This problem set counts towards the bonus if you hand it in by 8.30 on 18.5.11. (Note that handing it in jointly does not count.)

Problem 6.1 (20 points)

a) Define the generating functional $Z[J^i, J^{i*}]$ for n complex scalar fields $\phi^i, i = 1, \ldots, n$ via the path integral.

b) Compute Z explicitly for n free scalar fields (with the same mass) by appropriately completing the square.

c) Compute the free propagator by appropriately differentiating Z?

Problem 6.2 (20 points)

The Lagrangian for a massive photon is given by

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} m^2 A_\mu A^\mu$$

a) Compute the photon propagator in Fourier space using $A_\mu(x) = \int \frac{d^4k}{(2\pi)^4} \tilde{A}_\mu(k)\, e^{-ikx}$. What is the problem with the limit $m \to 0$?

b) Modify $\mathcal{L}$ by adding a real scalar field ϕ with couplings and a transformation law such that $\mathcal{L}$ is invariant under the gauge transformation $A_\mu \to A_\mu + \partial_\mu \alpha$.

<u>Problem 6.3</u> (20 points)

Consider the theory with the Lagragian

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}m^2\phi^2 - \tfrac{1}{3!}\lambda\phi^3 \ .$$

a) Determine the mass dimension of the couping λ.

b) Draw all superficially divergent Feynman diagrams for this theory.

   Hint: You should draw 6 diagrams.

c) Consider the two one-loop diagrams and evaluate them in $d = 4 - \epsilon$ dimensions in terms of Γ functions. (The overall normalization need not be computed.) What are their UV-divergences?

<u>Problem 6.4</u> (20 points)

Consider the massless Yukawa theory defined by the Lagrangian

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{4!}\lambda\phi^4 - g\bar{\psi}\psi\phi \ .$$

The counterterm for the scalar propagator is determined to be

$$\delta_{Z_\phi} = \lim_{d\to 4} \frac{4g^2(d-1)}{(4\pi)^{d/2}} \int_0^1 dx x(1-x) \frac{\Gamma(2-\frac{d}{2})}{\big(\Delta(p^2=-M^2)\big)^{2-\frac{d}{2}}}$$

for $\Delta(p^2) = -x(1-x)p^2$.

a) Compute the γ-function for the scalar field.

   Hint: You may asume that the β function does not contribute at this order in the Callan-Symanzik-equation.

b) The β-function is found at leading order to be

$$\beta = b_0 g^3 \ , \qquad b_0 = \frac{5}{16\pi^2} \ .$$

   Determine the running coupling constant $\bar{g}$ in terms of g and b_0.