Probing the electroweak scale (and beyond) with gravity waves

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References:

Grojean-Servant hep-ph/0607107, PRD

Randall-Servant hep-ph/0607158, JHEP

Caprini-Durrer-Servant astro-ph/0711.2593, PRD + one more in prep.

Gravitational Waves: A way to probe astrophysics
... and high energy particle physics.

Gravitational Waves interact very weakly and are not absorbed

direct probe of physical process of the very early universe

Small perturbations in FRW metric:

$$ds^2 = a^2(\eta)(d\eta^2 - (\delta_{ij} + 2h_{ij})dx^i dx^j) \qquad\qquad G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\ddot{h}_{ij}(\mathbf{k},\eta) + \frac{2}{\eta}\dot{h}_{ij}(\mathbf{k},\eta) + k^2 h_{ij}(\mathbf{k},\eta) = 8\pi G a^2(\eta)\Pi_{ij}(\mathbf{k},\eta)$$

Source of GW:
anisotropic stress

possible cosmological sources:

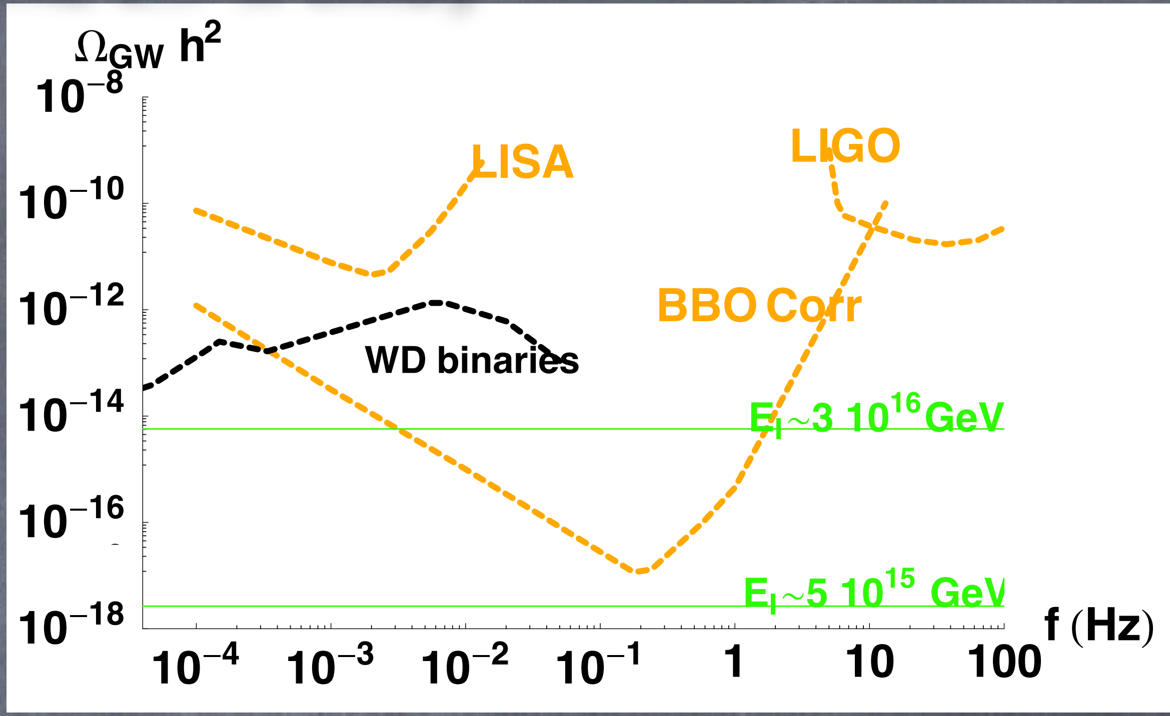
inflation, vibrations of topological defects, excitations of xdim modes, 1st order phase transitions...

frequency observed today:

$$f = f_* \frac{a_*}{a_0} = f_* \left(\frac{g_{s0}}{g_{s*}}\right)^{1/3} \frac{T_0}{T_*} \approx 6 \times 10^{-3}\mathrm{mHz} \left(\frac{g_*}{100}\right)^{1/6} \frac{T_*}{100\ \mathrm{GeV}} \frac{f_*}{H_*}$$

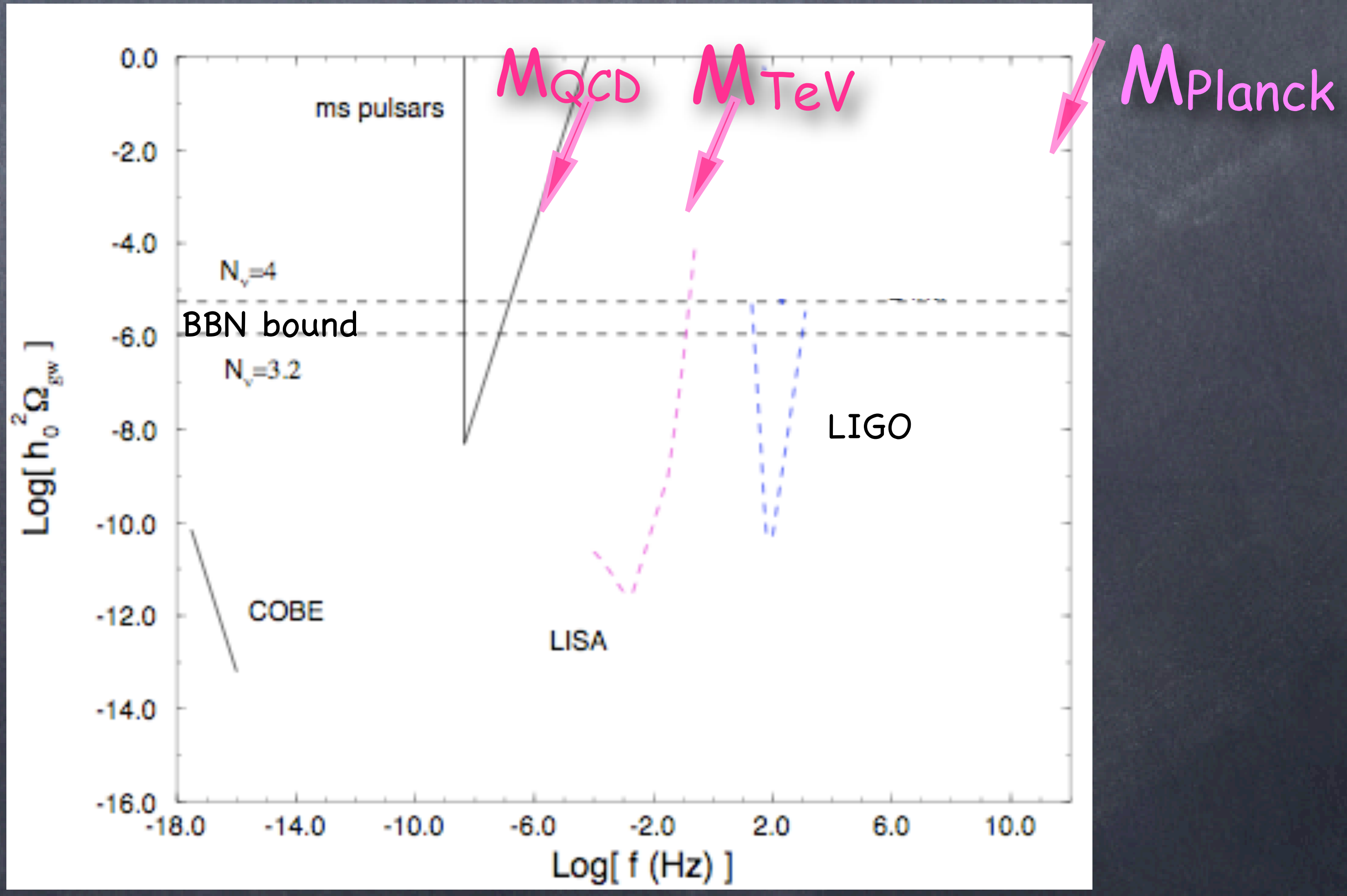
Beyond GW of astrophysical origin, another mission of GW astronomy will be to search for a stochastic background of gravitational waves of primordial origin (gravitational analog of the 2.7 K CMB)

Stochastic background: isotropic, unpolarized, stationary



GW energy density: $\Omega_G = \dfrac{\langle \dot{h}_{ij}\dot{h}^{ij}\rangle}{G\rho_c} = \displaystyle\int \frac{dk}{k}\frac{d\Omega_G(k)}{d\log(k)}$

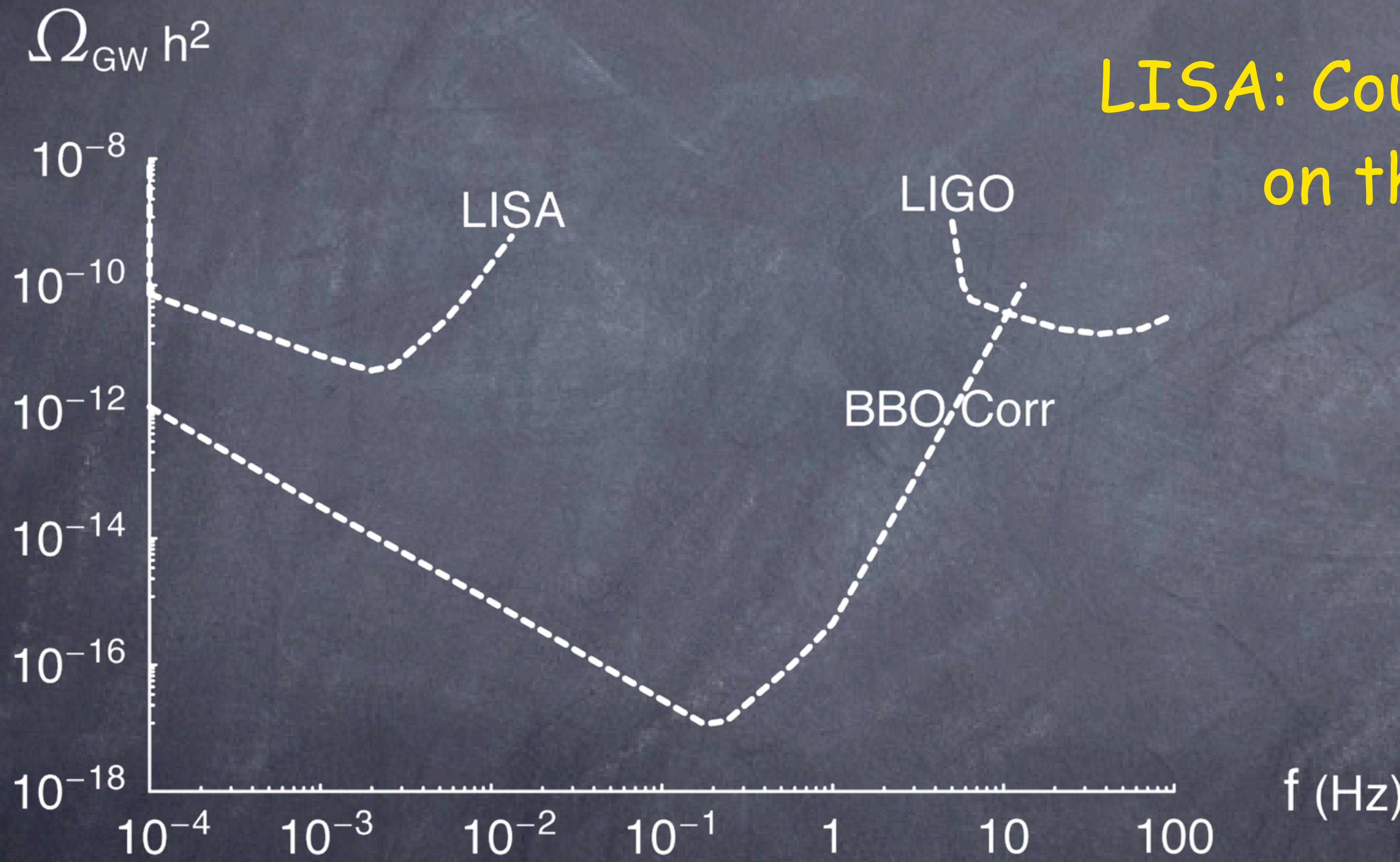
A huge range of frequencies



from Maggiore

Why should we be excited about mHZ freq.?

$$f = f_* \frac{a_*}{a_0} = f_* \left(\frac{g_{s0}}{g_{s*}}\right)^{1/3} \frac{T_0}{T_*} \approx 6 \times 10^{-3} \mathrm{mHz} \left(\frac{g_*}{100}\right)^{1/6} \frac{T_*}{100\ \mathrm{GeV}} \frac{f_*}{H_*}$$



LISA: Could be a new window on the Weak Scale

LISA band:

$10^{-4} - 10^{-2}$ Hz

complementary to collider informations

Which weak scale physics?

