Examples of $\mathcal{N} = 2$ to $\mathcal{N} = 1$ supersymmetry breaking

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Abstract

We consider the class of four-dimensional $\mathcal{N}=2$ supergravities from type II string compactifications with Abelian gaugings that admit $\mathcal{N}=1$ Minkowski vacua. We calculate explicitly the Kähler potential, superpotential and D-terms of the $\mathcal{N}=1$ low-energy effective theory for the STU and quantum STU models, i.e. the models where the special quaternionic-Kähler field space of the hypermultiplets is specified by a STU or quantum STU prepotential. For the STU model, the special quaternionic-Kähler manifold $\frac{SO(4,4)}{SO(4)\times SO(4)}$ descends to the Kähler manifold $\frac{SO(4,2)}{SO(4)\times SO(2)}$. We also determine how the superpotential is restricted by requiring supersymmetric vacua and find that in many cases where STU prepotentials are involved, supersymmetric vacua are only possible if the superpotential vanishes identically.

Contents

1 Introduction

The Standard Model (SM) of particle physics is a gauge theory with a spontaneously broken, non-abelian gauge group. It describes all forces of nature except gravity up to the energies and precision of current experiments. The SM cannot be valid up to arbitrary high energies, because gravity has to be taken into account at the Planck scale M_{P} [1]. It is thus believed to be a low-energy effective description of a more fundamental theory. The low mass of the Higgs boson of ~ 125 GeV [2, 3] indicates that the SM is only valid up to energies much smaller than M_{P}, because it obtains quadratically divergent quantum corrections which are only small enough with a cutoff far below M_{P} [1]. This problem is known as the hierarchy problem of the SM and can be solved by introducing a new symmetry: supersymmetry.

Supersymmetry is the only way to extend the symmetry algebra of the S-matrix, the Poincaré algebra, to a larger graded Lie algebra [4]. It also solves the hierarchy problem by protecting the Higgs mass from large quantum corrections and minimal supersymmetric extensions of the SM (MSSM) offer additional attractive features like gauge coupling unification and possible candidates for dark matter [1].

One can incorporate gravity into a supersymmetric gauge theory by gauging the super-Poincaré spacetime symmetry of the theory. The resulting theories are called supergravity theories. Due to their coupling constant with negative mass dimension, most supergravity theories are non-renormalizable.[1] Today it is widely believed that a consistent quantum theory of gravity should be a superstring theory, but in the low energy limit, superstring theory reduces to supergravity, so there are still many applications for supergravity.

The generators of supersymmetry Q are anticommutating spinors that change the spin of a state by 1/2[2]

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle\ , \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle\ . \tag{1.1}$$

It is possible to introduce multiple supersymmetry generators. The number of generators is denoted by $\mathcal{N}$. $\mathcal{N} = 1$ corresponds to a minimal amount of supersymmetry and the case $\mathcal{N} > 1$ is called extended supersymmetry. When comparing the amount of supersymmetry between theories in different dimensions, one has to take into account that different dimensions have different spinor representations. One has to compare the number of real supercharges, which is $\mathcal{N}$ times the dimension of the spinor representation.

The reason why $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking matters is the following. On the one hand, most superstring theories live in 10 dimensions and have at least 16 (type I and heterotic) or 32 (type II) supercharges [6]. On the other hand, it is phenomenologically compelling to try and construct the SM out of a four-dimensional theory with $\mathcal{N} = 1$ supersymmetry, which corresponds to 4 supercharges. This is because theories

[1]The finiteness of $\mathcal{N} = 8$ supergravity is currently under discussion.
[2]For an introduction to supersymmetry, see [5].

with more supersymmetry are not chiral. It is possible to perform the compactification from 10D superstring theory to a 4D effective theory in such a way that some of the original supersymmetries are broken, but there are cases where it is not possible to compactify to 4 supercharges directly. For example, fluxless compactifications of type II string theory on a Calabi-Yau threefold result in an effective 4D theory with 8 supercharges, i.e. $\mathcal{N} = 2$ supergravity [7]. Therefore, it is of interest whether a four-dimensional $\mathcal{N} = 2$ supergravity can have Minkowski vacua with $\mathcal{N} = 1$ supersymmetry.

There is a long-standing no-go theorem [8, 9] stating that $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking is not possible. It was later found in [10] that the no-go theorem did not use the most general $\mathcal{N} = 2$ Lagrangian. $\mathcal{N} = 2$ supergravities can be constructed from one gravitational multiplet with spin content $(2, 3/2, 1)$ and any number of vector multiplets $(1, 1/2, 0)$ and hypermultiplets $(1/2, 0)$. The scalar fields can be considered as coordinates on a manifold, the so-called target or scalar manifold. The target manifold of the vector multiplets must be a special-Kähler manifold, which is typically described in terms of a holomorphic prepotential $\mathcal{F}$. This description was also used for the no-go theorem about partial supersymmetry breaking. As it turns out, there are cases where such a prepotential does not exist. These cases allowed for examples of partially broken $\mathcal{N} = 2$ supergravities, some of which were presented in [11–13] by introducing magnetic Fayet-Iliopoulos terms. It was observed in [14] that the requirement that no prepotential exists can be replaced by the requirement that there must be both electric and magnetic charges. In addition, two particular isometries $\hat{k}_1$ and $\hat{k}_2$ must be gauged. In [14], partial supersymmetry breaking in the presence of prepotentials and magnetic charges was treated systematically. The low energy effective $\mathcal{N} = 1$ action of the theory was constructed in [15]. In the construction of the $\mathcal{N} = 1$ theory, the scalar field space of the hypermultiplets $\mathbf{M}_{\mathrm{h}}$ descends to a submanifold $\hat{\mathbf{M}}_{\mathrm{h}}$ of the quotient with respect to the two gauged isometries

$$\mathbf{M}_{\mathrm{h}} \qquad \to \qquad \hat{\mathbf{M}}_{\mathrm{h}} \subset \mathbf{M}_{\mathrm{h}}/\langle \hat{k}_1, \hat{k}_2 \rangle \ . \tag{1.2}$$

This quotient construction was studied in more detail in [16].

In supergravities arising from type II string compactifications, $\mathbf{M}_{\mathrm{h}}$ must be a special quaternionic-Kähler manifold, which is described in terms of a second prepotential $\mathcal{G}$. The number of scalars that are stabilized by $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking depend on the prepotentials $\mathcal{F}$ and $\mathcal{G}$. These numbers were determined for certain choices of prepotentials in [16]. We continue this work, giving the $\mathcal{N} = 1$ Kähler potentials, superpotentials and D-terms for two of these examples, the STU and the quantum STU models, in a Minkowski background. We find that for the STU model, $\hat{\mathbf{M}}_{\mathrm{h}}$ is given by

$$\mathbf{M}_{\mathrm{h}} = \frac{SO(4,4)}{SO(4) \times SO(4)} \qquad \to \qquad \hat{\mathbf{M}}_{\mathrm{h}} = \frac{SO(4,2)}{SO(4) \times SO(2)} \ . \tag{1.3}$$

Another result is that STU and quantum STU prepotentials heavily restrict the superpotential. If both $\mathcal{F}$ and $\mathcal{G}$ are (quantum) STU prepotentials, the superpotential must vanish.

Furthermore, we check under which conditions supersymmetric vacua exist if the prepotentials are taken from these two examples or match the extreme cases where they stabilize either all or no scalars of the corresponding sector. These extreme cases correspond to a sufficiently generic and a purely quadratic prepotential. We find that there cannot be non-trivial superpotentials with supersymmetric vacua for any combination of the prepotentials mentioned in this paragraph, except when both prepotentials are quadratic.

This thesis is organized as follows. Section 2 gives a short introduction to four-dimensional gauged $\mathcal{N}=2$ supergravities. In section 3, we review the main results from [14,15], restricted to the Minkowski case. Section 4 specializes these results further to supergravities from type II string compactifications, which contain a special quaternionic-Kähler manifold. We introduce our two choices for the prepotential $\mathcal{G}$, the STU and the quantum STU models, in section 5. Section 6 treats the question in which way scalars are stabilized in the process of partial supersymmetry breaking. In sections 7 and 8, we calculate the explicit expressions for the Kähler potential, superpotential and D-terms of the $\mathcal{N}=1$ theory in the STU and quantum STU models. Section 9 determines under which conditions the $\mathcal{N}=1$ theory has supersymmetric vacua. In appendix A, we give an introduction to the symplectic invariance inherent to the vector multiplets. Appendices B, C and D supplement section 3 by more detailed reviews of calculations from [14, 15]. Finally, we list some symmetries that are inherent to the Kähler potential of the STU model in appendix E.

2 Supergravity

2.1 Ungauged $\mathcal{N}=2$ supergravity in four dimensions

Let us begin with a short recapitulation of four-dimensional $\mathcal{N}=2$ supergravity.[3] We will recall the spectrum and couplings of the theory and, as partial supersymmetry breaking is mainly described in terms of the scalar fields, describe the scalar field space in detail.

The theory has the following field content.

- a gravitational multiplet

$$(g_{\mu\nu}, \Psi_{\mu\mathcal{A}}, A^0_\mu), \quad \mu,\nu=0,\ldots,3, \quad \mathcal{A}=1,2\ . \tag{2.1}$$

  The gravitational multiplet contains the spacetime metric $g_{\mu\nu}$, two gravitini $\Psi_{\mu\mathcal{A}}$ and the graviphoton A^0_μ.

[3] For a review, see e.g. [17].

- $n_{\rm v}$ vector multiplets

$$(A^i_\mu, \lambda^{i\mathcal{A}}, t^i), \quad i = 1, \ldots, n_{\rm v} \ . \tag{2.2}$$

  Each vector multiplet contains a vector A_μ, two gaugini $\lambda^{\mathcal{A}}$ and a complex scalar t.

- $n_{\rm h}$ hypermultiplets

$$(\zeta_\alpha, q^u), \quad \alpha = 1, \ldots, 2n_{\rm h}, \quad u = 1, \ldots, 4n_{\rm h} \ . \tag{2.3}$$

  A hypermultiplet contains two hyperini ζ_α and 4 real scalars q^u.

For $n_{\rm v}$ vector- and $n_{\rm h}$ hypermultiplets, there are a total of $2n_{\rm v} + 4n_{\rm h}$ real scalar fields and $2(n_{\rm v} + n_{\rm h})$ spin-$\frac{1}{2}$ fermions in the spectrum.

The bosonic matter Lagrangian of the ungauged theory contains Yang-Mills terms with kinetic matrix $\mathcal{N}_{IJ}$ for the vectors and two sigma models for the scalars

$$\mathcal{L} \;=\; -\mathrm{i}\mathcal{N}_{IJ}\, F^{I+}_{\mu\nu} F^{\mu\nu\, J+} + \mathrm{i}\overline{\mathcal{N}}_{IJ}\, F^{I-}_{\mu\nu} F^{\mu\nu\, J-} + g_{i\bar{j}}(t,\bar{t})\, \partial_\mu t^i \partial^\mu \bar{t}^{\bar{j}} + h_{uv}(q)\, \partial_\mu q^u \partial^\mu q^v \ . \tag{2.4}$$

The matrices describing the sigma models are restricted by supersymmetry: $g_{i\bar{j}}(t,\bar{t})$ is the metric of the $2n_{\rm v}$-dimensional special-Kähler manifold $\mathbf{M}_{\rm v}$ and $h_{uv}(q)$ is the metric of the $4n_{\rm h}$-dimensional quaternionic-Kähler manifold $\mathbf{M}_{\rm h}$. The total scalar field space is the direct product

$$\mathbf{M} = \mathbf{M}_{\rm v} \times \mathbf{M}_{\rm h} \ . \tag{2.5}$$

The metric of the special-Kähler manifold is given by

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}^{\rm v} \ , \quad \text{with Kähler potential} \quad \mathcal{K}^{\rm v} = -\ln \mathrm{i}\left(X^I \bar{\mathcal{F}}_I - \bar{X}^I \mathcal{F}_I\right) \ . \tag{2.6}$$

X^I, $I = 1, \ldots, n_{\rm v} + 1$ are homogeneous coordinates on $\mathbf{M}_{\rm v}$ and $X^I(t)$, $\mathcal{F}_I(t)$ are both functions of the scalars of the vector multiplets t^i. In the ungauged case, one can always choose these coordinates such that the $\mathcal{F}_I$ are the derivatives $\mathcal{F}_I = \partial\mathcal{F}/\partial X^I$ of a holomorphic prepotential $\mathcal{F}(X)$, which must be homogeneous of degree two. Also, the convenient choice $X^I = (t^i, 1)$ is always possible. The t^i are called special coordinates in this context.

We will also need that on a Kähler manifold there exists a complex structure J and a fundamental form K, which are related by

$$K_{i\bar{j}} = g_{i\bar{k}} J^{\bar{k}}_{\bar{j}} \ . \tag{2.7}$$

K is also called the Kähler two-form.

The $F^{I\pm}_{\mu\nu}$ that appear in the Lagrangian (2.4) are the self-dual and anti-self-dual parts of the usual field strengths. They include the field strengths of the gauge bosons of the vector multiplets and the graviphoton. Their kinetic matrix $\mathcal{N}_{IJ}$ is a function of the t^i

given by

$$\mathcal{N}_{IJ} = \bar{\mathcal{F}}_{IJ} + 2\,\mathrm{i}\,\frac{\mathrm{Im}\mathcal{F}_{IK}\mathrm{Im}\mathcal{F}_{JL}X^K X^L}{\mathrm{Im}\mathcal{F}_{LK}X^K X^L}\ , \tag{2.8}$$

where $\mathcal{F}_{IJ} = \partial_I \mathcal{F}_J$.

A $4n_\mathrm{h}$-dimensional quaternionic-Kähler manifold like $\mathbf{M}_\mathrm{h}$ is a Riemannian manifold with holonomy group contained in $Sp(1) \times Sp(n_\mathrm{h})$. It has a set of three almost complex structures $J^x, x = 1, 2, 3$ that satisfy the quaternionic algebra

$$J^x J^y = -\delta^{xy}\mathbf{1} + \epsilon^{xyz} J^z\ . \tag{2.9}$$

The metric h_{uv} is hermitian with respect to all three almost complex structures. Supersymmetry requires the existence of a principal $SU(2)$-bundle $\mathcal{SU}$ over $\mathbf{M}_\mathrm{h}$ [18]. Denoting the connection on the bundle by ω^x, the curvature form

$$K^x = \mathrm{d}\omega^x + \tfrac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z\ , \tag{2.10}$$

on $\mathcal{SU}$ is the analogue to the Kähler form of a Kähler manifold by virtue of the fact that

$$K^x = K^x_{uv}\mathrm{d}q^u \wedge \mathrm{d}q^v = h_{uw}(J^x)^w_v \mathrm{d}q^u \wedge \mathrm{d}q^v\ . \tag{2.11}$$

The global invariance group of the ungauged Lagrangian (2.4) is embedded into a product of $Sp(2n_\mathrm{v}, \mathbb{R})$ and the group of isometries on the hypermultiplet scalar manifold $Iso(\mathbf{M}_\mathrm{h})$ [17]

$$G_\mathrm{global} \subseteq Sp(2n_\mathrm{v}, \mathbb{R}) \times Iso(\mathbf{M}_\mathrm{h})\ . \tag{2.12}$$

The reason why the group of isometries on the vector multiplet target manifold is embedded into $Sp(2n_\mathrm{v}, \mathbb{R})$ is explained in appendix A.

2.2 Gauged $\mathcal{N} = 2$ supergravity

It is characteristic for ungauged supergravities like discussed in the previous subsection, that they exhibit exceptionally large global symmetry groups and Abelian gauge groups [19]. The matter fields are not charged under the gauge group and there exists a maximally supersymmetric Minkowski ground state in which all fields are massless.

Gauged supergravities are the ones where some of the isometries on the scalar manifold are gauged. Gauging gives charges and possibly masses to matter fields, thus the vacuum is further restricted and can have less than maximal or no supersymmetry. This is the reason why gauged supergravity has to be considered for $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking.

The gauge group G_0 must be embedded into the global invariance group G_global of the Lagrangian. This embedding is governed by the embedding tensor, which describes the gauge group generators δ_Λ in terms of the generators of G_{global}, δ_λ

$$\delta_\Lambda = \Theta_\Lambda{}^\lambda \delta_\lambda \ . \tag{2.13}$$

The embedding tensor formalism was developed in [20,21] and is capable of describing all possible gaugings. We will gauge only subgroups of $Iso(\mathbf{M}_\mathrm{h})$ here. This results in Abelian gaugings [22] and is implemented by introducing covariant derivatives for the q^u

$$D_\mu q^u = \partial_\mu q^u - A_\mu{}^I\,\Theta_I{}^\lambda\,\hat{k}^u_\lambda + B_{\mu I}\,\Theta^{I\lambda}\,\hat{k}^u_\lambda \ . \tag{2.14}$$

$\hat{k}_\lambda(q)$ are Killing vectors on $\mathbf{M}_\mathrm{h}$, $A_\mu{}^I$ are electric vectors and $B_{\mu I}$ their magnetic duals, and $(A_\mu{}^I, B_{\mu I})$ transforms under $Sp(2n_\mathrm{v}, \mathbb{R})$. The Lagrangian retains its full symmetry as long as the embedding tensor is treated as a spurionic object, i.e. it transforms according to its indices. This means that the index Λ of the embedding tensor in (2.13) is a symplectic one and $\Theta_I{}^\lambda$ and $\Theta^{I\lambda}$, the electric and magnetic parts of the embedding tensor, are combined into the symplectic object

$$\Theta_\Lambda{}^\lambda = (\Theta_I{}^\lambda, -\Theta^{I\lambda}) \ . \tag{2.15}$$

Analogously, the index λ of the embedding tensor transforms under the action of $Iso(\mathbf{M}_\mathrm{h})$. A gauge can be chosen by fixing the embedding tensor to a constant and breaking the symmetry that was sustained by the embedding tensor as long as it behaved spurionically. In the Abelian case under consideration, the embedding tensor is only restricted by the condition that electric and magnetic charges are mutually local [14], which reads

$$\Theta^{I[\lambda}\Theta_I{}^{\kappa]} = 0 \ . \tag{2.16}$$

For partial supersymmetry breaking, both electric and magnetic charges are required, as we will see in section 3.1. The embedding tensor approach provides a coherent formalism to describe such general charge configurations and is therefore ideally suited to describe partial supersymmetry breaking.

The reason why charged scalars from the hypermultiplets are considered is the following. We will see in our discussion of partial supersymmetry breaking in section 3 that the eigenvalues of the gravitino mass matrix $S_{\mathcal{AB}}$ have to be non-degenerate, $m_{\Psi_1} \neq m_{\Psi_2}$. To this end, the $SU(2)$ R-symmetry has to be broken by gauging scalars that are charged under this symmetry. The vector multiplet scalars are $SU(2)$ singlets. Only the scalars of the hypermultiplets carry the R-charges of the supersymmetry generators Q and Q^+.

