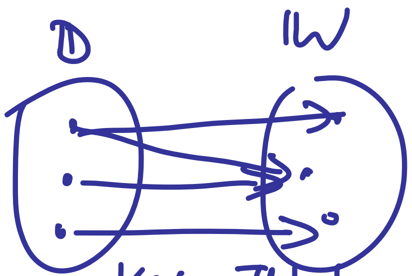
**Wdh:**

$$f : \mathbb{D} \to \mathbb{W} \qquad \mathbb{D}, \mathbb{W} \subset \mathbb{R}$$
$$x \mapsto f(x) = y$$

reelle Fkt., falls jedem $x \in \mathbb{D}$ genau ein $y = f(x) \in \mathbb{W}$ zugeordnet wird

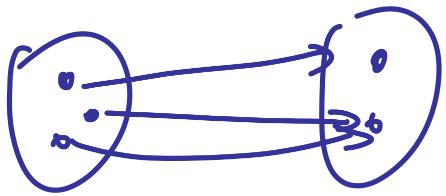
$f$ injektiv, falls jedes $y \in \mathbb{W}$ höchstens einmal angenommen wird

$f$ surjektiv, falls jedes $y \in \mathbb{W}$ mindestens einmal angenommen wird

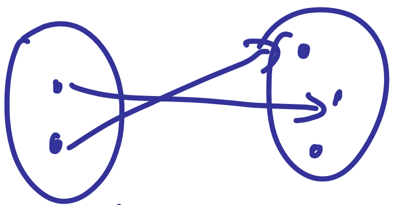



keine Fkt!

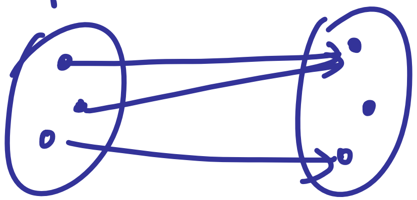
f keine Fkt.



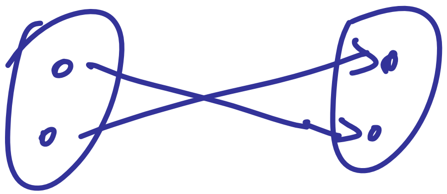
f surjektiv



f injektiv



f weder injektiv noch surjektiv



bijektiv

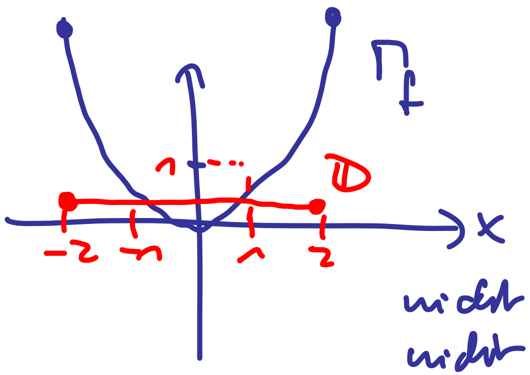
## Graph einer Funktion $f : \mathbb{D} \to \mathbb{W}$

Die Menge $\Gamma_f := \{ (x,y) \mid x \in \mathbb{D} \, , \, y = f(x) \in \mathbb{W} \}$

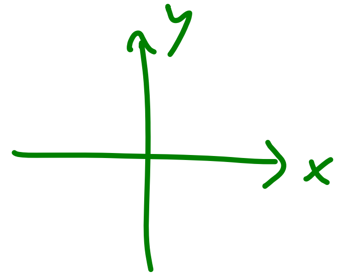
heißt Graph der Funktion $f$ — geordnetes Paar

**Bsp:** $\mathbb{D} = [-2, 2]$ ← alle reellen Zahlen $x$ mit $-2 \leq x \leq 2$

$\mathbb{W} = \mathbb{R}$




$f(x) = x^2$




nicht surjektiv
nicht injektiv

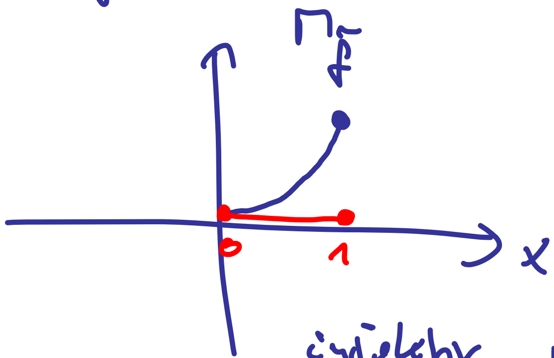
$\mathbb{R}^2 := \{ (x,y) \mid x, y \in \mathbb{R} \}$

