Problem 6.1

Consider the matrix

$$\mathcal{N} = \frac{1}{\tau_2}\begin{pmatrix} |\tau|^2 & \tau_1 \\ \tau_1 & 1 \end{pmatrix} \qquad \text{for} \quad \tau = \tau_1 + i\tau_2 \ .$$

a) Show that

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \qquad \text{with} \qquad ad - bc = 1$$

corresponds to

$$\mathcal{N} \rightarrow \mathcal{N}' = \Lambda \mathcal{N} \Lambda^T \qquad \text{for} \qquad \Lambda = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{pmatrix} \ .$$

b) Show

$$\mathcal{L} = \tfrac{1}{8}\mathrm{Tr}(\partial_\mu \mathcal{N}^{-1})\partial^\mu \mathcal{N} = -G_{\tau\bar{\tau}}\partial_\mu \tau \partial^\mu \bar{\tau}$$

and determine the metric $G_{\tau\bar{\tau}}$.

c) Give the Kähler potential for $G_{\tau\bar{\tau}}$.

Hint: : See problem 5.1.

Problem 6.2

a) Determine the decomposition of the massless $N = 8$ gravitational multiplet in terms of massless $N = 4$ multiplets.

b) Do an analogous decomposition in terms of massless $N = 2$ and $N = 1$ multiplets.