With the covariant derivatives (2.14), the action is no longer invariant under local supersymmetry. This is fixed by introducing the scalar potential V, which is given by [9,23]

$$V = -6 S_{\mathcal{AB}}\bar{S}^{\mathcal{AB}} + \frac{1}{2} g_{i\bar{\jmath}} W^{i\mathcal{AB}} W^{\bar{\jmath}}_{\mathcal{AB}} + N^{\mathcal{A}}_\alpha N^\alpha_{\mathcal{A}} \ . \tag{2.17}$$

$S_{\mathcal{AB}}$ is the mass matrix of the two gravitini. $W^{i\mathcal{AB}}$ and $N_\alpha^{\mathcal{A}}$ are related to the mass matrices of the spin-1/2 fermions. They are given by

$$\begin{aligned} S_{\mathcal{AB}} &= \tfrac{1}{2}\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2} V^\Lambda \Theta_\Lambda{}^\lambda P_\lambda^x (\sigma^x)_{\mathcal{A}}^{\ \mathcal{C}} \varepsilon_{\mathcal{CB}} \ , \\ W^{i\mathcal{AB}} &= \mathrm{i}\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2} g^{i\bar{\jmath}} (\nabla_{\bar{\jmath}} \bar{V}^\Lambda) \Theta_\Lambda{}^\lambda P_\lambda^x \varepsilon^{\mathcal{AC}} (\sigma^x)_{\mathcal{C}}^{\ \mathcal{B}} \ , \\ N_\alpha^{\mathcal{A}} &= 2\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2} \bar{V}^\Lambda \Theta_\Lambda{}^\lambda \mathcal{U}_{\alpha u}^{\mathcal{A}} \hat{k}_\lambda^u \ . \end{aligned} \tag{2.18}$$

V^Λ is a holomorphic symplectic vector defined by $V^\Lambda \equiv (X^I, \mathcal{F}_I)$. Its Kähler-covariant derivative is $\nabla_i V^\Lambda = \partial_i V^\Lambda + (\partial_i \mathcal{K}^{\mathrm{v}}) V^\Lambda$. $\varepsilon_{\mathcal{AB}}$ and $\varepsilon^{\mathcal{AB}}$ are the $SU(2)$ metric and its inverse and $(\sigma^x)_{\mathcal{B}}^{\mathcal{A}}$ are the standard Pauli matrices. The triplet of Killing prepotentials P_λ^x associated to an isometry $\hat{k}_\lambda^u$ on $\mathbf{M}_{\mathrm{h}}$ is defined by

$$-2\hat{k}_\lambda^u \, K_{uv}^x = \nabla_v P_\lambda^x = \partial_v P_\lambda^x + \epsilon^{xyz} \omega_v^y P_\lambda^z \ . \tag{2.19}$$

$\mathcal{U}_u^{\mathcal{A}\alpha}$ is the vielbein of $\mathbf{M}_{\mathrm{h}}$ and can be used to express the metric as

$$h_{uv} = \mathcal{U}_u^{\mathcal{A}\alpha} \varepsilon_{\mathcal{AB}} \mathcal{C}_{\alpha\beta} \mathcal{U}_v^{\mathcal{B}\beta} \ , \tag{2.20}$$

where $\mathcal{C}_{\alpha\beta}$ is the $Sp(n_{\mathrm{h}})$ invariant metric.

3 $\mathcal{N}=2$ to $\mathcal{N}=1$ supersymmetry breaking

3.1 Conditions from partial supersymmetry breaking

Partial supersymmetry breaking can be analyzed in terms of the scalar parts of the supersymmetry variations

$$\begin{aligned} \delta_\epsilon \Psi_{\mu\mathcal{A}} &= D_\mu \epsilon_{\mathcal{A}}^* - S_{\mathcal{AB}} \gamma_\mu \epsilon^{\mathcal{B}} + \dots \ , \\ \delta_\epsilon \lambda^{i\mathcal{A}} &= W^{i\mathcal{AB}} \epsilon_{\mathcal{B}} + \dots \ , \\ \delta_\epsilon \zeta_\alpha &= N_\alpha^{\mathcal{A}} \epsilon_{\mathcal{A}} + \dots \ . \end{aligned} \tag{3.1}$$

$\epsilon^{\mathcal{A}}$ is the $SU(2)$ doublet of spinors parametrising the $\mathcal{N}=2$ supersymmetry transformations. The ellipses indicate terms that vanish in a maximally symmetric ground state. $D_\mu \epsilon_{\mathcal{A}}^*$ vanishes in a Minkowski background. For supersymmetry to be spontaneously broken from $\mathcal{N}=2$ to $\mathcal{N}=1$, the supersymmetry variations (3.1) have to vanish for one linear combination of the supersymmetry variation parameters, say $\epsilon_1^{\mathcal{A}}$

$$W_{i\mathcal{AB}} \, \epsilon_1^{\mathcal{B}} = 0 \ , \qquad N_{\alpha\mathcal{A}} \, \epsilon_1^{\mathcal{A}} = 0 \qquad \text{and} \qquad S_{\mathcal{AB}} \, \epsilon_1^{\mathcal{B}} = 0 \ , \tag{3.2}$$

while the second generator $\epsilon_2^{\mathcal{A}}$ has to break supersymmetry, i.e. the supersymmetry variations are nonzero

$$W_{i\mathcal{A}\mathcal{B}}\,\epsilon_2^{\mathcal{B}} \neq 0 \qquad \text{or} \qquad N_{\alpha\mathcal{A}}\,\epsilon_2^{\mathcal{A}} \neq 0 \;, \qquad \text{and} \qquad S_{\mathcal{A}\mathcal{B}}\,\epsilon_2^{\mathcal{B}} \;\neq\; 0 \;. \tag{3.3}$$

The three conditions in (3.2) and (3.3) (in the order of their appearance) are called the gaugino, hyperino and gravitino conditions, according to the supersymmetry variations in (3.1) they stem from. The gaugings satisfying (3.2) and (3.3) were classified in [14]. We only give a short summary here and a more detailed recapitulation in appendix B. The gaugino and gravitino conditions of (3.2) and (3.3) imply

$$\begin{aligned} (\Theta_I{}^{\lambda} - \mathcal{F}_{IJ}\Theta^{J\lambda})\, P^x_\lambda \sigma^x_{\mathcal{A}\mathcal{B}}\epsilon_1^{\mathcal{B}} \;&=\; 0 \quad \text{for all } I \;, && (3.4)\\ (\Theta_I{}^{\lambda} - \bar{\mathcal{F}}_{IJ}\Theta^{J\lambda})\, P^x_\lambda \sigma^x_{\mathcal{A}\mathcal{B}}\epsilon_1^{\mathcal{B}} \;&\neq\; 0 \quad \text{for some } I \;. && (3.5) \end{aligned}$$

One immediately sees that (3.4) and (3.5) cannot be satisfied simultaneously, if there are no magnetic charges, $\Theta^{J\lambda} = 0$. They also cannot be both satisfied if only one isometry is gauged, because then (3.4) factorizes and the vanishing of either of the factors implies the vanishing of the left side of (3.5). Furthermore, (3.4) and (3.5) can only be satisfied for certain embedding tensors.

There is one condition left that must be enforced to make sure that the supersymmetry generated by $\epsilon_1^{\mathcal{A}}$ is unbroken, the hyperino condition in (3.2). A suitable set of Killing prepotentials to satisfy this condition is

$$P^3_{1,2} = 0 \;, \tag{3.6}$$

$$P^1_1 = -P^2_2 \;, \tag{3.7}$$

$$P^2_1 = P^1_2 \;. \tag{3.8}$$

To solve (3.4) and (3.5), the embedding tensor must take the form

$$\begin{aligned} \Theta_I{}^1 &= \mathrm{Re}\left(\mathcal{F}_{IJ}\, C^J\right) \;, \qquad & \Theta^{I1} &= \; \mathrm{Re}\, C^I \;, \\ \Theta_I{}^2 &= \mathrm{Im}\left(\mathcal{F}_{IJ}\, C^J\right) \;, \qquad & \Theta^{I2} &= \; \mathrm{Im}\, C^I \;, \end{aligned} \tag{3.9}$$

if (3.6-3.8) are assumed. C^I is an arbitrary complex vector. For the embedding tensor (3.9), the locality constraint (2.16) becomes a condition on the C^I

$$\bar{C}^I (\mathrm{Im}\,\mathcal{F})_{IJ} C^J = 0 \;. \tag{3.10}$$

This can easily be arranged, because $(\mathrm{Im}\,\mathcal{F})_{IJ}$ has signature $(n_{\mathrm{v}}, 1)$ [24].

Since the embedding tensor (3.9) has to be constant, a number of scalars may be stabilized in the $\mathcal{N}=1$ theory by the condition

$$\mathcal{F}_{IJ}C^J = \text{const.}\,. \tag{3.11}$$

This means that only a submanifold $\hat{\mathbf{M}}_{\rm v}$ of $\mathbf{M}_{\rm v}$ descends to the $\mathcal{N}=1$ theory. The condition (3.11) appears also in the hypermultiplet sector and will be discussed in that context. This discussion is in section 6.1 and applies without changes to (3.11).

This concludes our review of the requirements posed on a gauged supergravity by partial supersymmetry breaking. Both electric and magnetic charges and at least two gauged isometries are necessary to achieve partial supersymmetry breaking in a symplectic frame where a prepotential $\mathcal{F}$ exists. The gravitino and gaugino conditions only pose conditions on the charges, i.e. the embedding tensor. The hyperino condition can be solved by gauging isometries with Killing prepotentials satisfying (3.6-3.8). In this case, the embedding tensor takes the form (3.9).

3.2 Integrating out

One characteristic difference between $\mathcal{N}=2$ and $\mathcal{N}=1$ supergravity is that $\mathcal{N}=2$ supergravity has two gravitini, while in $\mathcal{N}=1$ supergravity there is only one. Indeed, the gravitino conditions in (3.2) and (3.3) imply that the eigenvalues of the gravitino mass matrix $S_{\mathcal{AB}}$ have to be non-degenerate, $m_{\Psi_1}=0\neq m_{\Psi_2}$. Thus, a low-energy effective theory valid up to the scale of partial supersymmetry breaking $m_{3/2}\equiv m_{\Psi_2}$ can be constructed by integrating out the heavy gravitino and all other particles of mass $\geq m_{3/2}$.

The unbroken $\mathcal{N}=1$ supersymmetry dictates that the heavy gravitino must be part of a $\mathcal{N}=1$ massive spin-3/2 multiplet with spin content $s=(3/2,1,1,1/2)$. This implies that there must be also two vectors and one spin-1/2 fermion of mass $m_{3/2}$. These fields have to be recruited from the massless $\mathcal{N}=2$ multiplets via a super-Higgs mechanism. The massive vectors consist of two massless vectors that eat one scalar each to acquire a longitudinal degree of freedom. The effect that integrating out the two massive vectors has on the scalar field space is discussed in section 3.3.1. The fermions do not affect the scalar field space and are not discussed further.

3.3 The $\mathcal{N}=1$ low-energy effective action

Integrating out the massive fields of $\mathcal{O}(m_{3/2})$ must lead to an effective $\mathcal{N}=1$ theory with a Lagrangian of the standard form [5, 25]

$$\hat{\mathcal{L}} = -\,K_{\hat{P}\hat{\bar{Q}}}D_\mu M^{\hat{P}}D^\mu\bar{M}^{\hat{\bar{Q}}} - \tfrac{1}{2}f_{\hat{I}\hat{J}}\,F^{\hat{I}-}_{\mu\nu}F^{\mu\nu\,\hat{J}-} - \tfrac{1}{2}\bar{f}_{\hat{I}\hat{J}}\,F^{\hat{I}+}_{\mu\nu}F^{\hat{J}+}_{\rho\sigma} - V\ , \tag{3.12}$$

where V is the scalar potential

$$V = V_F + V_{\mathcal{D}} = \mathrm{e}^{\mathcal{K}}\big(K^{\hat{P}\hat{\bar{Q}}} D_{\hat{P}}\mathcal{W} D_{\hat{\bar{Q}}}\bar{\mathcal{W}} - 3|\mathcal{W}|^2\big) + \tfrac{1}{2}\,(\mathrm{Re}\, f)_{\hat{I}\hat{J}}\mathcal{D}^{\hat{I}}\mathcal{D}^{\hat{J}}\ . \qquad (3.13)$$

$\hat{P}$ and $\hat{\bar{Q}}$ enumerate all scalars $M^{\hat{P}}$ in the theory, i.e. the ones originating from the $\mathcal{N}=2$ vector and hypermultiplet sectors. $\hat{I}$ and $\hat{J}$ run only over the scalars from the vector multiplets. The hats are meant to indicate $\mathcal{N}=1$ fields. $F^{\hat{I}+}_{\mu\nu}$ and $F^{\hat{I}-}_{\mu\nu}$ denote the self-dual and anti-self-dual $\mathcal{N}=1$ gauge field strengths. $f_{\hat{I}\hat{J}}$ is the holomorphic gauge kinetic function and given in (D.18). $\mathcal{K}$ and $K_{\hat{P}\hat{\bar{Q}}} = \partial_{\hat{P}}\bar{\partial}_{\hat{\bar{Q}}}\mathcal{K}$ are the Kähler potential and Kähler metric of the scalar field space, which is the direct product of two Kähler manifolds $\hat{\mathbf{M}}_{\mathrm{h}}$ and $\hat{\mathbf{M}}_{\mathrm{v}}$ descending from $\mathbf{M}_{\mathrm{h}}$ and $\mathbf{M}_{\mathrm{v}}$

$$\mathbf{M}^{\mathcal{N}=1} = \hat{\mathbf{M}}_{\mathrm{h}} \times \hat{\mathbf{M}}_{\mathrm{v}}\ . \qquad (3.14)$$

While $\hat{\mathbf{M}}_{\mathrm{v}}$ is determined by (3.11), we will discuss $\hat{\mathbf{M}}_{\mathrm{h}}$ in the following subsection 3.3.1. The $\mathcal{N}=1$ Kähler potential $\mathcal{K}$ is the sum of the Kähler potentials on $\hat{\mathbf{M}}_{\mathrm{h}}$ and $\hat{\mathbf{M}}_{\mathrm{v}}$, $\hat{\mathcal{K}}$ and $\mathcal{K}^{\mathrm{v}}$

$$\mathcal{K} = \hat{\mathcal{K}} + \mathcal{K}^{\mathrm{v}}\ . \qquad (3.15)$$

$\mathcal{W}$ is the superpotential and $D_{\hat{P}}\mathcal{W} = \partial_{\hat{P}}\mathcal{W} + (\partial_{\hat{P}}\mathcal{K})\mathcal{W}$ its Kähler-covariant derivative. $\mathcal{D}^{\hat{I}}$ are the D-terms. These terms will be discussed in more detail in the subsections 3.3.2 and 3.3.3. We will see that the superpotential and the D-terms vanish, if only the two isometries required for partial supersymmetry breaking are gauged. To get a nontrivial scalar potential, additional isometries have to be gauged at a scale $\tilde{m}$ below $m_{3/2}$. Additional Killing vectors either preserve the full $\mathcal{N}=2$ supersymmetry or break supersymmetry completely [14]. If they break supersymmetry, this breaking can be neglected if $\tilde{m} \ll m_{3/2}$ is assumed.

3.3.1 Quotient construction and Kähler potential

Integrating out the two heavy vector bosons of mass $m_{3/2}$ amounts to taking the quotient of $\mathbf{M}_{\mathrm{h}}$ with respect to the two gauged isometries $\hat{k}_1$ and $\hat{k}_2$. This was shown in [15]. Appendix C contains a review of this derivation and this section contains a short summary. When the two heavy gauge bosons are integrated out, the covariant derivatives in the kinetic term of the q^u in the Lagrangian (2.4) become partial derivatives, while the metric is modified

$$h_{uv}\, D_\mu q^u D^\mu q^v \quad \to \quad \hat{h}_{uv}\partial_\mu q^u \partial^\mu q^v\ . \qquad (3.16)$$

$\hat{h}_{uv}$ is the metric on $\mathbf{M}_{\mathrm{h}}/\langle \hat{k}_1, \hat{k}_2\rangle$, which is a Kähler manifold with metric, Kähler form and complex structure given by projections of objects related to the quaternionic-Kähler manifold $\mathbf{M}_{\mathrm{h}}$

$$\hat{h}_{uv} = \tilde{\pi}^w_u h_{wv}\ , \qquad \hat{K}_{uv} = \tilde{\pi}^w_u K^3_{wv}\ , \qquad \hat{J}^u_v = \tilde{\pi}^u_w J^{3w}_v\ . \qquad (3.17)$$

The projector $\tilde{\pi}^u_v$ is

$$\tilde{\pi}^u_v \equiv \delta^u_v - \frac{2\hat{k}^u_\lambda \hat{k}_{\lambda v}}{m^2} \ , \tag{3.18}$$

with

$$m^2 = 2\hat{k}^u_1 \hat{k}_{1u} = 2\hat{k}^u_2 \hat{k}_{2u} \ . \tag{3.19}$$

This projector projects onto the directions orthogonal to $\hat{k}_1$ and $\hat{k}_2$ and the two gauged isometries are annihilated by the metric and the Kähler two-form of $\hat{\mathbf{M}}_{\mathrm{h}}$.

$$\hat{h}_{uv}\hat{k}^v_{1,2} = 0 \ , \qquad \hat{k}^u_{1,2}\hat{K}_{uv} = 0 \ . \tag{3.20}$$

$\hat{\mathbf{M}}_{\mathrm{h}}$ is a quotient with respect to the gauged isometries $\mathbf{M}_{\mathrm{h}}/\langle \hat{k}_1, \hat{k}_2 \rangle$, or rather, since some of the scalars are integrated out themselves, a submanifold thereof

$$\hat{\mathbf{M}}_{\mathrm{h}} \subset \mathbf{M}_{\mathrm{h}}/\langle \hat{k}_1, \hat{k}_2 \rangle \ . \tag{3.21}$$

$\hat{K}$ is the differential of the connection ω^3, which appeared first in (2.10)

$$\hat{K} = \mathrm{d}\omega^3 \ . \tag{3.22}$$

Furthermore, the Kähler potential $\hat{\mathcal{K}}$ of $\hat{\mathbf{M}}_{\mathrm{h}}$ is defined by

$$\hat{K}_{s\bar{t}} = \mathrm{i}\, \partial_s \bar{\partial}_t \hat{\mathcal{K}} \ . \tag{3.23}$$

Since a Kähler two-form has no $(2,0)$ and $(0,2)$ parts, (3.22) reads in components

$$\hat{K}_{s\bar{t}} = \partial_s \bar{\omega}^3_{\bar{t}} - \bar{\partial}_{\bar{t}} \omega^3_s \ , \tag{3.24}$$

where ω^3_s and $\bar{\omega}^3_{\bar{s}}$ are the holomorphic and anti-holomorphic parts of ω^3. By comparing (3.23) and (3.24), one finds

$$\omega^3_s = -\tfrac{\mathrm{i}}{2}\partial_s \hat{\mathcal{K}} \ , \qquad \bar{\omega}^3_{\bar{s}} = \tfrac{\mathrm{i}}{2}\bar{\partial}_{\bar{s}} \hat{\mathcal{K}} \ . \tag{3.25}$$

3.3.2 The superpotential

The superpotential $\mathcal{W}$ appears in the $\mathcal{N}=1$ gravitino variations, so it can be obtained by comparing the remaining supersymmetry variation of (3.1) for ϵ_1 with the standard $\mathcal{N}=1$ variations [5]

$$\delta_\epsilon \Psi_{\mu\,1} = D_\mu \epsilon - S_{11}\gamma_\mu \bar{\epsilon} + \ldots \ = \ D_\mu \epsilon - \tfrac{1}{2} \mathrm{e}^{\frac{1}{2}\mathcal{K}^{\mathcal{N}=1}} \mathcal{W} \gamma_\mu \bar{\epsilon} + \ldots \ . \tag{3.26}$$

In this way, the following superpotential is found [15]

$$\mathcal{W} = 2\mathrm{e}^{-\frac{1}{2}\mathcal{K}^{\mathcal{N}=1}} S_{11} \;=\; \mathrm{e}^{-\hat{\mathcal{K}}/2} V^{\Lambda} \Theta_{\Lambda}{}^{\lambda} P_{\lambda}^{-} \;, \tag{3.27}$$

where $P_\lambda^- \equiv P_\lambda^1 - \mathrm{i}\, P_\lambda^2$. Since the superpotential appears in the $\mathcal{N} = 1$ supersymmetry variations, gaugings that lead to a non-vanishing superpotential break the remaining supersymmetry. The gauged isometries $\hat{k}_1$ and $\hat{k}_2$ are constructed to preserve this supersymmetry and thus do not contribute to the superpotential, i.e. the indices λ in (3.27) only run over $\lambda = 3, \dots, n$. In order to retain a $\mathcal{N} = 1$ supersymmetric theory, the gaugings $\hat{k}_\lambda,\ \lambda = 3, \dots, n$ are chosen to be at a scale $\tilde{m}$ much smaller than the partial breaking scale $m_{3/2}$.

