Problem 5.1

Consider an $N = 1$ supergravity with $n_c + 1$ chiral multiplets $T, \phi^i, i = 1, \ldots, n_c$ and a Kähler potential

$$K = -3 \ln Y \ , \qquad \text{where} \qquad Y \equiv (T + \bar{T} - \phi^i \delta_{i\bar{j}} \bar{\phi}^{\bar{j}}) \ , \quad \kappa = 1 \ .$$

a) Compute all components of the metric.

b) Show

$$K_I G^{I\bar{J}} K_{\bar{J}} = 3 \ ,$$

where I runs over all $n_c + 1$ chiral fields.

Hint: For $G^{I\bar{J}}$ use the Ansatz

$$G^{I\bar{J}} = \frac{Y}{3} \begin{pmatrix} X & Z\phi^i \\ Z\bar{\phi}^{\bar{j}} & \delta^{i\bar{j}} \end{pmatrix} \ ,$$

and determine X, Z.

c) Compute V in this theory for $W =$ constant.

Problem 5.2

Consider a globally supersymmetric $N = 2$ field theory with gauge group $G = SU(2)$ and prepotential $F = \frac{i}{2} \sum_{a=1}^{3} z^a z^a$.

a) Compute the metric, the covariant derivative $D_\mu z^a$ and the scalar potential V.

b) Show $\langle V \rangle = 0$ for $\langle z^3 \rangle$ arbitrary and $\langle z^1 \rangle = \langle z^2 \rangle = 0$.

c) Compute the mass matrix of the gauge bosons in this background. What is the unbroken gauge group?

d) Compute the mass of the three scalar fields z^a.