

Problem 7.1

Compute by Kaluza-Klein reduction the massless bosonic spectrum of the following theories:

- a) eleven-dimensional supergravity in a background

$$M_4 \times T^7 ,$$

- b) ten-dimensional type I supergravity in a background

$$M_5 \times T^5 .$$

Problem 7.2

The Chern-Simons three-form is defined as

$$\omega_3 := \text{tr}(A \wedge dA + \frac{2}{3}A \wedge A \wedge A) ,$$

where A is a connection one-form in the adjoint representation of a non-Abelian gauge group.

- a) Show

$$d\omega_3 = \text{tr}(F_2 \wedge F_2) , \quad \text{for } F_2 := dA + A \wedge A .$$

- b) Show

$$\delta\omega_3 = \text{tr } d(\Lambda F_2) , \quad \text{for } \delta A = d\Lambda + [A, \Lambda] .$$

- c) Determine the transformation law of B_2 which leaves $H_3 := dB - \omega_3$ invariant.