String theory and particle physics

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Abstract

We briefly review some selected aspects of the relation between string theory and particle physics.

1 Introduction

The Standard Model of Particle Physics is an extremely successful theory which has been tested experimentally to a high level of accuracy. After the discovery of the top quark the Higgs boson which is predicted to exist by the Standard Model is the only 'missing' ingredient that has not been directly observed yet.

However, a number of theoretical prejudices suggest that the Standard Model is not the 'final answer' of nature but rather an effective description valid up to the weak scale of order $O(100\ \text{GeV})$. The arbitrariness of the spectrum and gauge group, the large number of free parameters, the smallness of the weak scale compared to the Planck scale and the inability to turn on gravity suggest that at higher energies (shorter distances) a more fundamental theory will be necessary to describe nature.

Since the seventies string theory has been discussed as a possible candidate for a theory which unifies all known particle interactions including gravity. The concept of point-like particles is replaced by one-dimensional extended objects – strings – and the particles of the Standard Model appear as massless excitations of this string. One of the massless excitation necessarily carries spin two and can be identified with the graviton of General Relativity. Furthermore, within this framework it is possible to sensibly compute the perturbative quantum corrections to General Relativity. Apart from the graviton one also finds spin-1 gauge bosons as well as families of chiral fermions in anomaly free representation of some gauge group G among the massless modes. This fact led

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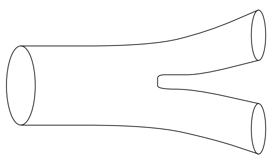
to the hope that string theory might unify all known interactions observed in nature.

Until recently, however, string theory has only been known in its perturbative regime. That is, the (particle) excitations of a string theory are computed in the free theory ($g_\mathrm{s} = 0$), while their scattering processes are evaluated in a perturbative series for $g_\mathrm{s} \ll 1$ [1]. The string coupling constant g_s is a free parameter of string theory but for $g_\mathrm{s} = O(1)$ no method of computing the spectrum or the interactions had been known. This situation dramatically changed during the past years. For the first time it became possible to go beyond the purely perturbative regime and to compute some of the non-perturbative properties of string theory [2]. The central point of these developments rests on the idea that the strong-coupling limit of a given string theory can be described in terms of another, weakly coupled, 'dual theory'. This dual theory can take the form of either a different string theory, or the same string theory with a different set of perturbative excitations, or a new theory termed M-theory.

More recently a very different type of duality has been proposed. Strongly coupled (super)-conformally invariant gauge theories are conjectured to be dual to string theories in an anti-de Sitter background [4]. This led to the hope that string theory might tell us something about QCD at low energies.

2 Perturbative string theory

In string theory the fundamental objects are one-dimensional strings which, as they move in time, sweep out a 2-dimensional worldsheet Σ [1]. Strings can be open or closed and their worldsheet is embedded in some D-dimensional target space which is identified with a Minkowskian spacetime. States in the target space appear as eigenmodes of the string and their scattering amplitudes are generalized by appropriate scattering amplitudes of strings. These scattering amplitudes are built from a fundamental vertex, which for closed strings is depicted in Fig. 1. It represents the splitting of a string or the joining



$\sim g_s$

Fig. 1. The fundamental closed string vertex.

of two strings and the strength of this interaction is governed by a dimensionless string coupling constant g_s. Out of the fundamental vertex one composes all possible closed string scattering amplitudes A, for example the four-point amplitude shown in Fig. 2. The expansion in the topology of the Riemann

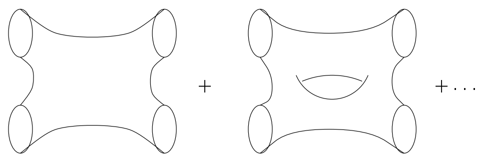


Fig. 2. The perturbative expansion of string scattering amplitudes. The order of $g_{\rm s}$ is governed by the number of holes in the world sheet.

