Problem Set 10 Quantum Field Theory I WS 10/11

This problem set is a former written exam. If you hand it in by 12.00h on 17.1.11, it counts towards the bonus. (Note that handing it in jointly does not count.)

Problem 10.1 (20 points)

a) Show that in d-space-time dimensions

$$\mathrm{Tr}\left(\not{p}_1\gamma^\mu\not{k}\gamma^\nu\not{p}_2\gamma_\nu\not{k}\gamma_\mu\right) \;=\; a\,(p_1\cdot k)(p_2\cdot k)$$

for $k^2=0$ and compute a.

Hint: Use $\not{k}\not{k}=k^2=0$ and note that $\mathrm{Tr}(\mathbf{1})=4$.

b) Prove in $d=4$ the identity

$$\bar{u}(p_1)\gamma^\mu u(p_2)=\bar{u}(p_1)\Big(\frac{(p_1+p_2)^\mu}{2m}+\frac{iS^{\mu\nu}(p_1-p_2)_\nu}{m}\Big)u(p_2)$$

for $S^{\mu\nu}=\frac{i}{4}[\gamma^\mu,\gamma^\nu]$.

Hint: Start the proof from the right hand side and use $(\not{p}-m)u(p)=\bar{u}(p)(\not{p}-m)=0$.

Problem 10.2 (20 points)

Consider a scalar field theory with an interaction $H_I=\frac{\lambda}{3!}\int d^4z\phi^3(z)$.

a) Draw all connected Feynman diagrams for $\langle\Omega|T\{\phi(x_1)\phi(x_2)\}|\Omega\rangle$ up to order $\mathcal{O}(\lambda^2)$ and give the associated expressions (including the symmetry factor) in terms of the Feynman propagators.

b) Which diagrams of a) are UV-divergent and what type of UV-divergence occurs?

c) Draw all connected Feynman diagrams for $\langle\Omega|T\{\phi(x_1)\phi(x_2)\phi(x_3)\}|\Omega\rangle$ up to order $\mathcal{O}(\lambda^3)$. (It is not necessary to give the expressions in terms of the Feynman propagators.)

Problem 10.3 (20 points)

a) Draw the two leading diagrams for the process $e^+e^- \to \gamma\gamma$ in QED.

b) Give $i\mathcal{M}$ for both diagrams in momentum space.

c) Give $i\mathcal{M}$ for the corresponding diagrams for the process $\mu^+\mu^- \to \gamma\gamma$ in momentum space.

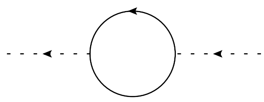
Problem 10.4 (20 points)

Consider the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m_\psi)\psi + \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - g\,\phi\bar{\psi}\psi\ ,$$

where ψ is a Dirac spinor and ϕ a real scalar field. Assume $g \ll 1$.

a) Show that $i\mathcal{M}$ for the diagram



is proportional to

$$i\mathcal{M} \sim g^2 \int \frac{d^4k}{(2\pi)^4}\,\frac{a\,k^2 + b\,k\cdot p + c}{AB}$$

and compute a, b, c, A, B.

Hint: The dashed line represents a ϕ, the solid line represents a ψ.

b) Introduce Feynman parameters and show

$$i\mathcal{M} \sim g^2 \int_0^1 dx \int \frac{d^4l}{(2\pi)^4}\,\frac{l^2 - \Theta}{(l^2 - \Delta)^m}\ .$$

Compute Θ, Δ, m. Which divergence occurs?

Hint: $[AB]^{-1} = \int_0^1 dx[xA + (1-x)B]^{-2}$.