3.3.3 D-terms

The D-terms appear in the $\mathcal{N} = 1$ gaugino variations and were found similarly to the superpotential, by comparing the $\mathcal{N} = 1$ variations with the expressions for $\mathcal{N} = 2$. Their derivation is reviewed in appendix D and the result is [15]

$$\mathcal{D}^{\hat{I}} = -\Pi^I_J \Gamma^J_K (\mathrm{Im}\, \mathcal{F})^{-1\ KL} \left(\Theta_L{}^{\lambda} - \bar{\mathcal{F}}_{LM} \Theta^{M\lambda} \right) P^3_\lambda \;. \tag{3.28}$$

Π^I_J and Γ^J_K are defined in (D.11) and (D.14). They are projectors that act on the field strengths to project out the heavy gauge bosons. As discussed in the previous section for the superpotential, only supersymmetry breaking gaugings give contributions to the D-terms and the λ in (3.28) only runs over $\lambda = 3, \dots, n$, as the other terms vanish.

4 Special quaternionic-Kähler manifolds

In the examples we want to study, $\mathbf{M}_\mathrm{h}$ is further restricted to be a special quaternionic-Kähler manifold. That means that it contains a $(2n_\mathrm{h} - 2)$-dimensional special-Kähler manifold $\mathbf{M}_\mathrm{sk}$ that has, like $\mathbf{M}_\mathrm{v}$, complex coordinates z^a, $a = 1, \dots, n_\mathrm{h} - 1$ and homogeneous coordinates $Z^A = (z^a, 1)$, $A = 1, \dots, n_\mathrm{h}$, a prepotential $\mathcal{G}(Z)$ that is homogeneous of degree two and a Kähler potential $\mathcal{K}^\mathrm{h}$

$$\mathcal{K}^\mathrm{h} = -\ln \mathrm{i} \left(Z^A \bar{\mathcal{G}}_A - \bar{Z}^A \mathcal{G}_A \right) \;. \tag{4.1}$$

Indices of $\mathcal{G}$ are derivatives $\mathcal{G}_A = \partial_A \mathcal{G}$ and we define $N_{AB} \equiv \mathrm{Im}\, \mathcal{G}_{AB}$ for later use. The remaining scalars are the $2n_\mathrm{h} + 2$ real fields $\phi, \tilde{\phi}, \xi^A, \tilde{\xi}_A$.[4] The construction of the metric on $\mathbf{M}_\mathrm{h}$ is called the c-map [26, 27].

We will use the following parametrization of the quaternionic vielbein $\mathcal{U}^{\mathcal{A}\alpha}$ of (2.20).

[4]In the context of type II string theory, ϕ corresponds to the dilaton, $\tilde{\phi}$ to the axion and $\xi^A, \tilde{\xi}_A$ are Ramond-Ramond scalars.

It was found in [27] and reads

$$\mathcal{U}^{\mathcal{A}\alpha} = \mathcal{U}_u^{\mathcal{A}\alpha} \mathrm{d}q^u = \tfrac{1}{\sqrt{2}} \begin{pmatrix} \bar{u} & \bar{e} & -v & -E \\ \bar{v} & \bar{E} & u & e \end{pmatrix} , \tag{4.2}$$

with the one-forms

$$\begin{aligned} u &= \mathrm{i}\, \mathrm{e}^{\mathcal{K}^{\mathrm{h}}/2+\phi} Z^A (\mathrm{d}\tilde{\xi}_A - \mathcal{M}_{AB} \mathrm{d}\xi^B) , \\ v &= \tfrac{1}{2} \mathrm{e}^{2\phi} \big[\mathrm{d}\mathrm{e}^{-2\phi} - \mathrm{i}(\mathrm{d}\tilde{\phi} + \tilde{\xi}_A \mathrm{d}\xi^A - \xi^A \mathrm{d}\tilde{\xi}_A) \big] , \\ E^{\underline{b}} &= -\tfrac{\mathrm{i}}{2} \mathrm{e}^{\phi - \mathcal{K}^{\mathrm{h}}/2} \Pi_A{}^{\underline{b}} N^{-1\,AB} (\mathrm{d}\tilde{\xi}_B - \mathcal{M}_{BC} \mathrm{d}\xi^C) , \\ \mathrm{e}^{\underline{b}} &= \Pi_A{}^{\underline{b}} \mathrm{d}Z^A . \end{aligned} \tag{4.3}$$

$\Pi_A{}^{\underline{b}} = (-e_a{}^{\underline{b}} Z^a, e_a{}^{\underline{b}})$ is defined using the vielbein $e_a{}^{\underline{b}}$ on $\mathbf{M}_{\mathrm{sk}}$ and $\mathcal{M}_{AB}$ is defined in terms of $(Z^A, \mathcal{G}_A)$ just as $\mathcal{N}_{IJ}$ is in terms of $(X^I, \mathcal{F}_I)$ in (2.8)

$$\mathcal{M}_{AB} = \bar{\mathcal{G}}_{AB} + 2\,\mathrm{i}\, \frac{N_{AC} N_{BD} Z^C Z^D}{N_{DC} Z^C Z^D} . \tag{4.4}$$

The metric on $\mathbf{M}_{\mathrm{h}}$ is

$$h = \big[v \otimes \bar{v} + u \otimes \bar{u} + E \otimes \bar{E} + e \otimes \bar{e} \big]_{\mathrm{sym}} . \tag{4.5}$$

In the most explicit form, the term in the Lagrangian (2.4) containing the metric reads [28]

$$\begin{aligned} h_{uv}(q)\, \partial_\mu q^u \partial^\mu q^v \;=\; & -(\partial\phi)^2 - e^{4\phi} (\partial\tilde{\phi} + \tilde{\xi}_A \partial\xi^A - \xi^A \partial\tilde{\xi}_A)^2 + g_{a\bar{b}} \partial z^a \partial \bar{z}^{\bar{b}} \\ & + e^{2\phi} \mathrm{Im}\mathcal{M}^{AB} (\partial\tilde{\xi} - \mathcal{M}\partial\xi)_A \overline{(\partial\tilde{\xi} - \mathcal{M}\partial\xi)}_B , \end{aligned} \tag{4.6}$$

where $g_{a\bar{b}}$ is the metric on $\mathbf{M}_{\mathrm{sk}}$.

The coordinates $(\phi, \tilde{\phi}, \xi^A, \tilde{\xi}_A)$ define a G-bundle over $\mathbf{M}_{\mathrm{sk}}$, where G is the semidirect product of a $(2n_{\mathrm{h}}+1)$-dimensional Heisenberg group with $\mathbb{R}$. This implies that the metric of $\mathbf{M}_{\mathrm{h}}$ has $(2n_{\mathrm{h}}+2)$ isometries generated by the Killing vectors

$$\begin{aligned} k_\phi &= \tfrac{1}{2} \frac{\partial}{\partial\phi} - \tilde{\phi} \frac{\partial}{\partial\tilde{\phi}} - \tfrac{1}{2} \xi^A \frac{\partial}{\partial\xi^A} - \tfrac{1}{2} \tilde{\xi}_A \frac{\partial}{\partial\tilde{\xi}_A} , \\ k_{\tilde{\phi}} &= -2 \frac{\partial}{\partial\tilde{\phi}} , \\ k_A &= \frac{\partial}{\partial\xi^A} + \tilde{\xi}_A \frac{\partial}{\partial\tilde{\phi}} , \\ \tilde{k}^A &= \frac{\partial}{\partial\tilde{\xi}_A} - \xi^A \frac{\partial}{\partial\tilde{\phi}} . \end{aligned} \tag{4.7}$$

They act transitively on the G-fiber coordinates and the subset $\{k_A, \tilde{k}^A, k_{\tilde{\phi}}\}$ spans a

Heisenberg algebra which is graded with respect to k_ϕ. The commutation relations are

$$\begin{aligned} [k_\phi, k_{\tilde\phi}] &= k_{\tilde\phi}\ , & [k_\phi, k_A] &= \tfrac{1}{2} k_A\ , \\ [k_\phi, \tilde k^A] &= \tfrac{1}{2}\tilde k^A\ , & [k_A, \tilde k^B] &= -\delta_A^B k_{\tilde\phi}\ ; \end{aligned} \tag{4.8}$$

all other commutators vanish. The $SU(2)$ connections ω^x are given by

$$\begin{aligned} \omega^1 &= \mathrm{i}(\bar u - u)\ , \qquad \omega^2 = u + \bar u\ , \\ \omega^3 &= \tfrac{\mathrm{i}}{2}(v - \bar v) - \mathrm{i}\, \mathrm{e}^{\mathcal{K}^{\mathrm{h}}} \left(Z^A (\operatorname{Im} N_{AB} \mathrm{d}\bar Z^B - \bar Z^A N_{AB} \mathrm{d} Z^B \right)\ . \end{aligned} \tag{4.9}$$

The Killing prepotentials P^x_λ of a special quaternionic-Kähler manifold take the simple form [29]

$$P^x_\lambda = \omega^x_u k^u_\lambda\ . \tag{4.10}$$

4.1 Killing vectors satisfying the hyperino condition

We will now review the construction of two commuting Killing vectors $\hat k_1, \hat k_2$ that fulfill the requirements for partial supersymmetry breaking in terms of the basis Killing vectors (4.7). It was shown in section 3.1, that the gravitino and gaugino conditions in (3.2) and (3.3) are fulfilled by choosing an appropriate embedding tensor, but the hyperino conditions pose the additional constraints (3.6-3.8) on the gauged isometries.

The gauged isometries are enumerated with the index $\lambda = 1, 2$ and the ansatz for expressing them in terms of the basis (4.7) is

$$\hat k_\lambda = r_\lambda{}^A k_A + s_{\lambda B} \tilde k^B + t_\lambda k_{\tilde\phi} + l_\lambda k_\phi\ , \tag{4.11}$$

with real parameters $r_\lambda{}^A, s_{\lambda B}, t_\lambda, l_\lambda$. We use the commutation relations (4.8) to calculate the commutator of the two Killing vectors

$$[\hat k_1, \hat k_2] = \left(r_{[2}{}^A s_{1]A} - l_{[2} t_{1]} \right) k_{\tilde\phi} - \frac{1}{2} l_{[2} r_{1]}{}^A k_A - \frac{1}{2} l_{[2} s_{1]B} \tilde k^B\ , \tag{4.12}$$

so a vanishing commutator requires

$$\begin{aligned} 0 &= r_{[2}{}^A s_{1]A} - l_{[2} t_{1]}\ , \\ 0 &= l_{[2} r_{1]}{}^A\ , \\ 0 &= l_{[2} s_{1]B}\ . \end{aligned} \tag{4.13}$$

If we assume $l_2 \neq 0$, (4.13) implies that $\hat k_1$ and $\hat k_2$ are equal up to rescaling, $\hat k_1 = \frac{l_1}{l_2} \hat k_2$. So it is only possible to construct two linearly independent commuting Killing vectors for

$$l_\lambda = 0\ . \tag{4.14}$$

The commutation conditions (4.13) reduce to

$$0 = r_{[\lambda}{}^{A} s_{\rho]A} \ , \tag{4.15}$$

with $\lambda, \rho = 1, 2$. Equation (4.15) is analogous to the locality constraint (2.16) of the embedding tensor, which emerges in the vector multiplet sector.

Using (4.3), (4.9), (4.10) and the fact that the Killing vectors only have components in the fiber directions, the first condition on the Killing prepotentials (3.6) becomes

$$0 = \tfrac{\mathrm{i}}{2}(v - \bar{v})\hat{k}_{1,2} = \tilde{\xi}_A r_{1,2}^{\ A} - \xi^B s_{1,2B} - t_{1,2} \ . \tag{4.16}$$

Similarly, the conditions (3.7) and (3.8) read

$$\begin{aligned} \mathrm{i}(\bar{u} - u)\hat{k}_1 &= -(u + \bar{u})\hat{k}_2 \ , \\ (u + \bar{u})\hat{k}_1 &= \mathrm{i}(\bar{u} - u)\hat{k}_2 \ . \end{aligned} \tag{4.17}$$

These equations can be rearranged into

$$\begin{aligned} u(\hat{k}_1 + \mathrm{i}\,\hat{k}_2) &= \bar{u}(\hat{k}_1 - \mathrm{i}\,\hat{k}_2) \ , \\ u(\hat{k}_1 + \mathrm{i}\,\hat{k}_2) &= -\bar{u}(\hat{k}_1 - \mathrm{i}\,\hat{k}_2) \ , \end{aligned} \tag{4.18}$$

or equivalently

$$0 = u(\hat{k}_1 + \mathrm{i}\,\hat{k}_2) \ . \tag{4.19}$$

We insert (4.3), (4.11) and (4.7) into (4.19) to obtain

$$0 = \mathrm{i}\, \mathrm{e}^{\mathcal{K}^{\mathrm{h}}/2+\phi} Z^A \left(-\mathcal{G}_{AB}(r_1{}^{B} + \mathrm{i}\, r_2{}^{B}) + (s_{1A} + \mathrm{i}\, s_{2A}) \right) \ , \tag{4.20}$$

where we used the identity

$$Z^A \mathcal{M}_{AB} = Z^A \mathcal{G}_{AB} \ , \tag{4.21}$$

that follows directly from the definition of $\mathcal{M}_{AB}$ (4.4). The solutions of (4.20) can be parametrized in terms of a complex vector D^A

$$\begin{aligned} D^A &= r_1{}^{A} + \mathrm{i}\, r_2{}^{A} \ , \\ D^A \mathcal{G}_{AB} &= s_{1B} + \mathrm{i}\, s_{2B} \ . \end{aligned} \tag{4.22}$$

To summarize (4.16) and (4.22), the two Killing vectors required for partial supersymmetry breaking are

$$\begin{aligned} \hat{k}_1 &= \mathrm{Re}\, D^A k_A + \mathrm{Re}(D^A \mathcal{G}_{AB})\tilde{k}^B + \mathrm{Re}\left(D^A(\tilde{\xi}_A - \mathcal{G}_{AB}\xi^B) \right) k_{\tilde{\phi}} \ , \\ \hat{k}_2 &= \mathrm{Im}\, D^A k_A + \mathrm{Im}(D^A \mathcal{G}_{AB})\tilde{k}^B + \mathrm{Im}\left(D^A(\tilde{\xi}_A - \mathcal{G}_{AB}\xi^B) \right) k_{\tilde{\phi}} \ . \end{aligned} \tag{4.23}$$

The same expressions were derived in [14] starting from the hyperino condition from (3.2) directly. Here, we took a shortcut by starting from a set of Killing prepotentials that is known to fulfill this hyperino condition. Inserting (4.22) into the condition for commuting Killing vectors (4.15) yields

$$0 = \bar{D}^A N_{AB} D^B \ . \tag{4.24}$$

So the vector D^A has to be null with respect to the matrix N_{AB}, analogous to (3.10).

5 Example prepotentials for the hypermultiplets

In the remainder of this thesis, we will consider some examples of special quaternionic-Kähler manifolds as scalar field spaces for the hypermultiplets. To this end, we specify the prepotential $\mathcal{G}$. We will consider some of the prepotentials given in [16] and use this section to introduce them.

5.1 *STU* model

The first example suggested in [16] is the STU (here $Z^1Z^2Z^3$) prepotential

$$\mathcal{G} = \frac{Z^1Z^2Z^3}{Z^4} \ . \tag{5.1}$$

$(Z^1, Z^2, Z^3, Z^4 = 1)$ are holomorphic coordinates on the special-Kähler manifold [16, 26]

$$\mathbf{M}_{\text{sk}} = \left(\frac{SU(1,1)}{U(1)}\right)^3 \ , \tag{5.2}$$

which is mapped by the c-map to the 16 dimensional symmetric space [16, 26]

$$\mathbf{M}_{\text{h}} = \frac{SO_0(4,4)}{SO(4) \times SO(4)} \ , \tag{5.3}$$

which contains $\mathbf{M}_{\text{sk}}$ as base space and a fiber with real coordinates $(\phi, \tilde{\phi}, \xi^A, \tilde{\xi}_A), A = 1, \ldots, 4$ as introduced in section 4.

The Hessian of the STU prepotential (5.1) will be used on multiple occasions, so we state it here

$$\mathcal{G}_{AB}|_{Z^4=1} = \begin{pmatrix} 0 & Z^3 & Z^2 & -Z^2Z^3 \\ Z^3 & 0 & Z^1 & -Z^1Z^3 \\ Z^2 & Z^1 & 0 & -Z^1Z^2 \\ -Z^2Z^3 & -Z^1Z^3 & -Z^1Z^2 & 2Z^1Z^2Z^3 \end{pmatrix} \ . \tag{5.4}$$

5.2 Quantum STU model

The quantum STU model is defined by the prepotential

$$\mathcal{G} = \frac{Z^1 Z^2 Z^3}{Z^4} + \frac{\alpha}{3}\frac{(Z^2)^3}{Z^4}, \qquad \alpha \in \mathbb{R} . \tag{5.5}$$

Its Hessian contains an additional term compared to the expression from the unperturbed STU model $\mathcal{G}_{AB}(\alpha = 0)$ given in (5.4)

$$\mathcal{G}_{AB}|_{Z^4=1} = \mathcal{G}_{AB}(\alpha = 0) + \alpha \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 2Z^2 & 0 & -(Z^2)^2 \\ 0 & 0 & 0 & 0 \\ 0 & -(Z^2)^2 & 0 & \frac{2}{3}(Z^2)^3 \end{pmatrix} . \tag{5.6}$$

6 Fixed scalars in the $\mathcal{N} = 1$ theory

In this section, we review the explicit construction of $\hat{\mathbf{M}}_{\mathrm{h}}$, which is the part of the $\mathcal{N} = 1$ scalar field space descending from $\mathbf{M}_{\mathrm{h}}$ when the heavy gravitino multiplet is integrated out.

For $\hat{k}_1$ and $\hat{k}_2$ to be Killing vectors, all prefactors in (4.23) have to be constant, i.e.

$$D^A = \text{const.} , \tag{6.1}$$

$$D^A \mathcal{G}_{AB} = \text{const.} , \tag{6.2}$$

$$D^A(\tilde{\xi}_A - \mathcal{G}_{AB}\xi^B) = \text{const.} . \tag{6.3}$$

The first condition (6.1) is trivially satisfied, but (6.2) may fix base coordinates and is discussed in section 6.1. The third condition (6.3) fixes two of the real coordinates on the fiber. Another two real fiber coordinates must be fixed by the quotient construction introduced in section 3.3.1. A way to satisfy both (6.3) and the quotient construction is discussed in section 6.2.

