= 1.055×10^{-34} kg m^2/s ----> GeV ~ 1.8 × 10^{-27} kg

The elementary blocks of matter

- Matter is made of molecules
- Molecules are built out of atoms
- Atoms are made of nuclei and electrons
- Nuclei are assemblies of protons and neutrons
- Protons and neutrons are quarks bound together...





The volume of an atom corresponds to 10^{24} times the volume of an electron

Classically, matter contains a lot of void

Quantum mechanically, this void is populated by pairs of virtual particles

1 TeV = 10^{12} eV

1 electron volt (eV) = *The energy of an electron accelerated by an electric potential difference of 1 volt. One eV is thus equal to ...* $1.6\,10^{-19}\mathrm{J}$

1 kg of sugar = 4000 kCalories= 17 millions of Joules ≈ 10^{14} TeV

but 1 kg sugar ≈ 10^{27} protons --> 0.1 eV / protons

To accelerate each proton contained in 1 kg of matter at 14 TeV, we would need the energy of of 10^{14} kg of sugar* i.e. 1% of the world energy production

*world annual production of sugar=150 millions of tons≈10^{11} kg

How impressive is this?

energies involved at CERN: 1 TeV = 1000 billions of eV=$10^{-24}\mathrm{kg}$

compared with the kinetic energy of a mosquito $10^{-3}\,J \sim 10^{16}\,\mathrm{eV} \sim 10^{4}\mathrm{TeV}$

...however, in terms of energy density... this corresponds to the mass of the Earth concentrated in a 1 mm^3 cube !

Classical versus quantum Collision

Compton wave length

$\lambda = h/mc$

strawberry : m~30 g ~10^{25} GeV/c^2 ➜ λ~10^{-40} m

classical: $\lambda \ll R$

e^- : m~9.1×10^{-31} kg ~0.5 MeV/c^2 ➜ λ~4×10^{-13} m

p : m~1.6×10^{-27} kg ~1 GeV/c^2 ➜ λ~10^{-16} m

quantum : $\lambda \gg R$

Why relativity

Particle physics is all about creating and annihilating particles.
This can only occur if we can convert mass to energy and vice-versa, which requires relativistic kinematics

A bit of history

Contemplating the unusual invariance of Maxwell's equations under Lorentz transformation, Einstein stated that Lorentz invariance must be the invariance of our space and time.

-> completely changed our view of space and time, so intertwined that it is now called spacetime, leading to exotic phenomena such as

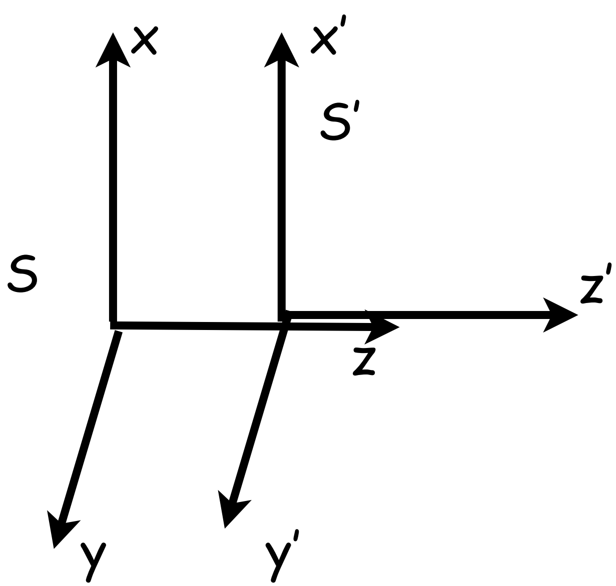
- time dilation
- length contraction
- prediction of antimatter when special relativity is married with quantum mechanics

Relativistic transformations

The two postulates of Special relativity

- Speed of light is the same in all reference frames

- Laws of physics are unchanged under a galilean transformation, i.e. in all reference frames moving at constant velocity with respect to each other



Look for coordinate transformations that satisfy these requirements

Unique choice: Lorentz transformations

$$\beta = \frac{v}{c}$$

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Implications of Lorentz transformation