⇛ Strong first order phase transitions

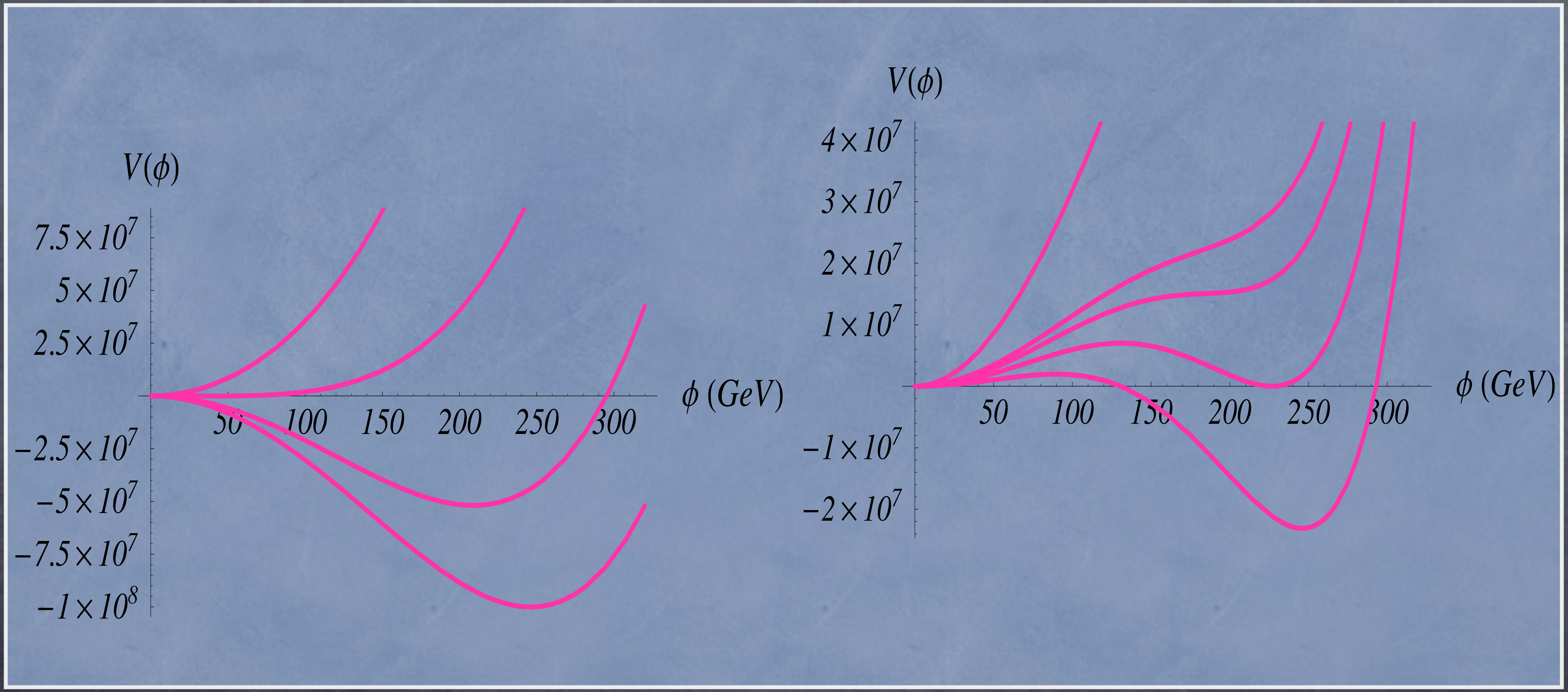
Examples:

⇛ "Standard" Electroweak Phase Transition

- test of the dynamics of the phase transition
  (quite important to analyze models of EW baryogenesis)
- reconstruction of the Higgs potential / study of new models of EW symmetry breaking (little Higgs, gauge-Higgs, composite Higgs, Higgsless...)

⇛ "3-brane" nucleation in Randall-Sundrum models

2nd order versus 1rst order



Is the electroweak phase transition 1rst or 2nd order?

LHC might answer as it will shed light on the Higgs sector

Question intensively studied within the Minimal Supersymmetric Standard Model (MSSM).

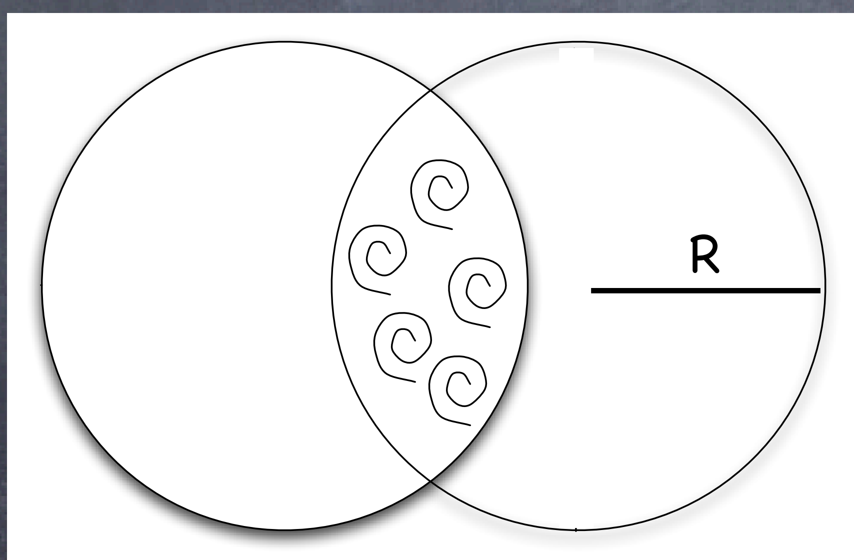
However, not so beyond the MSSM:

It is timely to investigate whether the recently proposed new models of Electroweak symmetry breaking (gauge-higgs unification in extra dimensions, composite Higgs, Little Higgs, Higgsless...) can lead to a first order electroweak phase transition.

GW from phase transitions

FIRST ORDER (e.g. ELECTROWEAK?):

- Collision of bubbles walls
- Turbulent motions
- Magnetic fields



$$R \simeq v_b \beta^{-1} \simeq 0.01 \mathcal{H}_*^{-1}$$

β^{-1} Duration of the phase transition

$v_b \leq 1$ Speed of the bubbles walls

$$\ddot{h}_{ij}(\mathbf{k},\eta) + \frac{2}{\eta}\dot{h}_{ij}(\mathbf{k},\eta) + k^2 h_{ij}(\mathbf{k},\eta) = 8\pi G a^2(\eta)\Pi_{ij}(\mathbf{k},\eta)$$

Source of GW: anisotropic stress

$$T_{ab}(\mathbf{x}) = (\rho + p)\frac{v_a(\mathbf{x})v_b(\mathbf{x})}{1 - v^2(\mathbf{x})}$$

A not so new subject...

first suggestion: Witten'84

- Early 90's, M. Turner & al studied the production of GW produced by bubble collisions. Not much attention since the LEP data excluded a 1st order phase transition within the SM.

Kosowsky, Turner, Watkins'92
Kamionkowski, Kosowsky, Turner '94

- '01-'02: Kosowsky et al. and Dolgov et al. computed the production of GW from turbulence ⇛ stronger signal. Application to the (N)MSSM where a 1st order phase transition is still plausible.

Kosowsky, Mack, Kahniashvili'02
Dolgov, Grasso, Nicolis'02
Caprini, Durrer '06

in 2006:

⇛ Model-independent analysis for detectability of GW from 1st order phase transitions

Grojean, Servant '06

⇛ Apply to Randall-Sundrum phase transition

Randall, Servant'06

⇛ Revisit the Turner et al original calculation

Caprini, Durrer, Servant'07

A two parameter problem...

A 1st order phase transition proceeds by nucleation of bubbles

kinetic energy of bubbles is transferred to GW either by

- bubble collisions
- injection of energy into the plasma fluid
  (creating a homogeneous, isotropic, fully developed and stationary turbulent regime).

Need to move large mass rapidly $\Rrightarrow$ detonation regime: bubble walls propagate faster than the speed of sound

the GW background is controlled by two quantities

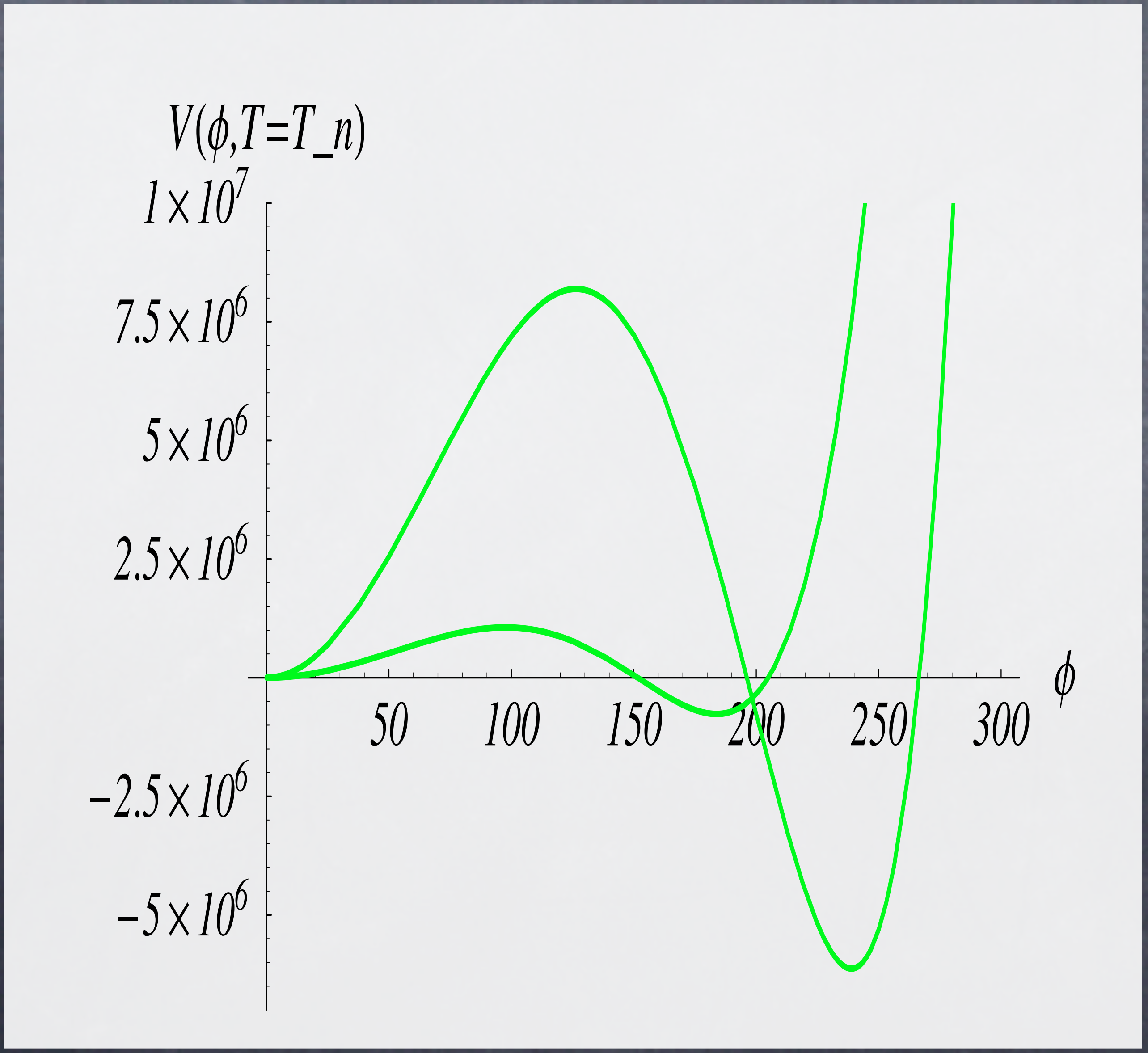
$$\alpha \sim \frac{\text{false vacuum energy density} = \text{latent heat}}{\text{plasma thermal energy density}}$$

$$\beta \sim \text{rate of time variation of the nucleation rate } \Gamma \ (\Gamma = \Gamma_0 e^{-\beta t})$$
$$\sim (\text{duration of transition})^{-1}$$

The stronger is the transition,
the larger is α and the smaller is β

Shape of the potential at the nucleation temperature

α and β : entirely determined by the effective scalar potential at high temperature



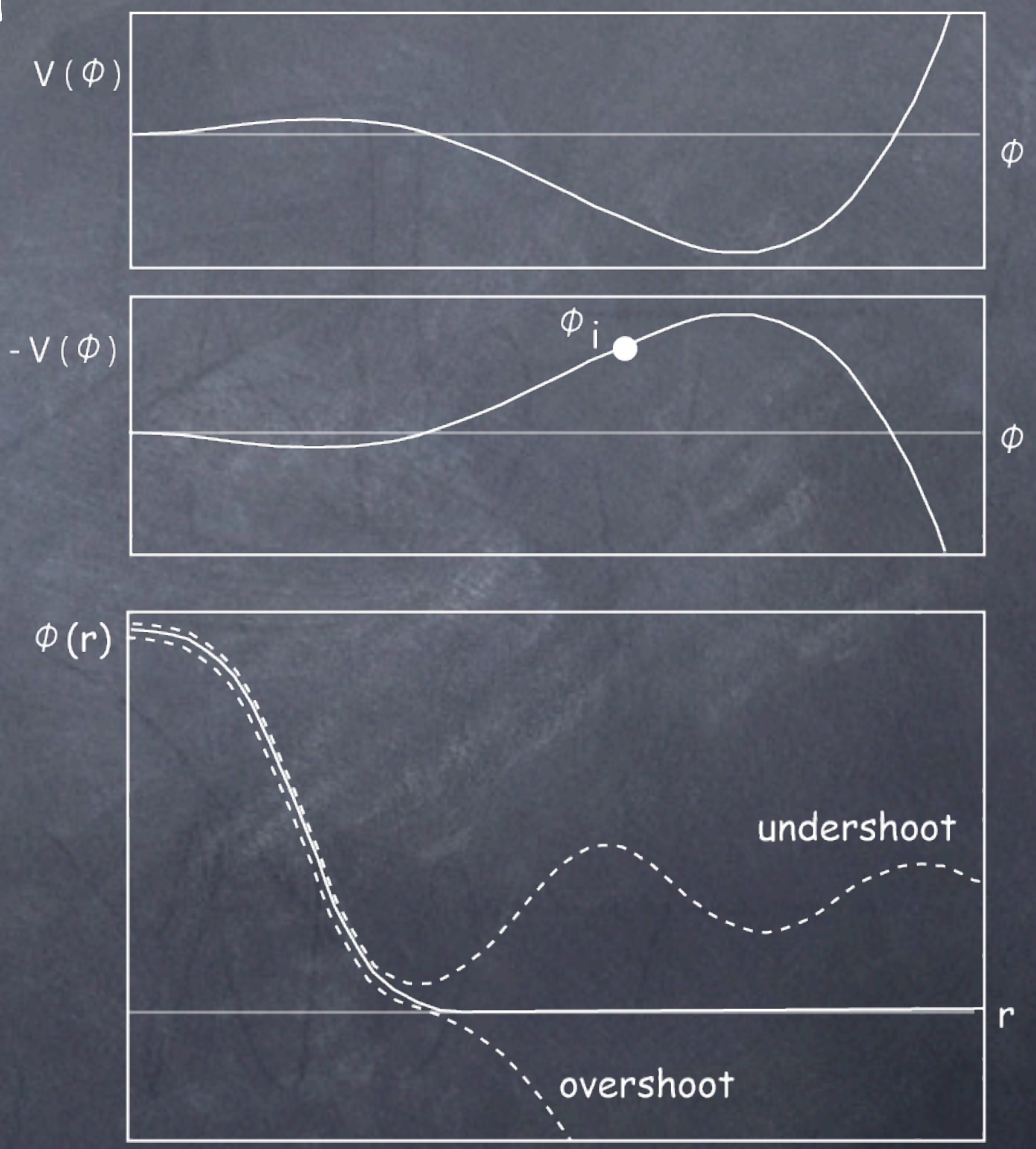
Compute rate of bubble nucleation $\Gamma \propto e^{-S_3(T)/T}$

$$S_3(T) = 4\pi \int dr\ r^2 \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$: euclidean action for a critical bubble