6.1 Base coordinates

We will discuss in this section how (6.2) and the choice of the prepotential $\mathcal{G}$ determine the fixing of scalars on the base manifold $\mathbf{M}_{\mathrm{sk}}$. The condition (6.2) does not imply that all fields appearing in $\mathcal{G}_{AB}$ are fixed, but rather that variations of these fields have to leave $D^A\mathcal{G}_{AB}$ invariant. This can be expressed by expanding (6.2) around any base point Z_0 for small variations $\delta Z \equiv Z - Z_0$

$$\mathcal{G}_{AB}D^B = \mathcal{G}_{AB}D^B|_{Z=Z_0} + \mathcal{G}_{ABC}D^B|_{Z=Z_0}\delta Z^C . \tag{6.4}$$

This implies that (6.2) is equivalent to [15]

$$\mathcal{G}_{ABC} D^B \delta Z^C = 0 \ . \tag{6.5}$$

It was pointed out in [16] that (6.5) freezes $\mathrm{rk}(\mathcal{G}_{ABC} D^B)$ of the $n_\mathrm{h} - 1$ complex coordinates on $\mathbf{M}_\mathrm{sk}$. The homogeneity of the prepotential implies $\mathcal{G}_{ABC} Z^C = 0$, so Z is a null eigenvector of the $n_\mathrm{h} \times n_\mathrm{h}$ matrix $\mathcal{G}_{ABC} D^B$, which therefore cannot have full rank

$$\mathrm{rk}(\mathcal{G}_{ABC} D^B) \leq n_\mathrm{h} - 1 \ . \tag{6.6}$$

We will denote the number of complex coordinates on $\mathbf{M}_\mathrm{sk}$ remaining free in the $\mathcal{N} = 1$ theory by $\hat{n}_\mathrm{h}$

$$\hat{n}_\mathrm{h} \equiv n_\mathrm{h} - 1 - \mathrm{rk}(\mathcal{G}_{ABC} D^B) \ . \tag{6.7}$$

Table 6.1 shows these numbers for some prepotentials. They are commented on in the following subsections.

$\mathcal{G}$	$\mathrm{rk}(\mathcal{G}_{ABC} D^B)$	$\hat{n}_\mathrm{h}$
generic	$n_\mathrm{h} - 1$	0
quadratic	0	$n_\mathrm{h} - 1$
(quantum) STU	2	1

Table 6.1: Number of base coordinates descending to $\hat{\mathbf{M}}_\mathrm{h}$ for some prepotentials

Note that the only equations that enter into this discussion are (6.2), (4.24) and (4.1). That implies that this whole section 6.1 also applies to the construction of the scalar field space descending from the vector multiplet sector, $\hat{\mathbf{M}}_\mathrm{v}$, because it is determined by the analogous equations (3.11), (3.10) and (2.6). We introduce $\hat{n}_\mathrm{v} \equiv \dim_\mathbb{C} \hat{\mathbf{M}}_\mathrm{v}$ for later use. To use table 6.1 for the vector multiplets, one has to substitute

$$\mathcal{G} \to \mathcal{F} \ , \qquad D^B \to C^J \ , \qquad n_\mathrm{h} - 1 \to n_\mathrm{v} \ , \qquad \hat{n}_\mathrm{h} \to \hat{n}_\mathrm{v} \ . \tag{6.8}$$

6.1.1 Generic prepotential

With generic prepotential, we mean a prepotential which is not specified in detail, but which is generic enough for (6.2) to fix all $n_\mathrm{h} - 1$ fields z^a.

6.1.2 Quadratic prepotential

If $\mathcal{G}$ is a quadratic function, its second derivatives $\mathcal{G}_{AB}$ are constant and (6.2) is trivially satisfied without fixing any fields.

6.1.3 *STU* model

The condition on the base coordinates (6.2) reads for a *STU* prepotential (5.1) [16]

$$
\begin{aligned}
(D^4 Z^2 - D^2)(D^4 Z^3 - D^3) &= \text{const.} \;, && (6.9)\\
(D^4 Z^1 - D^1)(D^4 Z^3 - D^3) &= \text{const.} \;, && (6.10)\\
(D^4 Z^1 - D^1)(D^4 Z^2 - D^2) &= \text{const.} \;, && (6.11)\\
2D^4 Z^1 Z^2 Z^3 - D^1 Z^2 Z^3 - D^2 Z^1 Z^3 - D^3 Z^1 Z^2 &= \text{const.} \;. && (6.12)
\end{aligned}
$$

If two of the three brackets appearing in (6.9-6.11) are set to zero, all four conditions are satisfied and the coordinate appearing in the third bracket remains free, i.e. $\hat{n}_\mathrm{h} = 1$. We will make this choice throughout this thesis, because the other case $\hat{n}_\mathrm{h} = 0$ is covered by the considerations about generic prepotentials. If Z^3 is chosen to be the remaining free coordinate, the conditions are

$$Z^1 = \frac{D^1}{D^4} \;, \qquad Z^2 = \frac{D^2}{D^4} \;, \qquad Z^3 \text{ arbitrary} \;. \tag{6.13}$$

For simplicity, we set

$$D^4 = 1 \;, \tag{6.14}$$

which is allowed, because D^A are homogeneous coordinates. Rescaling D has no effect on any of the considerations, if other variables are also rescaled accordingly, for example the right hand sides of (6.1-6.3).

The condition that D^A has to be null with respect to N_{AB} (4.24), reads with (6.13), (6.14) and the imaginary part of (5.4) inserted

$$
\begin{aligned}
0 =& 2\Big[\operatorname{Re}(D^2\bar{D}^3)\operatorname{Im} D^1 + \operatorname{Re}(D^3\bar{D}^1)\operatorname{Im} D^2 + \operatorname{Re}(D^1\bar{D}^2)\operatorname{Im} Z^3 \\
&- \operatorname{Re} D^1 \operatorname{Im}(D^2 Z^3) - \operatorname{Re} D^2 \operatorname{Im}(Z^3 D^1) - \operatorname{Re} D^3 \operatorname{Im}(D^1 D^2) + \operatorname{Im}(D^1 D^2 Z^3)\Big] \;.
\end{aligned} \tag{6.15}
$$

One can use the elementary identities

$$
\begin{aligned}
\operatorname{Re}(D^1\bar{D}^2)\operatorname{Im} Z^3 - \operatorname{Re} D^1 \operatorname{Im}(D^2 Z^3) - \operatorname{Re} D^2 \operatorname{Im}(Z^3 D^1) &= -\operatorname{Im}(D^1 D^2 Z^3) \;, && (6.16)\\
\operatorname{Re}(D^2\bar{D}^3)\operatorname{Im} D^1 + \operatorname{Re}(D^3\bar{D}^1)\operatorname{Im} D^2 - \operatorname{Re} D^3 \operatorname{Im}(D^1 D^2) &= 2\operatorname{Im} D^1 \operatorname{Im} D^2 \operatorname{Im} D^3 \;, && (6.17)
\end{aligned}
$$

to show that (6.15) can be simplified to read

$$0 = 4 \operatorname{Im} D^1 \operatorname{Im} D^2 \operatorname{Im} D^3 \;. \tag{6.18}$$

This calculation also produces the explicit result for $\mathrm{e}^{-\mathcal{K}^\mathrm{h}}$, which is according to (4.1)

$$\mathrm{e}^{-\mathcal{K}^\mathrm{h}} = \mathrm{i}\left(Z^A \bar{\mathcal{G}}_A - \bar{Z}^A \mathcal{G}_A\right) = 2 Z^A N_{AB} \bar{Z}^B \;. \tag{6.19}$$

By comparing (6.19) and (4.24) (and using (6.13) and (6.14)), we see that $e^{-\mathcal{K}^h}$ can be obtained by taking the right hand side of (6.18), replacing D^3 with Z^3 and multiplying by two

$$e^{-\mathcal{K}^h} = 8\,\mathrm{Im}\,D^1\,\mathrm{Im}\,D^2\,\mathrm{Im}\,Z^3\ . \tag{6.20}$$

The domain of the coordinates Z^A is restricted due to $e^{-\mathcal{K}^h} > 0$, so (6.20) dictates

$$0 \neq \mathrm{Im}\,D^1\ , \qquad 0 \neq \mathrm{Im}\,D^2\ , \qquad 0 \neq \mathrm{Im}\,Z^3\ . \tag{6.21}$$

This means that (6.18) becomes

$$0 = \mathrm{Im}\,D^3\ . \tag{6.22}$$

Let us summarize all conditions on the base coordinates (6.13), (6.14), (6.21) and (6.22)

$$\begin{gathered} Z^1 = D^1 \in \mathbb{C}\setminus\mathbb{R}\ , \qquad Z^2 = D^2 \in \mathbb{C}\setminus\mathbb{R}\ , \qquad Z^4 = D^4 = 1\ , \\ Z^3 \in \mathbb{C}\setminus\mathbb{R}\ , \qquad D^3 \in \mathbb{R}\ . \end{gathered} \tag{6.23}$$

6.1.4 Quantum STU model

For the prepotential (5.5), condition (6.2) reads [16]

$$(D^4Z^2 - D^2)(D^4Z^3 - D^3) = \text{const.}\ , \tag{6.24}$$

$$(D^4Z^1 - D^1)(D^4Z^3 - D^3) + \alpha(D^4Z^2 - D^2)^2 = \text{const.}\ , \tag{6.25}$$

$$(D^4Z^1 - D^1)(D^4Z^2 - D^2) = \text{const.}\ , \tag{6.26}$$

$$2D^4Z^1Z^2Z^3 - D^1Z^2Z^3 - D^2Z^1Z^3 - D^3Z^1Z^2 + \alpha(Z^2)^2(\tfrac{2}{3}D^4Z^2 - D^2) = \text{const.}\ . \tag{6.27}$$

Here (6.25) dictates that Z^2 must be fixed. One can arrange for one of the other coordinates to remain free in the same way as in the unperturbed STU model. For Z^3 to be free, (6.13) must again be imposed. We are still allowed to set $D^4 = 1$ as in (6.14).

Let us now determine the domain of the D^A and Z^A. When we insert (6.13), (6.14) and the imaginary part of (5.6) into (4.24), we obtain the result from the unperturbed model (6.18) plus the contribution proportional to α

$$\alpha\left[\bar{D}^2\,\mathrm{Im}\left(2D^2\right)D^2 + \left(\bar{D}^2 + D^2\right)\mathrm{Im}\left(-(D^2)^2\right) + \frac{2}{3}\,\mathrm{Im}\left((D^2)^3\right)\right] = \frac{4}{3}\alpha\left(\mathrm{Im}\,D^2\right)^3\ . \tag{6.28}$$

So the quantum STU version of (4.24) is

$$0 = 4\,\mathrm{Im}\,D^2\left[\mathrm{Im}\,D^1\,\mathrm{Im}\,D^3 + \frac{1}{3}\alpha\left(\mathrm{Im}\,D^2\right)^2\right]\ . \tag{6.29}$$

$e^{-\mathcal{K}^h}$ can be obtained in the same way as in the STU model (6.20), namely by replacing

D^3 by Z^3 in the right hand side of (6.29) and multiplying by two.

$$\mathrm{e}^{-\mathcal{K}^{\mathrm{h}}} = 8 \operatorname{Im} D^2 \left[\operatorname{Im} D^1 \operatorname{Im} Z^3 + \frac{1}{3}\alpha \left(\operatorname{Im} D^2\right)^2 \right] . \tag{6.30}$$

$\operatorname{Im} D^2$ has to be non-zero to keep (6.30) finite

$$0 \neq \operatorname{Im} D^2 . \tag{6.31}$$

This implies that the bracket in (6.29) must be zero

$$0 \neq \operatorname{Im} D^1 , \qquad \operatorname{Im} D^3 = -\frac{\alpha(\operatorname{Im} D^2)^2}{3 \operatorname{Im} D^1} . \tag{6.32}$$

Furthermore, the bracket in (6.30) must be non-zero

$$\operatorname{Im} Z^3 \neq -\frac{\alpha(\operatorname{Im} D^2)^2}{3 \operatorname{Im} D^1} . \tag{6.33}$$

We summarize the conditions (6.13), (6.14), (6.31), (6.32) and (6.33)

$$\begin{gathered} Z^1 = D^1 \in \mathbb{C} \setminus \mathbb{R} , \qquad Z^2 = D^2 \in \mathbb{C} \setminus \mathbb{R} , \qquad Z^4 = D^4 = 1 , \\ Z^3 + \mathrm{i}\,\frac{\alpha(\operatorname{Im} D^2)^2}{3 \operatorname{Im} D^1} \in \mathbb{C} \setminus \mathbb{R} , \qquad D^3 + \mathrm{i}\,\frac{\alpha(\operatorname{Im} D^2)^2}{3 \operatorname{Im} D^1} \in \mathbb{R} . \end{gathered} \tag{6.34}$$

This is the same result as in the *STU* model (6.23), except that the domains of Z^3 and D^3 are shifted by an imaginary constant.

6.2 Fiber coordinates

Four real fiber coordinates are fixed by first restricting to the submanifold defined by (6.3) and then applying the quotient construction of section 3.3.1. We start by introducing complex fiber coordinates. There is a set of coordinates (z^a, w^0, w_A) that was shown in [15] to be holomorphic on $\mathbf{M}_{\mathrm{h}}$ with respect to one of its almost complex structures, J^3. w^0 and w_A are given by

$$\begin{aligned} w^0 &= \mathrm{e}^{-2\phi} + \mathrm{i}(\tilde{\phi} + \xi^A(\tilde{\xi}_A - \mathcal{G}_{AB}\xi^B)) , \\ w_A &= -\mathrm{i}(\tilde{\xi}_A - \mathcal{G}_{AB}\xi^B) . \end{aligned} \tag{6.35}$$

The inverse transformation is

$$\phi = -\frac{1}{2} \ln \left(\operatorname{Re} w^0 + \operatorname{Re} w_A N^{-1\,AB} \operatorname{Re} w_B \right) , \tag{6.36}$$

$$\tilde{\xi}_A = -\operatorname{Re}(\mathcal{G}_{AB}(N_{BC})^{-1}\bar{w}_C) , \tag{6.37}$$

$$\xi^A = -\operatorname{Re}((N_{AB})^{-1}\bar{w}_B) . \tag{6.38}$$

(6.3) reads in complex coordinates

$$D^A w_A = \tilde{C} \ , \tag{6.39}$$

with a newly defined complex constant $\tilde{C}$. This condition can be used to fix one complex coordinate on the fiber.

The explicit construction of the quotient $\hat{\mathbf{M}}_{\mathrm{h}}$ was most elaborately described in [16] and goes as follows. Before taking the quotient by the action generated by the Killing vector fields $\hat{k}_1$ and $\hat{k}_2$, they are combined into one holomorphic vector field

$$\hat{k} \equiv \hat{k}_1 - \mathrm{i}\hat{k}_2 = \bar{D}^A \left(\frac{\partial}{\partial \xi^A} + \bar{\mathcal{G}}_{AB} \frac{\partial}{\partial \tilde{\xi}_B} - \left(\tilde{\xi}_A - \bar{\mathcal{G}}_{AB} \xi^B \right) \frac{\partial}{\partial \tilde{\phi}} \right) \ , \tag{6.40}$$

where (4.23) and (4.7) were inserted. One can use (6.35) to show that the action of $\hat{k}$ on w^0 and w_A is

$$\hat{k} w^0 = -4 \bar{D}^A \,\mathrm{Re}\, w_A \ , \tag{6.41}$$

$$\hat{k} w_A = -2 N_{AB} \bar{D}^B \ . \tag{6.42}$$

The quotient is taken by identifying points that lie on the same integral curves of $\hat{k}$

$$(w^0, w_A) \sim \left(1 + \lambda \hat{k}\right)(w^0, w_A) = (w^0 - 4\lambda \bar{D}^A \,\mathrm{Re}\, w_A, w_A - 2\lambda N_{AB} \bar{D}^B) \ , \tag{6.43}$$

with $\lambda \in \mathbb{C}$. It was shown in [14] that $Z^A N_{AB} \bar{D}^B \neq 0$. That guarantees that each equivalence class $[w^0, w_A]$ contains for each $\tilde{D} \in \mathbb{C}$ exactly one representative fulfilling

$$Z^A w_A = \tilde{D} \ , \tag{6.44}$$

i.e. the quotient is isomorphic to the submanifold obtained by fixing another coordinate on the fiber using (6.44). In total, 2 of the $n_{\mathrm{h}}+1$ complex fiber coordinates are fixed in the $\mathcal{N}=1$ theory. Including the $\hat{n}_{\mathrm{h}}$ remaining base coordinates, $\hat{\mathbf{M}}_{\mathrm{h}}$ has complex dimension $\hat{n}_{\mathrm{h}} + n_{\mathrm{h}} - 1$.

(6.39) and (6.44) are solved by switching to the coordinates (x^0, x_a), $a = 1, .., \hat{i}, .., \hat{j}, .., n_{\mathrm{h}}$ ($\hat{i}$ and $\hat{j}$ are omitted) by means of

$$\begin{aligned}
w^0 &\mapsto x^0 \ , \\
w_a &\mapsto x_a \ , \\
w_i &\mapsto \alpha \left((Z^j D^a - Z^a D^j) x_a - Z^j \tilde{C} + D^j \tilde{D} \right) \ , \\
w_j &\mapsto -\alpha \left((Z^i D^a - Z^a D^i) x_a - Z^i \tilde{C} + D^i \tilde{D} \right) \ ,
\end{aligned} \tag{6.45}$$

where α is defined as

$$\alpha \equiv \frac{1}{D^j Z^i - D^i Z^j} \ . \tag{6.46}$$

This is a generalization of a solution used in [16]. Since the complex structure $\hat{J}$ on $\hat{\mathbf{M}}_{\mathrm{h}}$ is a projection of J^3 (3.17) and the new coordinates (z^a, x^0, x_a) are a subset of the holomorphic coordinates on $\mathbf{M}_{\mathrm{h}}$, (z^a, x^0, x_a) are holomorphic with respect to $\hat{J}$.

