Phenomenological Aspects of String Theory

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Introduction

The Standard Model of Particle Physics is an extremely successful theory which has been tested experimentally to a high level of accuracy [1, 2]. After the discovery of the top quark the Higgs boson which is predicted to exist by the Standard Model is the only 'missing' ingredient that has not been directly observed yet.

However, a number of theoretical prejudices suggest that the Standard Model is not the 'final answer' of nature but rather an effective description valid up to the weak scale of order $\mathcal{O}(100\ \mathrm{GeV})$. The arbitrariness of the spectrum and gauge group, the large number of free parameters, the smallness of the weak scale compared to the Planck scale and the inability to turn on gravity suggest that at higher energies (shorter distances) a more fundamental theory will be necessary to describe nature. Over the past 20 years various extensions of the Standard Model such as Technicolor [3, 4], Grand Unified Theories [5, 6], Supersymmetry [7, 8] or String Theory [9] have been proposed.

In recent years supersymmetric extensions of the Standard Model became very popular also among experimentalists not necessarily because of their convincing solution of the above problems but rather because most other contenders have been (more or less) ruled out by now. Another reason for the popularity of supersymmetric theories among theorists is the fact that the low energy limit of superstring theory – a promising candidate for a unification of all interactions including gravity – is (by and large) supersymmetric.

Since the seventies string theory has been discussed as a possible candidate for a theory which unifies all known particle interactions including gravity. The concept of point-like particles is replaced by one-dimensional extended objects – strings – and the particles of the Standard Model appear as massless excitations of this string. One of the massless excitation necessarily carries spin two and can be identified with the graviton of General Relativity. Furthermore, within this framework it is possible to sensibly compute the perturbative quantum corrections to General Relativity. Apart from the graviton one also finds spin-1 gauge bosons as well as families of chiral fermions in anomaly free representation of some non-Abelian gauge group G among the massless modes. This fact led to the hope that string theory might unify all known interactions observed in nature.

Until recently, however, string theory has only been known in its perturbative regime. That is, the (particle) excitations of a string theory are computed in the free theory ($g_\mathrm{s} = 0$), while their scattering processes are evaluated in a perturbative series for $g_\mathrm{s} \ll 1$ [9]. The string coupling constant g_s is a free parameter of string theory but for $g_\mathrm{s} = \mathcal{O}(1)$ no method of computing the spectrum or the interactions had been known. This situation dramatically changed during the past years. For the first time it became possible to go beyond the purely perturbative regime and to compute some of the non-perturbative properties of string theory [10]. The central point of these developments rests on the idea that the strong-coupling limit of a given string theory can be described in terms of another, weakly coupled, 'dual theory'. This dual theory can take the form of either a different string theory, or the same string theory with a different set of perturbative excitations, or a new theory termed M-theory.

Since string theory is a candidate for a unified theory of all interactions it has always been a primary goal to identify the Standard Model of Particle Physics as the low energy limit of string theory. The massless spectrum of string theory can indeed accommodate families of chiral fermions transforming in appropriate representations of a non-Abelian gauge group as well as Higgs bosons necessary for the electro-weak symmetry breaking. Furthermore, most ground states of string theory studied so far are supersymmetric and have a universal gauge coupling constant at the leading order which is in very good agreement with the electroweak precision data of the past decade [1]. However, a more quantitative agreement with the Standard Model has so far not been achieved. The main obstacles seem to be a missing mechanism for spontaneously breaking supersymmetry at a scale hierarchically lower than the Planck scale $M_{\rm Pl}$ and the implementation of a Higgs mechanism in string theory which generates small masses of the light states and finally the lifting of an enormous vacuum degeneracy of string ground states. It is commonly believed that these deficiencies are due to our lack of understanding the non-perturbative structure of string theory. Thus it is of interest to study the phenomenological implications of the recent developments.

The first lecture gives an elementary introduction to the supersymmetric Standard Model. We start with some of the necessary background on generic supersymmetric field theories and develop supersymmetric extensions of the Standard Model and discuss spontaneous breaking of supersymmetry. We present extensions of the Standard Model with softly broken supersymmetry and discuss some of the phenomenological properties. The presentation follows earlier lectures given at the Saalburg summer-school 1996 [11] where also a more complete list of references can be found. Other standard reviews include refs. [12] – [18]. The second lecture introduces string theory and discusses some of its phenomenologically most interesting ground states. The third lectures addresses the problem of gauge unification, supersymmetry breaking and the stabilization of the dilaton.

Lecture 1

We start with the basic concepts of supersymmetry and the supersymmetry algebra in its simplest form. We introduce the two irreducible multiplets of the theory, the chiral supermultiplet and the vector supermultiplet. We review the Standard Model of particle physics and extend it to a supersymmetric version and show how it can be broken, both spontaneously as well as explicitly. At the end we will discuss the hierarchy and the naturalness problem and take a first look at a unification with gravity.

1.1 Introduction to supersymmetry

Supersymmetry is a symmetry between bosons and fermions or more precisely it is a symmetry between states of different spin [7]. For example, a spin-0 particle is mapped to a spin-$\frac{1}{2}$ particle under a supersymmetry transformation. Thus, the generators $Q_\alpha, \overline{Q}_{\dot\alpha}$ of the supersymmetry transformation must transform in the spin-$\frac{1}{2}$ representations of the Lorentz group. These new fermionic generators form together with the four-momentum P_m and the generators of the Lorentz transformations M^{mn} a graded Lie algebra which features in addition to commutators also anticommutators in their defining relations. The simplest ($N = 1$) supersymmetry algebra reads:

$$
\begin{aligned}
\{Q_\alpha, \overline{Q}_{\dot\beta}\} &= 2\sigma^m_{\alpha\dot\beta}\, P_m \\
\{Q_\alpha, Q_\beta\} &= \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \\
[\overline{Q}_{\dot\alpha}, P_m] &= [Q_\alpha, P_m] = 0 \\
[Q_\alpha, M^{mn}] &= \frac{1}{2}\,\sigma^{mn\beta}_{\ \ \alpha} Q_\beta \\
[\overline{Q}_{\dot\alpha}, M^{mn}] &= \frac{1}{2}\,\bar\sigma^{mn\dot\beta}_{\ \ \dot\alpha} Q_{\dot\beta}
\end{aligned}
\tag{1}
$$

where we use the notation and convention of ref. [8] which are also summarized in an appendix.

The particle states in a supersymmetric field theory form representations (supermultiplets) of the supersymmetry algebra (1). These supermultiplets have the following generic features:

(a) There is an equal number of bosonic degrees of freedom n_B and fermionic degrees of freedom n_F in a supermultiplet

$$n_B = n_F \ . \tag{2}$$

(b) The masses of all states in a supermultiplet are degenerate. In particular the masses of bosons and fermions are equal[1]

$$m_B = m_F \ . \tag{3}$$

(c) Q has mass dimension $\frac{1}{2}$ and thus the mass dimensions of the fields in a supermultiplet differ by $\frac{1}{2}$.

The two irreducible multiplets which are important for constructing the supersymmetric Standard Model are the chiral multiplet and the vector multiplet which are discussed in the following two sections.

[1] This follows immediately from the fact that P^2 is a Casimir operator of the supersymmetry algebra (1) $[P^2, Q] = [P^2, M^{mn}] = 0$.

1.2 The chiral supermultiplet

The chiral supermultiplet Φ [7] contains a complex scalar field $A(x)$ of spin 0 and mass dimension 1, a Weyl fermion $\psi_\alpha(x)$ of spin $\frac{1}{2}$ and mass dimension $\frac{3}{2}$ and an auxiliary complex scalar field $F(x)$ of spin 0 and mass dimension 2

$$\Phi = (A(x), \psi_\alpha(x), F(x)) \ . \tag{4}$$

Φ has off-shell four real bosonic degrees of freedom ($n_B = 4$) and four real fermionic degrees of freedom ($n_F = 4$) in accord with (2). The supersymmetry transformations act on the fields in the multiplet as follows:

$$\begin{aligned} \delta_\xi A &= \sqrt{2}\xi\psi \ , \\ \delta_\xi \psi &= \sqrt{2}\xi F + i\sqrt{2}\sigma^m\bar{\xi}\partial_m A \ , \\ \delta_\xi F &= i\sqrt{2}\bar{\xi}\bar{\sigma}^m\partial_m\psi \ . \end{aligned} \tag{5}$$

The parameters of the transformation ξ^α are constant, complex anticommuting Grassmann parameters obeying

$$\xi_\alpha\xi_\beta = -\xi_\beta\xi_\alpha. \tag{6}$$

The transformations (5) can be thought of as generated by the operator

$$\delta_\xi = \xi Q + \overline{\xi Q} \tag{7}$$

with Q and $\overline{Q}$ obeying (1). This can be explicitly checked by evaluating the commutators $[\delta_\xi,\ \delta_\eta]$ on the fields A, ψ and F.

The field F has the highest mass dimension of the members of the chiral multiplet and therefore is called the highest component. As a consequence it cannot transform into any other field of the multiplet but only into their derivatives. This is not only true for the chiral multiplet (as can be seen explicitly in (5)) but holds for any supermultiplet. This fact can be used to construct Lagrangian densities which transform into a total derivative under supersymmetry transformations leaving the corresponding actions invariant.

For the chiral multiplet a supersymmetric and renormalizable Lagrangian is given by [8]

$$\begin{aligned} \mathcal{L}(A,\psi,F) &= -i\bar{\psi}\bar{\sigma}^m\partial_m\psi - \partial_m\bar{A}\partial^m A + F\bar{F} \\ &\quad + m(AF + \bar{A}\bar{F} - \frac{1}{2}(\psi\psi + \bar{\psi}\bar{\psi})) \\ &\quad + Y(A^2F + \bar{A}^2\bar{F} - A\psi\psi - \bar{A}\bar{\psi}\bar{\psi}) \ , \end{aligned} \tag{8}$$

where m and Y are real parameters. This action has the peculiar property that no kinetic term for F appears. As a consequence the equations of motion for F are purely algebraic

$$\frac{\delta\mathcal{L}}{\delta\bar{F}} = F + m\bar{A} + Y\bar{A}^2 = 0, \qquad \frac{\delta\mathcal{L}}{\delta F} = \bar{F} + mA + YA^2 = 0.$$

Thus F is a non-dynamical, 'auxiliary' field which can be eliminated from the action algebraically by using its equation of motion. This yields

$$\begin{aligned}\mathcal{L}(A,\psi,F=-m\bar{A}-Y\bar{A}^2) &= -i\bar{\psi}\bar{\sigma}^m\partial_m\psi-\partial_m\bar{A}\partial^m A \\ &- \frac{m}{2}(\psi\psi+\bar{\psi}\bar{\psi})-Y(A\psi\psi+\bar{A}\bar{\psi}\bar{\psi})-V(A,\bar{A})\ ,\end{aligned} \tag{9}$$

where $V(A,\bar{A})$ is the scalar potential given by

$$\begin{aligned} V(A,\bar{A}) &= \mid mA+YA^2\mid^2 \\ &= m^2A\bar{A}+mY(A\bar{A}^2+\bar{A}A^2)+Y^2A^2\bar{A}^2 \\ &= F\bar{F}\mid_{\frac{\delta\mathcal{L}}{\delta F}=\frac{\delta\mathcal{L}}{\delta\bar{F}}=0}\ .\end{aligned} \tag{10}$$

As can be seen from (9) and (10) after elimination of F a standard renormalizable Lagrangian for a complex scalar A and a Weyl fermion ψ emerges. However (9) is not the most general renormalizable Lagrangian for such fields. Instead it satisfies the following properties:

- $\mathcal{L}$ only depends on two independent parameters, the mass parameter m and the dimensionless Yukawa coupling Y. In particular, the $(A\bar{A})^2$ coupling is not controlled by an independent parameter (as it would be in non-supersymmetric theories) but determined by the Yukawa coupling Y.
- The masses for A and ψ coincide in accord with (3).[2]
- V is positive semi-definite, $V\geq 0$.

