Problem Set 4 Introduction to Supersymmetry and Supergravity WS 15/16

Problem 4.1

Consider the scalar potential of $N = 1$ supergravity in the form

$$V = e^G\big(G^{i\bar{\jmath}}G_iG_{\bar{\jmath}} - 3\big) \ , \qquad G = K + \ln|W|^2$$

Compute the mass matrices in a Minkowskian background and show

$$M^2_{i\bar{\jmath}} = \langle(\nabla_iG_k\bar{\nabla}_{\bar{\jmath}}G^k - R_{i\bar{\jmath}k\bar{l}}G^kG^{\bar{l}} + G_{i\bar{\jmath}})e^G\rangle \ , \qquad M^2_{ij} = \langle(G^k\nabla_i\nabla_jG_k + \nabla_iG_j + \nabla_jG_i)e^G\rangle \ ,$$

where $\nabla_iG_j = \partial_iG_j - \Gamma^k_{ij}G_k$, $\nabla_iG_{\bar{\jmath}} = G_{i\bar{\jmath}}$.
Hint: : Use $\langle\nabla_iV\rangle = 0$ as the condition for a Minkowski minimum.

Problem 4.2

The Polonyi model is defined by

$$K = \phi\bar{\phi} \ , \qquad W_P = m^2(\phi + \beta) \ , \qquad m, \beta \in \mathbb{R}$$

a) For which β is supersymmetry spontaneously broken?

b) Check that $\kappa\phi = \pm(\sqrt{3} - 1)$, $\kappa\beta = \pm(2 - \sqrt{3})$ is a Minkowskian extremum of the potential V.

c) Compute the gravitino mass and $\langle F_\phi\rangle$.

Problem 4.3

Consider the situation where an observable sector is coupled to the Polonyi model with

$$K = \phi\bar{\phi} + Q^I\bar{Q}^I \ , \qquad W = \tfrac{1}{2}\mu_{IJ}\,Q^IQ^J + \tfrac{1}{3}Y_{IJL}\,Q^IQ^JQ^K + W_P(\phi) \ ,$$

where Q^I are the fields of the observable sector, m_{IJ}, Y_{IJL} are constant and W_P is given in problem 4.2.

a) Compute the soft scalar masses and the A and B terms assuming that $\langle F_\phi\rangle$ is the only non-vanishing F-term. Are they universal?

b) Compute the soft gaugino masses for the two cases of a gauge kinetic function $f(\phi)$ and $f =$ constant. Are they universal?

Hint: Use the formulas given in section 10 of the lecture notes.