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix} \qquad \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time dilation

consider time interval $\tau = t'_2 - t'_1$ in S', the rest frame of a particle located at $z'_1 = z'_2 = 0$.

then in frame S where the particle is moving: $t_2 - t_1 = \gamma\tau$

-->The observed lifetime of a particle is $\gamma \times \tau$

so it can travel over a distance $\beta c \gamma \tau$

-->muons which have a lifetime $\tau \sim 2 \times 10^{-6}$ s produced by reaction of cosmic rays with atmosphere at 15-20 km altitude can reach the surface

length contraction

an object at rest in S' has length $L_0 = z'_2 - z'_1$

It measures in S $z_2 - z_1 = L_0/\gamma$

--> densities increase $\rho_0 = \Delta n/(\Delta x' \Delta y' \Delta z')$ $\rho = \Delta n/(\Delta x \Delta y \Delta z) = \gamma\rho_0$

$\Delta x \Delta y \Delta z \Delta t$ is invariant

4-vectors

Time and space get mixed-up under Lorentz transformations. They are considered as different components of a single object, a four-component spacetime vector:

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$$

By construction: $x_\mu x^\mu = \mathbf{x}^2 = {x^0}^2 - \vec{x}^2$ is invariant

$$d\tau = \sqrt{dt^2 - d\vec{x}^2}$$ is invariant

Lorentz invariant action built with the proper time $d\tau$

$$S = -m \int d\tau = \int \mathcal{L} dt$$

$$\mathcal{L} = -m\sqrt{1 - \dot{x}^2}$$

$$\vec{p} = \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\gamma\vec{\beta}$$

$$E = \vec{p}.\dot{x} - \mathcal{L} = m\gamma$$

$$\vec{p} = E\vec{\beta}$$

Energy-momentum four-vector

so we find: $$m^2 = E^2 - \vec{p}^2$$

This suggests to define the four-vector $$p^\mu = (\frac{E}{c}, p_x, p_y, p_z)$$

$$m = 0 \to \beta = 1 \to \gamma = \infty \to \tau = \infty$$

a massless particle cannot decay

Conservation of energy-momentum

Consider collision between A and B

Define center of mass (CM) frame as where $\vec{p}_A + \vec{p}_B = 0$

Energy available in center of mass frame is an invariant: sqrt(s)= $E_* = E_A + E_B$ $\quad \mathbf{p}_{tot}^2 = E_*^2$

1) Collision on fixed target

B is at rest in lab frame, $E_B = m_B$ and E_A is energy of incident particle

$$E_*^2 = m_A^2 + m_B^2 + 2m_B E_A$$

2) Colliding beams

A and B travel in opposite directions

$$E_*^2 = m_A^2 + m_B^2 + 2(E_A E_B + |p_A||p_B|) \approx 4E_A E_B$$

if $m_A, m_B \ll E_A, E_B$

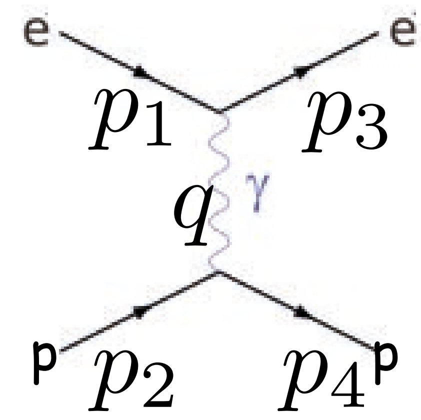
So for fixed target machine $E_* \sim \sqrt{2m_B E_A}$

while for colliding beam accelerators $E_* \sim 2E$

To obtain 2 TeV in the CM with a fixed proton target accelerator the energy of a proton beam would need to be 2000 TeV!