1.3 The vector supermultiplet

The vector supermultiplet V contains a gauge boson v_m of spin 1 and mass dimension 1, a Weyl fermion (called the gaugino) λ of spin $\frac{1}{2}$ and mass dimension $\frac{3}{2}$, and a real scalar field D of spin 0 and mass dimension 2

$$V=(v_m(x),\lambda_\alpha(x),D(x))\ . \tag{11}$$

Similar to the chiral multiplet also the vector multiplet has $n_B=n_F=4$.

The vector multiplet can be used to gauge the action of the previous section. An important consequence of the theorems of Refs. [19, 20] is the fact that the generators T^a of a compact gauge group G have to commute with the supersymmetry generators

$$[T^a,Q_\alpha]=[T^a,\overline{Q}_{\dot{\alpha}}]=0\ . \tag{12}$$

Therefore all members of a chiral multiplet (A,ψ,F) have to reside in the same representation of the gauge group. Similarly, the members of the vector multiplet have to

[2] As immediate consequence of this feature one notes that supersymmetry must be explicitly or spontaneously broken in nature.

transform in the adjoint representation of G and thus they all are Lie-algebra valued fields

$$v_m = v_m^a T^a \ , \qquad \lambda_\alpha = \lambda_\alpha^a T^a \ , \qquad D = D^a T^a \ . \tag{13}$$

The supersymmetry transformations of the components of the vector multiplet are [8]:

$$\begin{aligned} \delta_\xi v_m^a &= -i\bar\lambda^a \bar\sigma^m \xi + i\bar\xi \bar\sigma^m \lambda^a \ , \\ \delta_\xi \lambda^a &= i\xi D^a + \sigma^{mn}\xi F_{mn}^a \ , \\ \delta_\xi D^a &= -\xi\sigma^m D_m \bar\lambda^a - D_m \lambda^a \sigma^m \bar\xi \ . \end{aligned} \tag{14}$$

The field strength of the vector bosons F_{mn}^a and the covariant derivative $D_m\lambda^a$ are defined according to

$$\begin{aligned} F_{mn}^a &:= \partial_m v_n^a - \partial_n v_m^a - g f^{abc} v_m^b v_n^c \ , \\ D_m \lambda^a &:= \partial_m \lambda^a - g f^{abc} v_m^b \lambda^c \ , \end{aligned} \tag{15}$$

where f^{abc} are the structure constants of the Lie algebra and g is the gauge coupling. A gauge invariant, renormalizable and supersymmetric Lagrangian for the vector multiplet is given by

$$\mathcal{L} = -\frac{1}{4} F_{mn}^a F^{mn\,a} - i\bar\lambda^a \bar\sigma^m D_m \lambda^a + \frac{1}{2} D^a D^a \ . \tag{16}$$

As before the equation of motion for the auxiliary D-field is purely algebraic $D^a = 0$.

A gauge invariant, renormalizable Lagrangian containing a set of chiral multiplets (A^i, ψ^i, F^i) coupled to vector multiplets is found to be [8]

$$\begin{aligned} \mathcal{L}(A^i, \psi^i, F^i, v_m^a, \lambda^a, D^a) &= -\frac{1}{4} F_{mn}^a F^{mn\ a} - i\bar\lambda^a \bar\sigma^m D_m \lambda^a + \frac{1}{2} D^a D^a \\ &\quad - D_m A^i D^m \bar A^i - i\bar\psi^i \bar\sigma^m D_m \psi^i + \bar F^i F^i \\ &\quad + i\sqrt{2} g (\bar A^i T_{ij}^a \psi^j \lambda^a - \bar\lambda^a T_{ij}^a A^i \bar\psi^j) \\ &\quad + g D^a \bar A^i T_{ij}^a A^j - \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} \bar W_{ij} \bar\psi^i \bar\psi^j \\ &\quad + F^i W_i + \bar F^i \bar W_i \ , \end{aligned} \tag{17}$$

where the covariant derivatives are defined by

$$\begin{aligned} D_m A^i &:= \partial_m A^i + i g v_m^a T_{ij}^a A^i \ , \\ D_m \psi^i &:= \partial_m \psi^i + i g v_m^a T_{ij}^a \psi^j \ . \end{aligned} \tag{18}$$

W_i and W_{ij} in (17) are the derivatives of a holomorphic function $W(A)$ called the superpotential

$$\begin{aligned} W(A) &= \frac{1}{2} m_{ij} A^i A^j + \frac{1}{3} Y_{ijk} A^i A^j A^k \ , \\ W_i &\equiv \frac{\partial W}{\partial A^i} = m_{ij} A^j + Y_{ijk} A^j A^k \ , \\ W_{ij} &\equiv \frac{\partial^2 W}{\partial A^i \partial A^j} = m_{ij} + 2 Y_{ijk} A^k \ . \end{aligned} \tag{19}$$

By explicitly inserting (19) into (17) one observes that the m_{ij} are mass parameters while the Y_{ijk} are Yukawa couplings. Supersymmetry forces W to be a holomorphic function of the scalar fields A while renormalizability restricts W to be at most a cubic polynomial of A. Finally, the parameters m_{ij} and Y_{ijk} are further constrained by gauge invariance.

As before, F^i and D^a obey algebraic equations of motion which read

$$\begin{aligned} \frac{\delta\mathcal{L}}{\delta F} = 0 \;&\Rightarrow\; \bar{F}_i + W_i = 0\ , \\ \frac{\delta\mathcal{L}}{\delta \bar{F}} = 0 \;&\Rightarrow\; F_i + \bar{W}_i = 0\ , \\ \frac{\delta\mathcal{L}}{\delta D^a} = 0 \;&\Rightarrow\; D^a + g\bar{A}^i T^a_{ij} A^j = 0\ . \end{aligned} \tag{20}$$

They can be used to eliminate the auxiliary fields F^i and D^a from the Lagrangian (17) and one obtains

$$\begin{aligned} &\mathcal{L}(A^i, \psi^i, v^a_m, \lambda^a, F_i = -\bar{W}_i, D^a = -g\bar{A}^i T^a_{ij} A^j) = \\ &\quad -\frac{1}{4} F^a_{mn} F^{mn\ a} - i\bar{\lambda}^a \bar{\sigma}^m D_m \lambda^a - D_m A^i D^m \bar{A}^i - i\bar{\psi}^i \bar{\sigma}^m D_m \psi^i \\ &\quad + i\sqrt{2} g(\bar{A}^i T^a_{ij} \psi^j \lambda^a - \bar{\lambda}^a T^a_{ij} A^i \bar{\psi}^j) - \frac{1}{2} W_{ij} \psi^i \psi^j - \frac{1}{2} \bar{W}_{ij} \bar{\psi}^i \bar{\psi}^j - V(A, \bar{A}) \end{aligned} \tag{21}$$

where

$$\begin{aligned} V(A, \bar{A}) &= W_i \bar{W}_i + \frac{1}{2} g^2 (\bar{A}^i T^a_{ij} A^j)(\bar{A}^i T^a_{ij} A^j) \\ &= (F^i \bar{F}^i + \frac{1}{2} D^a D^a)\ |_{\frac{\delta\mathcal{L}}{\delta F}=0, \frac{\delta\mathcal{L}}{\delta D^a}=0} \\ &\geq 0\ . \end{aligned} \tag{22}$$

As before the scalar potential $V(A, \bar{A})$ is positive semi-definite.

1.4 The Standard Model

In this section we briefly review some basic features of the Standard Model. The Standard Model is a quantum gauge field theory with a chiral gauge group $G_{\rm SM} = SU(3) \times SU(2) \times U(1)_Y$. The spectrum of particles includes three families of quarks and leptons, the gauge bosons (gluons, $W^{\pm}, Z^0$, photon) of $G_{\rm SM}$ and one spin-0 Higgs doublet. In table 1 the particle content and their representations are displayed. The Lagrangian of the Standard Model reads

$$\begin{aligned} \mathcal{L} \;=\; & -\frac{1}{4} \sum_{(a)=1}^{3} \Big((F^b_{mn} F^{mn\ b})_{(a)} \Big) - D_m h D^m \bar{h} \\ & + \sum_{I=1}^{3} \Big(-i\bar{q}^I_L \not{D}\, q^I_L - i\bar{u}^I_R \not{D}\, u^I_R - i\bar{d}^I_R \not{D}\, d^I_R - i\bar{l}^I_L \not{D}\, l^I_L - i\bar{e}^I_R \not{D}\, e^I_R \Big) \\ & - \sum_{IJ=1}^{3} \Big((Y_u)_{IJ} \bar{h} q^I_L u^J_R + (Y_d)_{IJ} h q^I_L d^J_R + (Y_l)_{IJ} h l^I_L e^J_R + h.c. \Big) - V(h, \bar{h}), \end{aligned} \tag{23}$$

		SU(3)	SU(2)	$\mathrm{U}(1)_Y$	$\mathrm{U}(1)_{\mathrm{em}}$
quarks	$q_L^I = \begin{pmatrix} u_L^I \\ d_L^I \end{pmatrix}$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
	u_R^I	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
	d_R^I	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
leptons	$l_L^I = \begin{pmatrix} \nu_L^I \\ e_L^I \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	e_R^I	1	1	1	1
Higgs	$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
gauge bosons	G	8	1	0	0
	W	1	3	0	$(0, \pm 1)$
	B	1	1	0	0

Table 1: The particle content of the Standard Model. The index $I = 1, 2, 3$ labels the three families of chiral quarks q_L^I, u_R^I, d_R^I and chiral leptons l_L^I, e_R^I. All of them are Weyl fermions and transform in the $(\frac{1}{2}, 0)$ representation of the Lorentz group (they have an undotted spinor index α). The subscripts R, L do not specify the representation of the Lorentz group but instead are used to indicate the different transformation properties under the chiral gauge group $SU(2) \times U(1)$. This somewhat unconventional notation is used to make a smooth transition to the supersymmetric Standard Model later on. The electromagnetic charge listed in the last column is defined by $Q_{em} = T^3_{SU(2)} + Q_Y$.

where $\not{D} = \sigma^m D_m$ and the index (a) labels the 3 different factors in the gauge group. $V(h, \bar{h})$ is the scalar potential for the Higgs doublet which is chosen to be

$$V(h, \bar{h}) = \mu^2 h\bar{h} + \lambda(h\bar{h})^2 \ . \tag{24}$$

In order to have a bounded potential $\lambda > 0$ has to hold. For $\mu^2 < 0$ the electroweak gauge group $SU(2) \times U(1)_Y$ is spontaneously broken down to $U(1)_{\text{em}}$. In this case the minimum of the potential is not at $\langle h \rangle = 0$, but at $\langle h\bar{h} \rangle = -\frac{\mu^2}{2\lambda}$.

