Scalar perturbations in braneworld cosmology

Antonio Cardoso
University of Portsmouth

Takashi Hiramatsu, Kazuya Koyama, Sanjeev Seahra
University of Portsmouth, University of Tokyo

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Introduction

- The Randall-Sundrum (RS) braneworld model postulates that our observable universe is a thin 4D hypersurface residing in 5D anti-de Sitter (AdS) space
- The warping of AdS space allows us to recover ordinary general relativity (GR) at distances greater than the curvature radius of the bulk
- The equations of motion governing fluctuations differ from GR at early times: they acquire high-energy corrections similar to those found in the Friedmann equation
- Perturbations on the brane are coupled to fluctuations of the 5D bulk geometry ("Kaluza Klein" (KK) degrees of freedom)
- The only known way of tackling the problem on all scales simultaneously is by direct numerical solution of the equations

Background dynamics

- Bulk metric in Poincare coordinates

$$ds_5^2 = \frac{\ell^2}{z^2}(-d\tau^2 + \delta_{ij}dx^i dx^j + dz^2)$$

- Induced line element on the brane

$$ds_b^2 = a^2(-d\eta^2 + d\mathbf{x}^2) = -dt^2 + a^2 d\mathbf{x}^2 \qquad a(\eta) = \ell/z_b(\eta)$$

- Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1 + \frac{\rho}{2\sigma}\right) \qquad \sigma \gtrsim (\text{TeV})^4$$

- Conservation of stress-energy on the brane

$$\frac{d\rho}{dt} = -3(1+w)\rho H \qquad w = \frac{p}{\rho}$$

Scalar perturbations

Mukohyama, 2000; Kodama, Ishibashi, Seto, 2000

- Wave equation for bulk master variable

$$0 = -\frac{\partial^2 \Omega}{\partial \tau^2} + \frac{\partial^2 \Omega}{\partial z^2} + \frac{3}{z}\frac{\partial \Omega}{\partial z} + \left(\frac{1}{z^2} - k^2\right)\Omega$$

- Boundary condition on the brane

$$\left[\partial_n \Omega + \frac{1}{\ell}\left(1 + \frac{\rho}{\sigma}\right)\Omega + \frac{6\rho a^3}{\sigma k^2}\Delta\right]_{\mathrm{b}} = 0$$

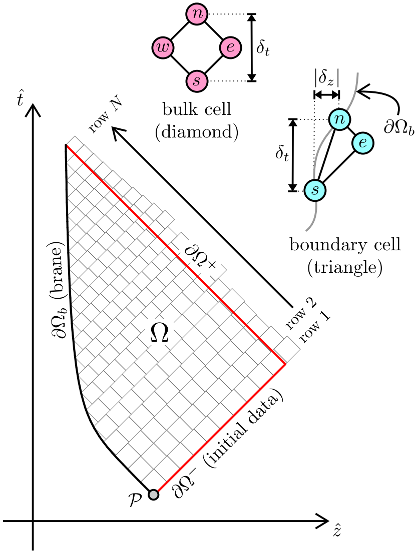
- Wave equation for density contrast on the brane

$$\frac{d^2\Delta}{d\eta^2} + (1 + 3c_s^2 - 6w)Ha\frac{d\Delta}{d\eta} + \left[c_s^2 k^2 + \frac{3\rho a^2}{\sigma \ell^2}A + \frac{3\rho^2 a^2}{\sigma^2 \ell^2}B\right]\Delta = -\frac{k^2 \Gamma}{\rho} + \frac{k^4(1+w)\Omega_{\mathrm{b}}}{3\ell a^3}$$

$$A = 6c_s^2 - 1 - 8w + 3w^2 \qquad B = 3c_s^2 - 9w - 4$$

Integration algorithm

Cardoso, Koyama, Mennim, Seahra, Wands, 2006



- Radiation dominated

$$w = c_s^2 = 1/3$$

- Dimensionless parameters

$$k = H_* a_* \qquad \hat{a} = \frac{a}{a_*}$$

- Critical epoch

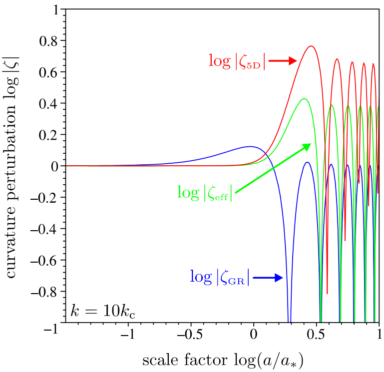
$$\hat{H}_c = H_c \ell = 1 \qquad k_c = H_c a_c$$