6.2.1 *STU* and quantum *STU* models

Two fiber coordinates are fixed by means of (6.45). With the identities (6.23) or (6.34), Z and D only differ in their third component, so the denominator in (6.46) is only non-zero if either i or j is 3. We set $i = 3$ and can now again use that Z and D are equal in their other components

$$Z^j = D^j \ , \qquad Z^a = D^a \ . \tag{6.47}$$

Now the mappings of w_i and w_j in (6.45) become

$$w_3 = \frac{-\tilde{C} + \tilde{D}}{Z^3 - D^3} \ , \qquad w_j = -\frac{(Z^3 - D^3) D^a x_a - Z^3 \tilde{C} + D^3 \tilde{D}}{D^j (Z^3 - D^3)} \ . \tag{6.48}$$

Since $\tilde{D}$ is an arbitrary constant, we are allowed to set $\tilde{D} = \tilde{C}$ to obtain simple expressions that are independent of base coordinates

$$w_3 = 0, \qquad w_j = \frac{\tilde{C} - D^a x_a}{D^j} \ . \tag{6.49}$$

The most convenient choice is $j = 4$ and we will use that in the following sections

$$(w^0, w_1, w_2) = (x^0, x_1, x_2), \qquad w_3 = 0, \qquad w_4 = \tilde{C} - D^1 x_1 - D^2 x_2 \ . \tag{6.50}$$

To obtain results for a different set of fiber coordinates, one can take any expression from the remainder of this thesis and use the last equation in (6.50) to perform a holomorphic coordinate transformation and eliminate x_1 or x_2 instead of w_4

$$x_1 \to \frac{\tilde{C} - D^2 x_2 - w_4}{D^1} \qquad \text{or} \qquad x_2 \to \frac{\tilde{C} - D^1 x_1 - w_4}{D^2} \ . \tag{6.51}$$

7 The $\mathcal{N} = 1$ Kähler potential

The Kähler potential of $\hat{\mathbf{M}}_{\mathrm{h}}$ was found in [15] by integrating

$$\begin{aligned} \mathrm{d}\hat{\mathcal{K}} &= \partial_s \hat{\mathcal{K}} \mathrm{d}y^s + \bar{\partial}_s \hat{\mathcal{K}} \mathrm{d}\bar{y}^s \\ &= 2\,\mathrm{i}\,\omega_s^3 \mathrm{d}y^s - 2\,\mathrm{i}\,\bar{\omega}_s^3 \mathrm{d}\bar{y}^s \ , \end{aligned} \tag{7.1}$$

where (3.25) was used and $y^s, s = 1, \dots, 2n_{\rm h} - 1$ are holomorphic coordinates on the quotient $\mathbf{M}_{\rm h}/\langle \hat{k}_1, \hat{k}_2 \rangle$. They are chosen in such a way that they coincide with the existing coordinates z^a on the base

$$y^s = z^s, \ s \leq n_{\rm h} - 1 \ . \tag{7.2}$$

Specifying the holomorphic fiber coordinates $y^s, s = n_{\rm h}, \dots, 2n_{\rm h} - 1$ is not necessary here. To find the holomorphic and antiholomorphic parts of ω^3 that appear in (7.1), it is crucial that v is holomorphic. This can be seen by comparing the metric (4.5) and the Kähler form as given in [15]

$$\hat{K} = {\rm i}(v \wedge \bar{v} + u \wedge \bar{u} + E \wedge \bar{E} - e \wedge \bar{e}) \ , \tag{7.3}$$

using (2.11) and (3.17). Knowing that v is holomorphic, ω^3 from (4.9) can now be written as the sum of its holomorphic and anti-holomorphic parts

$$\begin{aligned} \omega^3 &= \omega^3_s {\rm d}y^s + \bar{\omega}^3_s {\rm d}\bar{y}^s \\ &= \tfrac{\rm i}{2}(v_s - \partial_s \mathcal{K}^{\rm h}){\rm d}y^s - \tfrac{\rm i}{2}(\bar{v}_s - \bar{\partial}_s \mathcal{K}^{\rm h}){\rm d}\bar{y}^s \ . \end{aligned} \tag{7.4}$$

Inserting ω^3_s and $\bar{\omega}^3_s$ from (7.4) into (7.1), one gets

$$\begin{aligned} {\rm d}\hat{\mathcal{K}} &= -(v_s - \partial_s \mathcal{K}^{\rm h}){\rm d}y^s - (\bar{v}_s - \bar{\partial}_s \mathcal{K}^{\rm h}){\rm d}\bar{y}^s \\ &= -v - \bar{v} + {\rm d}\mathcal{K}^{\rm h} \ . \end{aligned} \tag{7.5}$$

Integrating this yields the result from [15]

$$\hat{\mathcal{K}} = 2\phi + \mathcal{K}^{\rm h} \ . \tag{7.6}$$

To obtain a form more suitable for further calculations, we exponentiate (7.6) and use (6.36) to express it in terms of complex fiber coordinates

$${\rm e}^{-\hat{\mathcal{K}}} = {\rm e}^{-\mathcal{K}^{\rm h}} \left({\rm Re}\, w^0 + {\rm Re}\, w_A N^{-1\,AB}\, {\rm Re}\, w_B \right) \ . \tag{7.7}$$

7.1 *STU* model

In order to calculate the $\mathcal{N} = 1$ Kähler potential (7.7) for the STU model, we first invert N_{AB}, which is the imaginary part of (5.4), with the help of Mathematica

$$(N_{AB})^{-1}|_{Z^4=1} = -\frac{1}{2\,{\rm Im}\, Z^1\, {\rm Im}\, Z^2\, {\rm Im}\, Z^3} \begin{pmatrix} |Z^1|^2 & {\rm Re}(Z^1 Z^2) & {\rm Re}(Z^1 Z^3) & {\rm Re}(Z^1) \\ {\rm Re}(Z^1 Z^2) & |Z^2|^2 & {\rm Re}(Z^2 Z^3) & {\rm Re}(Z^2) \\ {\rm Re}(Z^1 Z^3) & {\rm Re}(Z^2 Z^3) & |Z^3|^2 & {\rm Re}(Z^3) \\ {\rm Re}(Z^1) & {\rm Re}(Z^2) & {\rm Re}(Z^3) & 1 \end{pmatrix} . \tag{7.8}$$

We insert (7.8) and (6.20) into (7.7), already using $Z^1 = D^1$ and $Z^2 = D^2$

$$e^{-\hat{\mathcal{K}}} = 8\,\mathrm{Im}\,D^1\,\mathrm{Im}\,D^2\,\mathrm{Im}\,Z^3\,\mathrm{Re}\,w^0 - 4\left((\mathrm{Re}\,w_A)^2|Z^A|^2 + \sum_{A\neq B}\mathrm{Re}\,w_A\,\mathrm{Re}(Z^A Z^B)\,\mathrm{Re}\,w_B\right)\ . \tag{7.9}$$

The next step is to insert the full conditions fixing base (6.23) and fiber (6.50) coordinates into (7.9) to obtain

$$\begin{aligned} e^{-\hat{\mathcal{K}}} &= 8\,\mathrm{Im}\,D^1\,\mathrm{Im}\,D^2\,\mathrm{Im}\,Z^3\,\mathrm{Re}\,x^0 - 4\left((\mathrm{Re}\,x_a)^2|D^a|^2 + \sum_{a\neq b}\mathrm{Re}\,x_a\,\mathrm{Re}(D^a D^b)\,\mathrm{Re}\,x_b\right) \\ &\quad - 4\left((\mathrm{Re}(\tilde{C} - D^a x_a))^2 + 2\,\mathrm{Re}(\tilde{C} - D^a x_a)\,\mathrm{Re}\,D^b\,\mathrm{Re}\,x_b\right) \\ &= 8\,\mathrm{Im}\,D^1\,\mathrm{Im}\,D^2\,\mathrm{Im}\,Z^3\,\mathrm{Re}\,x^0 - 4|\,\mathrm{Im}\,D^1\bar{x}_1 - \mathrm{Im}\,D^2 x_2 - i\,\mathrm{Re}\,\tilde{C}|^2\ . \end{aligned} \tag{7.10}$$

The last equality in (7.10) is an elementary but slightly lengthy conversion, which was checked with Mathematica.

After the field redefinitions

$$\begin{aligned} -iZ^3 &\quad\rightarrow\quad Z^3\ , \\ 2\,\mathrm{Im}\,D^1\,\mathrm{Im}\,D^2 x^0 &\quad\rightarrow\quad x^0\ , \\ 2\,\mathrm{Im}\,D^1 x_1 + i\,\mathrm{Re}\,\tilde{C} &\quad\rightarrow\quad x_1\ , \\ -2\,\mathrm{Im}\,D^2 x_2 - i\,\mathrm{Re}\,\tilde{C} &\quad\rightarrow\quad x_2\ , \end{aligned} \tag{7.11}$$

(7.10) becomes

$$e^{-\hat{\mathcal{K}}} = (Z^3 + \bar{Z}^3)(x^0 + \bar{x}^0) - (x_1 + \bar{x}_2)(\bar{x}_1 + x_2)\ . \tag{7.12}$$

This Kähler manifold has dimension 8 and was identified in [30] as

$$\hat{\mathbf{M}}_{\mathrm{h}} = \frac{SO(4,2)}{SO(4)\times SO(2)}\ . \tag{7.13}$$

We saw in this section that the $\mathcal{N} = 1$ Kähler potential of the STU model can be brought into the simple form (7.12). This Kähler potential has a number of symmetries, which are listed in appendix E.

7.2 Quantum *STU* model

The Kähler potential for the quantum STU model is calculated analogously to the STU model. The inverse of the imaginary part of (5.6) was determined with Mathematica. It is

$$
\begin{aligned}
(N_{AB})^{-1}(\alpha) = g(Z)h(Z)\Big(&6(\operatorname{Im} Z^1 \operatorname{Im} Z^3)^2 (N_{AB})^{-1}(\alpha=0) \\
&+ \ \alpha \operatorname{Im} Z^2 \left\{ \operatorname{Re}(Z^A \bar{Z}^B) + 2 \operatorname{Re} Z^A \operatorname{Re} Z^B - 2\delta^{AB} (\operatorname{Im} Z^A)^2 \right. \\
&\left. - \ (\delta_1^A \delta_3^B + \delta_3^A \delta_1^B)(6 \operatorname{Im} Z^1 \operatorname{Im} Z^3 + 2\alpha (\operatorname{Im} Z^2)^2) \right\} \Big) \ ,
\end{aligned}
\tag{7.14}
$$

with

$$
g(Z) \equiv \left(6 \operatorname{Im} Z^1 \operatorname{Im} Z^3 + 2\alpha (\operatorname{Im} Z^2)^2 \right)^{-1} \ , \tag{7.15}
$$

$$
h(Z) \equiv \left(\operatorname{Im} Z^1 \operatorname{Im} Z^3 - \alpha (\operatorname{Im} Z^2)^2 \right)^{-1} \ . \tag{7.16}
$$

$(N_{AB})^{-1}(\alpha = 0)$ is the expression from the unperturbed *STU* model (7.8). When the coordinate fixing conditions (6.34) and (6.50) and the matrix (7.14) are inserted into (7.7), the third line of (7.14) does not contribute. It appears only multiplied with $\operatorname{Re} w_3$, which is set to zero. The first line of (7.14) is proportional to $(N_{AB})^{-1}(\alpha = 0)$ and contributes in the same way as in the unperturbed *STU* model. We see by comparing (7.7) and (7.10) that

$$
\operatorname{Re} w_A N^{-1\,AB}(\alpha=0) \operatorname{Re} w_B = \frac{-4|\operatorname{Im} D^1 \bar{x}_1 - \operatorname{Im} D^2 x_2 - i \operatorname{Re} \tilde{C}|^2}{\mathrm{e}^{-\mathcal{K}^{\mathrm{h}}}(\alpha=0)} \ , \tag{7.17}
$$

with $\mathrm{e}^{-\mathcal{K}^{\mathrm{h}}}(\alpha = 0)$ given in (6.20). The contribution of the second line of (7.14) is proportional to

$$
\begin{aligned}
&\operatorname{Re} w_A \left(\operatorname{Re}(Z^A \bar{Z}^B) + 2 \operatorname{Re} Z^A \operatorname{Re} Z^B - 2\delta^{AB} (\operatorname{Im} Z^A)^2 \right) \operatorname{Re} w_B \\
&= 3(\operatorname{Re} Z^A \operatorname{Re} w_A)^2 + (\operatorname{Im} Z^A \operatorname{Re} w_A)^2 - 2(\operatorname{Im} Z^A)^2 (\operatorname{Re} w_A)^2 \\
&= 3(\operatorname{Re} \tilde{C} + \operatorname{Im} D^a \operatorname{Im} x_a)^2 - (\operatorname{Im} D^1 \operatorname{Re} x_1 - \operatorname{Im} D^2 \operatorname{Re} x_2)^2 \ ,
\end{aligned}
\tag{7.18}
$$

where (6.34) and (6.50) were inserted in the last equality. Finally, we introduce the notation

$$
g(Z^3) \equiv g(Z^1 = D^1, Z^2 = D^2, Z^3) \ , \qquad h(Z^3) \equiv h(Z^1 = D^1, Z^2 = D^2, Z^3) \ . \tag{7.19}
$$

and note that $\mathrm{e}^{-\mathcal{K}^{\mathrm{h}}}$ (6.30) can be expressed in terms of $g(Z^3)$

$$
\mathrm{e}^{-\mathcal{K}^{\mathrm{h}}} = \frac{4}{3} \operatorname{Im} D^2 g(Z^3)^{-1} \ . \tag{7.20}
$$

The Kähler potential is now obtained by inserting (7.14), (7.17), (7.18) and (7.20) into (7.7)

$$\begin{aligned} e^{-\hat{\mathcal{K}}} = & \frac{4}{3}\operatorname{Im} D^2 g(Z^3)^{-1}\operatorname{Re} x^0 \\ & - 4h(Z^3)\Bigg(\operatorname{Im} D^1 \operatorname{Im} Z^3 |\operatorname{Im} D^1 \bar{x}_1 - \operatorname{Im} D^2 x_2 - i\operatorname{Re}\tilde{C}|^2 \\ & - \frac{\alpha}{3}(\operatorname{Im} D^2)^2\left(3(\operatorname{Re}\tilde{C} + \operatorname{Im} D^a \operatorname{Im} x_a)^2 - (\operatorname{Im} D^1 \operatorname{Re} x_1 - \operatorname{Im} D^2 \operatorname{Re} x_2)^2\right)\Bigg) . \end{aligned} \tag{7.21}$$

$\tilde{C}$ in (7.21) can again be redefined away by the same field redefinition of x_1 and x_2 as in (7.11)

$$\begin{aligned} 2\operatorname{Im} D^1 x_1 + i\operatorname{Re}\tilde{C} \quad &\rightarrow \quad x_1 , \\ -2\operatorname{Im} D^2 x_2 - i\operatorname{Re}\tilde{C} \quad &\rightarrow \quad x_2 , \end{aligned} \tag{7.22}$$

resulting in

$$\begin{aligned} e^{-\hat{\mathcal{K}}} = g(Z^3)^{-1}&\left[\frac{4}{3}\operatorname{Im} D^2 \operatorname{Re} x^0 - \frac{1}{6}h(Z^3)(x_1+\bar{x}_2)(\bar{x}_1+x_2)\right] \\ &- \frac{\alpha}{3}(\operatorname{Im} D^2)^2 h(Z^3)\Big[(x_1+\bar{x}_2)-(\bar{x}_1+x_2)\Big]^2 . \end{aligned} \tag{7.23}$$

To obtain an equation that is more comparable to (7.12), we first shift Z^3 in such a way that its domain (6.34) matches the one that would be applicable in the unperturbed *STU* model (6.23)

$$Z^3 \quad \rightarrow \quad Z^3 - \mathrm{i}\,\frac{\alpha(\operatorname{Im} D^2)^2}{3\operatorname{Im} D^1} . \tag{7.24}$$

Now the remaining field redefinitions from (7.11) can be applied

$$\begin{aligned} -iZ^3 \quad &\rightarrow \quad Z^3 , \\ 2\operatorname{Im} D^1 \operatorname{Im} D^2 x^0 \quad &\rightarrow \quad x^0 . \end{aligned} \tag{7.25}$$

The effect on $g(Z^3)$ and $h(Z^3)$ is

$$\begin{aligned} g(Z^3) \quad &\rightarrow \quad \left(6\operatorname{Im} D^1 \operatorname{Re} Z^3\right)^{-1} , \\ h(Z^3) \quad &\rightarrow \quad \left(\operatorname{Im} D^1 \operatorname{Re} Z^3 - \alpha\frac{4}{3}(\operatorname{Im} D^2)^2\right)^{-1} . \end{aligned} \tag{7.26}$$

With these field redefinitions, the Kähler potential finally takes the form of (7.12) plus a perturbative term

$$e^{-\hat{\mathcal{K}}} = (Z^3+\bar{Z}^3)(x^0+\bar{x}^0) - (x_1+\bar{x}_2)(\bar{x}_1+x_2) + \frac{\alpha\Big[(x_1+\bar{x}_2)+(\bar{x}_1+x_2)\Big]^2}{4\alpha - \frac{3\operatorname{Im} D^1}{2(\operatorname{Im} D^2)^2}\left(Z^3+\bar{Z}^3\right)} . \tag{7.27}$$

In the limit of large α, (7.27) can be approximated as

$$\mathrm{e}^{-\hat{\mathcal{K}}}|_{\alpha\to\infty} = (Z^3+\bar{Z}^3)(x^0+\bar{x}^0) + \frac{1}{4}\Big[(x_1+\bar{x}_2)-(\bar{x}_1+x_2)\Big]^2 . \tag{7.28}$$

The result (7.27) shares some of the symmetries with the Kähler potential of the unperturbed STU model. This is discussed in appendix E.

8 The $\mathcal{N}=1$ scalar potential

To get a non-zero scalar potential, let us now consider the case where $n>2$ linearly independent isometries are gauged. The gauge bosons are recruited among the graviphoton and the vectors of the vector multiplets, and the number of available gauge bosons limits the number of possible gaugings

$$n \le n_{\mathrm{v}} + 1 . \tag{8.1}$$

The commuting Killing vectors are parametrized as in (4.11), but now there are n of them, $\lambda = 1,\dots,n$. $\hat{k}_1$ and $\hat{k}_2$ are the two Killing vectors (4.23) that ensure partial supersymmetry breaking, while additional gaugings are assumed to have negligible effect on supersymmetry breaking, as discussed in section 3.3. $k_A, \tilde{k}^B, k_{\tilde{\phi}}$ form a $(2n_{\mathrm{h}}+1)$-dimensional Heisenberg algebra, which has maximal Abelian dimension $(n_{\mathrm{h}}+1)$ [31], thus the requested n commuting Killing vectors exist if and only if

$$n \le n_{\mathrm{h}} + 1 . \tag{8.2}$$

The conditions for commuting Killing vectors are (4.14) and (4.15). The condition (4.15) implies that subsequent gaugings are more and more restricted. The requirement that $\hat{k}_3$ must commute with $\hat{k}_1$ and $\hat{k}_2$ fixes two of the $2n_{\mathrm{h}}$ parameters ${r_3}^A, s_{3B}$. If we keep adding gaugings one by one, the ith gauging must be chosen such that it commutes with $\hat{k}_1$ up to $\hat{k}_{i-1}$ and is thus restricted by $(i-1)$ conditions.

The commutation condition (4.15) with $\lambda \ge 3$ and $\rho = 1,2$ can be brought into a more useful form by inserting (4.22)

$$(s_{\lambda B} - {r_\lambda}^A \mathcal{G}_{AB}) D^B = 0 . \tag{8.3}$$

Similarly, the embedding tensor constraint (2.16) with $\lambda \ge 3$ and $\kappa = 1,2$ reads

$$C^I ({\Theta_I}^\lambda - \mathcal{F}_{IJ} \Theta^{J\,\lambda}) = 0 , \tag{8.4}$$

when the explicit Θ_Λ^1 and Θ_Λ^2 from (3.9) are inserted.

With coordinates and a basis of Killing vectors on $\mathbf{M}_{\mathrm{h}}$ at hand, the superpotential and D-terms can be calculated more explicitly. This is what we will do in the following subsections. The only parts of the superpotential (3.27) and the D-terms (3.28) that depend on the fields of the hypermultiplets are the Killing prepotentials $e^{-\hat{\mathcal{K}}/2}P_\lambda{}^-$ and P^3_λ, so we will concentrate on calculating those. The rest depends only on the fields of the vector multiplets and is not affected by inserting a specific $\mathcal{G}$.

8.1 The superpotential

We begin with calculating the Killing prepotential $P_\lambda{}^-$, which appears in the superpotential (3.27)

$$P_\lambda{}^- = (\omega^1_u - i\omega^2_u)\hat{k}^u_\lambda = -2iu_u\left(r_\lambda{}^A k^u_A + s_{\lambda A}\tilde{k}^{Au} + t_\lambda k^u_{\tilde{\phi}}\right) , \tag{8.5}$$

where we use (4.10) in the first and (4.9), (4.11) and (4.14) in the second equality. We insert (4.7), (4.3) and (4.21) into (8.5) to arrive at

$$P_\lambda{}^- = 2e^{\hat{\mathcal{K}}/2}(s_{\lambda B} - r_\lambda{}^A\mathcal{G}_{AB})Z^B . \tag{8.6}$$

The resulting superpotential (3.27) is

$$\mathcal{W} = 2X^I(\Theta_I{}^\lambda - \mathcal{F}_{IJ}\Theta^{J\,\lambda})(s_{\lambda B} - r_\lambda{}^A\mathcal{G}_{AB})Z^B . \tag{8.7}$$

Since the constraints on the Killing vector coefficients (8.3) and the embedding tensor (8.4) are analogous formulae, the superpotential (8.7) is symmetric under the exchange of $(\mathcal{F}, X^I)$ and $(\mathcal{G}, Z^A)$. In symplectic form, (8.7) reads

$$\mathcal{W} = 2V^\Lambda\Theta_\Lambda{}^\lambda s_{\lambda\Sigma}U^\Sigma, \tag{8.8}$$

with the symplectic vectors $s_{\lambda\Sigma} \equiv (s_{\lambda A}, -r_\lambda{}^A)$ and $U^\Sigma \equiv (Z^A, \mathcal{G}_A)$. If we define the constant matrix $\Theta_{\Lambda\Sigma} \equiv \Theta_\Lambda{}^\lambda s_{\lambda\Sigma}$ and insert it into (8.8), we get the superpotential in the same form as given in [15]. The lesson we take away from rederiving the result in the form (8.8) is that the rank of $\Theta_{\Lambda\Sigma}$ is at most $n-2$. This can be seen as follows. We recall from section 3.3, that the first two gauged isometries do not contribute to the superpotential, which implies $\lambda = 3, \ldots, n$ in (8.8). Thus $\Theta_{\Lambda\Sigma}$ is the product of $(n-2)$-column matrix and a $(n-2)$-row matrix and its rank is bounded by $n-2$. The rank of $\Theta_{\Lambda\Sigma}$ is important for considerations regarding the vacuum of the $\mathcal{N} = 1$ theory, because it limits the number of constraints that are posed by formulae involving $\Theta_{\Lambda\Sigma}$.