Overshooting-undershooting method to search for the bounce solution

$$\frac{d^2\phi_b}{dr^2} + \frac{2}{r}\frac{d\phi_b}{dr} - \frac{\partial V}{\partial \phi_b} = 0$$

$$\left.\frac{d\phi_b}{dr}\right|_{r=0} = 0 \quad , \quad \phi_b|_{r=\infty} = 0$$



Nucleation occurs when the probability for the nucleation of 1 bubble per 1 horizon volume is ~ O(1)

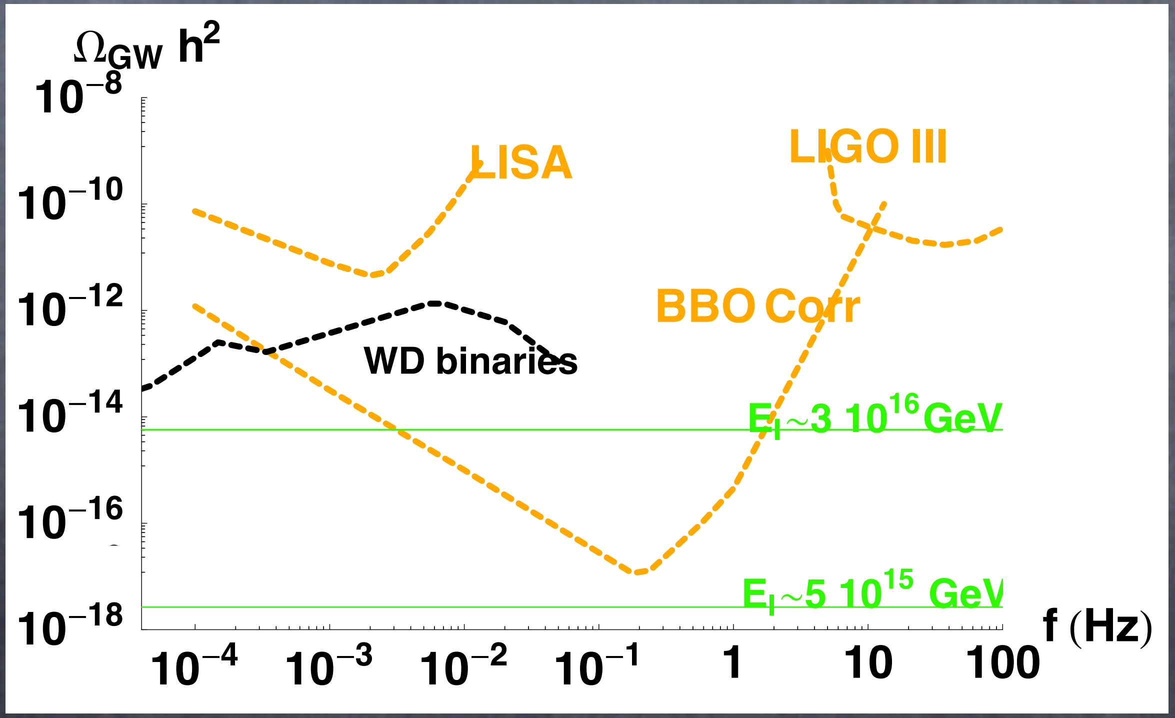
⇛ translates into $S_3(T_*)/T_* \simeq 140$

Fraction of the critical energy density in GW today

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_c} = \Omega_{GW*}\left(\frac{a_*}{a_0}\right)^4\left(\frac{H_*}{H_0}\right)^2 \simeq 1.67\times 10^{-5}h^{-2}\left(\frac{100}{g_*}\right)^{1/3}\Omega_{GW*}$$

has to be big ($\gtrsim 10^{-6}$ for LIGO/LISA and $\gtrsim 10^{-12}-10^{-9}$ for BBO)

where we used: $\rho_{GW} = \rho_{GW*}\left(\frac{a_*}{a_0}\right)^4$, $\rho_c = \rho_{c*}\frac{H_0^2}{H_*^2}$ and $H_0 = 2.1332\times h\times 10^{-42}\mathrm{GeV}$



Estimate of the GW energy density at the emission time

$$\rho_{GW} \sim \dot{h}^2 / 16\pi G$$

$$\delta G_{\mu\nu} = 8\pi\, G T_{\mu\nu} \Longrightarrow \beta^2 h \sim 8\pi G T \Longrightarrow \dot{h} \sim 8\pi G T/\beta$$

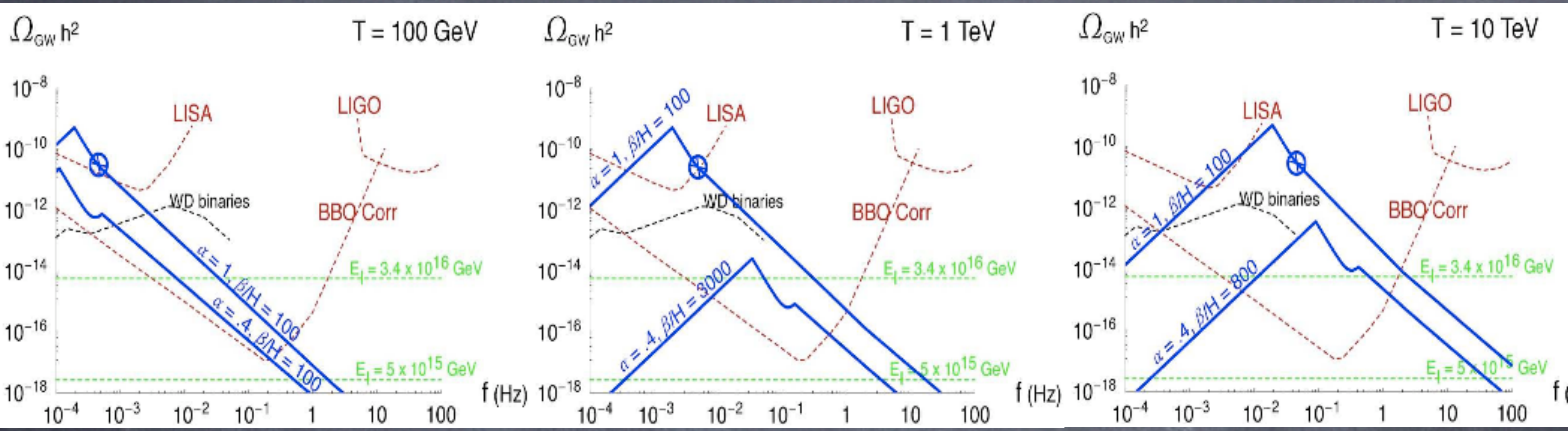
where $T \sim \rho_{kin} \sim \rho_{rad}\, v^2$

$$\Omega_{GW_*} = \frac{H_*^2}{\beta^2} \frac{\rho_{kin}{}^2}{\rho_{tot}{}^2} \quad \kappa^2\alpha^2 v^4$$

$$\Omega_{GW_*} \propto \frac{H_*^2}{\beta^2} v^4$$

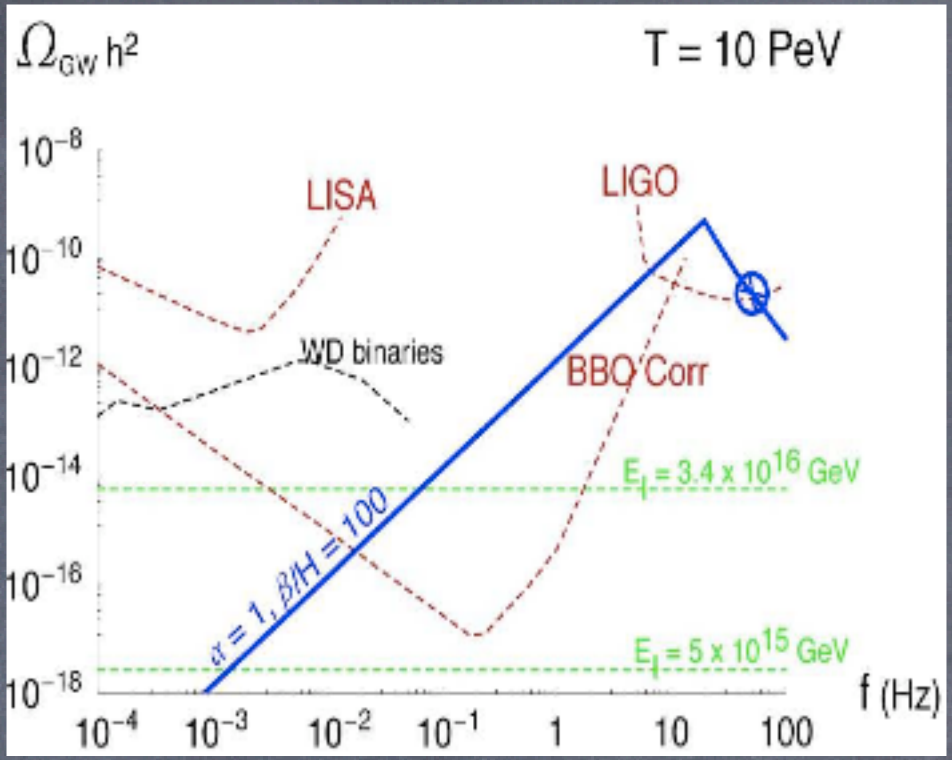
3 parameters:
α, β, v

Spectrum of gravitational waves produced at 1rst order phase transitions

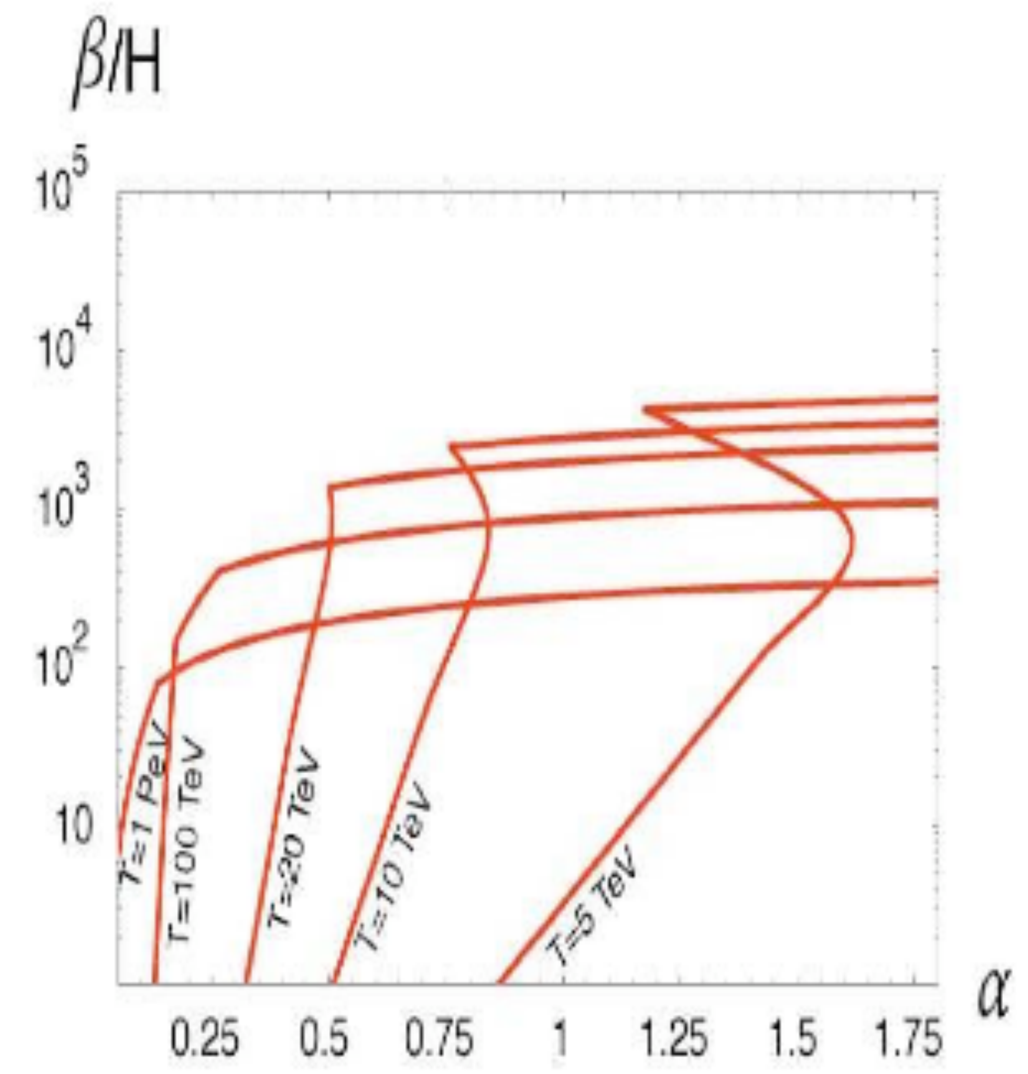


$$f_{\text{peak}} \sim 10^{-2}\,\text{mHz}\left(\frac{g_*}{100}\right)^{1/6}\frac{T_*}{100\text{ GeV}}\;\frac{\beta}{H_*}\;\frac{1}{v}$$