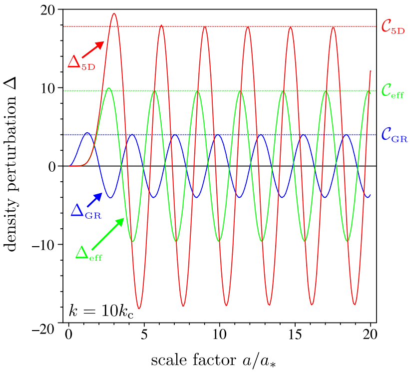
Enhancement factors and transfer functions

- Primordial value of curvature perturbation fixed by inflation

$$\zeta_{5D} \approx \zeta_{eff} \approx \zeta_{GR} \approx 1 \qquad a \ll a_*$$

$\zeta_{5D} \to$ simulation $\qquad \zeta_{eff} \to \mathcal{O}(\rho/\sigma)$ corrections $\qquad \zeta_{GR} \to$ GR





Enhancement factors and transfer functions

- Enhancement factors

$$\mathcal{Q}_{\text{eff}}(k) = \frac{\mathcal{C}_{\text{eff}}(k)}{\mathcal{C}_{\text{GR}}(k)} \qquad \mathcal{Q}_{\mathcal{E}}(k) = \frac{\mathcal{C}_{\text{5D}}(k)}{\mathcal{C}_{\text{eff}}(k)} \qquad \mathcal{Q}_{\text{5D}}(k) = \frac{\mathcal{C}_{\text{5D}}(k)}{\mathcal{C}_{\text{GR}}(k)}$$

- Transfer functions

$$T(k;\eta) = \frac{9}{4}\left[\frac{k}{H(\eta)a(\eta)}\right]^{-2}\frac{\Delta_k(\eta)}{\zeta_k^{\text{inf}}} \qquad T(k;\eta) \xrightarrow[0]{k} 1$$

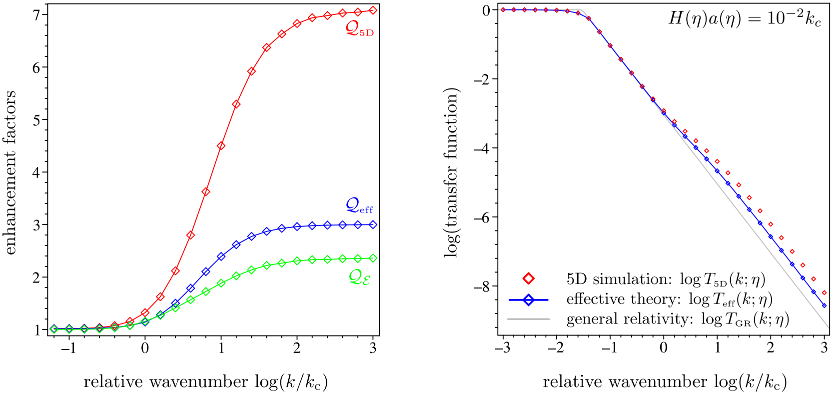
- We recover GR in the large scale limit

$$\Delta_k(\eta) \xrightarrow[0]{k} \frac{4}{9}\left[\frac{k}{H(\eta)a(\eta)}\right]^{2}\zeta_k^{\text{inf}} \qquad (a > a_c)$$

Enhancement factors and transfer functions

- Critical scale

$$\frac{a_0}{k_c} = 1.4\times10^{12}\left(\frac{\ell}{0.1\,\mathrm{mm}}\right)^{1/2}\left(\frac{g_c}{100}\right)^{1/12}\mathrm{m} \sim 10\,\mathrm{AU},\ \ell = 0.1\,\mathrm{mm}$$



Conclusions

- Amplitude of modes which enter the Hubble horizon during the high-energy regime gets enhanced over the standard GR result
- Corrections to background dynamics and influence of the KK modes give roughly equal contributions to the enhancement
- All tangible effects from the fifth dimension are on scales smaller than a critical value
- Not relevant to present-day/cosmic microwave background measurements of the matter power spectrum
- May have an important bearing on the formation of compact objects such as primordial black holes at very high energies