8.1.1 *STU* and quantum *STU* models

The Killing prepotential $P_\lambda{}^-$ (8.6) can be simplified using (8.3) and (6.23) or (6.34)

$$e^{-\hat{\mathcal{K}}/2}P_\lambda{}^- = 2(s_{\lambda B} - r_\lambda{}^A\mathcal{G}_{AB})(Z^B - D^B) = 2B^{STU}_\lambda(Z^3 - D^3) , \tag{8.9}$$

where we defined the constants

$$B_\lambda^{STU} = s_{\lambda 3} - r_\lambda{}^A \mathcal{G}_{A3} \ . \tag{8.10}$$

B_λ^{STU} are constants, because $\mathcal{G}_{A3}$ does not depend on Z^3

$$\mathcal{G}_{A3} = \partial_A \frac{Z^1 Z^2}{Z^4}\bigg|_{Z^1=D^1, Z^2=D^2, Z^4=1} = \text{const.} \ . \tag{8.11}$$

The superpotential is then

$$\mathcal{W} = 2V^\Lambda \Theta_\Lambda{}^\lambda B_\lambda^{STU} (Z^3 - D^3) \ . \tag{8.12}$$

Interestingly, the only term containing a hypermultiplet field $(Z^3 - D^3)$ factors out. This term can never vanish, because the domains of Z^3 and D^3 are disjunct, according to (6.23) or (6.34). We will see in section 9.1 that this implies a vanishing superpotential, if a (quantum) STU prepotential $\mathcal{G}$ is combined with certain other prepotentials $\mathcal{F}$.

8.2 The D-terms

P_λ^3 is calculated along the same lines as $P_\lambda{}^-$ was in section 8.1

$$P_\lambda^3 = \omega_u^3 \hat{k}_\lambda^u = \mathrm{e}^{2\phi} \left(r_\lambda{}^A \tilde{\xi}_A - s_{\lambda A} \xi^A - t_\lambda \right) \ . \tag{8.13}$$

We can use (6.37-6.38) and (7.6) to switch to holomorphic coordinates

$$P_\lambda^3 = \mathrm{e}^{\hat{\mathcal{K}} - \mathcal{K}^{\mathrm{h}}} \operatorname{Re}\left((s_{\lambda B} - r_\lambda{}^A \mathcal{G}_{AB})(N_{BC})^{-1} \bar{w}_C - t_\lambda \right) \ . \tag{8.14}$$

When this Killing prepotential is inserted, the D-terms (3.28) read

$$\begin{aligned} \mathcal{D}^{\hat{I}} = -\mathrm{e}^{\hat{\mathcal{K}} - \mathcal{K}^{\mathrm{h}}} \Pi^I_J \Gamma^J_K (\operatorname{Im} \mathcal{F})^{-1\ KL} \left(\Theta_L{}^\lambda - \bar{\mathcal{F}}_{LM} \Theta^{M\lambda} \right) \\ \cdot \operatorname{Re}\left((s_{\lambda B} - r_\lambda{}^A \mathcal{G}_{AB})(N_{BC})^{-1} \bar{w}_C - t_\lambda \right) \ . \end{aligned} \tag{8.15}$$

8.2.1 *STU* model

P_λ^3 (8.14) can only be made more explicit by inserting $N^{-1}_{AB} \bar{w}_B$. In order to calculate this, we first write down (7.8) in components

$$\begin{aligned} N^{-1}_{BA} &= -4\mathrm{e}^{\mathcal{K}^{\mathrm{h}}} \left(\delta^{BA} |Z^A|^2 + (1 - \delta^{BA}) \operatorname{Re}(Z^B Z^A) \right) \\ &= -4\mathrm{e}^{\mathcal{K}^{\mathrm{h}}} \left(\operatorname{Re} Z^B \operatorname{Re} Z^A + \delta^{BA} (\operatorname{Im} Z^A)^2 - (1 - \delta^{BA}) \operatorname{Im} Z^B \operatorname{Im} Z^A \right) \ . \end{aligned} \tag{8.16}$$

As a first step, we use (8.16) and insert $Z^4 = 1$ and (6.50) to calculate

$$N^{-1}_{BA}\bar{w}_A = -4e^{\mathcal{K}^{\mathrm{h}}} \operatorname{Re} Z^B(\bar{\tilde{C}} - \bar{D}^a \bar{x}_a) + N^{-1}_{Ba}\bar{x}_a \ . \tag{8.17}$$

Then we insert (8.16) also for the remaining N^{-1}_{Ba}

$$\begin{aligned} N^{-1}_{BA}\bar{w}_A &= -4e^{\mathcal{K}^{\mathrm{h}}} \Big(\operatorname{Re} Z^B \bar{\tilde{C}} - \operatorname{Re} Z^B (\operatorname{Re} D^a - \mathrm{i} \operatorname{Im} D^a)\bar{x}_a\Big) + N^{-1}_{Ba}\bar{x}_a \\ &= -4e^{\mathcal{K}^{\mathrm{h}}} \Big(\operatorname{Re} Z^B \bar{\tilde{C}} + \left\{\mathrm{i} \operatorname{Re} Z^B \operatorname{Im} D^a - \operatorname{Im} Z^B \operatorname{Im} D^a + 2\delta^{Ba}(\operatorname{Im} D^a)^2\right\} \bar{x}_a\Big) \ . \end{aligned} \tag{8.18}$$

Using (8.3) again, $\mathcal{D}^{\hat{I}}$ is given by (3.28) with Killing prepotential (8.14)

$$\begin{aligned} P^3_\lambda = & -4e^{\hat{\mathcal{K}}} \operatorname{Re}\left\{B^{STU}_\lambda (Z^3 - D^3)(i \operatorname{Im} D^a \bar{x}_a)\right. \\ & \left. +(s_{\lambda B} - r_\lambda{}^A \mathcal{G}_{AB}) \Big(\operatorname{Re} Z^B \bar{\tilde{C}} + 2\left[\delta^{B1}(\operatorname{Im} D^1)^2 \bar{x}_1 + \delta^{B2}(\operatorname{Im} D^2)^2 \bar{x}_2\right]\Big)\right\} - e^{\hat{\mathcal{K}} - \mathcal{K}^{\mathrm{h}}} t_\lambda \ . \end{aligned} \tag{8.19}$$

8.2.2 Quantum STU model

To calculate $N^{-1}_{BA}\bar{w}_A$ for the quantum STU model, we first multiply the expression from the second line of (7.14) with $\bar{w}_A$ and insert (6.50)

$$\begin{aligned} & \Big(\operatorname{Re}(Z^B \bar{Z}^A) + 2 \operatorname{Re} Z^B \operatorname{Re} Z^A - 2\delta^{BA}(\operatorname{Im} Z^A)^2\Big) \bar{w}_A \\ &= 3 \operatorname{Re} Z^B(\bar{\tilde{C}} - \bar{D}^a \bar{x}_a) + \left\{3 \operatorname{Re} Z^B \operatorname{Re} D^a + \operatorname{Im} Z^B \operatorname{Im} D^a - 2\delta^{Ba}(\operatorname{Im} D^a)^2\right\} \bar{x}_a \\ &= 3 \operatorname{Re} Z^B \bar{\tilde{C}} + \left\{3 \mathrm{i} \operatorname{Re} Z^B \operatorname{Im} D^a + \operatorname{Im} Z^B \operatorname{Im} D^a - 2\delta^{Ba}(\operatorname{Im} D^a)^2\right\} \bar{x}_a \ . \end{aligned} \tag{8.20}$$

We can now use (8.20) and the full N^{-1}_{BA} (7.14) to obtain

$$\begin{aligned} N^{-1}_{BA}\bar{w}_A = g(Z^3)h(Z^3)\Big[&6(\operatorname{Im} Z^1 \operatorname{Im} Z^3)^2 (N^{-1}_{BA}\bar{w}_A)(\alpha = 0) \\ &+ \alpha \operatorname{Im} Z^2 \Big(3 \operatorname{Re} Z^B \bar{\tilde{C}} + \left\{3 \mathrm{i} \operatorname{Re} Z^B \operatorname{Im} D^a + \operatorname{Im} Z^B \operatorname{Im} D^a - 2\delta^{Ba}(\operatorname{Im} D^a)^2\right\} \bar{x}_a\Big)\Big] \ . \end{aligned} \tag{8.21}$$

$(N^{-1}_{BA}\bar{w}_A)(\alpha = 0)$ is the result from the unperturbed STU model (8.18). All the terms in the second line of (8.21) already appear in (8.18), and when $(N^{-1}_{BA}\bar{w}_A)(\alpha = 0)$ is inserted into (8.21), the terms combine to cancel either $g(Z^3)$ or $h(Z^3)$

$$N^{-1}_{BA}\bar{w}_A = -\frac{3g(Z^3)}{\operatorname{Im} D^2} \operatorname{Re} Z^B \Big(\bar{\tilde{C}} + \mathrm{i} \operatorname{Im} D^a \bar{x}_a\Big) + \frac{h(Z^3)}{2 \operatorname{Im} D^2} \Big(\operatorname{Im} Z^B \operatorname{Im} D^a - 2\delta^{Ba}(\operatorname{Im} D^a)^2\Big) \bar{x}_a \ . \tag{8.22}$$

The corresponding P^3_λ can be calculated by inserting (8.22) into (8.14).

9 Supersymmetric vacua of the $\mathcal{N}=1$ theory

In this section, we check if the scalar potential (3.13) admits supersymmetric vacua. This is the case, if we can set $\langle D_{\hat{P}}\mathcal{W}\rangle = 0$ and $\langle \mathcal{D}^{\hat{I}}\rangle = 0$ [5].

9.1 Minima of V_F

Throughout this thesis, we have assumed a Minkowski space, which implies $\langle\mathcal{W}\rangle = 0$. We begin by showing as a consistency check that $\langle D_{\hat{P}}\mathcal{W}\rangle = 0$ also requires $\langle\mathcal{W}\rangle = 0$. The V_F part of the scalar potential is minimized for

$$\langle D_{\hat{P}}\mathcal{W}\rangle = \langle(\partial_{\hat{P}} + \partial_{\hat{P}}\mathcal{K})\mathcal{W}\rangle = 0 \ , \tag{9.1}$$

with $\mathcal{K} = \hat{\mathcal{K}} + \mathcal{K}^{\text{v}}$. Since $\mathcal{W}$ (3.27) and $\mathcal{K}^{\text{v}}$ (2.6) are independent of w^0 and w_A, the partial derivatives with respect to fiber coordinates vanish and the corresponding covariant derivatives are

$$D_w\mathcal{W} = (\partial_w\hat{\mathcal{K}})\mathcal{W} \ . \tag{9.2}$$

The derivative of the Kähler potential given by (7.7) with respect to w^0 is non-zero

$$\partial_{w^0}\hat{\mathcal{K}} = -\frac{1}{\operatorname{Re} w^0 + \operatorname{Re} w_A (N_{AB})^{-1} \operatorname{Re} w_B} = -\mathrm{e}^{\hat{\mathcal{K}}-\mathcal{K}^{\text{h}}} \neq 0 \ , \tag{9.3}$$

so $\langle D_{w^0}\mathcal{W}\rangle = 0$ implies $\langle\mathcal{W}\rangle = 0$.

Let us now turn to the covariant derivatives with respect to the base coordinates $X^{\hat{I}}$ and $Z^{\hat{A}}$. The indices $\hat{I}$ and $\hat{A}$ run over the $\mathcal{N}=1$ fields descending from the vector and hypermultiplets respectively. That means $\hat{A}$ takes $\hat{n}_{\text{h}}$ values and $\hat{I}$ takes $\hat{n}_{\text{v}}$ values. For a (quantum) *STU* prepotential $\mathcal{G}$, $\hat{A} = 3$. A quadratic $\mathcal{G}$ does not fix any base coordinates and $\hat{A} = 1, \dots, n_{\text{h}} - 1$, while a generic $\mathcal{G}$ fixes all fields and there are no $\hat{A}$ indices.

The two factors $X^I({\Theta_I}^\lambda - \mathcal{F}_{IJ}\Theta^{J\,\lambda})$ and $(s_{\lambda B} - {r_\lambda}^A\mathcal{G}_{AB})Z^B$ that together form the superpotential (8.7) are both linear in the fields $X^{\hat{I}}$ or $Z^{\hat{A}}$, for all prepotentials appearing in table 6.1. For (quantum) *STU* prepotentials, this is explained by (8.9), for quadratic prepotentials $\mathcal{G}_{AB}$ or $\mathcal{F}_{IJ}$ is constant and generic prepotentials fix all fields so that $X^I({\Theta_I}^\lambda - \mathcal{F}_{IJ}\Theta^{J\,\lambda})$ or $(s_{\lambda B} - {r_\lambda}^A\mathcal{G}_{AB})Z^B$ is constant altogether.

To make this more concrete, we introduce complex constants A^λ and B_λ for the parts that are constant

$$X^I({\Theta_I}^\lambda - \mathcal{F}_{IJ}\Theta^{J\,\lambda}) \equiv \begin{cases} (X^3 - C^3)A^\lambda_{STU} \ , & \mathcal{F} \text{ (quantum) } STU \ , \\ X^I A^\lambda_{\text{quad},I} \ , & \mathcal{F} \text{ quadratic} \ , \\ A^\lambda_{\text{gen}} \ , & \mathcal{F} \text{ generic} \ , \end{cases} \tag{9.4}$$

$$(s_{\lambda B} - r_\lambda{}^A \mathcal{G}_{AB}) Z^B \equiv \begin{cases} B_\lambda^{STU}(Z^3 - D^3) \;, & \mathcal{G} \text{ (quantum) } STU \;, \\ B^{\text{quad}}_{\lambda B} Z^B \;, & \mathcal{G} \text{ quadratic} \;, \\ B^{\text{gen}}_\lambda \;, & \mathcal{G} \text{ generic} \;. \end{cases} \tag{9.5}$$

Combinations of STU, quantum STU and generic prepotentials imply a zero superpotential, because in these cases the superpotential consists only of a constant factor $A^\lambda B_\lambda$ and for (quantum) STU prepotentials factors $(X^3 - C^3)$ or $(Z^3 - D^3)$ that are non-zero due to (6.23) or (6.34). Here $\langle \mathcal{W} \rangle = 0$ implies $A^\lambda B_\lambda = 0$ and thus $\mathcal{W} = 0$. In these cases the superpotential must be set to zero by choosing an appropriate embedding tensor and gauged Killing vectors.

If STU, quantum STU and generic prepotentials are combined with a quadratic prepotential, say $\mathcal{F}$ quadratic and $\mathcal{G}$ generic or (quantum) STU, the situation is similar. With the superpotential vanishing in the vacuum, the covariant derivatives are just partial derivatives, which implies either for $\mathcal{G}$ generic

$$0 = \langle \partial_{X^I} \mathcal{W} \rangle = \langle 2 A^\lambda_{\text{quad},I} B^{\text{gen}} \rangle \;, \tag{9.6}$$

or for $\mathcal{G}$ (quantum) STU

$$0 = \langle \partial_{X^I} \mathcal{W} \rangle = \langle 2 A^\lambda_{\text{quad},I} B^{STU} (Z^3 - D^3) \rangle \;. \tag{9.7}$$

$(Z^3 - D^3)$ is non-zero in the (quantum) STU case, so $A^\lambda_{\text{quad},I} B^{STU/\text{gen}}$ must be set to zero in either case, which implies that the superpotential vanishes identically. More generally speaking, a STU, quantum STU or generic prepotential $\mathcal{G}$ allows only for non-trivial superpotentials, if the other prepotential $\mathcal{F}$ is such that $X^I(\Theta_I{}^\lambda - \mathcal{F}_{IJ}\Theta^{J\,\lambda})$ is not just linear in the fields X^I.

The last remaining case is that both prepotentials are quadratic. We impose 9.1 and use $\langle \mathcal{W} \rangle = 0$

$$0 = \langle \partial_{X^I} \mathcal{W} \rangle = \langle 2 A^\lambda_{\text{quad},I} B^{\text{quad}}_{\lambda A} Z^A \rangle \;, \tag{9.8}$$

$$0 = \langle \partial_{Z^A} \mathcal{W} \rangle = \langle 2 X^I A^\lambda_{\text{quad},I} B^{\text{quad}}_{\lambda A} \rangle \;. \tag{9.9}$$

$A^\lambda_{\text{quad},I} B^{\text{quad}}_{\lambda A}$ is the product of a $(n_{\text{v}} + 1) \times (n-2)$ and a $(n-2) \times n_{\text{h}}$ matrix, so its rank is at most $\min(n_{\text{v}} + 1, n_{\text{h}}, n-2)$. The reason why λ takes only $n-2$ values is because the first two gauged isometries do not contribute to the superpotential. We know from (8.3) and (8.4) that both $A^\lambda_{\text{quad},I}$ and $B^{\text{quad}}_{\lambda A}$ have a non-trivial null eigenvector

$$C^I A^\lambda_{\text{quad},I} = 0 \;, \qquad B^{\text{quad}}_{\lambda A} D^A = 0 \;, \tag{9.10}$$

which restricts the possible rank of $A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A}$ further

$$\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A}) \leq \min(n_{\text{v}}, n_{\text{h}} - 1, n - 2) \ . \tag{9.11}$$

The conditions (9.8) and (9.9) fix $\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A})$ of the fields Z^A and X^I respectively. If $\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A}) = n_{\text{h}} - 1$, then D is the only non-trivial null eigenvector of the matrix and (9.8) fixes Z to be proportional to D. This solution however does not lie in the domain of the Z^A, because it creates a contradiction between (4.24) and $2Z^A N_{AB}\bar{Z}^B = \mathrm{e}^{-\mathcal{K}^{\text{h}}} > 0$. The same argument would hold for $\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A}) = n_{\text{v}}$, but that is not possible due to (8.1) and (9.11). Thus, a non-trivial superpotential is only allowed for two quadratic prepotentials if

$$\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A}) < n_{\text{h}} - 1 \ . \tag{9.12}$$

In this case the vacuum retains $n_{\text{h}} - 1 + n_{\text{v}} - 2\text{rk}(A^{\lambda}_{\text{quad},I}B^{\text{quad}}_{\lambda A})$ complex moduli from the X^I and Z^A.

Let us summarize the results of this section. When considering STU, quantum STU, generic or purely quadratic prepotentials, the only case admitting a non-trivial superpotential with supersymmetric vacua is when $\mathcal{G}$ and $\mathcal{F}$ are both quadratic and (9.12) holds.