Conservation of energy-momentum

Consider the interaction e + p -> e + p due to exchange of electromagnetic field



a photon is massless, however, for a short amount of time, an "exchanged" photon γ_* (virtual photon) can have a mass (Heisenberg inequalities)

$$p_1 + p_2 = p_3 + p_4$$

$$q = p_1 - p_3 = p_4 - p_2$$

q is the transfer energy-momentum four-vector

$$q^2 = m^2_{\gamma_*}$$: invariant, can be computed in any frame

in frame where proton is at rest: $p_2 = \begin{pmatrix} m \\ \vec{0} \end{pmatrix}$ $p_4 = \begin{pmatrix} E_4 \\ \vec{p_4} \end{pmatrix}$

$$E_4 = m + T$$

kinetic energy

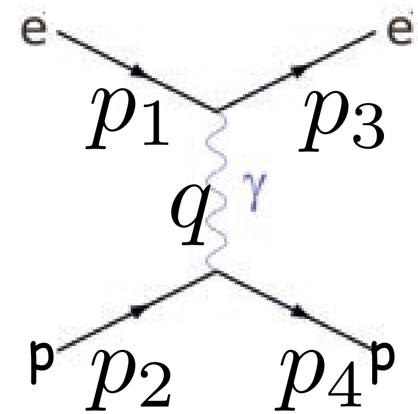
$$T \approx p^2/2m$$

$$q^2 = (p_4 - p_2)^2 = p_4^2 + p_2^2 - 2p_4.p_2$$

$$q^2 = m^2 + m^2 - 2E_4\ m = -2mT$$ < 0 virtual photon

Range of an interaction

$$R = \frac{\hbar c}{|m_*|}$$



reminder: $\hbar c \sim 200 \text{ MeV fm}$

To probe the proton, we need

$$R \ll R_{proton} \sim 1\text{fm}^{-1}$$

$$|m_*| \gg 200 \text{ MeV}$$

Next step : marry quantum mechanics and relativity

non relativistic

(Schrodinger's equation cannot account for creation/annihilation of particles)

Schrödinger Equation (1926): $$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\Delta-V\right)\Phi=0$$

$$E=\frac{p^2}{2m}+V$$

classical ↔ quantum correspondance

$E\to i\hbar\frac{\partial}{\partial t}$ & $p\to i\hbar\frac{\partial}{\partial x}$

relativistic

Klein-Gordon Equation (1927): $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\Delta+\frac{m^2c^2}{\hbar^2}\right)\Phi=0$$

$$\frac{E^2}{c^2}=p^2+m^2c^2$$

negative energies $$E=\pm(p^2+m^2)^{1/2}$$

and

does not admit a positive probability density

and does not describe fermions

Antimatter and Dirac equation

Dirac Equation (1928):

$$\left(i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right)\Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2c^2 + m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2 + m^2c^4} & \text{antimatter} \end{cases}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$$