1.5 Supersymmetric extensions of the Standard Model

Let us now turn to the supersymmetric generalization of the Standard Model.[3] The idea is to promote the Lagrangian (23) to a supersymmetric Lagrangian. As we learned in the previous section supersymmetry requires the presence of additional states which form supermultiplets with the known particles. Since all states of a supermultiplet carry the same gauge quantum numbers we need at least a doubling of states: For every field of the SM one has to postulate a superpartner with the exact same gauge quantum numbers and a spin such that it can form an appropriate supermultiplet. More specifically, the quarks and leptons are promoted to chiral multiplets by adding scalar (spin-0) squarks $(\tilde{q}_L^I, \tilde{u}_R^I, \tilde{d}_R^I)$ and sleptons $(\tilde{l}_L^I, \tilde{e}_R^I)$ to the spectrum. The gauge bosons are promoted to vector multiplets by adding the corresponding spin-$\frac{1}{2}$ gauginos $(\tilde{G}, \tilde{W}, \tilde{B})$ to the spectrum. Finally, the Higgs boson is also promoted to a chiral multiplet with a spin-$\frac{1}{2}$ Higgsino superpartner. However, the supersymmetric version of the Standard Model cannot 'live' with only one Higgs doublet and at least a second Higgs doublet has to be added. This can be seen from the fact that one cannot write down a supersymmetric version of the Yukawa interactions of the Standard Model without introducing a second Higgs doublet. The reason is that the superpotential W is a holomorphic function and therefore does not contain factors $\bar{h}$. The precise spectrum of the supersymmetric Standard Model is summarized in table 2.

The Lagrangian for the supersymmetric Standard Model has to be of the form (17) with an appropriate superpotential W. It has to be chosen such that the Lagrangian of the non-supersymmetric Standard Model (23) is contained. This is achieved by

$$W = \sum_{IJ} \left((Y_u)_{IJ} h_u \tilde{q}_L^I \tilde{u}_R^J + (Y_d)_{IJ} h_d \tilde{q}_L^I \tilde{d}_R^J + (Y_l)_{IJ} h_d \tilde{l}_L^I \tilde{l}_R^I \right) + \mu\, h_u h_d \ . \tag{25}$$

Once W it specified also the scalar potential is fixed. Of particular importance is the scalar potential for the Higgs fields since it controls the electroweak symmetry breaking. Using (22) and (25) one derives the Higgs potential for the two neutral Higgs fields h_d^0, h_u^0 by setting all other scalars to zero

$$V(h_d^0, h_u^0) = |\mu|^2 \left(|h_d^0|^2 + |h_u^0|^2\right) + \frac{1}{8}\left(g_1^2 + g_2^2\right)\left(|h_u^0|^2 - |h_d^0|^2\right)^2 \ . \tag{26}$$

[3]See also [12] - [18].

	supermultiplet	F	B	SU(3)	SU(2)	$U(1)_Y$	$U(1)_{em}$
quarks	$Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix}$	q_L^I	$\tilde{q}_L^I$	3	2	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
	U_R^I	u_R^I	$\tilde{u}_R^I$	$\bar{3}$	1	$-\frac{2}{3}$	$-\frac{2}{3}$
	D_R^I	d_R^I	$\tilde{d}_R^I$	$\bar{3}$	1	$\frac{1}{3}$	$\frac{1}{3}$
leptons	$L_L^I = \begin{pmatrix} \mathcal{N}_L^I \\ E_L^I \end{pmatrix}$	l_L^I	$\tilde{l}_L^I$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	E_R^I	e_R^I	$\tilde{e}_R^I$	1	1	1	1
Higgs	$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^0 \\ \tilde{h}^- \end{pmatrix}$	$\begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	1	2	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
	$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$	$\begin{pmatrix} \tilde{h}^+ \\ \tilde{h}^0 \end{pmatrix}$	$\begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	1	2	$\frac{1}{2}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
gauge bosons	G	$\tilde{G}$	G	8	1	0	0
	W	$\tilde{W}$	W	1	3	0	$(0, \pm 1)$
	B	$\tilde{B}$	B	1	1	0	0

Table 2: Particle content of the supersymmetric Standard Model. The column below 'F' ('B') denotes the fermionic (bosonic) content of the model.

The coupling of the terms quartic in the Higgs fields is not an independent parameter but instead determined by the gauge couplings g_1 of $U(1)_Y$ and g_2 of $SU(2)$. Thus it seems that the number of parameters is reduced. However, now there are two possible vacuum expectation values $\langle h_u^0\rangle, \langle h_d^0\rangle$ – one more than in the Standard Model.

In the last section we learned that the potential of any supersymmetric theory is positive semi-definite and the Higgs potential of Eq. (26) is no exception as can be seen explicitly: $|\mu|^2$ cannot be chosen negative. Thus the minimum of V necessarily sits at $\langle h_u^0\rangle = \langle h_d^0\rangle = 0$ which corresponds to a vacuum with unbroken $SU(2)\times U(1)$. Therefore, the supersymmetric version of the Standard Model as it is defined so far – the spectrum of table 2 with interactions specified by the Lagrangian (17) with the W of (25) – cannot accommodate a vacuum with spontaneously broken electroweak symmetry. A second phenomenological problem is the presence of all the new supersymmetric states which have the same mass as their superpartners but are not observed in nature. Therefore supersymmetry itself necessarily has to appear in its broken phase and as we will see electroweak symmetry breaking is closely tied to the breakdown of supersymmetry.

1.6 Spontaneous breaking of supersymmetry

In the previous section we learned that in the simplest supersymmetric extension of the Standard Model the electroweak symmetry is unbroken. However, so far we constructed a manifestly supersymmetric extension but from the mass degeneracy of each multiplet (3) it is already clear that supersymmetry cannot be an exact symmetry in nature but has to be either spontaneously or explicitly broken. Therefore we now turn to the question of spontaneous supersymmetry breaking and return to the electroweak symmetry breaking afterwards.

Let us first recall the order parameter for supersymmetry breaking. Multiplying the anticommutator $\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\sigma^m_{\alpha\dot\alpha}P_m$ of the supersymmetry-algebra (1) with $\bar\sigma^n$ and using $Tr(\sigma^m\bar\sigma^n) = -2\eta^{mn}$ results in

$$\bar\sigma^{n\alpha\dot\alpha}\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = -4P^n \ .$$

Thus the Hamiltonian H of a supersymmetric theory is expressed as the 'square' of the supercharges

$$H = P_0 = \frac{1}{4}\left(Q_1\overline{Q}_1 + \overline{Q}_1 Q_1 + Q_2\overline{Q}_2 + \overline{Q}_2 Q_2\right) \ . \tag{27}$$

This implies that H is a positive semi-definite operator on the Hilbert space

$$\langle\psi|H|\psi\rangle \ \geq 0 \quad \forall\psi \ . \tag{28}$$

Supersymmetry is unbroken if the supercharges annihilate the vacuum $Q_\alpha|0\rangle = \overline{Q}_{\dot\alpha}|0\rangle = 0$. From (27) we learn that also H annihilates a supersymmetric vacuum $H|0\rangle = 0$.

This in turn implies that the scalar potential V of a supersymmetric field theory which has a supersymmetric ground state has to vanish at its minimum

$$\langle H\rangle = 0 \quad \Rightarrow \quad \langle V\rangle \equiv V(A,\bar{A})|_{\min} = 0\ . \tag{29}$$

The general form of the scalar potential $V = F^i\bar{F}^i + \frac{1}{2}D^aD^a$ was given in (22). Since V is positive semi-definite one immediately concludes from (29) that in a supersymmetric ground state

$$\langle F^i\rangle \equiv F^i|_{\min} = 0 \quad \text{and} \quad \langle D^a\rangle \equiv D^a|_{\min} = 0 \tag{30}$$

has to hold. The converse is also true

$$\langle F^i\rangle \neq 0 \quad \text{or} \quad \langle D^a\rangle \neq 0 \quad \Rightarrow \quad V|_{min} > 0 \quad \Rightarrow \quad Q_\alpha|0> \neq 0 \tag{31}$$

and supersymmetry is spontaneously broken. Thus $\langle F^i\rangle$ and $\langle D^a\rangle$ are the order parameters of supersymmetry breaking in that non-vanishing F- or D-terms signal spontaneous supersymmetry breaking.

Specific potentials which do lead to non-vanishing D- or F-terms have been constructed [21, 22]. For such theories the mass degeneracy is lifted indeed. However, the following sum rule continues to hold [23]:

$$\mathrm{Str}M^2 \equiv \sum_{J=0}^{1}(-)^{2J}(2J+1)\mathrm{Tr}M_J^2 = 0\ , \tag{32}$$

where J is the spin of the particles. This sum rule is problematic for the supersymmetric Standard Model. Since non of the supersymmetric partners has been observed yet they must be heavier than the particles of the Standard Model. Close inspection of (32) shows that this cannot be arranged within a spontaneously broken supersymmetric Standard Model.

To summarize, the lesson of this section is that also spontaneously broken supersymmetry runs into phenomenological difficulties. The only way out is an explicit breaking of (global) supersymmetry.

1.7 Local supersymmetry — supergravity

As we have seen in the previous section models with spontaneously broken supersymmetry are phenomenologically not acceptable. For example the mass formula (32), generally valid in such cases, forbids that all supersymmetric particles acquire masses large enough to make them invisible in present experiments. One way to overcome those difficulties is to allow explicit supersymmetry breaking. The corresponding supersymmetry breaking terms in the Lagrangian can be motivated by local supersymmetric theories that are considered in this section.

Ultimately one has to couple the supersymmetric Standard Model to gravity. This requires the promotion of global supersymmetry to a local symmetry, that is the parameter of the supersymmetry transformation $\xi_\alpha = \xi_\alpha(x)$ is no longer constant but depends on the space-time coordinates x [8, 24]. This demands the presence of an additional massless fermionic gauge field (the gravitino) $\Psi_{m\alpha}$ with spin 3/2 and an inhomogeneous transformation law

$$\delta_\xi \Psi_{m\alpha} = -\partial_m \xi_\alpha + \dots \ . \tag{33}$$

(The necessity of this transformation law can be seen for example from the supersymmetry transformation of $\partial_m A$ which now has an extra contribution $\partial_m \delta_\xi A \propto \partial_m \xi\psi = \xi\partial_m\psi + (\partial_m\xi)\psi$.) Together with the metric g_{mn} and 6 auxiliary fields $b_m, M, \bar{M}$ the gravitino $\Psi_{m\alpha}$ forms the supergravity multiplet $(g_{mn}, \Psi_{m\alpha}, b_m, M, \bar{M})$.

