Problem 2.1

Solve the constraint

$$\bar{D}_{\dot\alpha} f(x,\theta,\bar\theta) = 0 \ , \qquad \text{for} \qquad \bar{D}_{\dot\alpha} = -\partial_{\dot\alpha} - i\theta^\beta \sigma^\mu_{\beta\dot\alpha}\partial_\mu \ ,$$

and a general complex superfield $f(x,\theta,\bar\theta)$.

Problem 2.2

a) Compute the anticommutation relations of the operators

$$Q_\alpha = \partial_\alpha - i\sigma^\mu_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \ , \qquad \overline{Q}_{\dot\alpha} = -\partial_{\dot\alpha} + i\theta^\beta\sigma^\mu_{\beta\dot\alpha}\partial_\mu \ .$$

b) Compute the supersymmetry transformations of the lowest and θ component for a general and a chiral superfield using

$$\delta_\xi f(x,\theta,\bar\theta) = (\xi Q + \bar\xi\bar Q) f(x,\theta,\bar\theta) \ .$$

Problem 2.3

a) Show that the superalgebra is left invariant by the transformation

$$Q \to Q' = e^{-i\alpha} Q \ , \qquad \bar Q \to \bar Q' = e^{i\alpha}\bar Q \ , \qquad \alpha \in \mathbb{R} \ . \qquad (*)$$

b) Assume that chiral superfields Φ^i transform under the symmetry transformation $(*)$ as

$$\Phi^i \to \Phi^{i\prime} = e^{iq_i\alpha}\Phi^i \ , \qquad q_i \in \mathbb{R} \ .$$

Compute the transformation law of the component fields ϕ^i, χ^i, F^i.

c) Derive the necessary transformation law of the superpotential $W(\Phi)$ such that one obtains an invariant Lagrangian.

d) Consider the superpotentials

$$W_1 = m\Phi^2 \ , \quad m \in \mathbb{R} \ , \qquad W_2 = y\Phi^3 \ , \quad y \in \mathbb{R} \ , \qquad W_3 = W_1 + W_2 \ .$$

Which Ws satisfy the transformation laws derived in c) and under which conditions.