Problem Set 2 Introduction to Supersymmetry and Supergravity WS 13/14

Problem 2.1

Solve the constraint $\bar{D}_{\dot{\alpha}} f(x,\theta,\bar{\theta}) = 0$ for $\bar{D}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} - i\theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu}$ and a general complex superfield $f(x,\theta,\bar{\theta})$.

Problem 2.2

a) Compute the anticommutation relations of the operators

$$Q_{\alpha} = \partial_{\alpha} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \ , \qquad \overline{Q}_{\dot{\alpha}} = -\partial_{\dot{\alpha}} + i\theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu} \ .$$

b) Compute the supersymmetry transformations of the lowest and θ component for a general and a chiral superfield using

$$\delta_{\xi} f(x,\theta,\bar{\theta}) = (\xi Q + \bar{\xi}\bar{Q}) f(x,\theta,\bar{\theta}) \ .$$

Problem 2.3

Consider a supersymmetric $U(1)$ gauge theory with a vector multiplet V and two chiral multiplets $\Phi_{\pm}$ with charges $q_{\pm} = \pm 1$.

a) Give the gauge transformation of the superfields $V, \Phi_{\pm}$.

b) Which gauge invariant and renormalizable terms are allowed in W?
Is this W R-invariant and if yes give the R-charges of $\Phi_{\pm}$.

c) Give the Lagrangian in superspace and in components using the formulas given in class.

d) Determine the minimum of the scalar potential.

Problem 2.4

Compute the Higgs potential for the supersymmetric Standard Model and show

$$\begin{aligned} V_{\text{Higgs}} = {} & |\mu|^2 \left(|h_u^+|^2 + |h_d^-|^2 + |h_u^0|^2 + |h_d^0|^2 \right) + \tfrac{1}{2} g_2^2 \left| h_u^+ \bar{h}_d^0 + h_u^0 \bar{h}_d^- \right|^2 \\ & + \tfrac{1}{8} \left(g_1^2 + g_2^2 \right) \left(|h_u^0|^2 + |h_u^+|^2 - |h_d^0|^2 - |h_d^-|^2 \right)^2 \ . \end{aligned}$$