This supergravity multiplet can be coupled to vector and chiral multiplets. The bosonic terms of the most general gauge invariant supergravity Lagrangian with only chiral and vector multiplets and no more than two derivatives is given by [25, 8]

$$\begin{aligned}\mathcal{L} \; &= \; -\sqrt{g}\Big(\frac{1}{2\kappa^2}R + G_{i\bar{j}}\, D_m \bar{A}^{\bar{j}} D^m A^i + V(A,\bar{A}) \\ &\quad + \sum_{(a)} \Big(\frac{1}{4g_{(a)}^2}(F_{mn}F^{mn})_{(a)} + \frac{\theta_{(a)}}{32\pi^2}(F\tilde{F})_{(a)}\Big) \\ &\quad + \ \text{fermionic terms}\Big) \ ,\end{aligned} \tag{34}$$

where $\kappa^2 = 8\pi M_{\rm Pl}^{-2}$. Supersymmetry imposes constraints on the couplings of $\mathcal{L}$ in Eq. (34). The metric $G_{i\bar{j}}$ of the manifold spanned by the complex scalars A^i is necessarily a Kähler metric and therefore obeys

$$G_{i\bar{j}} = \frac{\partial}{\partial A^i}\frac{\partial}{\partial \bar{A}^{\bar{j}}}\, K(A,\bar{A}) \ , \tag{35}$$

where $K(A,\bar{A})$ is the Kähler potential. It is an arbitrary real function of A and $\bar{A}$. The gauge group G is in general a product of simple group factors $G_{(a)}$ labeled by an index (a), i.e.

$$G = \prod_{(a)} G_{(a)} \ . \tag{36}$$

With each factor $G_{(a)}$ there is an associated gauge coupling $g_{(a)}$ which can depend on the A^i. However, supersymmetry constrains the possible functional dependence and demands that the (inverse) gauge couplings $g_{(a)}^{-2}$ are the real part of holomorphic functions $f_{(a)}(A)$ called the gauge kinetic functions. The imaginary part of the $f_a(\phi)$ are (field-dependent) θ-angles. One finds

$$\begin{aligned} g_{(a)}^{-2} &= {\rm Re} f_{(a)}(A) \ , \\ \theta_{(a)} &= -8\pi^2\, {\rm Im} f_{(a)}(A) \ . \end{aligned} \tag{37}$$

The scalar potential $V(A,\bar{A})$ is still determined by the superpotential $W(A)$ but in a modified form

$$V = e^{\kappa^2 K}\Big(D_i W G^{i\bar{j}} \bar{D}_{\bar{j}} \bar{W} - 3\kappa^2 |W|^2\Big) + \frac{1}{2} D^a D^a \ , \tag{38}$$

where $D_i W := \frac{\partial W}{\partial A^i} + \kappa^2 \frac{\partial K}{\partial A^i} W$. To summarize, $\mathcal{L}$ is completely determined by three functions of the chiral multiplets, the real Kähler potential $K(A, \bar{A})$, the holomorphic superpotential $W(A)$ and the holomorphic gauge kinetic functions $f_{(a)}(A)$.

The limit $\kappa^2 \to 0$ corresponds to turning off gravity and in this limit one obtains indeed $V \to \frac{\partial W}{\partial A^i}\frac{\partial \bar{W}}{\partial \bar{A}^i} + \frac{1}{2} D^a D^a$ in accord with (22). Local supersymmetry is spontaneously broken if $D_i W|_{min} \neq 0$ for some i. This can be achieved by introducing a hidden sector which only couples via non-renormalizable interactions to the observable sector of the supersymmetric Standard Model and which has a superpotential $W_{hid}(\phi)$ suitably chosen to ensure $D_\phi W|_{min} \neq 0$ [12, 26]. In this case the gravitino becomes massive through a supersymmetric Higgs effect whereas the graviton stays massless [25].

Expanding the Lagrangian (34) in powers of $m_{3/2}/M_{Pl}$ where $m_{3/2}$ is the gravitino mass one gets at leading order the Lagrangian of an effective (renormalizable) theory

$$\mathcal{L} = \mathcal{L}_{\text{susy}} + \mathcal{L}_{\text{soft}} \tag{39}$$

where $\mathcal{L}_{\text{susy}}$ is given by (21) and

$$\mathcal{L}_{\text{soft}} = -m^2_{ij} A^i \bar{A}^j - (b_{ij} A^i A^j - a_{ijk} A^i A^j A^k + \text{h.c.}) - \frac{1}{2}(\tilde{m}_{ab} \lambda^a \lambda^b + \text{h.c.}) \ . \tag{40}$$

$\mathcal{L}_{\text{soft}}$ breaks global supersymmetry explicitly but softly. That is, no quadratic divergences are introduced into the theory (we return to this feature shortly). The parameters m^2_{ij}, $(b_{ij}, a_{ijk}, \tilde{m}_{ab}$ are expressed in terms of the coupling functions K, W, f of the hidden sector fields. The sum rule (32)) of global supersymmetry is modified and reads in supergravity

$$\sum_{J=0}^{1} (-)^{2J} (2J+1) \text{Tr} M_J^2 = m_{3/2} \ . \tag{41}$$

As a consequence all superpartners in the observable sector can be heavier than the particles of the Standard Model. Hence, the supersymmetric Standard Model coupled to spontaneously broken supergravity in leading order in $m_{3/2}/M_{Pl}$ leads to an explicitly but softly broken supersymmetric theory which is no longer in obvious conflict with current experimental data.

1.8 The hierarchy and naturalness problem

Before we continue in our endeavor to construct a phenomenologically viable extension of the Standard Model let us briefly review what is called the hierarchy and naturalness problem in the Standard Model.

Consider the following (non-supersymmetric) Lagrangian of a complex scalar A and a Weyl fermion χ

$$\begin{aligned} \mathcal{L} = & - \partial_m \bar{A} \partial^m A - i \bar{\chi} \bar{\sigma}^m \partial_m \chi - \frac{1}{2} m_f (\chi\chi + \bar{\chi}\bar{\chi}) - m_b^2 \bar{A} A \\ & - Y (A \chi\chi + \bar{A} \bar{\chi}\bar{\chi}) \ - \ \lambda (\bar{A} A)^2 \ . \end{aligned} \tag{42}$$

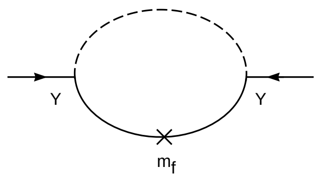

Figure 1: The one-loop correction to the fermion mass.

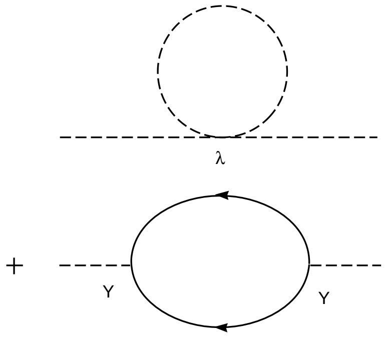

Figure 2: The one-loop corrections to the boson mass.

From (9) we learn that this Lagrangian is supersymmetric if $m_f = m_b$ and $Y^2 = \lambda$ but let us not consider this choice of parameters at first. $\mathcal{L}$ has a chiral symmetry for $m_f = 0$ given by

$$A \to e^{-2i\alpha} A\ , \qquad \chi \to e^{i\alpha} \chi\ . \tag{43}$$

This symmetry prohibits the generation of a fermion mass by quantum corrections. For $m_f \neq 0$ the fermion mass does receive radiative corrections, but all possible diagrams have to contain a mass insertion as can be seen from the one-loop diagram shown in Fig. 1. Since the propagator of the boson (upper dashed line in the diagram) is $\sim \frac{1}{k^2}$ while the propagator of the fermion (lower solid line) is $\sim \frac{1}{k}$ one obtains a mass correction which is proportional to m_f

$$\delta m_f \sim Y^2 m_f \ln \frac{m_f^2}{\Lambda^2}\ , \tag{44}$$

where Λ is the ultraviolet cutoff. Hence the mass of a chiral fermion does not receive large radiative corrections if the bare mass is small. For that reason ‘t Hooft calls fermion masses ‘natural’ – an extra symmetry appears when the mass is set to zero which in turn leads to a protection of the fermion mass by an approximate chiral symmetry [27].

This state of affairs is different for scalar fields. The diagrams giving the one-loop corrections to m_b are shown in Fig. 2. Both diagrams are quadratically divergent but they have an opposite sign because in the second diagram fermions are running in the loop. One finds

$$\delta m_b^2 \sim (\lambda - \ Y^2)\, \Lambda^2\ . \tag{45}$$

Thus, in non-supersymmetric theories scalar fields receive large mass corrections (even if the bare mass is set to zero) and small scalar masses are 'unnatural' [27, 28, 3]. They can only be arranged by delicately fine-tuning the bare mass and the couplings λ, Y. This problem becomes apparent in extensions of the Standard Model which apart from the weak scale M_Z do have a second larger scale, say $M_{\rm GUT}$ with $M_{\rm GUT} \gg M_Z$ [28, 3]. In such theories the mass of the scalar boson is naturally of the order of the largest mass parameter in the theory. This discussion applies to the Higgs boson of the Standard Model and it is difficult to understand the smallness of M_Z and how it can be kept stable against quantum corrections whenever the Standard Model is the low energy limit of a theory with a large mass scale.

A concrete example of this problem occurs in Grand Unified Theories (GUTs) [6] where the Standard Model is embedded into a single simple gauge group $G_{\rm GUT}$ (e.g. $G_{\rm GUT} = SU(5)$). The GUT gauge symmetry is broken by a Higgs mechanism to the gauge group of the Standard Model and one has the following pattern of symmetry breaking

$$G_{\rm GUT} \stackrel{M_{\rm GUT}}{\rightarrow} SU(3) \times SU(2) \times U(1) \stackrel{M_Z}{\rightarrow} SU(3) \times U(1)_{\rm em} \, , \tag{46}$$

where $M_{\rm GUT} \approx 10^{15}$ GeV and thus $M_{\rm GUT} \gg M_Z$.

This problem is solved in supersymmetric theories where the Higgs boson is elementary but the quadratic divergence in (45) exactly cancels due to the supersymmetric relation $Y^2 = \lambda$.

The cancelation of quadratic divergences is a general feature of supersymmetric quantum field theories and a consequence of a more general non-renormalization theorem: The superpotential W of a supersymmetric quantum field theory is not renormalized in perturbation theory [29] and all quantum corrections solely arise from the gauge coupling and wavefunction renormalization. The non-renormalization theorem or in other words the 'taming' of the quantum corrections is one of the attractive features of supersymmetric quantum field theories. It leads (among other things) to the possibility of stabilizing the weak scale M_Z.

In that sense supersymmetry solves the naturalness problem in that it allows for a small and stable weak scale without fine-tuning. However, supersymmetry does not solve the hierarchy problem in that it does not explain why the weak scale is small in the first place.

The 'attractive' feature of supersymmetric field theories to solve the naturalness problem can be maintained in theories with explicitly broken supersymmetry if the supersymmetry breaking terms are of a particular form. Such terms which break supersymmetry explicitly and generate no quadratic divergences are called 'soft breaking terms' and in particular the terms resulting from supergravity considered in the previous section are of this 'soft' type.