9.2 Minima of $V_{\mathcal{D}}$

Next we try to set the D-terms (8.15) to zero $\langle \mathcal{D}^{\hat{I}} \rangle = 0$, which is another requirement for a supersymmetric ground state.

First of all, the projectors Π^I_J and Γ^J_K in (8.15) cause $\mathcal{D}^{\hat{I}}$ to be orthogonal to the vectors $\bar{X}^J \,\text{Im}(\mathcal{F})_{JI}$ and $\bar{C}^{(P)\,J}\,\text{Im}(\mathcal{F})_{JI}$ due to (D.12) and (D.14)

$$\bar{X}^J \,\text{Im}(\mathcal{F})_{JI}\mathcal{D}^I = 0 \ , \qquad \bar{C}^{(P)\,J}\,\text{Im}(\mathcal{F})_{JI}\mathcal{D}^I = 0 \ . \tag{9.13}$$

This implies that at most $n_{\text{v}} - 1$ of the $n_{\text{v}} + 1$ D-terms can be independent. The $n_{\text{v}} - 1$ complex conditions $\langle \mathcal{D}^{\hat{I}} \rangle = 0$ can be satisfied by solving the smaller set of $n - 2$ real conditions $\langle P^3_{\lambda} \rangle = 0$, where P^3_{λ} (8.14) is just one of the factors occurring in $\mathcal{D}^{\hat{I}}$. This is always a smaller number of conditions due to (8.1). Let us now try to solve these $n-2$ real conditions. The term $(\text{Im}\,\mathcal{F})^{-1\ KL}\left(\Theta_L{}^{\lambda} - \bar{\mathcal{F}}_{LM}\Theta^{M\lambda}\right)$ in (8.15) depends only on the fields via $\mathcal{F}_{IJ}$. It was argued in the previous subsection that $\mathcal{F}_{IJ}$ is constant for quadratic and generic prepotentials and depends only on X^3 for (quantum) STU prepotentials. Analogously, $(s_{\lambda B} - r_{\lambda}{}^A\mathcal{G}_{AB})(N_{BC})^{-1}$ depends only on $\mathcal{G}_{AB}$, which in turn depends on Z^3 for (quantum) STU prepotentials and is constant otherwise. So there are at most four real base coordinates available to solve $n-2$ equations. On the other hand, P^3_{λ} contains the fiber coordinates $w_A = w_A(x_a)$ which depend on the $n_{\text{h}} - 2$ complex fields x_a. These constitute $2(n_{\text{h}} - 2)$ real unknowns to solve $n-2$ equations. Because of (8.2), there are always enough unknowns to solve the equations, if $n_{\mathrm{h}} \geq 3$. After solving $\langle P^3_\lambda \rangle = 0$ by fixing fiber coordinates x_a, there are at least $2n_{\mathrm{h}} - n - 2$ real moduli from the x_a left. The two degrees of freedom of x^0 remain untouched.

10 Conclusion

By calculating the $\mathcal{N} = 1$ Kähler potential for the STU model (7.12), we showed that the quotient and submanifold construction that corresponds to integrating out the heavy gravitino multiplet can be performed in such a way that the special quaternionic-Kähler manifold $\frac{SO(4,4)}{SO(4)\times SO(4)}$ descends to the Kähler manifold $\frac{SO(4,2)}{SO(4)\times SO(2)}$. The deformed $\mathcal{N} = 1$ Kähler potential that emerges from the quantum STU prepotential (5.5) was also determined (7.27).

For both the perturbed and unperturbed STU prepotentials, two of the three fields S, T, U are stabilized in the $\mathcal{N} = 1$ theory. The superpotential has a very simple dependency on the remaining field, a non-zero factor depending on the field factors out. As a consequence, the superpotential must vanish if both prepotentials are (quantum) STU or generic. Also, a combination of a (quantum) STU or generic prepotential with a quadratic prepotential is only compatible with a supersymmetric vacuum, if the superpotential vanishes identically. An example with non-trivial superpotential and supersymmetric vacua is the model where both prepotentials $\mathcal{F}$ and $\mathcal{G}$ are quadratic and the number of gauged isometries does not exceed the number of hypermultiplets.

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A Symplectic invariance

Since symplectic objects appear throughout this thesis, we introduce the symplectic invariance that is inherent to the vector multiplets. This discussion follows the review [17]. The dual magnetic field strengths are defined as

$$G_I^{\mu\nu\pm} = \pm\frac{\mathrm{i}}{2}\frac{\partial\mathcal{L}}{\partial F^{I\pm}_{\mu\nu}} \ . \tag{A.1}$$

Spacetime indices are suppressed from now on. One can read off from Lagrangian (2.4)

$$G_I^+ = \mathcal{N}_{IJ}\, F^{J+} \ , \qquad G_I^- = \overline{\mathcal{N}}_{IJ}\, F^{J-} \ . \tag{A.2}$$

The field equations and Bianchi identities of the vectors that are associated with the Lagrangian are

$$\begin{aligned} \partial^\mu \operatorname{Im} F^{I\pm}_{\mu\nu} &= 0 \ , \\ \partial^\mu \operatorname{Im} G^{I\pm}_{\mu\nu} &= 0 \ . \end{aligned} \tag{A.3}$$

These equations are not affected by the rotation

$$\begin{pmatrix} F^+ \\ G^+ \end{pmatrix}' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F^+ \\ G^+ \end{pmatrix} \ , \qquad \begin{pmatrix} F^- \\ G^- \end{pmatrix}' = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F^- \\ G^- \end{pmatrix} \ , \tag{A.4}$$

with $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \equiv \Lambda \in GL(2n_{\mathrm{v}}, \mathbb{R})$. The transformations are further restricted by the fact that the definition of $G^{\pm\prime}$ (A.1) must still lead to a symmetric kinetic matrix $\mathcal{N}$. Inserting the primed quantities into the first equation of (A.2) gives

$$\mathcal{N}' = (C + D\mathcal{N})\,(A + B\mathcal{N})^{-1} \tag{A.5}$$

Requiring $\mathcal{N}' = \mathcal{N}'^T$ results in

$$(C + D\mathcal{N})\,(A + B\mathcal{N})^{-1} = (A + B\mathcal{N})^{-1T}\,(C + D\mathcal{N})^T \ , \tag{A.6}$$

or equivalently

$$(A + B\mathcal{N})^T\,(C + D\mathcal{N}) = (C + D\mathcal{N})^T\,(A + B\mathcal{N}) \ . \tag{A.7}$$

The requirements on Λ can be extracted from (A.7) in powers of $\mathcal{N}$.

$$\begin{aligned} A^T C &= C^T A \ , \\ \mathcal{N} B^T D \mathcal{N} &= \mathcal{N} D^T B \mathcal{N} \ , \\ (A^T D - C^T B)\mathcal{N} &= \mathcal{N}(D^T A - B^T C) \ . \end{aligned} \tag{A.8}$$

The last equation implies that $(A^T D - C^T B)$ is proportional to the identity matrix. If rescalings are neglected, (A.8) can be brought into the familiar form of conditions for symplectic matrices

$$\begin{aligned} A^T C - C^T A &= 0 \ , \\ B^T D - D^T B &= 0 \ , \\ A^T D - C^T B &= 1 \ . \end{aligned} \tag{A.9}$$

This means Λ is an element of the symplectic group $Sp(2n_{\mathrm{v}},\mathbb{R})$. The symplectic symmetry extends to the scalars of the vector multiplets due to (A.5) and the fact that $\mathcal{N}$ is a function of these scalars (2.8). In order to find a true symmetry of the Lagrangian, the symplectic duality transformations on the field strengths have to be accompanied by a suitable transformation of the scalars that provides for the correct transformation (A.5) of $\mathcal{N}$. This leads ultimately to the requirement that the group of isometries on $\mathbf{M}_{\mathrm{v}}$ that leave the Lagrangian invariant must be embedded into $Sp(2n_{\mathrm{v}},\mathbb{R})$ [17]. As a result, the global invariance group of the ungauged Lagrangian (2.4) is embedded into a product of $Sp(2n_{\mathrm{v}},\mathbb{R})$ and the group of isometries on the hypermultiplet scalar manifold $Iso(\mathbf{M}_{\mathrm{h}})$

$$G_{\mathrm{global}} \subseteq Sp(2n_{\mathrm{v}},\mathbb{R}) \times Iso(\mathbf{M}_{\mathrm{h}}) \ . \tag{A.10}$$

B Supersymmetry breaking conditions

This appendix follows a more general calculation from [14]. Here, a shortened derivation that is only applicable in a Minkowski background is provided. It will be shown that in the scenario at hand, where a prepotential $\mathcal{F}$ exists, partial supersymmetry breaking is only possible in the presence of magnetic charges and restricts the embedding tensor.

(2.18) is inserted into the first equation of (3.2) to get

$$\begin{aligned} W_{i\mathcal{A}\mathcal{B}}\,\epsilon_1^{\mathcal{B}} &= -\,\mathrm{i}\,\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2}(\nabla_i X^I \Theta_I{}^{\lambda} - \nabla_i \mathcal{F}_I \Theta^{I\lambda}) P^x_{\lambda} \varepsilon^{\mathcal{A}\mathcal{C}} (\sigma^x)^{*}{}_{\mathcal{C}}{}^{\mathcal{B}} \epsilon_1^{\mathcal{B}} \\ &= \mathrm{i}\,\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2}(\partial_i X^I \Theta_I{}^{\lambda} - \partial_i \mathcal{F}_I \Theta^{I\lambda}) P^x_{\lambda} (\sigma^x)_{\mathcal{A}}{}^{\mathcal{C}} \varepsilon_{\mathcal{C}\mathcal{B}} \epsilon_1^{\mathcal{B}} + 2\,\mathrm{i}\,\mathcal{K}^{\mathrm{v}}_i S_{\mathcal{A}\mathcal{B}} \epsilon_1^{\mathcal{B}} \\ &= \mathrm{i}\,\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2}(\Theta_i{}^{\lambda} - \mathcal{F}_{iI} \Theta^{I\lambda}) P^x_{\lambda} (\sigma^x)_{\mathcal{A}}{}^{\mathcal{C}} \varepsilon_{\mathcal{C}\mathcal{B}} \epsilon_1^{\mathcal{B}} = 0 \ , \end{aligned} \tag{B.1}$$

where the vanishing gravitino variation of (3.2) and the following identity were used

$$\varepsilon^{\mathcal{A}\mathcal{C}} (\sigma^x)^{*}{}_{\mathcal{C}}{}^{\mathcal{B}} = -(\sigma^x)_{\mathcal{A}}{}^{\mathcal{C}} \varepsilon_{\mathcal{C}\mathcal{B}} \ . \tag{B.2}$$

With (B.1), the vanishing gravitino variation of (3.2) implies

$$\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2}(\partial_0 X^I \Theta_I{}^{\lambda} - \partial_0 \mathcal{F}_I \Theta^{I\lambda}) P^x_{\lambda} (\sigma^x)_{\mathcal{A}}{}^{\mathcal{C}} \varepsilon_{\mathcal{C}\mathcal{B}} \epsilon_1^{\mathcal{B}} = 0 \ , \tag{B.3}$$

Combining (B.1) and (B.3) yields

$$(\Theta_I{}^{\lambda} - \mathcal{F}_{IJ} \Theta^{J\lambda})\, P^x_{\lambda} \sigma^x_{\mathcal{A}\mathcal{B}} \epsilon_1^{\mathcal{B}} \ = \ 0 \quad \text{for all } I \ , \tag{B.4}$$

Taking the complex conjugate of the broken gravitino variation of (3.3) and inserting

$$(\epsilon_2^{\mathcal{A}})^{*} = \varepsilon_{\mathcal{A}\mathcal{B}} \epsilon_1^{\mathcal{B}} \ , \tag{B.5}$$

yields

$$(S_{\mathcal{AB}}\epsilon_2^{\mathcal{B}})^* = \tfrac{1}{2}\mathrm{e}^{\mathcal{K}^{\mathrm{v}}/2}\bar{V}^{\Lambda}\Theta_{\Lambda}{}^{\lambda}P_{\lambda}^{x}(\sigma^x)^{*\mathcal{C}}_{\mathcal{A}}\varepsilon_{\mathcal{CB}}\varepsilon_{\mathcal{BD}}\epsilon_1^{\mathcal{D}} \neq 0 \ . \tag{B.6}$$

Now (B.2) can be used again to show that (B.6) implies

$$(\Theta_I{}^{\lambda} - \bar{\mathcal{F}}_{IJ}\Theta^{J\lambda})\, P_{\lambda}^{x}\sigma^{x}_{\mathcal{AB}}\epsilon_1^{\mathcal{B}} \neq 0 \quad \text{for some } I \ . \tag{B.7}$$

One immediately sees that (B.4) and (B.7) cannot be satisfied simultaneously, if there are no magnetic charges, $\Theta^{J\lambda} = 0$. They also cannot be both satisfied if only one isometry is gauged, because then (B.4) factorizes and the vanishing of either of the factors implies the vanishing of the left side of (B.7). Furthermore, (B.4) and (B.7) can only be satisfied for certain embedding tensors.

The next step that was done in [15] was to choose a SU(2) frame to simplify the analysis, namely $P^3_{1,2} = 0$. For $\epsilon_1^{\mathcal{A}} = (\epsilon_1^1, 0)$, (B.4) and (B.7) then become

$$P_1^-(\Theta_I{}^1 - \mathcal{F}_{IJ}\Theta^{J1}) + P_2^-(\Theta_I{}^2 - \mathcal{F}_{IJ}\Theta^{J2}) = 0 \quad \text{for all } I \ , \tag{B.8a}$$

$$P_1^-(\Theta_I{}^1 - \bar{\mathcal{F}}_{IJ}\Theta^{J1}) + P_2^-(\Theta_I{}^2 - \bar{\mathcal{F}}_{IJ}\Theta^{J2}) \neq 0 \quad \text{for some } I \ , \tag{B.8b}$$

where

$$P_{\lambda}^{\pm} \equiv P_{\lambda}^1 \pm \mathrm{i}\, P_{\lambda}^2 \ . \tag{B.9}$$

(B.8) is solved by

$$\begin{aligned} \Theta_I{}^1 &= -\operatorname{Im}(P_2^+\mathcal{F}_{IJ}C^J) \ , \qquad & \Theta^{I1} &= -\operatorname{Im}(P_2^+C^I) \ , \\ \Theta_I{}^2 &= \operatorname{Im}(P_1^+\mathcal{F}_{IJ}C^J) \ , \qquad & \Theta^{I2} &= \operatorname{Im}(P_1^+C^I) \ , \end{aligned} \tag{B.10}$$

for an arbitrary complex vector C^I.

There is one condition left that must be enforced to make sure that the supersymmetry generated by $\epsilon_1^{\mathcal{A}}$ is really unbroken, the hyperino condition in (3.2)

$$0 = N_{\alpha\mathcal{A}}\epsilon_1^{\mathcal{A}} = N_{\alpha}^2\varepsilon_{21}\epsilon_1^1 \ . \tag{B.11}$$

Inserting N_{α}^2 from (2.18) results in

$$\hat{k}^u\,\mathcal{U}^2_{\alpha u} \ = \ 0 \ , \tag{B.12}$$

where

$$\hat{k}^u = V^{\Lambda}\big(\Theta_{\Lambda}{}^1\hat{k}_1^u + \Theta_{\Lambda}{}^2\hat{k}_2^u\big) \ . \tag{B.13}$$

At this point a change of basis is performed. $\hat{k}^u_{1,2}$ denote the real and imaginary parts of (B.13) from now on. This does not affect the form of Lagrangian, if the embedding tensor is changed accordingly. One can contract (B.12) with $\mathcal{U}_v^{\mathcal{B}\alpha}$ and use the formula [17, 23]

$$\mathcal{U}_{\alpha u}^{\mathcal{A}}\mathcal{U}_v^{\mathcal{B}\alpha} = -\tfrac{\mathrm{i}}{2}K_{uv}^x\sigma^{x\mathcal{AB}} - \tfrac{1}{2}h_{uv}\epsilon^{\mathcal{AB}} \ , \tag{B.14}$$

to obtain

$$\hat{k}^u\left(J^1{}_u{}^v - \mathrm{i}\,J^2{}_u{}^v\right) = 0 \ , \qquad \hat{k}^u J^3{}_u{}^v = \mathrm{i}\,\hat{k}^v \ . \tag{B.15}$$

The relation between the almost complex structures (2.9) can be used to show that these two conditions are equivalent on a quaternionic-Kähler manifold. The second equation in (B.15) just means that $\hat{k}^u$ is holomorphic with respect to J^3. Written in real and imaginary parts of $\hat{k}^u$ it reads

$$J^3{}_u{}^v\hat{k}_1^u = -\hat{k}_2^v \ , \qquad J^3{}_u{}^v\hat{k}_2^u = \hat{k}_1^v \ . \tag{B.16}$$

Contracting the first equation of (B.15) with the metric and reading off real and imaginary parts, one obtains

$$\hat{k}_1^u K_{uv}^1 = -\hat{k}_2^u K_{uv}^2 \ , \qquad \hat{k}_1^u K_{uv}^2 = \hat{k}_2^u K_{uv}^1 \ . \tag{B.17}$$

Having in mind the definition of the Killing prepotentials (2.19), (B.17) can be arranged by choosing prepotentials

$$P_1^1 = -P_2^2 \ , \qquad P_1^2 = P_2^1 \ . \tag{B.18}$$

This, together with the aforementioned

$$P_1^3 = P_2^3 = 0 \ , \tag{B.19}$$

is a sufficient condition to fulfill the hyperino constraint. With (B.18) and after a redefinition of the C^I, the embedding tensor (B.10) becomes

$$\begin{aligned} \Theta_I{}^1 &= \mathrm{Re}\left(\mathcal{F}_{IJ}\,C^J\right) \ , \qquad & \Theta^{I1} &= \ \mathrm{Re}\,C^I \ , \\ \Theta_I{}^2 &= \mathrm{Im}\left(\mathcal{F}_{IJ}\,C^J\right) \ , \qquad & \Theta^{I2} &= \ \mathrm{Im}\,C^I \ . \end{aligned} \tag{B.20}$$

One can conclude that (B.18-B.20) are sufficient conditions for partial supersymmetry breaking.

C Kähler potential

This appendix reviews the discussion of the quotient construction of [15]. It is shown that integrating out the two heavy gauge bosons corresponds to taking the quotient $\mathbf{M}_{\mathrm{h}}/\langle\hat{k}_1, \hat{k}_2\rangle$. The resulting target space is a Kähler manifold, consistent with $\mathcal{N} = 1$ supersymmetry, and the Kähler potential is determined.