surface (i.e. the number of holes in the surface) coincides with a power series expansion in the string coupling constant formally written as

$$A \,=\, \sum_{n=0}^{\infty} g_{\rm s}^{-\chi}\, A^{(n)} \ , \tag{1}$$

where $A^{(n)}$ is the scattering amplitude on a Riemann surface of genus n and $\chi(\Sigma)$ is the Euler characteristic of the Riemann surface

$$\chi(\Sigma) = \frac{1}{4\pi}\int_{\Sigma} R^{(2)} = 2 - 2n - b \ . \tag{2}$$

$R^{(2)}$ is the curvature scalar on Σ and b the number of boundaries of the Riemann surface. (For the four-point amplitude of Fig. 2 one has $b = 4$.) [1]

In all string theories there is a massless scalar field ϕ called the dilaton which couples to $R^{(2)}$ and therefore its vacuum-expectation value determines the size of the string coupling; one finds

$$g_{\rm s} \,=\, e^{\langle\phi\rangle} \ . \tag{3}$$

$g_{\rm s}$ is a free parameter since ϕ is a flat direction (a modulus) of the effective potential. Thus, string perturbation theory is defined in that region of the parameter space (which is also called the moduli space) where $g_{\rm s} < 1$ and the tree-level amplitude (genus-0) is the dominant contribution with higher-loop amplitudes suppressed by higher powers of $g_{\rm s}$. Until 1995 this was the only regime accessible in string theory.

Unitarity in spacetime imposes a restriction on the maximal number of spacetime dimensions and the spacetime spectrum. All supersymmetric string theories necessarily have $D \leq 10$ and they are particularly simple in their maximal possible dimension $D = 10$. In $D = 10$ there are only five consistent spacetime supersymmetric string theories: type-IIA, type-IIB, heterotic $E_8 \times E_8$ (HE8), heterotic SO(32) (HSO) and the type-I SO(32) string.

[1] For open strings different diagrams contribute at the same order of the string loop expansion [1].

Lower dimensional theories can be obtained by compactifying the $D = 10$ theories on an internal, 'curled up' compact manifold Y

$$M^{(10)} = M^{(D)} \times Y^{(10-D)} \ . \tag{4}$$

If one demands some 'left-over' supersymmetry in lower dimensions Y has to be a Calabi–Yau manifold [1]. Calabi–Yau manifolds are Ricci-flat Kähler manifolds of vanishing first Chern class ($c_1(Y) = 0$) with holonomy group $SU(M)$ where M is the complex dimension of Y. For $D = 4$ three-dimensional Calabi–Yau manifolds Y_3 are particularly important. However, it is also possible to construct string backgrounds directly in $D = 4$ without going through a geometrical picture but instead using an appropriate conformal field theory on the string worldsheet. In either case there exist a large variety of allowed constructions and this is what is generally called vacuum degeneracy: Every Calabi–Yau manifold or every appropriate conformal field theory defines a consistent string background.

Calabi-Yau manifolds have generically continuous deformations parameterized by so called moduli which preserve the Calabi-Yau condition. Thus there is a continuous degeneracy of possible string groundstates. The moduli parameters correspond to vacuum expectation value of scalar fields which are flat directions of the effective potential.

Since string theory is a candidate for a unified theory of all interactions it has always been a primary goal to identify the Standard Model of Particle Physics as the low energy limit of string theory [3]. The massless spectrum of string theory can indeed accommodate families of chiral fermions transforming in appropriate representations of a non-Abelian gauge group as well as Higgs bosons necessary for the electro-weak symmetry breaking. Furthermore, most ground states of string theory studied so far are supersymmetric and have a universal gauge coupling constant at the leading order which is in very good agreement with the electroweak precision data of the past decade. However, a more quantitative agreement with the Standard Model has not been achieved so far. The main obstacles seem to be a missing mechanism for spontaneously breaking supersymmetry at a scale hierarchically lower than the Planck scale $M_{\rm Pl}$. Closely related is the lack of a viable Higgs mechanism in string theory which generates small masses of the light states. Finally, lifting the (enormous) vacuum degeneracy of possible string backgrounds is the third problem faced by phenomenological studies of string theory.

It is commonly believed that these deficiencies are due to our lack of understanding the non-perturbative structure of string theory. Thus one has started to study the phenomenological implications of the non-perturbative properties of string theory.