1.9 Back to the supersymmetric Standard Model coupled to supergravity

In the equations (39) and (40) the most general Lagrangian of a softly broken supersymmetric gauge theory was presented. For $\mathcal{L}_{\rm susy}$ we continue to take (21) together with the superpotential specified in (25). For $\mathcal{L}_{\rm soft}$ only gauge invariance and R-parity is imposed. This leads to the following possible soft terms [12, 13, 15, 16, 18]:

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\Big((a_u)_{IJ}h_u\tilde{q}^I_L\tilde{u}^J_R+(a_d)_{IJ}h_d\tilde{q}^I_L\tilde{d}^J_R+(a_e)_{IJ}h_d\tilde{l}^I_L\tilde{e}^J_R+bh_uh_d+\text{ h.c. }\Big)\\ &\quad-\sum_{\text{all scalars}} m^2_{ij}A^i\bar{A}^j-(\frac{1}{2}\sum_{(a)=1}^{3}\tilde{m}_{(a)}(\lambda\lambda)_{(a)}+\text{ h.c. })\ .\end{aligned} \tag{47}$$

Obviously a huge number of new parameters is introduced via $\mathcal{L}_{\rm soft}$. The parameters of $\mathcal{L}_{\rm susy}$ are the Yukawa couplings Y and the parameter μ in the Higgs potential. The Yukawa couplings are determined experimentally already in the non-supersymmetric Standard Model. In the softly broken supersymmetric Standard Model the parameter space is enlarged by

$$\Big(\mu,(a_u)_{IJ},(a_d)_{IJ},(a_e)_{IJ},b,m^2_{ij},\tilde{m}_{(a)}\Big)\ . \tag{48}$$

A much more constrained version (with less free parameters) became known as the Minimal Supersymmetric Standard Model (MSSM). The MSSM was motivated by the success of Grand Unified Theories combined with a simple, flavor blind mechanism of supersymmetry breaking in a hidden sector [30]. Over the last 15 years this model went through a few alterations but today it is a well defined model with a very particular set of soft supersymmetry breaking terms which are flavor blind and in some sense the minimal choice of free parameters [12, 13, 15, 16]. One imposes that *all* scalar masses are the same $m^2_{ij}=m^2_0\delta_{ij}$, all gaugino masses are the same $\tilde{m}_1=\tilde{m}_2=\tilde{m}_3=\tilde{m}$, all a-parameters are proportional to the Yukawa couplings with the same universal proportionality constant a_0 and finally that the b-parameter is of a specific form. Altogether one has

$$\begin{aligned} m^2_{ij} &= m^2_0\,\delta_{ij}\ , \qquad \tilde{m}_1=\tilde{m}_2=\tilde{m}_3=\tilde{m}\ , \qquad b=b_0\,m_0\,\mu\ ,\\ (a_u)_{IJ} &= a_0\,(Y_u)_{IJ}\ , \qquad (a_d)_{IJ}=a_0\,(Y_d)_{IJ}\ , \qquad (a_l)_{IJ}=a_0\,(Y_l)_{IJ}\ .\end{aligned} \tag{49}$$

Thus, the parameter space of the MSSM is spanned by the 5 parameters

$$(m_0,\tilde{m},a_0,b_0,\mu)\ . \tag{50}$$

This model is not ruled out and in particular in specific regions of the parameter space it is consistent with the electroweak precision data. It will be further tested in future experiments such as the LHC.

1.10 Electroweak symmetry breaking in the MSSM

In section 1.5 we noticed that for unbroken or spontaneously broken supersymmetry the electroweak symmetry remains intact in the supersymmetric version of the Standard Model. Let us now review the situation in the presence of soft breaking terms [31]. The Higgs sector of the supersymmetric Standard Model consists of two $SU(2)$-doublets

$$h_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \quad , \qquad h_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix} \quad ,$$

which carry eight real degrees of freedom, four of them neutral and four charged. Like in the Standard Model $SU(2)_L \times U(1)_Y$ will be broken to $U(1)_{\text{em}}$ by non-vanishing VEVs of the neutral Higgs bosons h_u^0 and h_d^0.

It turns out that the electroweak symmetry is unbroken at tree level. However, the Higgs potential has flat directions and electroweak symmetry breaking is induced radiatively by the soft terms and the top Yukawa coupling. This symmetry breaking mechanism results in three light particles corresponding to $W^\pm$ and Z and five remaining degrees of freedom that yield the physical Higgs bosons of the model:

$H^\pm$	charged Higgs boson pair
A^0	CP-odd neutral Higgs boson
H^0, h^0	CP-even neutral Higgs bosons .

Their masses obey the relations

$$m_{H^\pm} \geq M_W \ , \qquad m_{H^0} \geq M_Z \ , \qquad m_{h^0} \leq M_Z \ . \tag{51}$$

Physically the most interesting is the last inequality since it predicts the existence of a light Higgs boson. This 'prediction' can be directly traced to the fact the quartic couplings in the Higgs potential are fixed by the (measured) gauge couplings and are not free parameters as in the Standard Model. However, radiative corrections for this lightest Higgs boson mass can be large and after taking into account quantum corrections the upper bound on m_{h^0} is pushed up to about 150 GeV [32]. However, the prediction of one light neutral Higgs boson remains and is one of the characteristic features of the supersymmetric two-doublet Higgs sector. It even holds in the limit that all masses of the supersymmetric particles are sent to infinity. In this limit one recovers the non-supersymmetric Standard Model – albeit with a light Higgs.

Minimization of the Higgs potential results in the Z mass which is expressed in terms of the soft parameters

$$M_Z = M_Z(\mu, m_0, b_0, a_0, \tilde{m}_0) \ . \tag{52}$$

If these soft parameters are much larger than 100GeV fine tuning is necessary to adjust this mass. Therefore the solution of the naturalness problem requires the soft parameters to be at the weak scale or at most at about 1TeV. This 'absence of fine-tuning' is one of the theoretical reasons to expect supersymmetry to be observable in the next generation of accelerator experiments.

Lecture 2

In this lecture perturbative string theory is introduced. We focus on string backgrounds with four space-time dimensions and outline the derivation of the effective low-energy action.

2.1 Perturbative string theory

In string theory the fundamental objects are one-dimensional strings which, as they move in time, sweep out a 2-dimensional worldsheet Σ [9]. Strings can be open or closed and their worldsheet is embedded in some D-dimensional target space which is identified with a Minkowskian spacetime. States in the target space appear as eigenmodes of the string and their scattering amplitudes are generalized by appropriate scattering amplitudes of strings. These scattering amplitudes are built from a fundamental vertex, which for closed strings is depicted in Fig. 3. It represents the splitting of a string or the

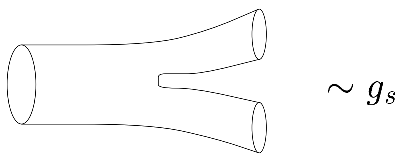

Figure 3: The fundamental closed string vertex.

joining of two strings and the strength of this interaction is governed by a dimensionless string coupling constant $g_{\rm s}$. Out of the fundamental vertex one composes all possible closed string scattering amplitudes $\mathcal{A}$, for example the four-point amplitude shown in Fig. 4. The expansion in the topology of the Riemann surface (i.e. the number of holes

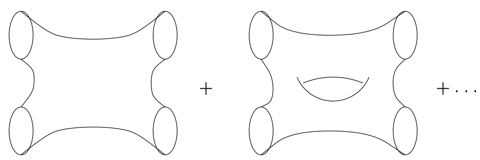

Figure 4: The perturbative expansion of string scattering amplitudes. The order of $g_{\rm s}$ is governed by the number of holes in the world sheet.

in the surface) coincides with a power series expansion in the string coupling constant formally written as

$$\mathcal{A} = \sum_{n=0}^{\infty} g_{\rm s}^{-\chi} \mathcal{A}^{(n)} \ , \tag{53}$$

where $\mathcal{A}^{(n)}$ is the scattering amplitude on a Riemann surface of genus n and $\chi(\Sigma)$ is the Euler characteristic of the Riemann surface

$$\chi(\Sigma) = \frac{1}{4\pi}\int_{\Sigma} R^{(2)} = 2 - 2n - b \ . \tag{54}$$

$R^{(2)}$ is the curvature scalar on Σ and b the number of boundaries of the Riemann surface. (For the four-point amplitude of Fig. 4 one has $b = 4$.)[4]

In all string theories there is a massless scalar field ϕ called the dilaton which couples to $R^{(2)}$ and therefore its vacuum-expectation value determines the size of the string coupling. One finds [9, 33]

$$g_s = e^{\langle\phi\rangle} . \tag{55}$$

g_s is a free parameter since ϕ is a flat direction (a modulus) of the effective potential, which is called dilaton. Thus, string perturbation theory is defined in that region of the parameter space (which is also called the moduli space) where $g_s < 1$ and the tree-level amplitude (genus-0) is the dominant contribution with higher-loop amplitudes suppressed by higher powers of g_s. Until 1995 this was the only regime accessible in string theory.

Conformal invariance on the worldsheet (or equivalently unitarity in spacetime) imposes a restriction on the maximal number of spacetime dimensions and the spacetime spectrum. All supersymmetric string theories necessarily have $D \leq 10$ and they are particularly simple in their maximal possible dimension $D = 10$.[5]

In $D = 10$ there are only five consistent spacetime supersymmetric string theories: type-IIA, type-IIB, heterotic $E_8 \times E_8$ (HE8), heterotic SO(32) (HSO) and the type-I SO(32) string. In the type-IIA theory the massless bosonic spectrum contains the graviton $g_{\mu\nu}$, an antisymmetric tensor $b_{\mu\nu}$, the dilaton ϕ, an abelian vector V_μ and a 3-form $C_{\mu\nu\rho}$. In type-IIB one also has $g_{\mu\nu}, b_{\mu\nu}$ and ϕ together with a 2-form $b'_{\mu\nu}$, an additional scalar ϕ' and a 4-form $c^*_{\mu\nu\rho\sigma}$ whose field strength is self dual.

Type II string theories have a unique massless supergravity multiplet while type I and the heterotic string feature a gravitational multiplet consisting of the metric $g_{\mu\nu}$, an antisymmetric tensor $b_{\mu\nu}$ and the dilaton ϕ together with vector multiplets in anomaly free representations of the gauge group. In $D = 10$ anomaly cancelation is very restrictive and only allow $E_8 \times E_8$ or $SO(32)$ as possible gauge groups.

2.2 String theory with $D < 10$

So far we discussed the various string theories in 10 spacetime dimensions. Lower dimensional theories can be obtained by compactifying the $D = 10$ theories on an internal, 'curled up' compact manifold Y

$$M^{(10)} = M^{(D)} \times Y^{(10-D)} . \tag{56}$$

If one demands some 'left-over' supersymmetry in lower dimensions Y has to be a Calabi–Yau manifold [9]. Calabi–Yau manifolds are Ricci-flat Kähler manifolds of

[4]For open strings different diagrams contribute at the same order of the string loop expansion [9].

[5]For closed strings an additional constraint arises from the requirement of modular invariance of one-loop amplitudes which results in an anomaly-free spectrum of the corresponding low-energy effective theory [34]. For open strings anomaly cancelation is a consequence of the the absence of tadpole diagrams [9].

vanishing first Chern class ($c_1(Y) = 0$) with holonomy group SU(M) where M is the complex dimension of Y. For $D = 4$ three-dimensional Calabi–Yau manifolds Y_3 are particularly important. However, it is also possible to construct string backgrounds directly in $D = 4$ without going through a geometrical picture but instead using an appropriate conformal field theory on the string worldsheet. In either case there exist a large variety of allowed constructions and this is what is generally called vacuum degeneracy: Every Calabi–Yau manifold or every appropriate conformal field theory defines a consistent string background, corresponding to consistent string vacua.

Calabi-Yau manifolds have generically continuous deformations parameterized by so called moduli which preserve the Calabi-Yau condition. Thus there is a continuous degeneracy of possible string groundstates. As we will see shortly the moduli parameters correspond to the vacuum expectation value of some scalar fields.

To solve this problem one could either try to classify the sting vacua and try to find some adequate selection principle, or one could study only the space of phenomenological interesting string vacua (string phenomenology).

2.3 Vacuum cleaning

String phenomenology focuses on the low energy limit of string theory and asks to what extent the Standard Model emerges as this low energy effective theory. Thus, only those string theories have to be considered which can possibly accommodate the Standard Model. This process of choosing (by hand) a subset of all string theories and within a string theory only a subspace of the space of ground states is sometimes termed 'vacuum cleaning'.[6]

The criteria for this selection process are somewhat ambiguous but the following necessary conditions should hold: [7]

1. $D = 4$
   The spacetime should have four flat Minkowski dimensions.

