Problem 7.1

Consider a (real) scalar field theory with interaction $H_I = \frac{\lambda}{4!}\int d^4z\, \phi^4(z)$ and $\lambda \ll 1$.

a) Draw all fully connected and amputated Feynman-diagrams for the amplitude $\langle\Omega|T\{\phi(x_1)\ldots\phi(x_4)\}|\Omega\rangle$ to order λ^2.

b) Give for each diagram the associated expression in position space in terms of the Feynman propagators.

c) Insert the explicit expression for the Feynman propagator and perform the integral over the internal positions (z and z, z'). Compare the resulting expression with the momentum space Feynman rules.

d) Which diagrams are divergent and what type of divergence occurs?

Problem 7.2

Consider the Lagrangian

$$\mathcal{L} \;=\; \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi \;+\; \tfrac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m_\phi^2\phi^2 - g\,\phi\bar{\psi}\psi\;,$$

where ψ is a Dirac spinor and ϕ a real scalar field. Assume that $g \ll 1$ holds.

a) What is H_I for this theory?

b) Draw the Feynman diagrams at leading order in g for the scattering process

$$\text{fermion} + \text{fermion} \quad\rightarrow\quad \text{fermion} + \text{fermion}$$

and determine $i\mathcal{M}$. (Use a solid line for a fermion and a dashed line for the scalar.)

Hint: Use the Feynman rules of QED but replace the photon propagator with a scalar propagator.

c) What is $i\mathcal{M}$ if anti-fermions scatter?

d) Draw the Feynman diagrams at leading order in g for the scattering process

$$\text{fermion} + \text{anti-fermion} \quad\rightarrow\quad \text{scalar} + \text{scalar}$$

and determine $i\mathcal{M}$.

Problem 7.3

Show

$$\int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\vec{x}}}{\vec{q}^2 + m^2} \;=\; \frac{1}{4\pi|\vec{x}|}\, e^{-m|\vec{x}|} \;.$$

Hint:

Use spherical coordinates and first perform the angular integrals. The left over radial integral can be computed using the residue formula.