3 Non-perturbative developments

Since 1995 string theory has seen spectacular progress in that for the first time it has been possible to also control a subset of the interaction in the strong coupling regime. This is due to the observation that many (if not all) of the perturbatively distinct string theories are related when all quantum corrections are taken into account [2]. In particular it has been observed that often the strong coupling regime of one string theory can be mapped to the weak coupling regime of another, perturbatively different string theory. This situation is termed duality among string theories and it offers the compelling picture that the known perturbative string theories are merely different regions in the moduli space of one underlying theory termed 'M-theory'.

The precise nature of the strong-coupling limit sensitively depends on the number of (Minkowskian) spacetime dimensions and the amount of supersymmetry. Supersymmetry has played a major role in the recent developments in two respects. First of all, it is difficult (and it has not been satisfactorily accomplished) to rigorously prove a string duality, since it necessitates a full non-perturbative formulation, which is not yet available. Nevertheless it has been possible to perform nontrivial checks of the conjectured dualities for quantities or couplings whose quantum corrections are under (some) control. It is a generic property of supersymmetry that it protects a subset of the couplings and implies a set of non-renormalization theorems. The recent developments heavily rely on the fact that the mass (or tension) of BPS-multiplets is protected and that holomorphic couplings obey a non-renormalization theorem. Thus, they can be computed in the perturbative regime of string theory and, under the assumption of unbroken supersymmetry, reliably extrapolated into the non-perturbative region. It is precisely for these BPS-states and holomorphic couplings that the conjectured dualities have been successfully verified.

Second of all, for a given spacetime dimension D and a given representation of supersymmetry there can exist perturbatively different string theories. For example, the heterotic SO(32) string in $D = 10$ and the type-I string in $D = 10$ share the same supersymmetry, but their interactions are different in perturbation theory. However, once non-perturbative corrections are taken into account, it is believed that the two theories are identical and merely different perturbative limits of the same underlying quantum theory. A similar phenomenon is encountered with other string theories in different dimensions and the moduli space of string theory is much smaller than was previously assumed.

Of particular interest is the strong coupling limit of string theories with $D = 4$ and $N = 1$ supersymmetry. They are related to M-theory compactified on Calabi–Yau threefolds times an interval or F-theory compactified on elliptic

Calabi–Yau fourfolds. Let us discuss these two cases in turn.

It turns out that not all strong-coupling limits are governed by a perturbatively different string theory. Instead it is possible that the strong-coupling limit of a given theory is something entirely new, not any of the other string theories [5]. The prime example of this situation is the strong-coupling limit of the type-IIA theory in $D = 10$. It has a Kaluza-Klein BPS spectrum with masses

$$M^{\text{KK}} \sim \frac{|n|}{g_{\text{s}}} \;, \tag{5}$$

where n is an arbitrary integer. These KK-states are not part of the perturbative type-IIA spectrum since they become heavy in the weak-coupling limit $g_{\text{s}} \to 0$. However, in the strong-coupling limit $g_{\text{s}} \to \infty$ they become light and can no longer be neglected in the effective theory. This infinite number of light states (which can be identified with D-particles of type-IIA string theory, or extremal black holes of IIA supergravity) signals that the theory effectively decompactifies where g_{s} is related to the radius R_{11} of a new (11-th) dimension[5]

$$R_{11} \sim l_{11}\, g_{\text{s}}^{\frac{2}{3}} \;. \tag{6}$$

l_{11} is the characteristic length scale of the 11th dimension which is related to the 11-dimensional Planck scale via $\kappa_{11}^2 \sim l_{11}^9$. Supersymmetry is unbroken in this limit and hence the KK-states assemble in supermultiplets of the 11-dimensional supergravity. Since there is no string theory which has 11-dimensional supergravity as the low-energy limit, the strong-coupling limit of type-IIA string theory has to be a new theory, called M-theory, which cannot be a theory of (only) strings.

There exists a conjecture according to which the degrees of freedom of M-theory are captured in $\text{U}(N)$ supersymmetric matrix models in the $N \to \infty$ limit [6]. These matrix models have been known for some time and were also known to describe supermembranes in the lightcone gauge [7]. The same quantum-mechanical models describe the short-distance dynamics of N D-particles, caused by the exchange of open strings.

