Problem Set 3 Introduction to Supersymmetry and Supergravity WS 15/16

Problem 3.1

Consider a theory with superpotential

$$W = \lambda A_0 + m A_1 A_2 + Y A_0 A_1^2 \ , \qquad m^2 > 2\lambda Y \ .$$

a) Determine the minimum of the scalar potential V.

b) Compute the mass spectrum for all bosons and all fermions for $\langle A_0 \rangle = 0$.

c) Verify the sum rule $\mathrm{Str} M^2 = 0$.

Problem 3.2

Consider a supersymmetric $U(1)$ gauge theory with gauge coupling g and two massive chiral multiplets $\Phi_\pm$ of opposite $U(1)$ charge, superpotential $W = m\Phi_+\Phi_-$ and with a non-vanishing FI-term $\xi_{\rm FI}$.

a) Give the Lagrangian in superspace and in components using the formulas given in class.

b) Determine the minimum of V for $|m|^2 > \xi_{\rm FI} g$. Is the $U(1)$ spontaneously broken? Is supersymmetry spontaneously broken?

c) Determine the minimum of V for $|m|^2 < \xi_{\rm FI} g$. Which symmetries are spontaneously broken? Is there a choice of parameters where supersymmetry is unbroken and the $U(1)$ is broken?

Problem 3.3

a) Compute all Christoffel symbols and all components of the Riemann curvature tensor for a Kähler manifold with the metric $G_{i\bar{j}} = \partial_i \partial_{\bar{j}} K$.

b) Show

$$[D_i, \bar{D}_{\bar{j}}] v_k = R_{i\bar{j}k}{}^{l} v_l \ ,$$

where $D_{\bar{j}} v_k = \partial_{\bar{j}} v_k$, $D_i v_k = \partial_i v_k - \Gamma^l_{ik} v_l$.

c) Show that the Ricci tensor is given by

$$R_{i\bar{j}} = \partial_i \partial_{\bar{j}} \ln \det(G_{l\bar{k}}) \ .$$