$\Gamma_f \subset \mathbb{R}^2$

**Bsp:** $\tilde{f}(x) = x^2$

$\tilde{\mathbb{D}} = [0, 1]$

$\mathbb{W} = \mathbb{R}$

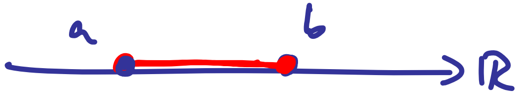



injektiv, nicht surjektiv

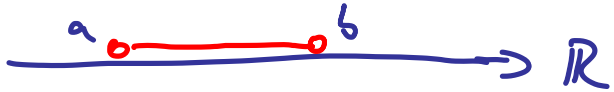
$f$ und $\tilde{f}$ haben die gleiche Funktionsvorschrift, sind aber nicht dieselben Funktionen (denn $\mathbb{D} \neq \tilde{\mathbb{D}}$)

## Intervalle

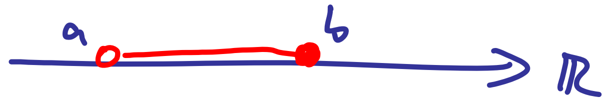
seien $a, b \in \mathbb{R}$, $a \leq b$



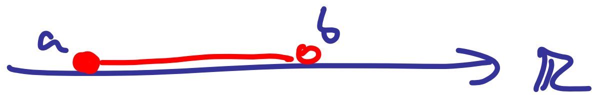

- $[a,b] := \{ x \in \mathbb{R} \mid a \leq x \leq b \}$ geschlossenes Intervall
- $(a,b) = ]a,b[ := \{ x \in \mathbb{R} \mid a < x < b \}$ offenes Intervall




- $(a,b] = ]a,b] := \{ x \in \mathbb{R} \mid a < x \leq b \}$




- $[a,b) = [a,b[ := \{ x \in \mathbb{R} \mid a \leq x < b \}$




halboffene Intervalle

# 1.2 Eigenschaften von Funktionen

## Nullstellen

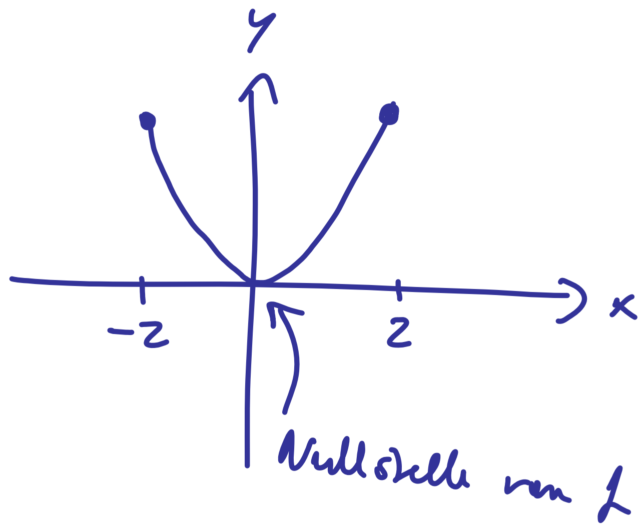
Sei $f : \mathbb{D} \to \mathbb{W}$ eine Funktion und sei $x_0 \in \mathbb{D}$ mit

$$f(x_0) = 0$$

dann heißt $x_0$ eine <u>Nullstelle</u> von $f$, und

$$N_f := \{ x \in \mathbb{D} \mid f(x) = 0 \}$$

die Menge der Nullstellen von $f$.



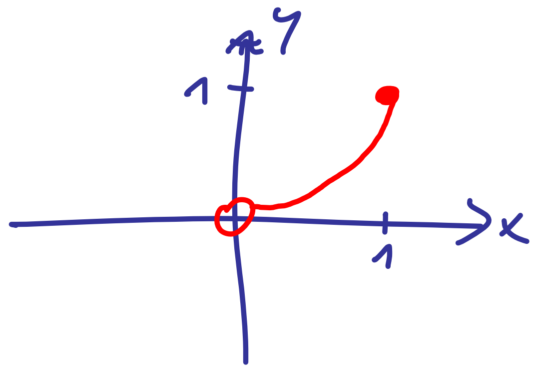

<u>Bsp:</u> $f : \underset{\substack{\shortparallel \\ \mathbb{D}}}{[-2,2]} \to \underset{\substack{\shortparallel \\ \mathbb{W}}}{\mathbb{R}} \qquad f(x) = x^2$

$$N_f = \{0\}$$

Bsp: $f: ]0,1] \to \mathbb{R}$ $f(x) = x^2$

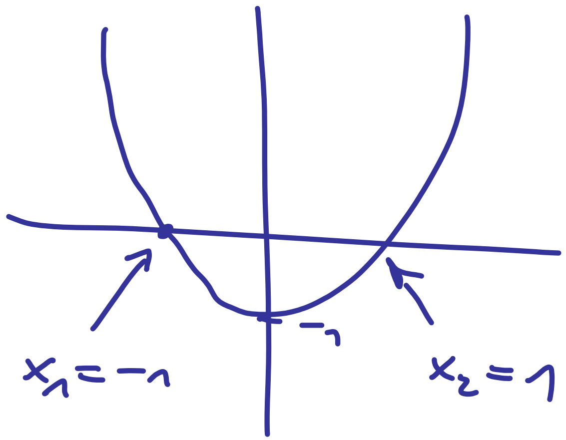
$N_f = \{ \ \} = \emptyset$

"leere Menge"