A phase transition at $T \sim 10^7$ GeV could be observed both at LIGO and BBO:



GW from phase transitions could entirely mask the GW signal expected from inflation:



What to expect for the EW phase transition

In the SM, a 1rst-order phase transition can occurr due to thermally generated cubic Higgs interactions:

$$V(\phi,T) \approx \frac{1}{2}(-\mu_h^2 + cT^2)\phi^2 + \frac{\lambda}{4}\phi^4 - ET\phi^3$$

$$-ET\phi^3 \subset -\frac{T}{12\pi}\sum_i m_i^3(\phi)$$

Sum over all bosons which couple to the Higgs

In the SM: $\sum_i \simeq \sum_{W,Z}$ $\Rightarrow$ not enough

mh<35 GeV would be needed to get Φ/T>1 and for mh >72 GeV, the phase transition is 2nd order

Strength of the transition in the SM:

$$\langle\phi(T_c)\rangle = \frac{2\ E\ T_c}{\lambda} \quad \Rrightarrow \quad \frac{\langle\phi(T_c)\rangle}{T_c} = \frac{2\ E\ v_0^2}{\lambda\ v_0^2} = \frac{4\ E\ v_0^2}{m_h^2}$$

$$v_0 \approx 246 \text{ GeV} \quad \text{and} \quad E = \frac{2}{3}\frac{2m_W^3 + m_Z^3}{4\pi v_0^3} \sim 6.3 \times 10^{-3}$$

$$\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1 \quad \longrightarrow \quad m_h \lesssim 47 \text{ GeV}$$

Grojean-Servant-Wells '04

Our approach:

- ✔ Does not rely on the thermally generated negative self cubic Higgs interactions
- ✔ Instead, we add a non-renormalizable Φ^6 term to the SM Higgs potential and allow a negative quartic coupling :

$$V(\Phi) = \mu_h^2|\Phi|^2 - \lambda|\Phi|^4 + \frac{|\Phi|^6}{\Lambda^2}$$

$\Rrightarrow$ Can induce a strong 1rst-order phase transition if $\boxed{\Lambda \lesssim 1 \text{ TeV}}$

Strength of the transition in the SM:

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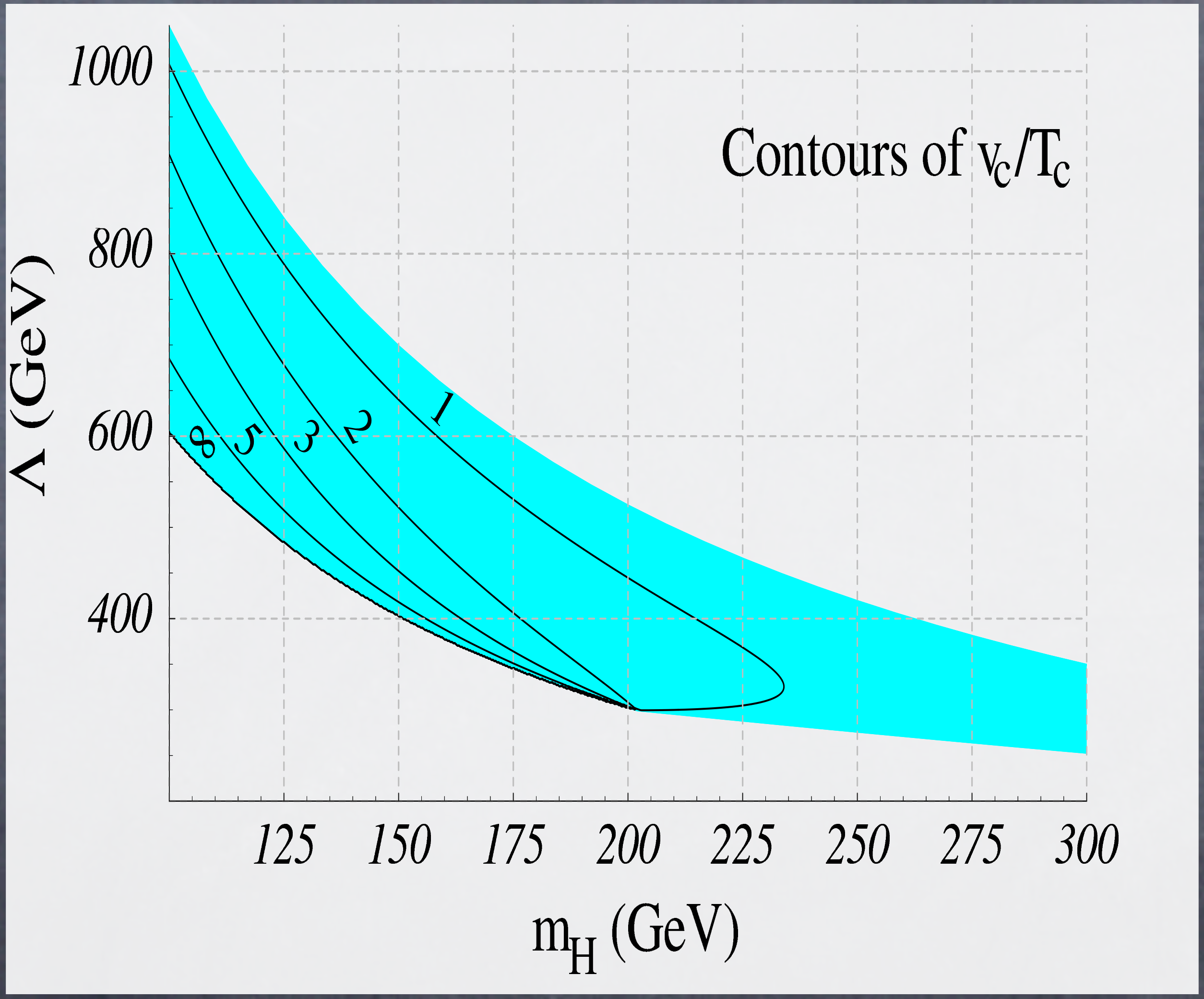
$$\frac{\langle\phi(T_c)\rangle}{T_c} \gtrsim 1 \quad \longrightarrow \quad m_h \lesssim 47\ \text{GeV}$$

and using our potential:

$$\langle\phi^2(T_c)\rangle = \frac{3}{2}v_0^2 - \frac{m_h^2\Lambda^2}{2v_0^2} \qquad \text{and} \qquad T_c^2 = \frac{\Lambda^4 m_h^4 + 2\Lambda^2 m_h^2 v_0^4 - 3v_0^8}{16c\Lambda^2 v_0^4}$$

A strongly 1rst order phase transition compatible with large Higgs masses



The blue region corresponds to a first order phase transition

A concrete example of the possible origin of the $|\Phi^6|$ term

$\Rrightarrow$ Decouple a massive scalar singlet coupled to the Higgs via

$$\Delta V = \frac{1}{2} m_s^2 \phi_s^2 + m \phi_s \Phi^\dagger \Phi + \frac{1}{2} a \phi_s^2 \Phi^\dagger \Phi .$$

Assuming $m \sim m_s > v_0 \Rrightarrow$

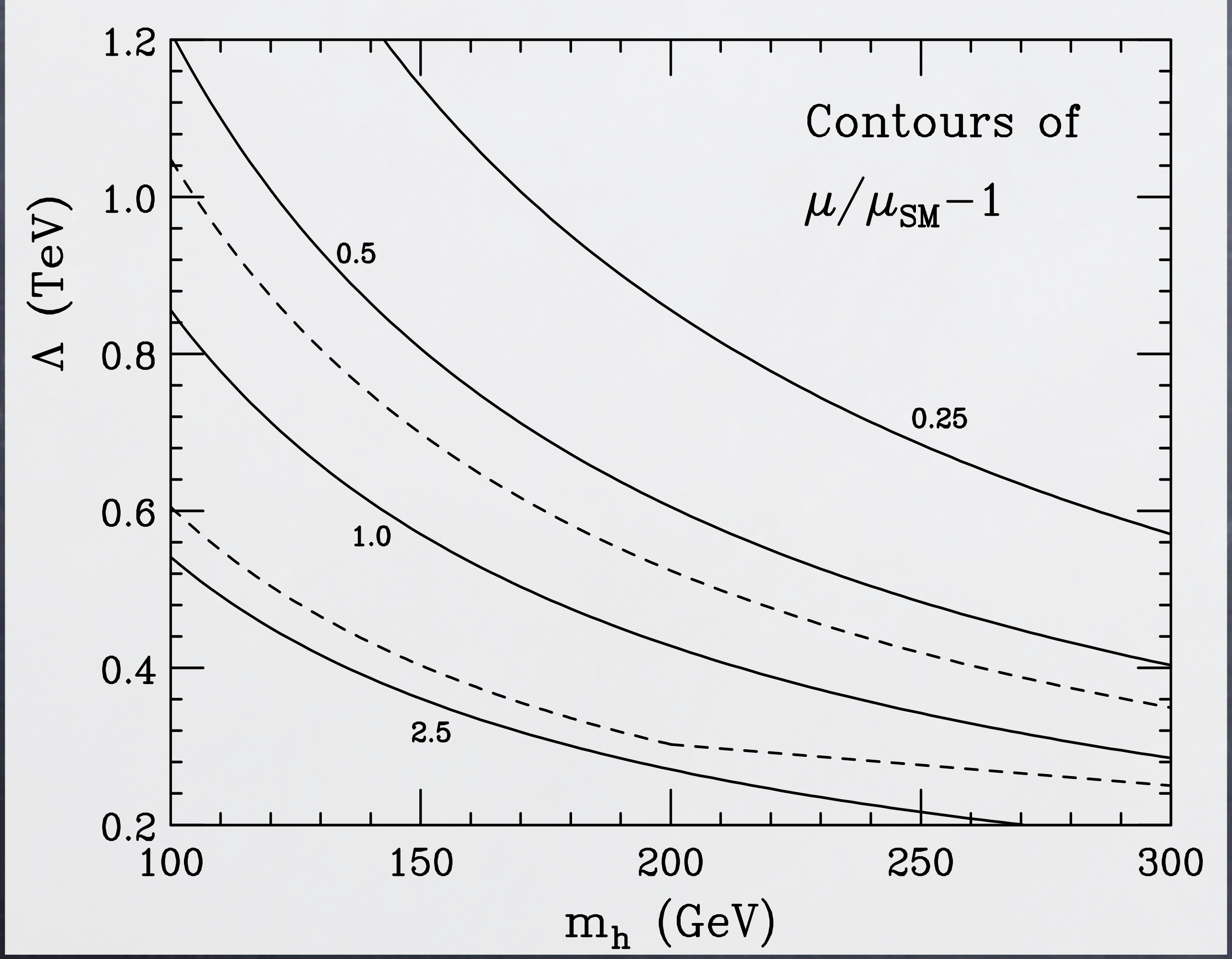
$$V_{\rm new} = -\frac{m^2}{2 m_s^2} |\Phi|^4 + \frac{a m^2}{2 m_s^4} |\Phi|^6 + \mathcal{O}\left(\frac{a^2 m^4 |\Phi|^8}{m_s^6} \right)$$

This scenario predicts large deviations to the Higgs self-couplings

$$\mathcal{L} = \frac{m_H^2}{2} H^2 + \frac{\mu}{3!} H^3 + \frac{\eta}{4!} H^4 + \ldots \quad \textit{where}$$