The Lagrangian (2.4) with covariant derivatives (2.14) has the gauge boson mass terms

$$h_{uv}\, D_\mu q^u D^\mu q^v = \ldots + h_{uv}\, \left(A_\mu{}^I\, \Theta_I{}^\lambda - B_{\mu I}\, \Theta^{I\lambda}\right) \hat{k}^u_\lambda \left(A^{\mu I}\, \Theta_I{}^\rho - B^\mu{}_I\, \Theta^{I\rho}\right) \hat{k}^v_\rho\ . \tag{C.1}$$

The two heavy gauge bosons can be identified as

$$A^\lambda_\mu \equiv A^\Lambda_\mu \Theta^\lambda_\Lambda = A^I_\mu \Theta^\lambda_I - B_{\mu I}\Theta^{I\lambda}\ , \tag{C.2}$$

and the following mass matrix can be shown to be diagonal using (B.16) and $J^3{}_u{}^v J^3{}_v{}^w = -\delta^w_u$

$$m^2_{\lambda\rho} = 2\hat{k}^u_\lambda h_{uv} \hat{k}^v_\rho = m^2\, \delta_{\lambda\rho}\ , \tag{C.3}$$

where

$$m^2 = 2\hat{k}^u_1 \hat{k}_{1u} = 2\hat{k}^u_2 \hat{k}_{2u}\ . \tag{C.4}$$

In the limit $p \ll m_{3/2}$, the kinetic terms of the massive gauge bosons can be neglected in order to obtain the algebraic field equations

$$\frac{\partial \mathcal{L}}{\partial A^\lambda_\mu} = -2\hat{k}^v_\lambda h_{uv} D_\mu q^u = -2\hat{k}^v_\lambda h_{uv} \partial_\mu q^u + m^2_{\lambda\rho} A^\rho_\mu = 0\ , \qquad \lambda, \rho = 1, 2\ , \tag{C.5}$$

which are used to eliminate the gauge fields. The gauge fields only appear in the covariant derivatives (2.14) and can be eliminated using (C.5)

$$D_\mu q^u = \tilde{\pi}^u_v \partial_\mu q^v\ , \quad \text{with the projector } \tilde{\pi}^u_v \equiv \delta^u_v - \frac{2\hat{k}^u_\lambda \hat{k}_{\lambda v}}{m^2}\ , \tag{C.6}$$

The Lagrangian now reads

$$\hat{\mathcal{L}} = \hat{h}_{uv} \partial_\mu q^u \partial^\mu q^v\ , \tag{C.7}$$

with

$$\hat{h}_{uv} = \tilde{\pi}^w_u h_{wr} \tilde{\pi}^r_v = \tilde{\pi}^w_u h_{wv}\ . \tag{C.8}$$

The last equality can be easily verified using (C.4). The two gauged isometries are annihilated by the metric

$$\hat{h}_{uv} \hat{k}^v_{1,2} = 0\ , \tag{C.9}$$

so they are orthogonal to the $\mathcal{N} = 1$ field space. This implies that the resulting manifold is the quotient with respect to the gauged isometries.

The Kähler two-form $\hat{K}_{uv}$ on the quotient was calculated in a similar way, by imposing an auxiliary two-dimensional σ-model with metric K^3_{uv} and performing analogous steps. For this purpose, the following auxiliary Lagrangian is used

$$\mathcal{L}_{K^3} = K^3_{uv} D_\alpha q^u D_\beta q^v \epsilon^{\alpha\beta}\ , \quad \alpha, \beta = 1, 2\ . \tag{C.10}$$

The mass matrix is now

$$m^2_{\lambda\rho} = 2\hat{k}^u_\lambda K^3_{uv} \hat{k}^v_\rho = m^2\, \epsilon_{\lambda\rho} \ , \tag{C.11}$$

with m^2 still being the one from (C.4). This time, the field equation in the low-energy limit is

$$\frac{\partial \mathcal{L}_{K^3}}{\partial A^\lambda_\alpha} = -2\hat{k}^u_\lambda K^3_{uv} D_\beta q^v \epsilon^{\alpha\beta} = -2\hat{k}^u_\lambda K^3_{uv} \partial_\beta q^v \epsilon^{\alpha\beta} + m^2_{\lambda\rho} A^\rho_\beta \epsilon^{\alpha\beta} = 0 \ , \qquad \lambda, \rho = 1, 2 \ . \tag{C.12}$$

One can show using (C.11) that the this field equation leads again to the same covariant derivative (C.6). One obtains the $\mathcal{N} = 1$ Lagrangian

$$\hat{\mathcal{L}}_{K^3} = \hat{K}^3_{uv} D_\alpha q^u D_\beta q^v \epsilon^{\alpha\beta} \ , \tag{C.13}$$

with

$$\hat{K}_{uv} = \tilde{\pi}^w_u K^3_{wr} \tilde{\pi}^r_v = K^3_{uv} - \frac{2\hat{k}_{\lambda u} \epsilon_{\lambda\sigma} \hat{k}_{\sigma v}}{m^2} = \tilde{\pi}^w_u K^3_{wv} \ . \tag{C.14}$$

The next step is to show that the $\hat{K}$ is the differential of the connection ω^3

$$\hat{K} = \mathrm{d}\omega^3 \ . \tag{C.15}$$

To this end, it is used that for two commuting isometries $\hat{k}_1$ and $\hat{k}_2$ [17]

$$2\hat{k}^u_1 \hat{k}^v_2 K^x_{uv} + \epsilon^{xyz} P^y_1 P^z_2 = 0 \ , \tag{C.16}$$

so that the squared mass m^2 in (C.11) is

$$m^2 = 2\hat{k}^u_1 K^3_{uv} \hat{k}^v_2 = P^1_1 P^2_2 - P^1_2 P^2_1 \ . \tag{C.17}$$

Contracting (B.16) with the metric and using the definition of the prepotentials (2.19) yields

$$\begin{aligned} \hat{k}_{2v} &= \hat{k}^u_1 K^3_{uv} &= -\frac{1}{2}\left(\omega^2_v P^1_1 - \omega^1_v P^2_1\right) \ , \\ \hat{k}_{1v} &= -\hat{k}^u_2 K^3_{uv} &= -\frac{1}{2}\left(\omega^1_v P^2_2 - \omega^2_v P^1_2\right) \ . \end{aligned} \tag{C.18}$$

When (C.17) and (C.18) are inserted into (C.14), the result is

$$\hat{K}_{uv} = K^3_{uv} + \frac{1}{2}(\omega^2_u \omega^1_v - \omega^1_u \omega^2_v) = \partial_u \omega^3_v - \partial_v \omega^3_u \ , \tag{C.19}$$

where (2.10) was used in the second step. So $\hat{K} = \mathrm{d}\omega_3$ is a closed fundamental two-form.

The almost complex structure on the quotient is defined by the equivalent to (2.7) $K_{uv} = h_{uw} J^w_v$. It is

$$\hat{J}^u_v = \tilde{\pi}^u_w J^{3w}_v \ , \tag{C.20}$$

the projected complex structure J^3.

All that is left to do now to show that $\mathbf{M}_{\rm h}/\langle \hat{k}_1, \hat{k}_2\rangle$ is a Kähler manifold is to show that it is a complex manifold, i.e. that the Nijenhuis-tensor vanishes, which is indeed the case [15].

D D-terms

The $\mathcal{N}=1$ D-terms were derived in [15]. This appendix follows this derivation. The D-terms were found similarly to the superpotential in section 3.3.2. They appear in the $\mathcal{N}=1$ gaugino variations. In this case, it is necessary to identify the $\mathcal{N}=1$ gaugino among the available gauginos first. To this end, the following $\mathcal{N}=2$ gaugino variation [17] is considered

$$\delta_\epsilon \lambda^{i\mathcal{A}} = \gamma^\mu \partial_\mu t^i \epsilon^{\mathcal{A}} - \tilde{G}^{i-}_{\mu\nu}\gamma^{\mu\nu}\varepsilon^{\mathcal{A}\mathcal{B}}\epsilon_{\mathcal{B}} + W^{i\mathcal{A}\mathcal{B}}\epsilon_{\mathcal{B}} + \dots \ . \tag{D.1}$$

Here $\tilde{G}^{i-}_{\mu\nu}$ are the 'dressed' anti-self-dual field strengths [17]

$$\tilde{G}^{i-}_{\mu\nu} = -\mathrm{e}^{\frac{\mathcal{K}^{\rm V}}{2}} g^{i\bar{\jmath}} \nabla_{\bar{\jmath}} \bar{X}^I \mathrm{Im}\mathcal{N}_{IJ} F^{J-}_{\mu\nu} + \dots \ . \tag{D.2}$$

The ellipses denote higher-order fermionic contributions. Inserting the preserved supersymmetry variation $\epsilon_1 = (\epsilon, 0),\ \epsilon_2 = 0$, (D.1) becomes

$$\delta_\epsilon \lambda^{i1} \ = \gamma^\mu \partial_\mu t^i \bar{\epsilon} + W^{i11}\epsilon \ + \dots \ , \tag{D.3}$$

$$\delta_\epsilon \lambda^{i2} \ = \ -\tilde{G}^{i-}_{\mu\nu}\gamma^{\mu\nu}\epsilon + W^{i21}\epsilon \ + \dots \ . \tag{D.4}$$

The standard $\mathcal{N}=1$ variations are [5, 25]

$$\delta_\epsilon \lambda^{\hat{I}} = F^{\hat{I}-}_{\mu\nu}\gamma^{\mu\nu}\epsilon + \mathrm{i}\mathcal{D}^{\hat{I}}\epsilon \ + \dots \ , \tag{D.5}$$

So λ^{i2} are candidates, but before the D-terms can be read off, one first has to know how the 'dressed' magnetic field strengths $\tilde{G}^{i-}_{\mu\nu}$ compare to the $\mathcal{N}=1$ field strengths $F^{\hat{I}-}_{\mu\nu}$.

First, an expression for the $\mathcal{N}=1$ field strengths is introduced. Integrating out the heavy gauge bosons (C.2) corresponds to fixing them to constants, $\partial_\mu A^{1,2}_\nu = 0$, which implies for the field strengths

$$\Theta^\lambda_I\, F^{I\pm}_{\mu\nu} - \Theta^{\lambda I} G^{\ \pm}_{I\mu\nu} = 0 \ , \qquad \lambda = 1, 2 \ . \tag{D.6}$$

This is easily checked by inserting the familiar definition of the Abelian (anti-)self-dual field strengths

$$
\begin{aligned}
&\Theta_I^\lambda F^{I\pm} - \Theta^{\lambda I} G^{\pm}_I \\
&= \frac{1}{2}\left(\Theta_I^\lambda \partial_{[\mu} A^I_{\nu]} - \Theta^{\lambda I}\partial_{[\mu} B_{\nu]I}\right) \pm \frac{\mathrm{i}}{4}\epsilon_{\mu\nu\rho\sigma}\left(\Theta_I^\lambda \partial_{[\rho} A^I_{\sigma]} - \Theta^{\lambda I}\partial_{[\rho} B_{\sigma]I}\right) \\
&= \frac{1}{2}\partial_{[\mu} A^\lambda_{\nu]} \pm \frac{\mathrm{i}}{4}\epsilon_{\mu\nu\rho\sigma}\partial_{[\rho} A^\lambda_{\sigma]} = 0 \ .
\end{aligned} \tag{D.7}
$$

Inserting complex combinations of the embedding tensor solution (3.9) into (D.6) and using (A.2), one obtains

$$
C^I(\mathcal{F}_{IJ} - \bar{\mathcal{N}}_{IJ})F^{J-} = 0 \ , \tag{D.8}
$$

$$
\bar{C}^I(\bar{\mathcal{F}}_{IJ} - \bar{\mathcal{N}}_{IJ})F^{J-} = 0 \ . \tag{D.9}
$$

When (2.8) is inserted and after omitting a non-zero factor, (D.8) becomes

$$
\bar{X}^I \,\mathrm{Im}\,(\mathcal{F}_{IJ})\, F^{J-} = 0 \ . \tag{D.10}
$$

To get a field strength satisfying (D.10), one defines the projector

$$
\Pi^I_J \equiv \delta^I_J - 2\mathrm{e}^{\mathcal{K}^{\mathrm{v}}} X^I \bar{X}^K \,\mathrm{Im}(\mathcal{F})_{KJ} \ . \tag{D.11}
$$

The projected field strength $\Pi^I_J F^{J-}$ satisfies (D.10) automatically, because

$$
\bar{X}^I \,\mathrm{Im}\,(\mathcal{F}_{IJ})\, \Pi^J_K = 0 \ . \tag{D.12}
$$

Analogously, (D.9) yields

$$
\bar{C}^{(P)\,I}\,\mathrm{Im}(\mathcal{F})_{IJ} F^{J-} = 0 \ , \tag{D.13}
$$

with $C^{(P)\,I} \equiv \Pi^I_J C^J$. The term in (D.13) which is proportional to $\bar{X}^I \,\mathrm{Im}\,(\mathcal{F}_{IJ})\, F^{J-}$ vanishes because of (D.10), but the projection on C^I is necessary to make sure that Π^I_J commutes with the projector induced by (D.13)

$$
\Gamma^I_J \equiv \delta^I_J - \frac{C^{(P)\,I}\bar{C}^{(P)\,K}\,\mathrm{Im}(\mathcal{F})_{KJ}}{C^{(P)\,M}\,\mathrm{Im}(\mathcal{F})_{MN}\bar{C}^{(P)\,N}} \ . \tag{D.14}
$$

Indeed, one can check that thanks to the definition of $C^{(P)\,I}$

$$
\Pi^I_K \Gamma^K_J = \Pi^I_J + \Gamma^I_J - \delta^I_J = \Gamma^I_K \Pi^K_J \ . \tag{D.15}
$$

One can also easily check that Γ^I_J and Π^I_J are both idempotent

$$
\Pi^I_K \Pi^K_J = \Pi^I_J \ , \qquad \Gamma^I_K \Gamma^K_J = \Gamma^I_J \ . \tag{D.16}
$$

The light field strengths are now the projected ones

$$F^{\hat{I}-} \equiv F^{I-}\Big|_{\mathcal{N}=1} = \Pi^I_J \Gamma^J_K F^{K-} \ . \tag{D.17}$$

When the $\mathcal{N} = 1$ field strengths are inserted into the term $\mathrm{i}\overline{\mathcal{N}}_{IJ}\, F^{I-}_{\mu\nu} F^{\mu\nu\, J-}$ of the Lagrangian (2.4), the second term in the definition of $\overline{\mathcal{N}}_{IJ}$ (2.8) drops out due to (D.10) and we are left with the gauge coupling for the Lagrangian (3.12) [15]

$$f_{IJ} = \mathrm{i}\bar{\mathcal{N}}_{IJ}\Big|_{\mathcal{N}=1} = \mathrm{i}\mathcal{F}_{IJ} \ . \tag{D.18}$$

Let us now come back to the D-terms. The term $-\tilde{G}^{i-}_{\mu\nu}$ in (D.4) is mapped onto the one with $F^{\hat{I}-}_{\mu\nu} = \Pi^I_J \Gamma^J_K F^{K-}$ in (D.5) if the following relation holds

$$\delta_\epsilon \lambda^{\hat{I}} = -2\mathrm{e}^{\frac{\mathcal{K}^{\mathrm{v}}}{2}} \Pi^K_J \Gamma^J_I \nabla_i X^I \delta_\epsilon \lambda^{i2} \ . \tag{D.19}$$

This can be shown by using (D.2) and the relation [17]

$$\nabla_i X^I g^{i\bar{\jmath}}\, \nabla_{\bar{\jmath}} \bar{X}^J = -\tfrac{1}{2}\mathrm{e}^{-\mathcal{K}^{\mathrm{v}}} (\mathrm{Im}\mathcal{N})^{-1\ IJ} - X^I \bar{X}^J \ , \tag{D.20}$$

to calculate

$$\begin{aligned} & 2\mathrm{e}^{\frac{\mathcal{K}^{\mathrm{v}}}{2}} \Pi^K_J \Gamma^J_I \nabla_i X^I \tilde{G}^{i-}_{\mu\nu} \\ = & -2\mathrm{e}^{\mathcal{K}^{\mathrm{v}}} \Pi^K_J \Gamma^J_I \nabla_i X^I g^{i\bar{\jmath}} \nabla_{\bar{\jmath}} \bar{X}^L \mathrm{Im}\mathcal{N}_{LM} F^{M-}_{\mu\nu} \\ = & \Pi^K_J \Gamma^J_I F^{J-}_{\mu\nu} + 2\mathrm{e}^{\mathcal{K}^{\mathrm{v}}} \Pi^K_J \Gamma^J_I X^I \bar{X}^L \mathrm{Im}\mathcal{N}_{LM} F^{M-}_{\mu\nu} \ . \end{aligned} \tag{D.21}$$

The last term in the last line vanishes, because $\Pi^I_J X^J = 0$. One can now use (D.19) to compare (D.5) and (D.4) and read of the D-terms

$$\begin{aligned} \mathcal{D}^{\hat{I}} = & 2\mathrm{i}\mathrm{e}^{\frac{\mathcal{K}^{\mathrm{v}}}{2}} \Pi^I_J \Gamma^J_K \nabla_i X^K W^{i21} \\ = & -2\mathrm{e}^{\mathcal{K}^{\mathrm{v}}} \Pi^I_J \Gamma^J_K \nabla_i X^K g^{i\bar{\jmath}} \nabla_{\bar{\jmath}} \bar{X}^L \left(\Theta_L{}^\lambda - \bar{\mathcal{F}}_{LM} \Theta^{M\lambda}\right) P^3_\lambda \end{aligned} \tag{D.22}$$

where the definition of W^{i21} (2.18) was put to use. Inserting (D.20) again and using (D.18) finally results in

$$\mathcal{D}^{\hat{I}} = -\Pi^I_J \Gamma^J_K (\mathrm{Im}\,\mathcal{F})^{-1\ KL} \left(\Theta_L{}^\lambda - \bar{\mathcal{F}}_{LM} \Theta^{M\lambda}\right) P^3_\lambda \ . \tag{D.23}$$

E Symmetries of the STU model Kähler potentials

We use this appendix to give some symmetries of the Kähler potentials (7.12) and (7.27). Kähler potentials that are related by a Kähler transformation $\hat{\mathcal{K}} \to \hat{\mathcal{K}} + F + \bar{F}$ with F holomorphic describe the same Kähler manifold. Some of the presented symmetries come with such a Kähler transformation F. Complex parameters are denoted by α and real ones by a, b, c, d.

The $\mathcal{N} = 1$ Kähler potentials of the STU (7.12) and the quantum STU (7.27) models both depend only on $\mathrm{Re}\, Z^3$, $\mathrm{Re}\, x^0$ and $(x_1 + \bar{x}_2)$, so they are invariant under shifts of $(x_1 - \bar{x}_2)$ and of the imaginary parts of Z^3 and x^0

$$\begin{aligned}
&Z^3 \to Z^3 + \mathrm{i}\, a \;, \\
&x^0 \to x^0 + \mathrm{i}\, b \;, \\
&x_1 \to x_1 + \alpha \;, \\
&x_2 \to x_2 - \bar{\alpha} \;, \qquad\qquad F = 0 \;.
\end{aligned} \tag{E.1}$$

We state some more symmetries of the STU Kähler potential (7.12) that were discovered in [30]. Since the Kähler potential is invariant under exchange of Z^3 and x^0, the same symmetries with these coordinates exchanged also apply

$$\begin{aligned}
&Z^3 \to Z^3 \;, \\
&x^0 \to x^0 + a^2 Z^3 + a(x_1 + x_2) \;, \\
&x_1 \to x_1 + aZ^3 \;, \\
&x_2 \to x_2 + aZ^3 \;, \qquad\qquad F = 0 \;,
\end{aligned} \tag{E.2}$$

$$\begin{aligned}
&Z^3 \to \frac{-x^0}{-Z^3 x^0 + x_1 x_2} \;, \\
&x^0 \to \frac{-Z^3}{-Z^3 x^0 + x_1 x_2} \;, \\
&x_1 \to \frac{x_1}{-Z^3 x^0 + x_1 x_2} \;, \\
&x_2 \to \frac{x_2}{-Z^3 x^0 + x_1 x_2} \;, \qquad\qquad F = \ln(Z^3 x^0 - x_1 x_2) \;,
\end{aligned} \tag{E.3}$$

$$\begin{aligned}
&Z^3 \to \frac{aZ^3 - ib}{icZ^3 + d} \;, \\
&x^0 \to x^0 - ic\frac{x_1 x_2}{icZ^3 + d} \;, \\
&x_1 \to \frac{x_1}{icZ^3 + d} \;, \\
&x_2 \to \frac{x_2}{icZ^3 + d} \;, \qquad\qquad F = \ln(icZ^3 + d) \;,
\end{aligned} \tag{E.4}$$

with $ad - bc = 1$.

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Erklärung

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