A second and maybe even more surprising result shows that also the strong-coupling limit of the heterotic $\text{E}_8 \times \text{E}_8$ string is captured by M-theory. In this case, 11-dimensional supergravity is not compactified on a circle but rather on a Z_2 orbifold of the circle [8]. In this case there is an E_8 gauge factor on each hyperplane at the end of the interval. Just as in the type-IIA case one has $R_{11} \sim g_{\text{s}}^{2/3}$ and thus weak coupling corresponds to small R_{11} and the two 10-dimensional hyperplanes sit close to each other; in the strong-coupling limit the two 10-dimensional hyperplanes move far apart (to the end of the world).

The strong coupling limit of type-IIB theory in 10 spacetime dimensions is believed to be governed by type-IIB itself. This is accomplished by an exact $\mathrm{SL}(2,Z)$ quantum symmetry which is a generalization of a strong-weak coupling duality. This fact led Vafa to propose that the type-IIB string could be viewed as the toroidal compactification of a twelve-dimensional theory, called F-theory [9]. Apart from having a geometrical interpretation of the $\mathrm{SL}(2,Z)$ symmetry this proposal led to the construction of new, non-perturbative string vacua in lower spacetime dimensions. In order to preserve the $\mathrm{SL}(2,Z)$ quantum symmetry the compactification manifold cannot be arbitrary but has to be what is called an elliptic fibration. That is, the manifold is locally a fiber bundle with a two-torus T^2 over some base B but there are a finite number of singular points where the torus degenerates. As a consequence nontrivial closed loops on B can induce an $\mathrm{SL}(2,Z)$ transformation of the fiber. This implies that the dilaton is not constant on the compactification manifold, but can have $\mathrm{SL}(2,Z)$ monodromy. It is precisely this fact which results in nontrivial (non-perturbative) string vacua inaccessible in string perturbation theory.

It is believed that the heterotic string compactified on a Calabi–Yau threefold Y_3 is quantum equivalent to F-theory compactified on an elliptic Calabi–Yau fourfold. Calabi-Yau fourfolds are Calabi-Yau manifolds of complex dimension four and holonomy group $\mathrm{SU}(4)$. Compactification of F-theory on Calabi–Yau fourfolds is not yet well understood and the phenomenological investigations are only at the beginning.

4 AdS/CFT Correspondence

Recently, a very different type of duality relating a string theory to a (strongly coupled) gauge field theory has been conjectured. It was put forward that IIB string theory in the background $AdS_5 \times S_5$ is equivalent or dual to a supersymmetric $N = 4$ $SU(n)$ gauge theory [4].

AdS_5 is the five-dimensional Anti-de-Sitter space which is a space of negative constant curvature ($R < 0$) whose boundary at spatial infinity is the $D = 4$ Minkowski space. The isometry group of AdS_5 is $SO(2,4)$ which coincides with the conformal group of Minkowski space. S_5 is the five-dimensional sphere which has positive curvature $R > 0$ and the isometry group $SO(6)$. In the AdS/CFT correspondence this is identified with the R-symmetry of $N = 4$ supersymmetry. The parameters of the dual theories are related by

$$g_{\mathrm{YM}}^2 \sim g_s \,, \qquad (rM_{\mathrm{ST}})^4 \sim g_s\, n \,, \tag{7}$$

where g_{YM} is the Yang-Mills gauge coupling, M_{ST} is the mass scale of string theory, r is the radius of S_5 and n denotes the n of $SU(n)$. Type IIB supergravity is a valid approximation for

$$g_s < 1 , \qquad rM_{\rm ST} \gg 1 . \tag{8}$$

In terms of the gauge theory this corresponds to

$$g_{\rm YM} < 1 , \qquad n \to \infty \tag{9}$$

The 't Hooft limit

$$g_{\rm YM} < 1 , \qquad n \to \infty , \qquad g_{\rm YM}^2 n \ \text{ fixed} \tag{10}$$

corresponds to classical string theory without string loop corrections. In that sense it can be viewed as a realization of Witten's 'master field'.

To check or prove this duality is problematic since the two dual theories have no common range of validity. Thus only quantities with simple quantum properties have been successfully matched. Specifically, global symmetries and correlation functions related to anomalies have been shown to agree. Furthermore, the gauge invariant (chiral) operators of the $SU(n)$ gauge theory are mapped to the Kaluza-Klein excitation of IIB-supergravity.

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