2. $SU(3) \times SU(2) \times U(1) \subset G$
   The gauge group G should be big enough to contain the gauge group of the Standard Model and account for its fermion content.

3. $n_g \geq 3$
   The number of light chiral generations n_g should be at least three.

[6] This terminology was coined by L. Dixon.

[7] One might contemplate to impose additional constraints. For example, one could demand the gauge group to be precisely the gauge group of the SM $G = SU(3) \times SU(2) \times U(1)$ or the number of light generations to be exactly three $n_g = 3$. However, none of these two is obviously true since they strongly depend on the physics which governs the energy range between the weak scale M_{weak} and M_{Pl}. Similarly, demanding no fast proton decay or a reasonable fermion mass hierarchy is difficult to impose without further knowledge of the physics just above M_{weak}.

In addition to 1.–3. one further condition is usually imposed:

4. $N = 1$ spacetime supersymmetry
   The low energy limit should be $N = 1$ supersymmetric.

This last condition is much more questionable than the first three. After all there are no experimental signs for supersymmetry yet. However, it seems difficult to understand how a hierarchy of scales $M_{\text{weak}}/M_{\text{Pl}} \ll 1$ can be generated and kept stable without something like supersymmetry. Furthermore, among the known, consistent string backgrounds almost all display low energy supersymmetry. It is for these two reasons that most phenomenological investigations in string theory have concentrated on supersymmetric backgrounds and we follow here the same assumption.[8] However, it might be worthwhile at some point to relax condition 4 and study non-supersymmetric ground states in more detail [35]. Once we accept condition 4 we eventually have to face the problem of how to break supersymmetry near M_{weak}. This question will occupy a later section.

The bosonic string (open or closed) is tachyonic and cannot accommodate spacetime fermions. Thus it does not obey conditions 2,3 and will be immediately discarded. Type II string theories are tachyon-free and do have spacetime fermions in its massless spectrum. However, Dixon, Kaplunovsky, and Vafa showed that the particular fermion representation of the Standard Model can never appear in the massless spectrum [36]. Therefore also the perturbative type II strings have been discarded. Finally, the heterotic string and the type I string have no obvious deficiency and have been extensively studied. In fact until recently it was the heterotic string which was the prime target of string phenomenology [37]. On the one hand it is easier to accommodate chiral representations in the massless spectrum of the heterotic string and on the other hand the construction of consistent four-dimensional ground states is considerably simpler in the heterotic string. Only recently the phenomenological properties of the type I string have been investigated [38].

2.4 The low energy effective action

The spacetime spectrum of a string theory contains a finite number of massless modes, which we denote as L, and an infinite number of massive modes H. The mass of H is an (integer) multiple of M_{s}. To derive the low energy effective action $\mathcal{L}_{\text{eff}}(L)$ which only depends on the light modes L one considers scattering processes of L with external momenta p much smaller than M_{s}, i.e. $p^2/M_{\text{s}}^2 \ll 1$. A systematic procedure for computing $\mathcal{L}_{\text{eff}}(L)$ has been developed and is often referred to as the S-matrix approach [39, 40, 41]. One computes the S-matrix elements for a given string vacuum as a perturbative power series in g_s. At the lowest order (tree level) an S-matrix element typically

[8] In fact one needs precisely $N = 1$ supersymmetry since theories with $N > 1$ have difficulties accommodating the chiral structure of the Standard Model.

has a pole in the external momentum which corresponds to the exchange of a massless mode L. The finite part is a power series in p^2/M_s^2 and corresponds to the exchange of the whole tower of massive H-modes. $\mathcal{L}_{\text{eff}}$ is then constructed to reproduce the string S-matrix elements in the limit $p^2/M_s^2 \ll 1$ with S-matrix elements constructed entirely from the effective field theory of the L-modes. In this low energy effective theory the exchange of the H-modes in the string scattering is replaced by an effective interaction of the L-modes. For a four-point amplitude this procedure is schematically sketched in figure 5. The first row denotes the string scattering amplitude and its separation

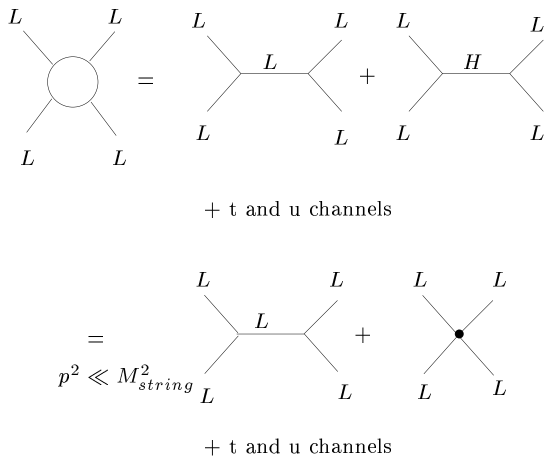

Figure 5: The S-matrix approach.

in a 'pole piece' (exchange of a massless mode) and the finite piece (exchange of the heavy modes). The second row indicates ordinary field-theoretical Feynman diagrams computed from the effective Lagrangian. The pole piece is reproduced by the same exchange of the massless modes while the finite part is identified with an effective interaction. Using this procedure $\mathcal{L}_{\text{eff}}$ can be systematically constructed as a power series in both p^2/M_s^2 and g_s. The power of p^2 counts the number of spacetime derivatives in $\mathcal{L}_{\text{eff}}$; at order $(p^2/M_s^2)^0$ one finds the effective potential while the order (p^2/M_s^2) corresponds to the two-derivative kinetic terms.[9]

The selection criteria 1–4 already significantly reduce the number of string vacua for which the low energy effective theory has to be computed. Further simplification of the S-matrix approach comes from the use of all symmetries a string vacuum might have in that one does not have to compute separately S-matrix elements which are related by a symmetry.[10] One only has to determine those couplings in the effective theory which are not related by general coordinate transformations, gauge transformations

[9]Instead of using the S-matrix approach one can alternatively construct the effective action by computing the β-functions of the two-dimensional σ-model and interpreting them as the equations of motion of string theory. The effective action is then constructed to reproduce these equations of motion [33].

[10]Except to check the consistency of the procedure.

and $N = 1$ supersymmetry. Concretely this means that one only has to compute the function K, W, f of the supersymmetric Lagrangian given in Eq. (34).

Generically, these function are 'model-dependent that is, they depend on the specific Calabi-Yau manifold under consideration. However, it is also possible to determine a few generic properties. Already in supersymmetric field theories the holomorphicity of $W(A)$ and $f_{(a)}(A)$ leads to two perturbative non-renormalization theorems: W receives no perturbative corrections [29] while $f_{(a)}$ is only corrected at one-loop order but has no further perturbative corrections [42]. Altogether one has

$$\begin{aligned} W &= W^{(0)} + W^{(\mathrm{NP})} \,, \\ f &= f^{(0)} + f^{(1)} + f^{(\mathrm{NP})} \,, \\ K &= \sum_{n=0}^{\infty} K^{(n)} + K^{(\mathrm{NP})} \,, \end{aligned} \tag{57}$$

where the superscript (NP) indicates possible non-perturbative corrections.

Lecture 3

In this lecture we discuss the unification of the gauge couplings, the problem of supersymmetry breaking and the stabilization of the dilaton. Finally we consider non-perturbative developments, M-theory and F-theory together with some of their phenomenological aspects.

3.1 Unification of gauge couplings

Let us first briefly recall the unification of the gauge couplings in the supersymmetric Standard Model. Grand Unified Theories (GUTs) predict a universal gauge coupling g_{GUT} at some scale M_{GUT}. The observable low energy gauge couplings of the Standard Model are then obtained as the 'running' gauge couplings from the relation

$$g_{(a)}^{-2}(\mu) = \frac{1}{g_{\mathrm{GUT}}^2} + \frac{b_{(a)}}{8\pi^2} \ln \frac{M_{\mathrm{GUT}}}{\mu} + \Delta_{(a)} \,, \tag{58}$$

where $b_{(a)}$ are the one-loop coefficients of the β-function and $\Delta_{(a)}$ are infrared finite 'threshold' correction. In the Standard Model (58) are three equations for two unknowns $(g_{\mathrm{GUT}}, M_{\mathrm{GUT}})$ and thus a successful fit is non-trivial. Indeed such a fit fails for the Standard Model while the supersymmetric Standard Model does satisfy (58) for $M_{\mathrm{GUT}} \approx 3 \cdot 10^{16}$ GeV and $g_{\mathrm{GUT}}^2 \approx \frac{4\pi}{23}$ [43].

In string theory all couplings become the vacuum expectation value (VEV) of a complex dilaton S or a set of moduli scalars T^i.[11] Using the method outlined in the

[11] One combines the dilaton ϕ with the dual axion a of the antisymmetric tensor $b_{\mu\nu}$ into a complex superfield $S = e^{-2\phi} + i\,a$.

last lecture one determines in heterotic string perturbation theory

$$\begin{aligned} f_{(a)} &= S + f^{(1)}_{(a)}(T) \ , \\ \Delta_{(a)}(T,\bar{T}) &= \mathrm{Re} f^{(1)}_{(a)}(T) + \mathcal{A}(T,\bar{T}) \ , \end{aligned} \tag{59}$$

where $\mathcal{A}$ is a non-harmonic term known as the holomorphic anomaly [37, 44, 45, 46]. The important point is that at the string tree level $f_{(a)}$ and thus $g_{(a)}$ are universal functions of the dilaton. Thus the heterotic string automatically reproduces the feature of a universal gauge coupling at leading order. However, the fundamental scale of string theory M_{s} is not an arbitrary scale but related to M_{Pl} and g_{s} by [47]

$$M_{\mathrm{s}} = \frac{2 \cdot 3^{-\frac{3}{4}}\, e^{\frac{1}{2}(1-\gamma)}}{\sqrt{2\pi\alpha'}} = \frac{3^{-\frac{3}{4}}\, e^{\frac{1}{2}(1-\gamma)}}{4\pi}\ g_{\mathrm{s}} M_{\mathrm{Pl}} \approx g_{\mathrm{s}} \cdot 5 \cdot 10^{17} \mathrm{GeV}\, . \tag{60}$$

Thus the string scale is roughly one order of magnitude bigger than M_{GUT}. The perturbative heterotic string does reproduce the experimental situation of a unified gauge coupling. However, the unification occurs not quite at the right scale. The mismatch between M_{s} and M_{GUT} needs an explanation but the fact that it comes so close is one of the attractive model independent features of the perturbative heterotic string.

In the past a number of attempts to overcome this mismatch have been suggested and we briefly review some of them here. One of the early suggestions has been that maybe the compactification scale of Calabi–Yau manifolds can be chosen lower than M_{s} and therefore serve as M_{GUT}. However, within the perturbative heterotic string this suggestion is problematic [48]. Since this argument partially breaks down in non-perturbative string theory let us go through it in slightly more detail.