plane wave solution $\Psi(x,t) = u(p)e^{i(p.x - Et)/\hbar}$

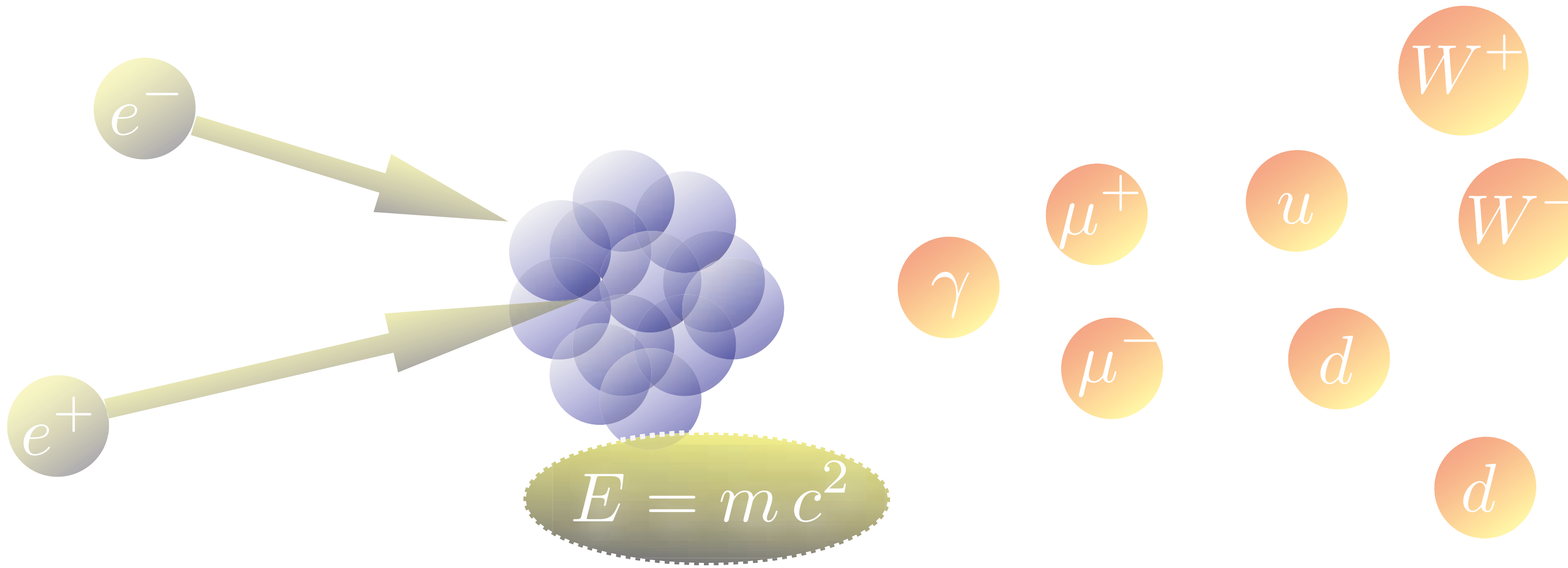
a particle of energy -E travelling backward in time -> antiparticle

positron (e^+) discovered by C. Anderson in 1932

conservation of fermion number: +1 for particles and -1 for antiparticles. fermions can only be created or destroyed in pairs

The necessity to introduce fields for a multiparticle description

Relativistic processes cannot be explained in terms of a single particle. Even if there is not enough energy for creating several particles, they can still exist for a short amount of time because of uncertainty principle



We need a theory that can account for processes in which the number and type of particles changes like in most nuclear and particle reactions

quantization of a single relativistic particles does not work, we need quantization of fields -> Quantum Field Theory (QFT)

Quantum Field Theory

We want to describe $A \to C_1 + C_2$ or $A+B \to C_1 + C_2 + \ldots$

1) Associate a field to a particle

2) Write action $$S = \int d^4x \mathcal{L}(\phi_i, \partial_\mu \phi_i)$$

3) $\mathcal{L}$ invariant under Poincaré (Lorentz+translations) tranformations and internal symmetries

The symmetries of the lagrangian specify the interactions

4) Quantization of the fields

Symmetries and conservation laws: the backbone of particle physics

Noether's theorem (from classical field theory) :
A continuous symmetry of the system <-> a conserved quantity

I- Continuous global space-time symmetries:

translation invariance in space <-> momentum conservation
translation invariance in time <-> energy conservation
rotational invariance <-> angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu \qquad \phi(x) \rightarrow \phi'(x')$$

$$\phi'(x) = \phi(x)$$ scalar

$$V^\mu \rightarrow \Lambda^\mu_\nu V^\nu$$ vector

$$\psi(x) \rightarrow \exp(-\frac{i}{2}\omega_{\mu\nu}J^{\mu\nu})\psi(x)$$ spinor

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)

A field transforms under the Lorentz transformations in a particular way.

Picking a particular representation of the
Lorentz transformation specifies the spin.