$$\mu = 3\frac{m_H^2}{v_0} + 6\ \frac{v_0^3}{\Lambda^2}$$

$$\eta = 3\frac{m_H^2}{v_0^2} + 36\ \frac{v_0^2}{\Lambda^2}$$

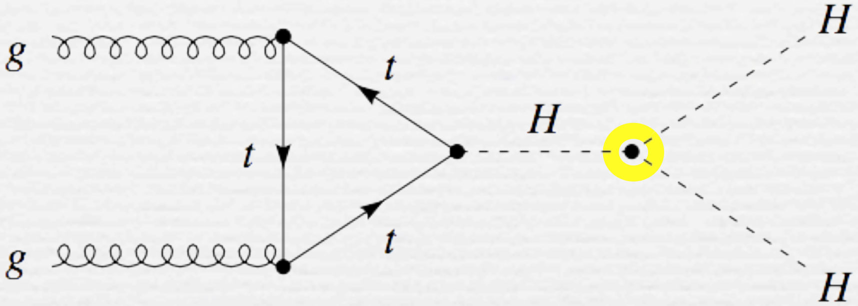


The dotted lines delimit the region for a strong 1rst order phase transition

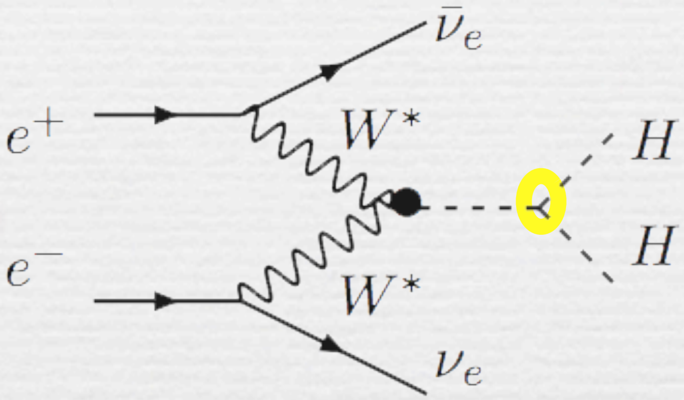
deviations between a factor 0.7 and 2

Experimental tests of the Higgs self-coupling

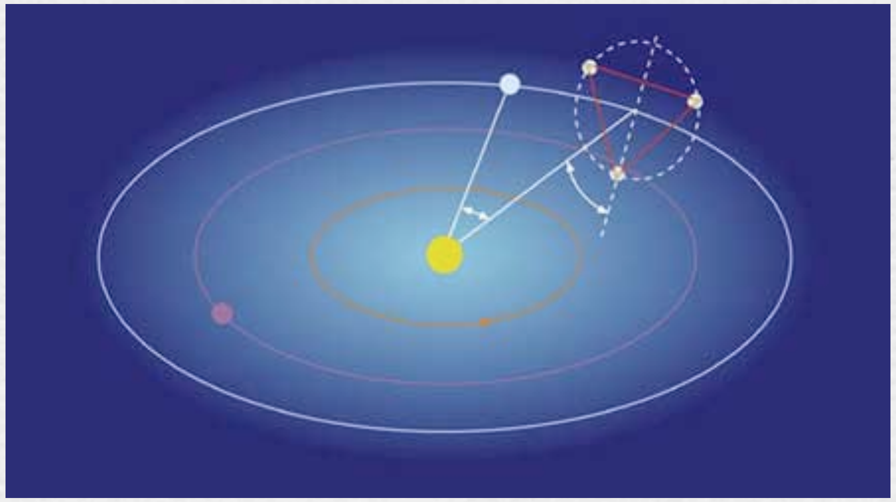
at a Hadron Collider

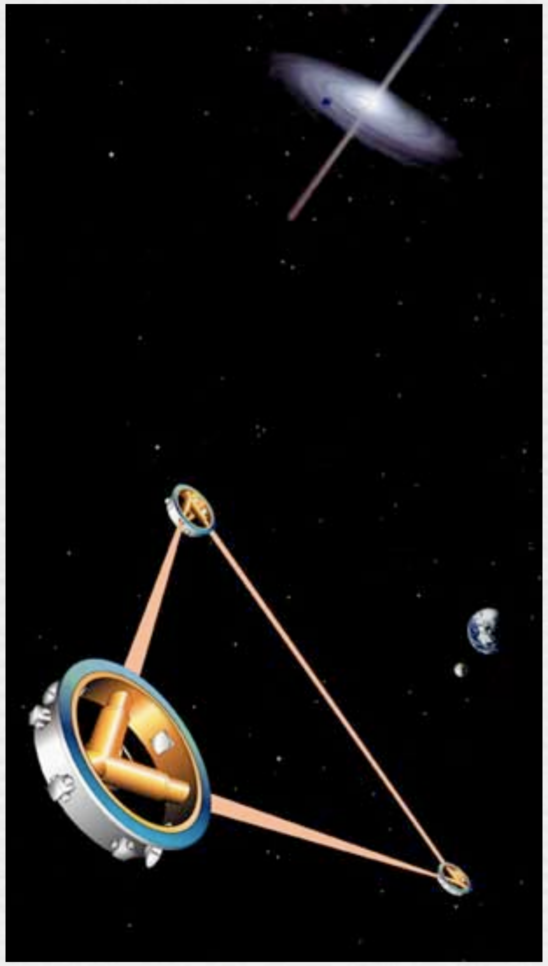


at an $e^+ e^-$ Linear Collider



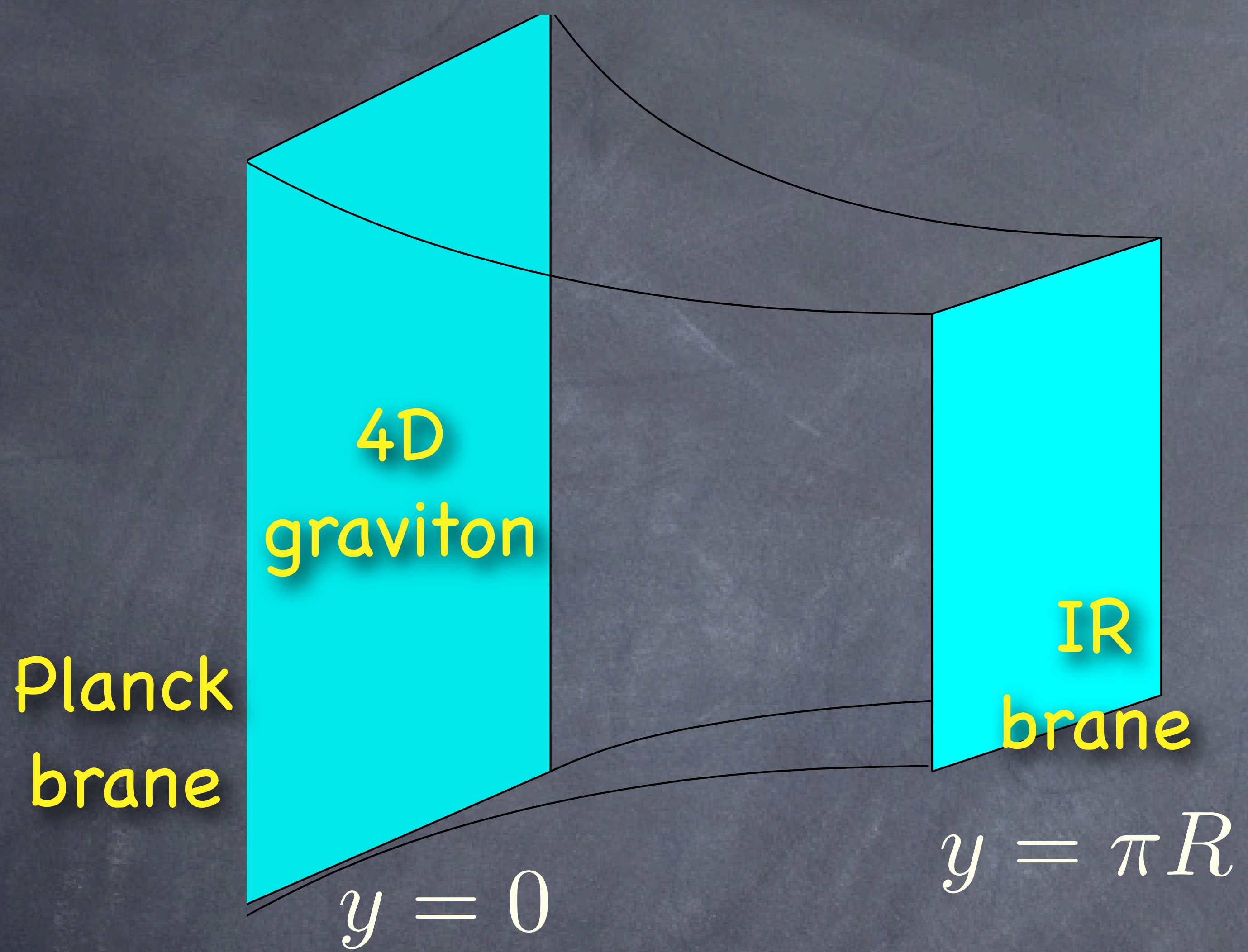
... or at the gravitational wave detector LISA





Gravitational Waves from Warped Extra-Dimensional Geometry

Space-time is a slice of AdS_5



$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

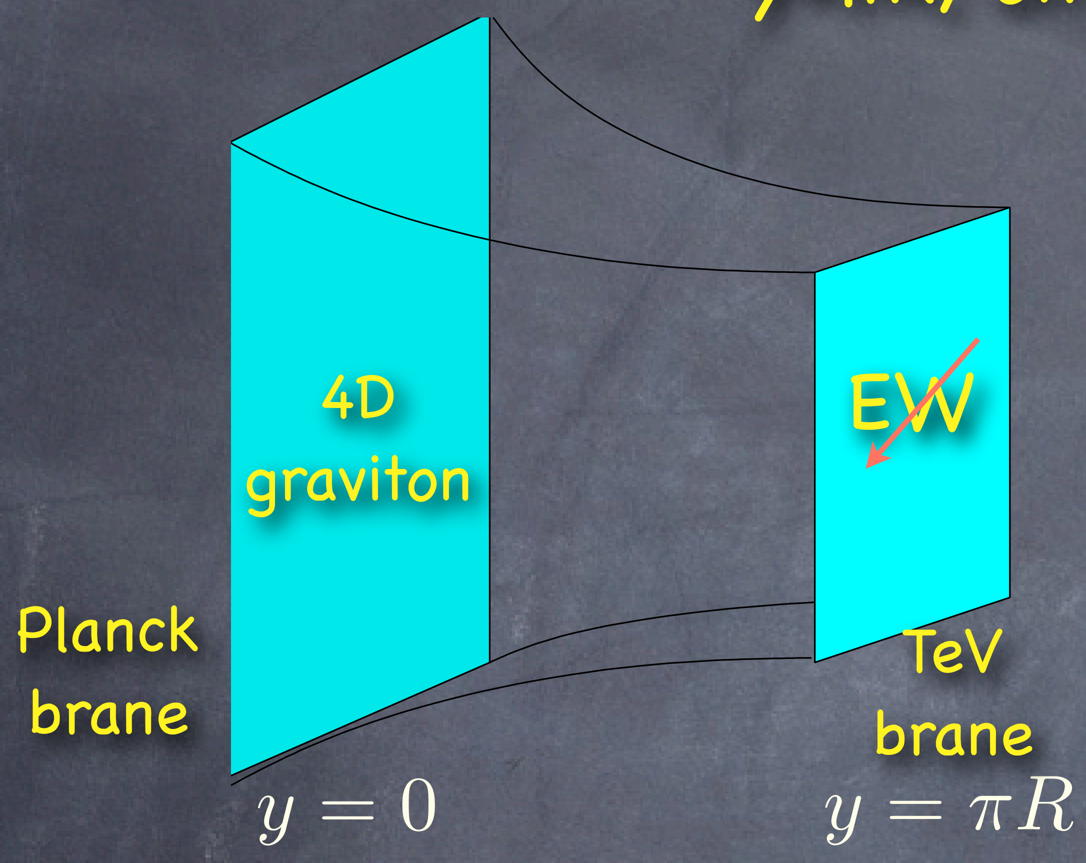
$$M_{Pl}^2 \sim \frac{M_5^3}{k}$$

The effective 4D energy scale varies with position along 5th dimension

RS1 (has two branes) versus RS2 (only Planck brane)

Solution to the Planck/Weak scale hierarchy

The Higgs (or any alternative EW breaking) is localized at $y=\pi R$, on the TeV (IR) brane



After canonical normalization of the Higgs:

$$v_{\rm eff} = v_0 e^{-k\pi R}$$

parameter in the 5D lagrangian

$$k\pi R \sim \log\left(\frac{M_{Pl}}{\rm TeV}\right)$$

Exponential hierarchy from O(10) hierarchy in the 5D theory

One Fondamental scale : $M_5 \sim M_{Pl} \sim k \sim \Lambda_5/k \sim r^{-1}$

Radius stabilisation using bulk scalar (Goldberger-Wise mechanism)

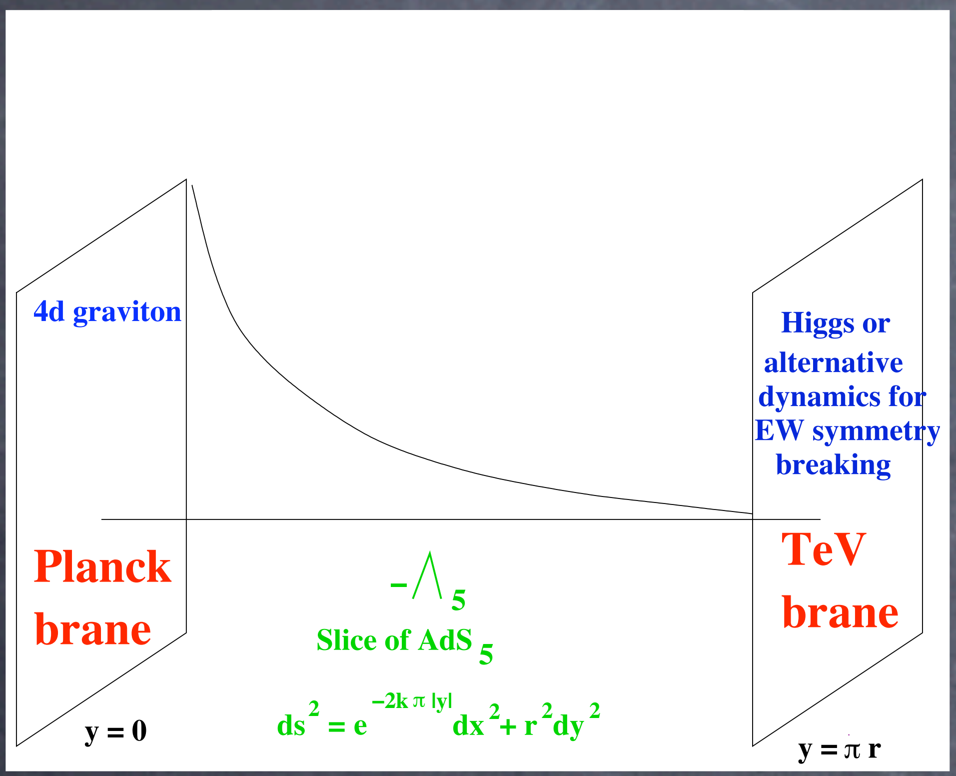
$$kr = \frac{4}{\pi}\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right] \sim 10$$

Warped hierarchies are radiatively stable as cutoff scales get warped down near the IR brane

Cosmology of the Randall-Sundrum model

At high T: AdS-Schwarzchild BH solution with event horizon shielding the TeV brane
Temperature is too high to experience weak scale phenomena

At low T: usual RS solution with stabilized radion and TeV brane



Natural stabilisation of radius à la Goldberger-Wise :

$$kr = \frac{4}{\pi}\frac{k^2}{m^2}\ln\left[\frac{v_h}{v_v}\right] \sim 10$$

Randall-Sundrum phase transition

Creminelli-Nicolis-Rattazzi '01

Assuming the universe started at T>> Tc, the PT has to take place if we want a RS set-up at low T.