Compactification of the ten-dimensional effective field theory on a Calabi–Yau threefold yields a relation between the string coupling $g_{\mathrm{s}}^{(4)}$ in the four-dimensional action and the string coupling $g_{\mathrm{s}}^{(10)}$ in the ten-dimensional action which involves the volume V_6 of the Calabi–Yau threefold

$$(g_{\mathrm{s}}^{(4)})^{-2} = (g_{\mathrm{s}}^{(10)})^{-2}\, V_6\, l_{\mathrm{s}}^{-6}\ , \tag{61}$$

where $l_{\mathrm{s}} \equiv \sqrt{\alpha'}$ is the string length. The volume V_6 is in principle an independent scale in the problem, the compactification scale. The perturbative decompactification limit sends $V_6\, l_{\mathrm{s}}^{-6} \to \infty$ and demands that the string coupling stays in the perturbative regime, i.e. $g_{\mathrm{s}}^{(10)}$ is kept fixed and small. Eq. (61) then implies in this limit $g_{\mathrm{s}}^{(4)} \to 0$. On the other hand, the measured gauge couplings do not allow an arbitrarily small gauge coupling as a consequence of (58). Instead one roughly has to have

$$\frac{1}{23} = \frac{g_{\mathrm{GUT}}^2}{4\pi} \approx \frac{(g_{\mathrm{s}}^{(4)})^2}{4\pi} = \frac{(g_{\mathrm{s}}^{(10)})^2}{4\pi}\frac{l_{\mathrm{s}}^6}{V_6} < \frac{l_{\mathrm{s}}^6}{4\pi V_6} \quad \text{for} \quad g_{\mathrm{s}}^{(10)} < 1\ . \tag{62}$$

We learn that V_6 cannot be arbitrarily big but has to obey $V_6 l_{\mathrm{s}}^{-6} < \frac{23}{4\pi} \approx 2$ which implies $V_6 = \mathcal{O}(l_{\mathrm{s}}^6)$. Thus V_6 cannot be used as an independent scale or tuned to be M_{GUT}.

The same problem in another disguise can be seen from Eq. (58). One might hope to find string vacua where $\Delta_{(a)}(T^i)$ is large [49, 50]. If $\Delta_{(a)}(T^i)$ has the form

$$\Delta_{(a)}(T^i) = -\frac{b_{(a)}}{8\pi^2}\ln\delta(T^i) + \tilde{\Delta}_{(a)}(T^i) \tag{63}$$

one would have a "redefinition of the GUT-scale"

$$g_{(a)}^{-2}(\mu) = \frac{1}{g_{\rm s}^2} + \frac{b_{(a)}}{8\pi^2}\ln\frac{M_{\rm GUT}}{\mu} + \tilde{\Delta}_{(a)}(T^i)\ , \tag{64}$$

with $M_{\rm GUT} = \frac{M_{\rm s}}{\delta(T^i)}$. In order to have $M_{\rm GUT} \approx 3\cdot 10^{16} GeV$ one needs $\delta = 20$ or $\ln\delta \approx 3$. Thus the mismatch of scales puts a strong constraint on sign, coefficient and size of $\Delta_{(a)}(T^i)$. However, generically one finds $\ln\delta = O(1)$ which is just another way to observe the perturbative decompactification limit.

As an alternative scenario one can envisage a GUT-group $G_{\rm GUT}$ at $M_{\rm Pl}$ which breaks by an appropriate Higgs mechanism at $M_{\rm GUT}$ to $G_{\rm SM}$. However, now one is in need of an explanation why this breaking occurs precisely at $M_{\rm GUT}$.

3.2 Supersymmetry breaking and stabilizing the dilaton

Let us now turn to the question of lifting the vacuum degeneracy and supersymmetry breaking which is another and more serious problem shared by all perturbative heterotic string vacua. In particular we need to address the following points:

- What determines the vacuum expectation values $\langle S\rangle$ and $\langle T^i\rangle$ of the dilaton and moduli fields and what are their values? As a consequence of (58) one needs for the dilaton

  $$\langle {\rm Re} S\rangle \simeq g_s^{-2} \simeq \frac{23}{4\pi}\ . \tag{65}$$

  For the case of a geometrical compactification the moduli T^i parameterize the size and shape of the Calabi-Yau threefold. Thus, as a consequence of (62) one needs to arrange

  $$\langle T^i\rangle = O(l_{\rm s}^2)\ . \tag{66}$$

- What is the mechanism for supersymmetry breaking and at what scale does the breaking occur? The naturalness problem of the Standard Model and the unification of the gauge couplings result in the theoretical prejudice of unbroken supersymmetry almost all the way down to $M_{\rm weak}$.

- Independently of the previous points one needs an explanation of the hierarchy $\frac{M_{\rm weak}}{M_{\rm Pl}}$.

None of these issues has a satisfactory answer within the perturbative heterotic string and thus the hope has been that non-perturbative effects come to rescue. Without a non-perturbative formulation of the heterotic string it is difficult to address non-perturbative properties and in fact only those of the effective field theory can be sensibly studied [51, 52]. These field-theoretic effects certainly do occur in string theory but to what extent they dominate over 'stringy' non-perturbative contributions remains open.[12]

One considers an asymptotically free supersymmetric gauge theory which becomes strong at the scale

$$\Lambda = M_{\rm Pl}\, e^{-\frac{8\pi^2}{bg^2}} . \tag{67}$$

Thus a hierarchy $\frac{\Lambda}{M_{\rm Pl}} \ll 1$ is generated if g and/or b are small. In addition supersymmetry can be broken and the scale of the breaking (often parameterized by the gravitino mass $m_{3/2}$) is found to be

$$m_{3/2} \approx \frac{\Lambda^3}{M_{\rm Pl}^2} . \tag{68}$$

Thus for $\Lambda \sim 10^{13} - 10^{14}$ GeV one obtains $m_{3/2} \sim 10^1 - 10^3$ GeV which is the 'desired' mass scale.[13] In string theory asymptotically free gauge theories do exist but their gauge couplings g are necessarily tied to the dilaton $g = g_s$ and the unification of all gauge couplings (58) implies $g^{-2} = \frac{23}{4\pi}$.

The strong gauge forces generate a potential which looks like

$$V \cong \frac{|\Lambda(S)|^6}{M_{\rm Pl}^2} . \tag{69}$$

Inserting (67) one finds that the minimum of V occurs at $g_{\rm s} = 0, \langle S\rangle \to \infty$ (see fig. 6) unacceptable for realistic phenomenology. This is a generic problem of all heterotic string vacua and is known as the dilaton problem [54].

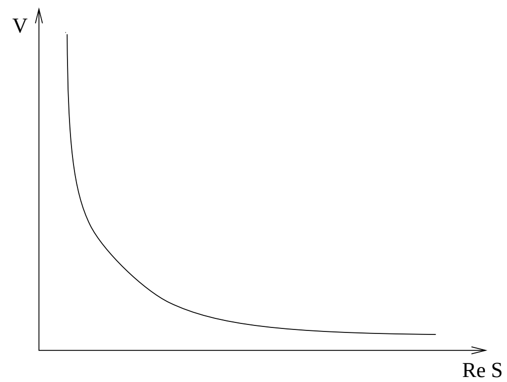

Figure 6: The dilaton potential

[12] Conversely, under the assumption that the field-theoretic effects are the dominant non-perturbative contribution the following analysis is legitimate.

[13] Once supersymmetry is broken it can induce radiatively the breakdown of the electroweak symmetry by a supersymmetric version of the Coleman–Weinberg mechanism [53].

It is surprisingly difficult to contemplate solutions of the dilaton problem. Within the perturbative heterotic string essentially only one scenario has been proposed [55]. One considers two (or more) confining gauge groups each with different one-loop corrections to the gauge couplings (this situation does exist in string theory). The appropriate condensation scale for each group factor reads

$$\Lambda_{(a)} = M_{\rm Pl}\, e^{-\frac{8\pi^2}{b_{(a)}}(S+f^{(1)}_{(a)}(T^i))}. \tag{70}$$

Self-consistent expansion in $\Lambda/M_{\rm Pl}$ gives at leading order

$$V \approx \frac{1}{M_{\rm Pl}^2}\left|\Lambda_1^3 + \Lambda_2^3\right|^2 \tag{71}$$

with a minimum at $|\Lambda_1| = |\Lambda_2|$. Inserting (70) one obtains

$$\langle {\rm Re} S\rangle \approx \frac{b_1 b_2}{b_2 - b_1}\left(\frac{f_1^{(1)}}{b_1} - \frac{f_2^{(1)}}{b_2}\right) , \tag{72}$$

where in our conventions the size of $f^{(1)}$ is generically $f^{(1)} = O(\frac{b}{24\pi^2})$. Estimating the order of magnitudes one has $4\pi\langle{\rm Re}S\rangle = O(\frac{b}{6\pi})$. Thus $4\pi\langle{\rm Re}S\rangle = 23$ can only be arranged if

i) b is large ($b \approx 400$),

ii) $f^{(1)}$ is large ($f^{(1)} \approx 2$),

iii) one 'fine-tunes' $b_1 \simeq b_2$.

The first two options are impossible in the perturbative heterotic string. $b \approx 400$ is incompatible with rank(G) ≤ 22 and $f^{(1)} \approx 2$ is in conflict with the consistency condition $T^i = O(l_{\rm s}^2)$.[14] Only the last option (iii) seems barely possible in the perturbative heterotic string. For example $G_1 = SU(8), G_2 = SU(9), f^{(1)} \approx \frac{2}{9}, {\rm Re}T \approx 2$ does not contradict any perturbative constraint. However, arranging $4\pi{\rm Re}S = 23$ is only one requirement and one needs to simultaneously generate the hierarchy $\Lambda \sim 10^{13} - 10^{14} GeV$. As we already noted this demands b and/or g to be small. Inserting the appropriate numbers one finds that within the perturbative heterotic string it is almost impossible to arrange the correct $g_{\rm GUT}$ and simultaneously generate a hierarchy.

3.3 Non-perturbative developments

Since 1995 string theory has seen spectacular progress in that for the first time it has been possible to also control a subset of the interaction in the strong coupling regime. This is due to the observation that many (if not all) of the perturbatively distinct string theories are related when all quantum corrections are taken into account [10].

[14] As we will see shortly both options are available in the non-perturbative heterotic string.

In particular it has been observed that often the strong coupling regime of one string theory can be mapped to the weak coupling regime of another, perturbatively different string theory. This situation is termed duality among string theories and it offers the compelling picture that the known perturbative string theories are merely different regions in the moduli space of one underlying theory termed 'M-theory'.

The precise nature of the strong-coupling limit sensitively depends on the number of (Minkowskian) spacetime dimensions and the amount of supersymmetry. Supersymmetry has played a major role in the recent developments in two respects. First of all, it is difficult (and it has not been satisfactorily accomplished) to rigorously prove a string duality, since it necessitates a full non-perturbative formulation, which is not yet available. Nevertheless it has been possible to perform nontrivial checks of the conjectured dualities for quantities or couplings whose quantum corrections are under (some) control. It is a generic property of supersymmetry that it protects a subset of the couplings and implies a set of nonrenormalization theorems. The recent developments heavily rely on the fact that the mass (or tension) of BPS-multiplets is protected and that holomorphic couplings obey a nonrenormalization theorem. Thus, they can be computed in the perturbative regime of string theory and, under the assumption of unbroken supersymmetry, reliably extrapolated into the non-perturbative region. It is precisely for these BPS-states and holomorphic couplings that the conjectured dualities have been successfully verified.

Second of all, for a given spacetime dimension D and a given representation of supersymmetry there can exist perturbatively different string theories. For example, the heterotic SO(32) string in $D = 10$ and the type-I string in $D = 10$ share the same supersymmetry, but their interactions are different in perturbation theory. However, once non-perturbative corrections are taken into account, it is believed that the two theories are identical and merely different perturbative limits of the same underlying quantum theory. A similar phenomenon is encountered with other string theories in different dimensions and the moduli space of string theory is much smaller than was previously assumed.