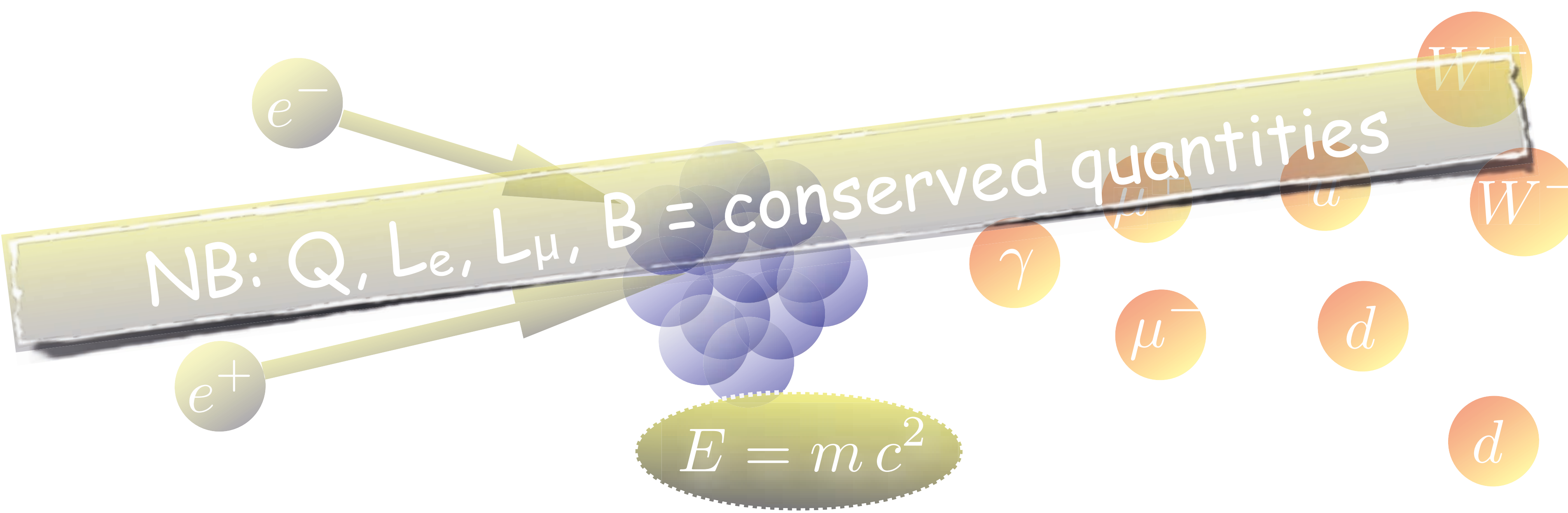
After quantizing the field, you find that the field operator can create or annihilate a particle of definite spin

The spin is part of the field

II- Global (continuous) internal symmetries:

acting only on fields

conservation of baryon number and lepton number



NB: Q, L_e, L_μ, B = conserved quantities

Quantum numbers and Conservation laws

When the positron was discovered, it raised a naive question: why can't a proton decay into a positron and a photon $p \to e^+\gamma$?

This process would conserve momentum, energy, angular momentum, electric charge and even parity

This can be understood if we impose conservation of baryon number

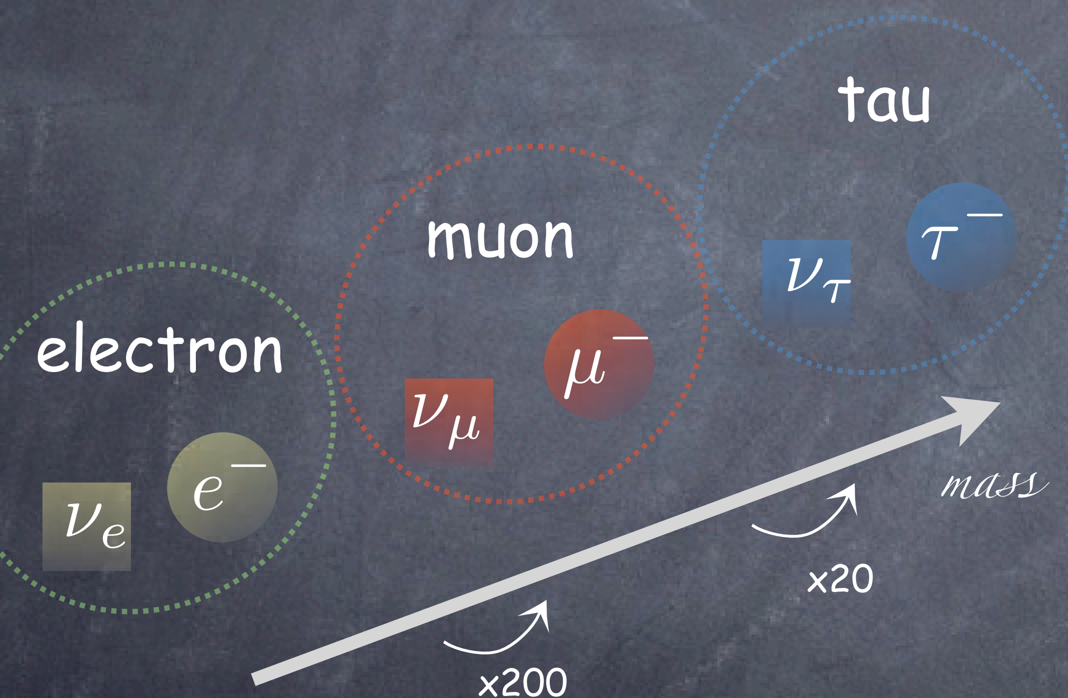
Similarly, when the muon was discovered, it raised the question: why doesn't a muon decay as $\mu^- \to e^-\gamma$?