Start with a black brane, nucleate "gaps" in the horizon which then grow until they take over the entire horizon.

High-T Phase: AdS-S Black hole

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{\rho_h^4/L^2}{\rho^2}\right) dt^2 + \frac{d\rho^2}{\frac{\rho^2}{L^2} - \frac{\rho_h^4/L^2}{\rho^2}} + \frac{\rho^2}{L^2}\sum_i dx_i^2$$

reduces to pure AdS metric for $\rho_h = 0$

$$T_h \equiv \frac{\rho_h}{\pi L^2}$$

$$\boxed{F_{\text{AdS-S}} = -2\pi^4 (ML)^3 T^4}$$

both local minima of free energy

by holography:

$(ML)^3 = N^2/16\pi^2$

Low-T Phase : RS1 geometry

Radion field determines spacing between branes

Require that radion is stabilized around TeV

$$\mu = e^{-k\pi r} M_{Pl}$$

$$F_{RS} = (4+2\epsilon)\mu^4 (v_1 - v_0(\mu/\mu_0)^\epsilon)^2 \\ -\epsilon v_1^2 \mu^4 + \delta T_1 \mu^4 + \mathcal{O}(\mu^8/\mu_0^4)$$

$$\boxed{V_{min} \approx -\epsilon^{3/2} v_1^2 \mu_{\text{TeV}}^4}$$

Second brane emerges at T~TeV

i.e. radion starts at $\mu = 0$

and evolves to $\mu = \mu_{\text{TeV}}$

Key is stabilising mechanism

$$T_c = \left(\frac{-8V_{min}}{\pi^2 N^2}\right)^{1/4}$$

Below T_c, expect first-order phase transition

From 4D perspective , expect transition through bubble nucleation

From 5D perspective , spherical brane patches on horizon

Goldberger-Wise mechanism

Start with the bulk 5d theory $\mathcal{L} = \int dx^4 dz \sqrt{-g}[2M^3\mathcal{R} - \Lambda_5]$ $\quad \Lambda_5 = -24M^3k^2$

The metric for RS1 is
$$ds^2 = (kz)^{-2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \quad \text{where } k = L^{-1} \text{ is the AdS curvature}$$
$$= e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \qquad z = k^{-1}e^{ky}$$

and the orbifold extends from $z=z_0=L$ (Planck brane) to $z=z_1$ (TeV brane)

Which mechanism naturally selects $z_1 \gg z_0$? simply a bulk scalar field φ can do the job:

$$\int d^4x dz \left(\sqrt{g}[-(\partial\phi)^2 - m^2\phi^2] + \delta(z-z_0)\sqrt{g_0}L_0(\phi(z)) + \delta(z-z_1)\sqrt{g_1}L_1(\phi(z))\right)$$

φ has a bulk profile satisfying the 5d Klein-Gordon equation

$$\phi = Az^{4+\epsilon} + Bz^{-\epsilon} \qquad \text{where} \qquad \epsilon = \sqrt{4+m^2L^2} - 2 \approx m^2L^2/4$$

Plug this solution into $V_{eff} = \int_{z_0}^{z_1} dz\sqrt{g}[-(\partial\phi)^2 - m^2\phi^2]$

$$V_{\rm GW} = z_1^{-4}\left[(4+2\epsilon)\left(v_1 - v_0\left(\frac{z_0}{z_1}\right)^\epsilon\right)^2 - \epsilon v_1^2\right] + \mathcal{O}(z_0^4/z_1^8) = z_1^{-4}P(z^{-\epsilon})$$

⇨
$$z_1 \approx z_0\left(\frac{v_0}{v_1}\right)^{1/\epsilon}$$

Similar to Coleman-Weinberg mechanism

Completion of the phase transition

a five-dimensional set-up
but we can treat this as bubble nucleation in four dimensions

Low energies: radion dominates potential

High energies: holography

$$(M/k)^3 \sim N^2/16\pi^2$$

Need N large

Computation of the tunneling rate $\Gamma \sim T_c^4 e^{-S}$

under the approximation $T_c \ll \mu_{TeV}$: (i.e large N)

only the radion mode contributes significantly to the action

justified if v_1 and ϵ are small: in which case potential is shallow and most of the action comes from the RS side as μ changes from 0 to μ_{TeV}

The contribution from the AdS-S side to the thermal bounce is neglected

Phase transition only completes in borderline perturbativity region

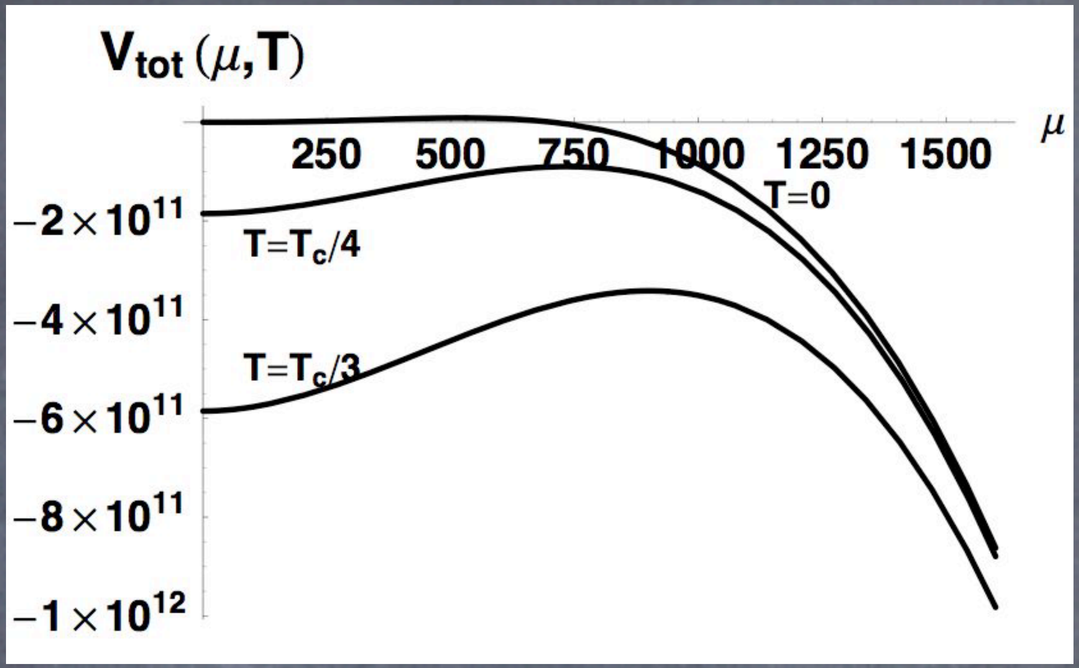
k large , ϵ large , $v_1 \sim N$

Transition rate $\Gamma \sim T_c^4 e^{-N^2}$ $\Longrightarrow$ $N \lesssim 2\sqrt{\log \frac{M_{Pl}}{T_c}}$

i.e if $T_c \sim 1$ TeV $\rightarrow$ $N \lesssim 12$

$$(ML)^3 = N^2/16\pi^2$$

Evolution of radion potential with temperature

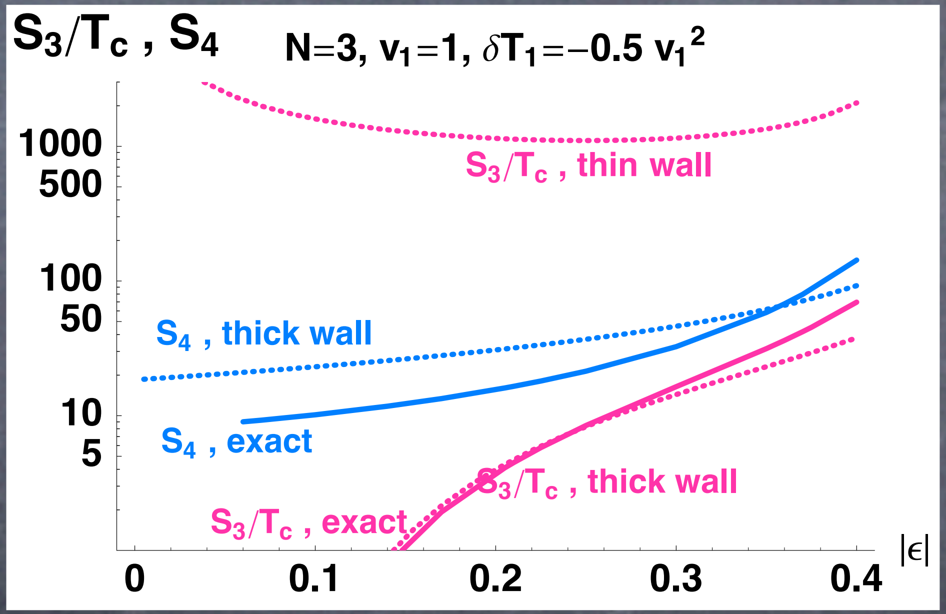


The transition does not take place until the temperature is sufficiently low so that we enter in the thick wall regime.

Calculation of S: $S_3 \propto \frac{\mu_{\rm TeV}^3}{\sqrt{|V|}} N^3$

- in thin wall and thick wall ($\mu_{nucl} \neq \mu_{TeV}$) approximations

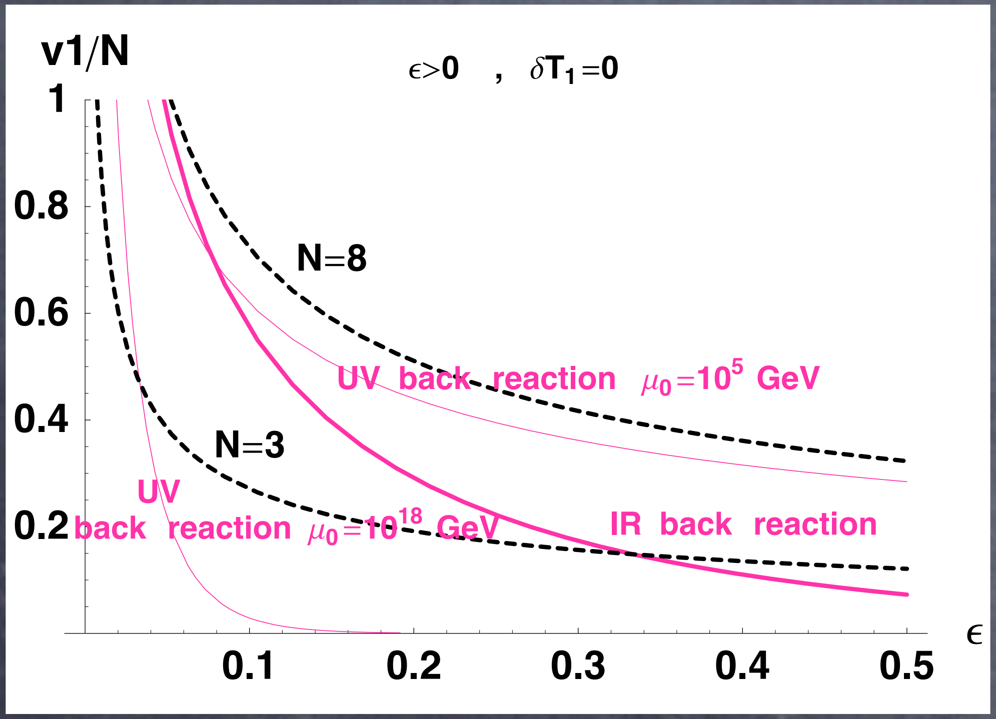
Comparison of thin wall and thick wall approximations with exact solution:

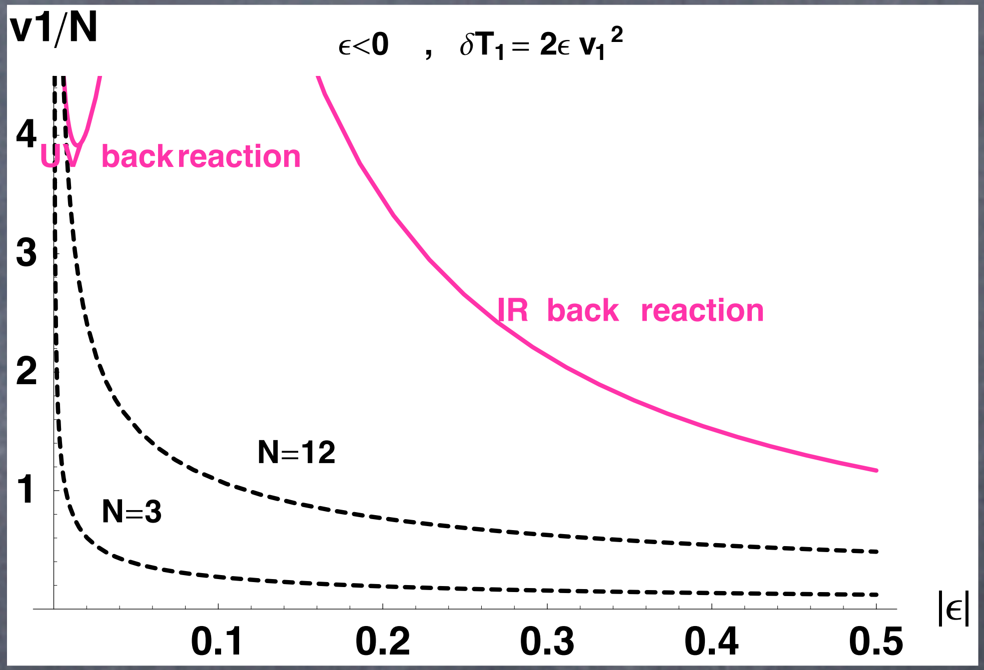


Calculation of S:

$$S_3 \propto \frac{\mu_{\rm TeV}^3}{\sqrt{|V|}} N^3$$

- in thin wall and thick wall ($\mu_{nucl} \neq \mu_{TeV}$) approximations
- for both $\epsilon>0$ and $\epsilon<0$
- for large supercooling: O(4) symmetric bubbles
- effect of modified TeV brane tension