We are particularly interested in the strong coupling limit of string theories with $D = 4$ and $N = 1$ supersymmetry. They are related to M-theory compactified on Calabi–Yau threefolds times an interval or F-theory compactified on elliptic Calabi–Yau fourfolds. Let us discuss these two cases in turn.

3.4 M-theory

It turns out that not all strong-coupling limits are governed by a perturbatively different string theory. Instead it is possible that the strong-coupling limit of a given theory is something entirely new, not any of the other string theories [56]. The prime example of this situation is the strong-coupling limit of the type-IIA theory in $D = 10$. It has a Kaluza-Klein BPS spectrum with masses

$$M^{\rm KK} \sim \frac{|n|}{g_{\rm s}} \,, \tag{73}$$

where n is an arbitrary integer. These KK-states are not part of the perturbative type-IIA spectrum since they become heavy in the weak-coupling limit $g_{\rm s} \to 0$. However, in the strong-coupling limit $g_{\rm s} \to \infty$ they become light and can no longer be neglected in the effective theory. This infinite number of light states (which can be identified with D-particles of type-IIA string theory, or extremal black holes of IIA supergravity) signals that the theory effectively decompactifies where $g_{\rm s}$ is related to the radius R_{11} of a new (11-th) dimension[56]

$$R_{11} \sim l_{11}\, g_{\rm s}^{\frac{2}{3}} \,. \tag{74}$$

l_{11} is the characteristic length scale of the 11th dimension which is related to the 11-dimensional Planck scale via $\kappa_{11}^2 \sim l_{11}^9$. Supersymmetry is unbroken in this limit and hence the KK-states assemble in supermultiplets of the 11-dimensional supergravity. Since there is no string theory which has 11-dimensional supergravity as the low-energy limit, the strong-coupling limit of type-IIA string theory has to be a new theory, called M-theory, which cannot be a theory of (only) strings.[15]

A second and maybe even more surprising result shows that also the strong-coupling limit of the heterotic $E_8 \times E_8$ string is captured by M-theory. In this case, 11-dimensional supergravity is not compactified on a circle but rather on a Z_2 orbifold of the circle [63]. In this case there is an E_8 gauge factor on each hyperplane at the end of the interval. Just as in the type-IIA case one has $R_{11} \sim g_{\rm s}^{2/3}$ and thus weak coupling corresponds to small R_{11} and the two 10-dimensional hyperplanes sit close to each other; in the strong-coupling limit the two 10-dimensional hyperplanes move far apart (to the end of the world).

3.5 F-theory

The strong coupling limit of type-IIB theory in 10 spacetime dimensions is believed to be governed by type-IIB itself. This is accomplished by an exact $SL(2, Z)$ quantum symmetry which is a generalization of a strong-weak coupling duality. This fact led Vafa to propose that the type-IIB string could be viewed as the toroidal compactification of a twelve-dimensional theory, called F-theory [64]. Apart from having a geometrical interpretation of the $SL(2, Z)$ symmetry this proposal led to the construction of new, non-perturbative string vacua in lower spacetime dimensions. In order to preserve the $SL(2, Z)$ quantum symmetry the compactification manifold cannot be arbitrary

[15] There exists a conjecture according to which the degrees of freedom of M-theory are captured in $U(N)$ supersymmetric matrix models in the $N \to \infty$ limit [57]. These matrix models have been known for some time [58] and were also known to describe supermembranes [59] in the lightcone gauge [60]. The same quantum-mechanical models describe the short-distance dynamics of N D-particles, caused by the exchange of open strings [61, 62].

but has to be what is called an elliptic fibration. That is, the manifold is locally a fiber bundle with a two-torus T^2 over some base B but there are a finite number of singular points where the torus degenerates. As a consequence nontrivial closed loops on B can induce an $\mathrm{SL}(2,Z)$ transformation of the fiber. This implies that the dilaton is not constant on the compactification manifold, but can have $\mathrm{SL}(2,Z)$ monodromy [65]. It is precisely this fact which results in nontrivial (non-perturbative) string vacua inaccessible in string perturbation theory.

It is believed that the heterotic string compactified on a Calabi–Yau threefold Y_3 is quantum equivalent to F-theory compactified on an elliptic Calabi–Yau fourfold [64]. Calabi-Yau fourfolds are Calabi-Yau manifolds of complex dimension four and holonomy group SU(4). Compactification of F-theory on Calabi–Yau fourfolds is not yet well understood also the phenomenological investigations are only at the beginning. Let us point out a few features which seem to emerge from the study of non-perturbative heterotic string vacua.

- One finds that generically new massless gauge bosons appear which have no tree level coupling to the dilaton [66]. The total gauge group is thus a sum

  $$G = G_{(\mathrm{P})} + G_{(\mathrm{NP})}, \tag{75}$$

  where $G_{(\mathrm{NP})}$ denotes the non-perturbative factors. The gauge couplings for these factors are governed by some moduli other than the dilaton, i.e. $g^{-2}_{(\mathrm{NP})} = \mathrm{Re}T + \ldots$. Thus g_a^{-2} is no longer universal at tree level but only the perturbative gauge factors of $G_{(\mathrm{P})}$ are universal. Furthermore, $\mathrm{rank}(G)$ is no longer bounded to be smaller than 23 but can be almost arbitrarily big. The current record gauge group has $\mathrm{rank}(G) = 302896$ with 251 simple factors and $SO(7232)$ being the biggest of them [67].

  Thus, there seems to be no problem in such vacua to have a hidden sector with $b \simeq 400$; for example $SO(140)$ could nicely do the job. Furthermore, one can have many gauge factors participating in the stabilization of the dilaton and hierarchical supersymmetry breaking. Now it is easy to have a different sector which breaks supersymmetry and another sector which stabilizes the dilaton and the moduli [68].

  However, now also the gauge couplings of the Standard Model are no longer necessarily universal and one needs a mechanism to ensure it. This might point towards a GUT-unification within string theory.

- A different issue concerns non-perturbative corrections to the superpotential. Due to the non-renormalization theorem one only has $W = W^{(0)} + W^{(\mathrm{NP})}$ but within the perturbative heterotic string the corrections to $W^{(\mathrm{NP})}$ have to be of the form $W^{(\mathrm{NP})} \sim e^{-f(S,T)}$. However, an asymptotically free gauge factor of $G_{(\mathrm{NP})}$ generates terms of the form $W^{(\mathrm{NP})} \sim e^{-\gamma f(T)}$ with no dilaton dependence. In this case

$W^{(\mathrm{NP})}$ is indistinguishable from $W^{(0)}$ or in other words non-perturbative effects can 'compete' with perturbative effects due to their unusual dilaton dependence.

This fact has been used to 'explain' the conifold singularities of the tree level superpotential [69, 70]. In all Calabi–Yau compactifications one has

$$W^{(0)} = Y_{ijk} Q^i Q^j Q^k + \dots \ , \tag{76}$$

where Q^i are charged matter multiplets and the Y_{ijk} are their moduli dependent Yukawa couplings. The moduli dependence is always singular on subspaces of the moduli space, i.e.

$$Y_{ijk} \sim \frac{1}{z(T)} \ , \tag{77}$$

where $z(T)$ has a zero at a conifold singularity. Singularities of holomorphic, not renormalized quantities should have a physical mechanism behind them. It has been suggested that a non-perturbative gauge group with a quantum modified moduli space confines and generates the conifold singularity [69, 70].

Appendix - conventions and notation

In these lectures the notation and conventions of ref. [8] are used. The four-dimensional Lorentz metric is chosen as

$$\eta_{mn} = \mathrm{diag}(-1,1,1,1) \ . \tag{78}$$

Lorentz indices are labeled by Latin indices $m, n, \dots$ which run from 0 to 3. Greek indices are used to denote spinors. A two-component Weyl spinor can transform under the $(\frac{1}{2},0)$ or the complex conjugate $(0,\frac{1}{2})$–representation of the Lorentz group and dotted or undotted indices are used to distinguish between these representations. ψ_α denotes a spinor transforming under the $(\frac{1}{2},0)$ representation while $\bar{\chi}_{\dot\alpha}$ transforms under the $(0,\frac{1}{2})$ representation of the Lorentz group. The spinor indices α and $\dot\alpha$ can take the values 1 and 2. These indices can be raised and lowered using the skew-symmetric $SU(2)$– invariant tensor $\epsilon^{\alpha\beta}$ or $\epsilon_{\alpha\beta}$.

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta \ , \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta \ , \tag{79}$$

where

$$\epsilon_{21} = -\epsilon_{12} = 1, \quad \epsilon_{11} = \epsilon_{22} = 0, \quad \epsilon_{\alpha\gamma}\epsilon^{\gamma\beta} = \delta_\alpha^\beta \ .$$

For dotted indices the analogous equations hold. The product $\epsilon^{\beta\alpha}\psi_\alpha\chi_\beta = \psi^\beta\chi_\beta$ is a Lorentz scalar. Spinors are anticommuting objects and one has the following summation convention:

$$\begin{aligned} \psi\chi &= \psi^\alpha\chi_\alpha = -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \ , \\ \bar\psi\bar\chi &= \bar\psi_{\dot\alpha}\bar\chi^{\dot\alpha} = -\bar\psi^{\dot\alpha}\bar\chi_{\dot\alpha} = \bar\chi_{\dot\alpha}\bar\psi^{\dot\alpha} = \bar\chi\bar\psi \ . \end{aligned} \tag{80}$$

The convention for the conjugate spinors are chosen such that it is consistent with the conjugation of scalars:

$$(\psi\chi)^\dagger = (\psi^\alpha \chi_\alpha)^\dagger = \bar{\psi}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} = \bar{\psi}\bar{\chi} = \bar{\chi}\bar{\psi} \ . \tag{81}$$

The σ-matrices $\sigma^m_{\alpha\dot{\alpha}}$ are given by:

$$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} , \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,$$
$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{82}$$

The invariant ϵ–tensor raises and lowers their indices:

$$\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} \sigma^m_{\beta\dot{\beta}} \tag{83}$$

and we have:

$$\bar{\sigma}^0 = \sigma^0 \ , \qquad \bar{\sigma}^{1,2,3} = -\sigma^{1,2,3} \ . \tag{84}$$

The generators of the Lorentz group in the spinor representation are given by

$$\sigma^{nm} = \frac{1}{4}(\sigma^n \bar{\sigma}^m - \sigma^m \bar{\sigma}^n) \ , \qquad \bar{\sigma}^{nm} = \frac{1}{4}(\bar{\sigma}^n \sigma^m - \bar{\sigma}^m \sigma^n) \ . \tag{85}$$

The Dirac-γ–matrices can be written in terms of Weyl matrices:

$$\gamma^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix} \tag{86}$$

which fulfill

$$\{\gamma^m, \gamma^n\} = -2\, \eta^{mn} \ . \tag{87}$$

A four component Dirac spinor contains two Weyl spinors

$$\Psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \ . \tag{88}$$

Its conjugate is

$$\bar{\Psi}_D = \Psi_D^\dagger \gamma^0 = (\chi^\alpha, \bar{\psi}_{\dot{\alpha}}) \ . \tag{89}$$

The Dirac equation describing relativistic spin-$\frac{1}{2}$ particles reads:

$$(i\, \gamma^n \partial_n + m)\, \Psi_D = 0. \tag{90}$$

It can be decomposed into two Weyl equations

$$\begin{aligned} i\sigma^n \partial_n \, \bar{\chi} + m\psi &= 0 \ , \\ i\bar{\sigma}^n \partial_n \psi + m\bar{\chi} &= 0 \ . \end{aligned} \tag{91}$$

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