This led to propose another quantum number: lepton family number

The Standard Model: matter

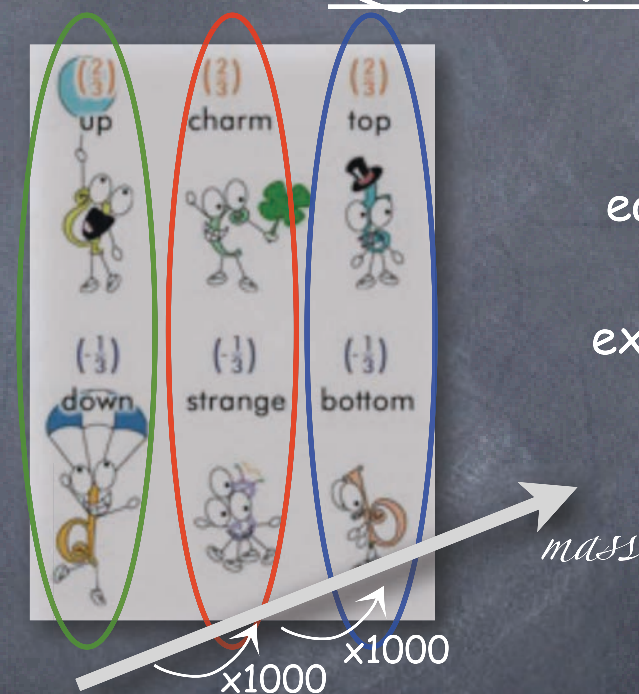
the elementary blocks:

LEPTONS



no composite states made of leptons

QUARKS



each of the 6 quarks exists in three colors

composite states (white objects

- baryons — proton $p = (u, u, d)$, neutron $n = (u, d, d)$
- mesons

+ antiparticles

The following processes have not been seen.
Explain which conservation law forbids each of them

$$n \rightarrow p\mu^-\bar{\nu}_\mu$$

$$\mu^- \rightarrow e^-e^-e^+$$

$$n \rightarrow p\nu_e\bar{\nu}_e$$

$$p \rightarrow e^+\pi^0$$

$$\tau^- \rightarrow \mu\gamma$$

$$K^0 \rightarrow \mu^+e^-$$

$$\mu^- \rightarrow \pi^-\nu_\mu$$

The following processes have not been seen.
Explain which conservation law forbids each of them

$n \rightarrow p\mu^-\bar{\nu}_\mu$	energy
$\mu^- \rightarrow e^-e^-e^+$	muon number or electron number
$n \rightarrow p\nu_e\bar{\nu}_e$	electric charge
$p \rightarrow e^+\pi^0$	baryon number or electron number
$\tau^- \rightarrow \mu\gamma$	tau number or muon number
$K^0 \rightarrow \mu^+e^-$	muon number or electron number
$\mu^- \rightarrow \pi^-\nu_\mu$	energy