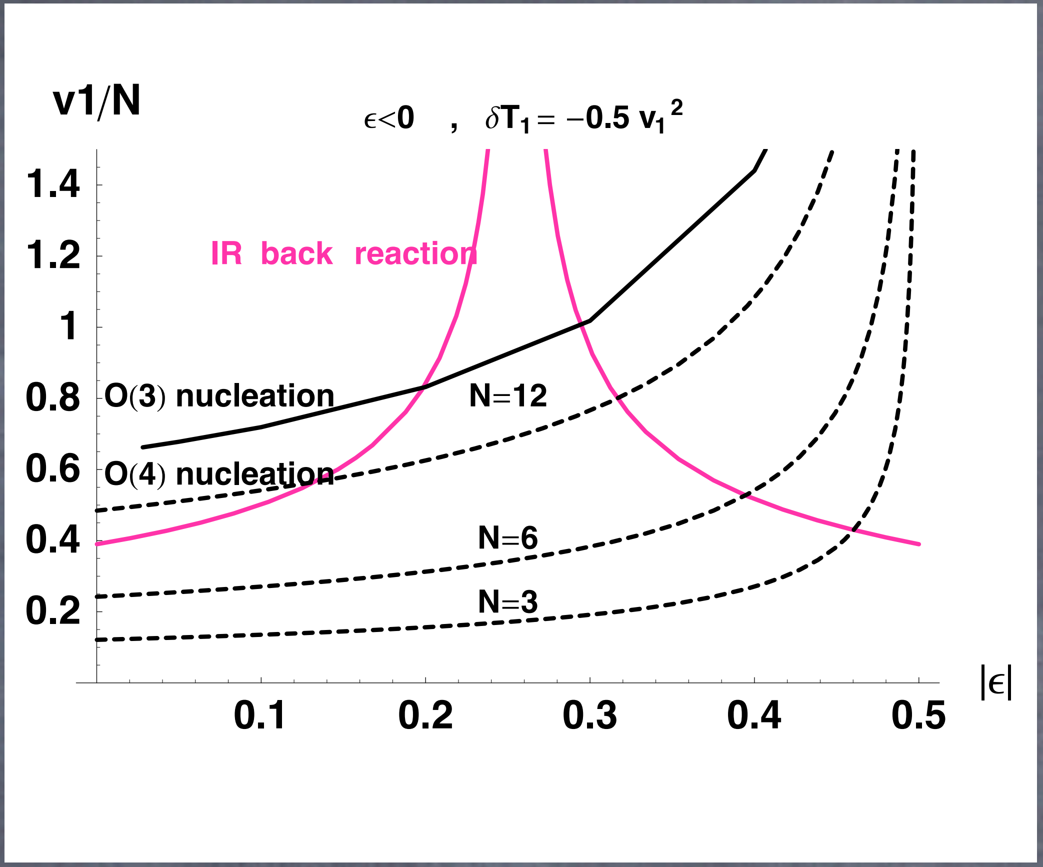




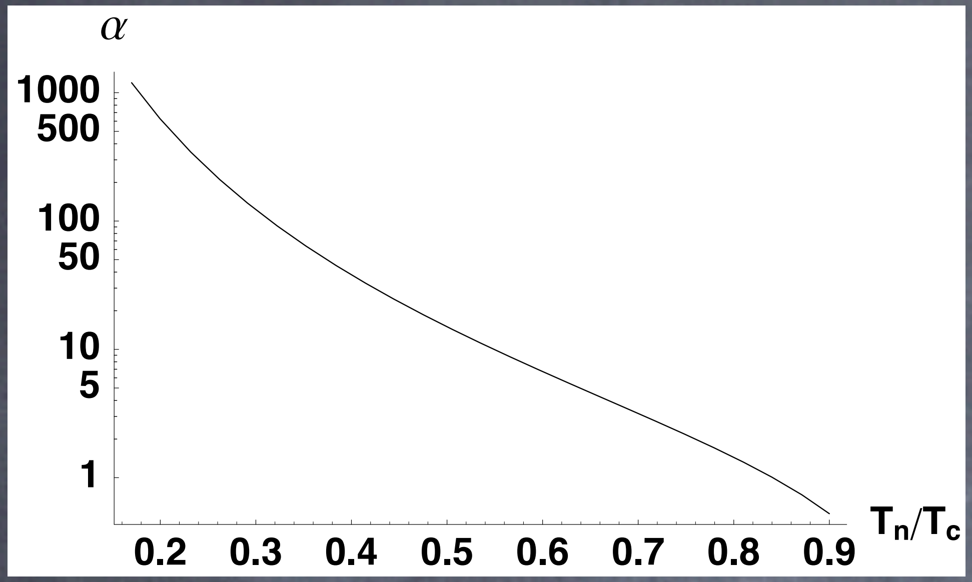
In region where transition can take place :

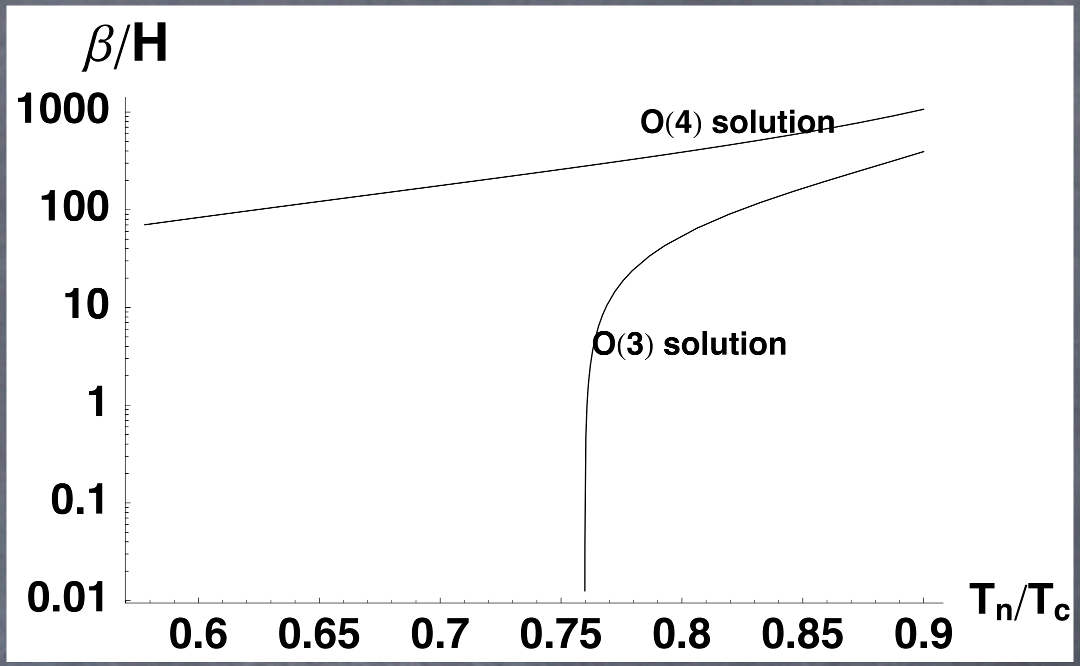
strong gravity wave signal

Effect of increasing N



Gravitational Wave signal as a function of nucleation temperature





$$\alpha = \frac{|\Delta V_{\pm}|}{\pi^2 N^2 T^4/8} = \frac{T_c^4 - T_n^4}{T_n^4}$$

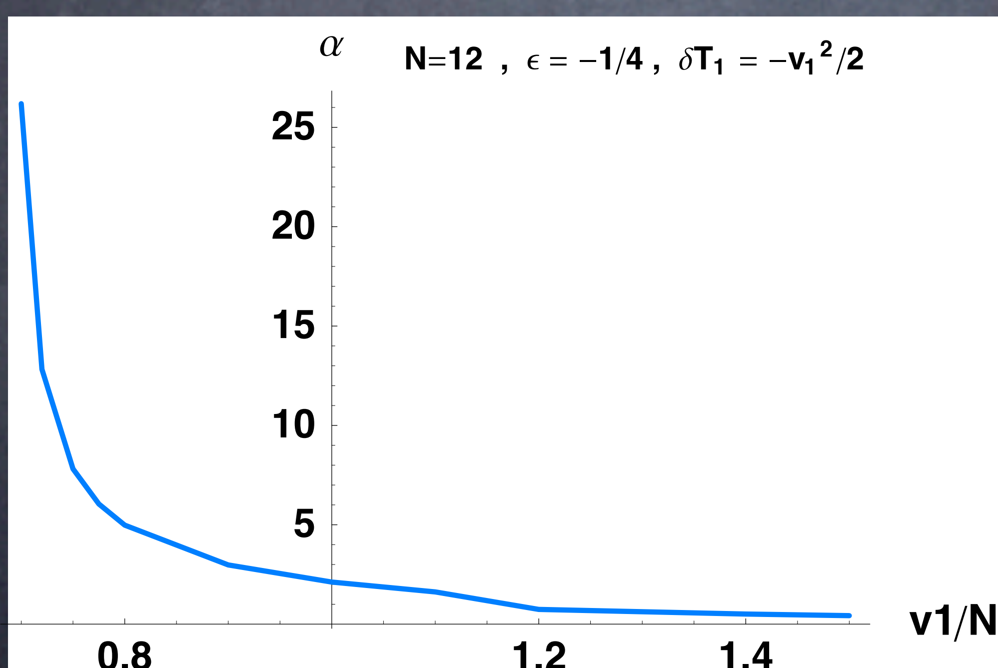
$$\frac{\beta}{H} = \begin{cases} \frac{S_3(T_n)}{T_n} \times \frac{3a^4-1}{1-a^4} \approx 140 \times \frac{3a^4-1}{1-a^4} & \text{for } O(3) \text{ sol.} \\ S_4(T_n) \times \frac{4a^4}{1-a^4} \approx 140 \times \frac{4a^4}{1-a^4} & \text{for } O(4) \text{ sol.} \end{cases}$$

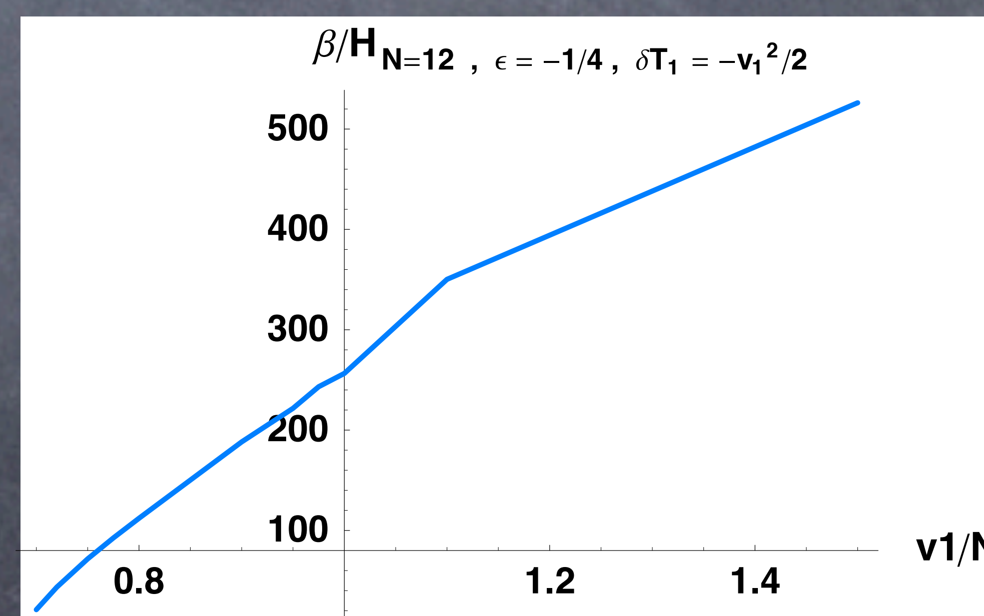
We can play the same game that we played with the Higgs potential

and determine the values of **α** *and* **β/H**

as a function of the parameters describing the radion potential

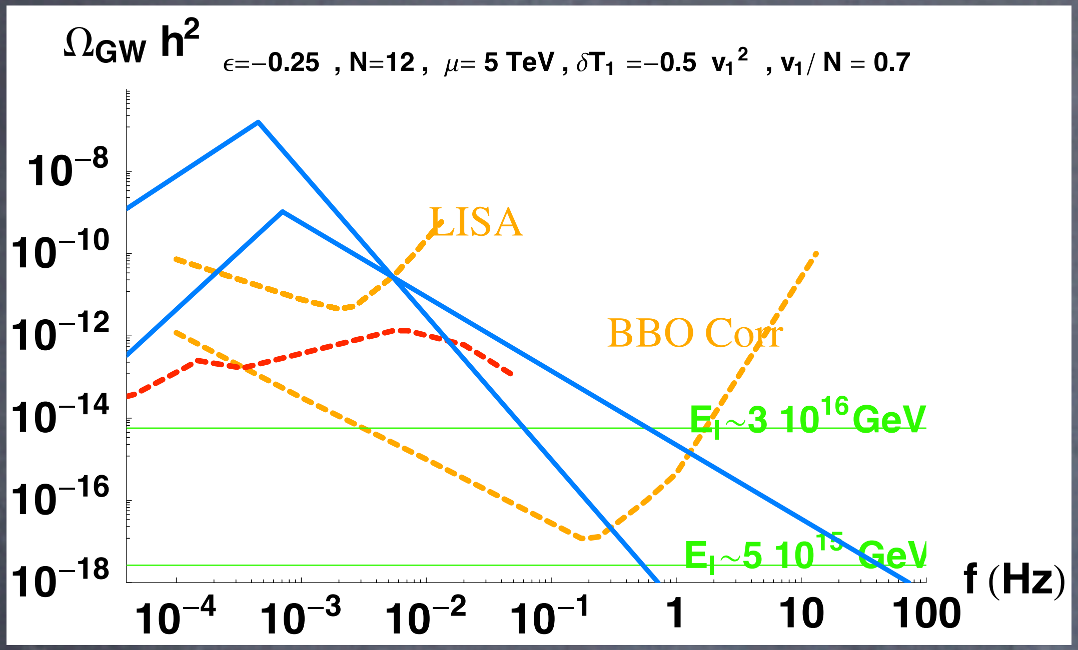
Particular model:

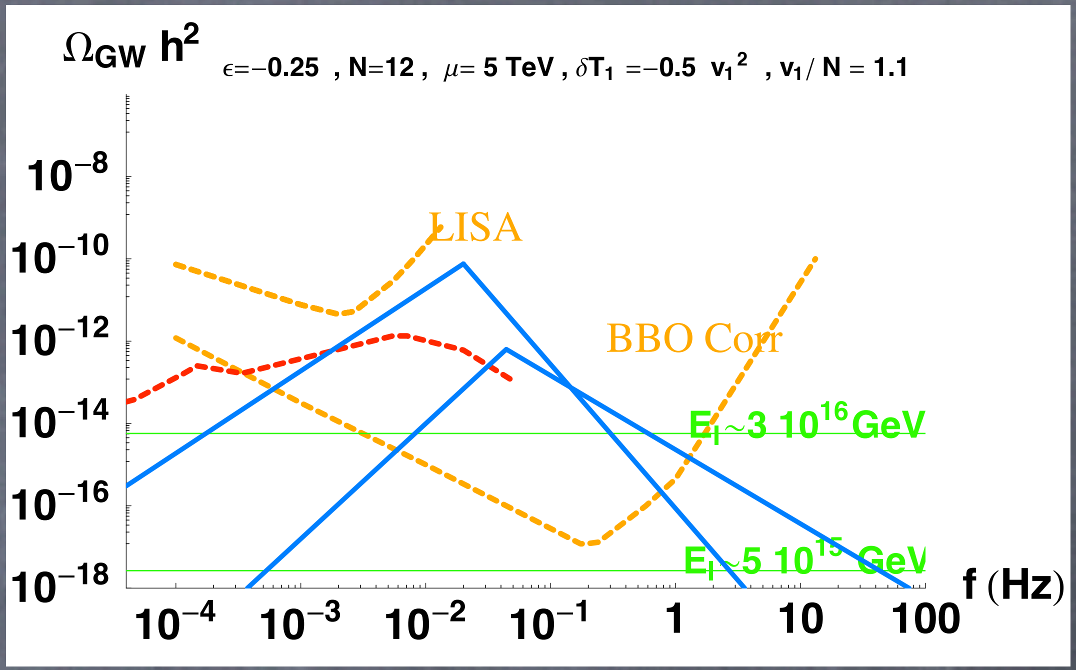




Gravitational Waves from "3-brane" nucleation:

Signal versus LISA's sensitivity





Signature in GW is generic,

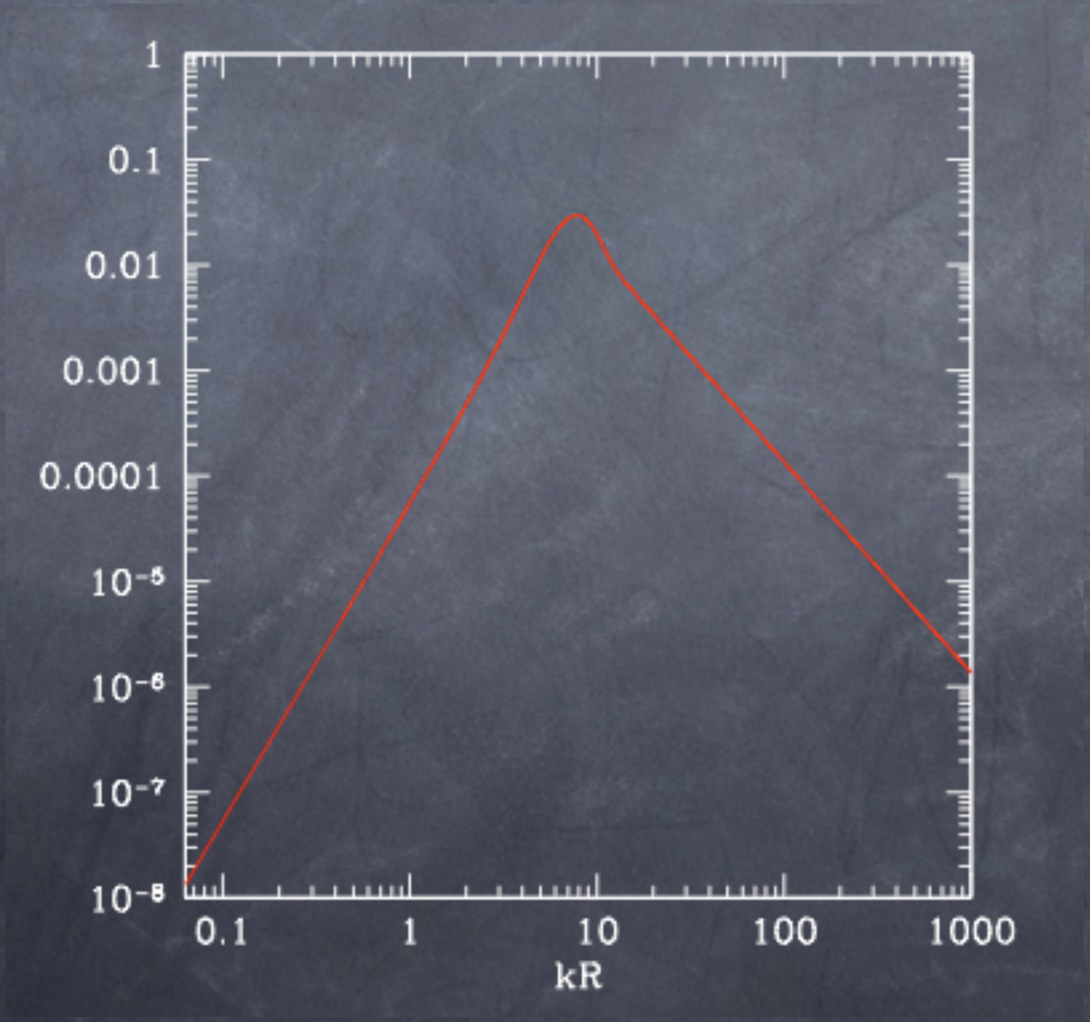
i.e. does not depend whether Standard Model is in bulk or on TeV brane

but crucially depends on the radion properties

Conclusion 1

We might be learning something about the Higgs/radion by looking at the sky

Gravity wave spectrum from bubble collisions



Caprini-Durrer-Servant
astro-ph/0711.2593

Stochastic background of GWs

GW power spectrum:

$$\frac{d\Omega_G}{d\ln k} = \frac{k^3|\dot{h}|^2}{G\rho_c} \qquad \langle \dot{h}_{ij}(\mathbf{k},\eta)\dot{h}^*_{ij}(\mathbf{q},\eta)\rangle = \delta(\mathbf{k}-\mathbf{q})|\dot{h}|^2(k,\eta)$$

Wave equation: $$h_{ij}(\mathbf{k},\eta) = \int_{\eta_{\rm in}}^{\eta} d\tau \mathcal{G}(\tau,\eta)\Pi_{ij}(\mathbf{k},\tau)$$

Anisotropic stress power spectrum: $$\langle \Pi_{ij}(\mathbf{k},\tau_1)\Pi^*_{ij}(\mathbf{q},\tau_2)\rangle = \delta(\mathbf{k}-\mathbf{q})\Pi(k,\tau_1,\tau_2)$$

Energy momentum tensors: $\Pi_{ij} = \mathcal{P}^{lm}_{ij}T_{lm}$ $\Longrightarrow$ energy squared

$$T^B_{ij} = \frac{1}{8\pi}\int d^3\mathbf{q} B_i(|\mathbf{k}-\mathbf{q}|)B_j(\mathbf{q})$$

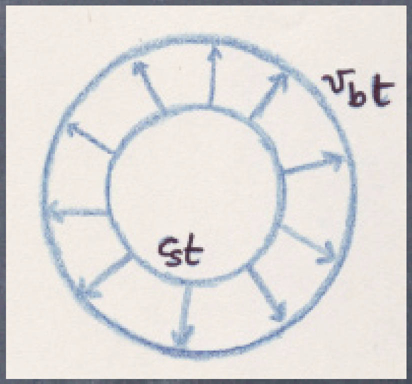
4 point correlation function $\Longrightarrow$ Gaussianity $\Longrightarrow$ Power spectra

$$\ddot{h}_{ij}(\mathbf{k},\eta) + \frac{2}{\eta}\dot{h}_{ij}(\mathbf{k},\eta) + k^2 h_{ij}(\mathbf{k},\eta) = 8\pi G a^2(\eta)\Pi_{ij}(\mathbf{k},\eta)$$

Source of GW: anisotropic stress

transverse traceless component of energy-momentum tensor

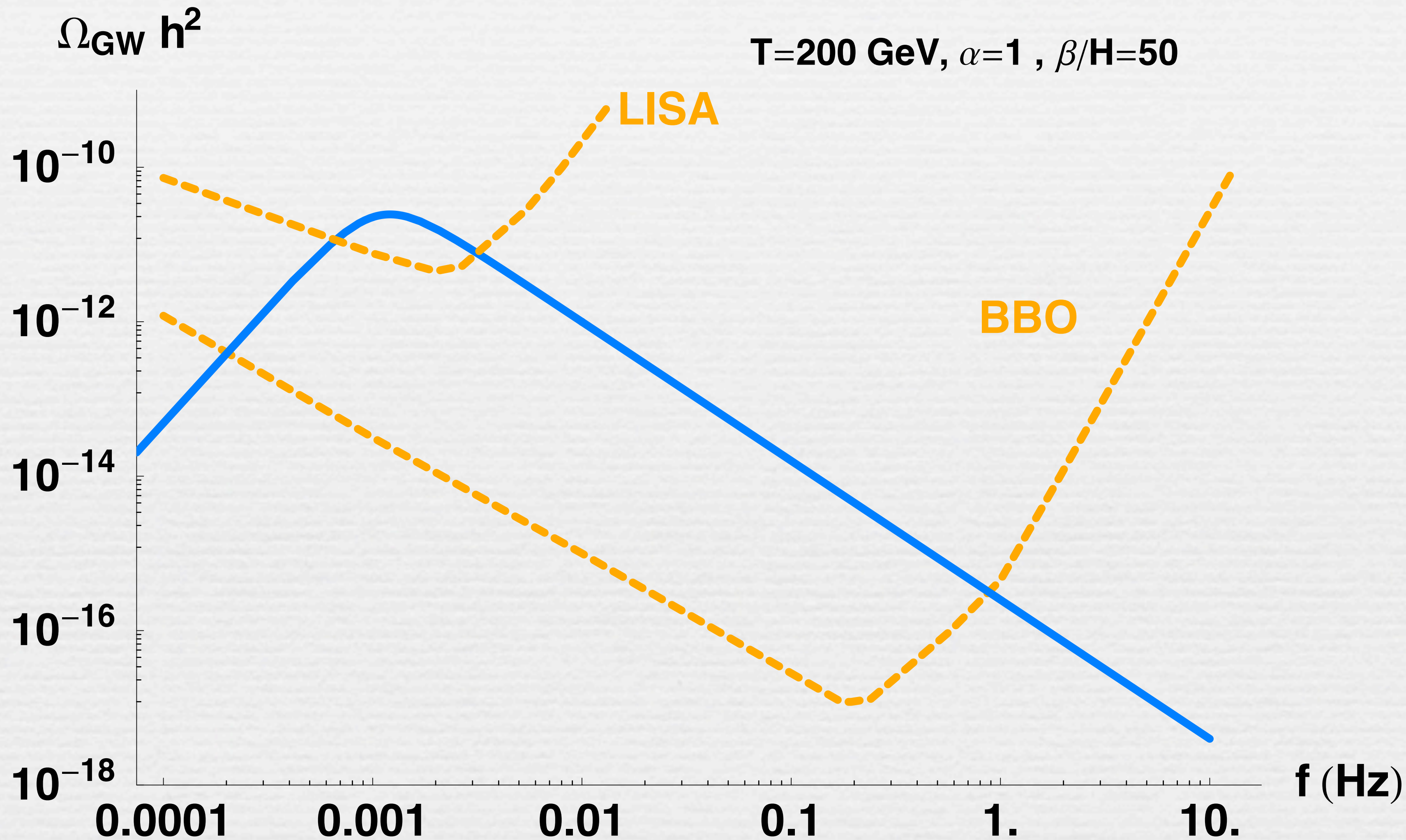
Caprini-Durrer-Servant astro-ph/0711.2593, PRD:



- intrinsically stochastic approach
- provide a model for the bubble velocity power spectrum
- peak frequency is higher by a factor 2/v

Peak frequency is associated with the bubble size, i.e.the length scale at which the velocity correlation function goes to zero. The same length scale determines the peak in the GW power spectrum

- resolution of wave equation, shape of spectrum derived
- extension to deflagration case (velocity shell)
- signal from just collisions is typically not observable unless β/H~10



Ω_{GW} h^2
T=200 GeV, α=1 , β/H=50
LISA
BBO
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
0.0001
0.001
0.01
0.1
1.
10.
f (Hz)

Conclusion 2

Still a bit too premature to apply existing formulae for the GW spectrum from phase transitons to your favorite particle physics model. Wait until

1) spectrum from MHD turbulence better under control

2) reliable bubble wall velocity calculation available