Problem 8.1

The BRST operator Q of a non-Abelian gauge theory is defined by the action

$$\begin{aligned} QA^a_\mu &= \partial_\mu c^a + g f^{abc} A^b_\mu c^c \ , \\ Q\psi &= igc^a t^a \psi \ , \\ Qc^a &= -\tfrac{1}{2} g f^{abc} c^b c^c \ , \\ Q\bar{c}^a &= B^a \ , \\ QB^a &= 0 \ , \end{aligned}$$

where Q anticommutes with the ghost fields, i.e. $Qc^a c^b = (Qc^a)c^b - c^a(Qc^b)$.

a) Show

$$Q^2 = 0 \ .$$

b) Show

$$B^a \partial^\mu A^a_\mu - \bar{c}^a \partial^\mu (D_\mu c^a) + \tfrac{\xi}{2} B^a B^a = QX \ ,$$

and determine X.

Problem 8.2

Consider

$$S_{int} = g f^{abc} \int d^4x (\partial_\nu A^a_\mu) A^{\nu b} A^{\mu c} \ .$$

a) Give S_{int} in Fourier space by using $A_\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \tilde{A}_\mu(k)$ and show

$$\mathcal{L}_{int} \sim \delta(k+p+q) \tilde{A}^a_\mu(k) \tilde{A}^b_\nu(p) \tilde{A}^c_\rho(q) D^{\mu\nu\rho}_{abc}$$

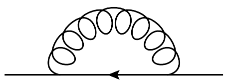
for $D^{\mu\nu\rho}_{abc} = g k^\nu \eta^{\mu\rho} f^{abc}$.

b) Use the antisymmetry of f^{abc} to show that $D^{\mu\nu\rho}_{abc}$ can be rewritten as

$$D^{\mu\nu\rho}_{abc} = \tfrac{1}{6} g f^{abc} \big((k-q)^\nu \eta^{\mu\rho} + (p-k)^\rho \eta^{\mu\nu} + (q-p)^\mu \eta^{\rho\nu} \big) \ .$$

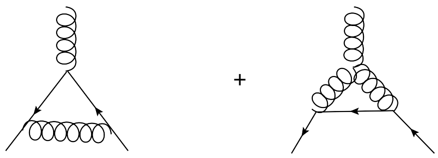
Problem 8.3

a) Compute the one-loop fermion propagtor correction Σ_2 in a non-Abelian gauge theory with massless fermions using the Feynman gauge for the gauge boson propagator.



Hint: Prove (and use) $t^b t^a t^b = (c_2(r) - \frac{1}{2}c_2(G))t^a$.

b) Compute the divergent part of the two diagrams



in a non-Abelian gauge theory with massless fermions in the limit that the loop momentum is much larger than any of the external momenta.