Bsp: $f: \mathbb{R} \to \mathbb{R}$ $f(x) = x^2 - 1$

$N_f = \{1, -1\} = \{-1, 1\}$



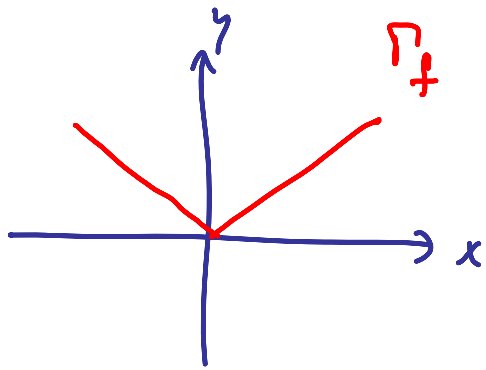

Nullstellen findet man durch Auflösen der Gleichung $f(x) = 0$ nach $x$

Bsp: $x^2 - 1 = 0 \iff x^2 = 1 \iff x = 1$ oder $x = -1$

## Symmetrie einer Funktion

Eine Funktion $f: \mathbb{D} \to \mathbb{R}$ heißt gerade, falls für alle $x \in \mathbb{D}$ gilt, dass $f(x) = f(-x)$

$\Gamma_f$ ist spiegelsymmetrisch zur y-Achse




Bsp: $f(x) = |x|$

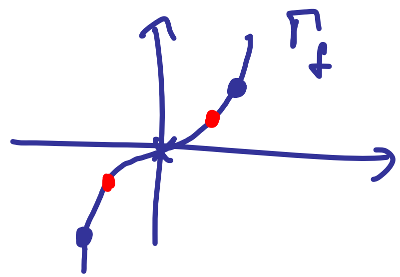
$f(-x) = |-x| = |x| = f(x)$ ✓

$(\forall x \in \mathbb{D})$

---

Eine Funktion $f$ heißt ungerade, falls $\forall x \in \mathbb{D}: f(x) = -f(-x)$

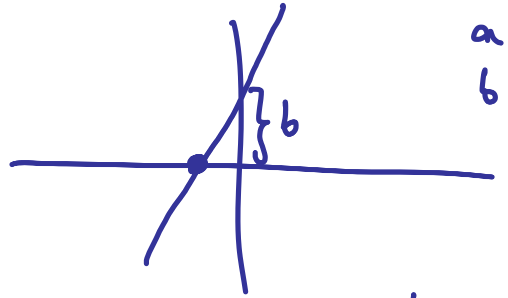
$\Gamma_f$ ist punktsymmetrisch zum Punkt $(0,0)$




Bsp: $f(x) = x^3$

Bsp: $f(x) = ax + b$

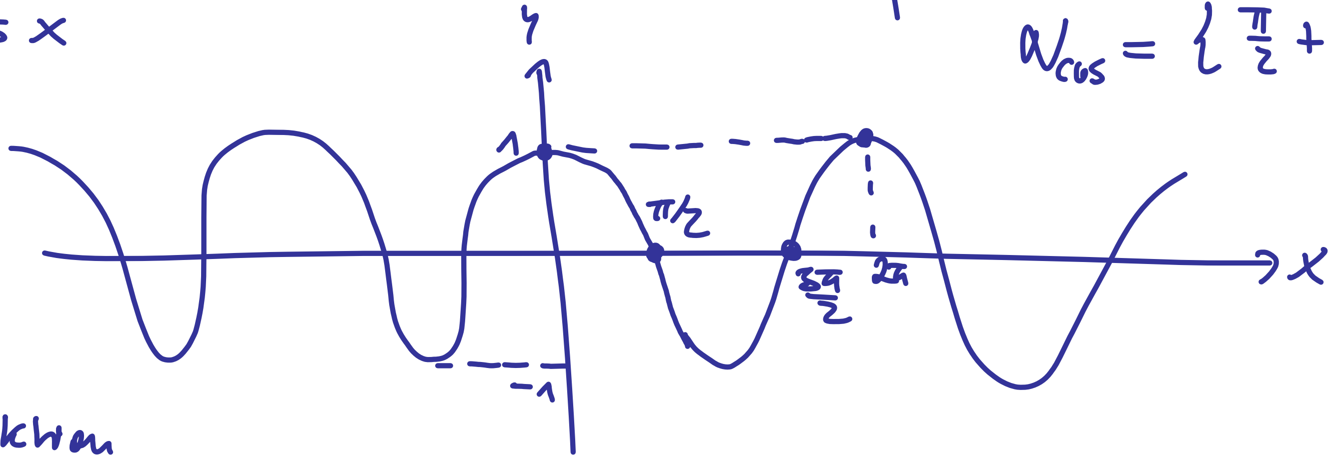
weder gerade noch ungerade




$a > 1$
$b > 0$

Bsp: $f(x) = \cos x$

$N_{\cos} = \{ \frac{\pi}{2} + k \cdot \pi \mid k \in \mathbb{Z} \}$

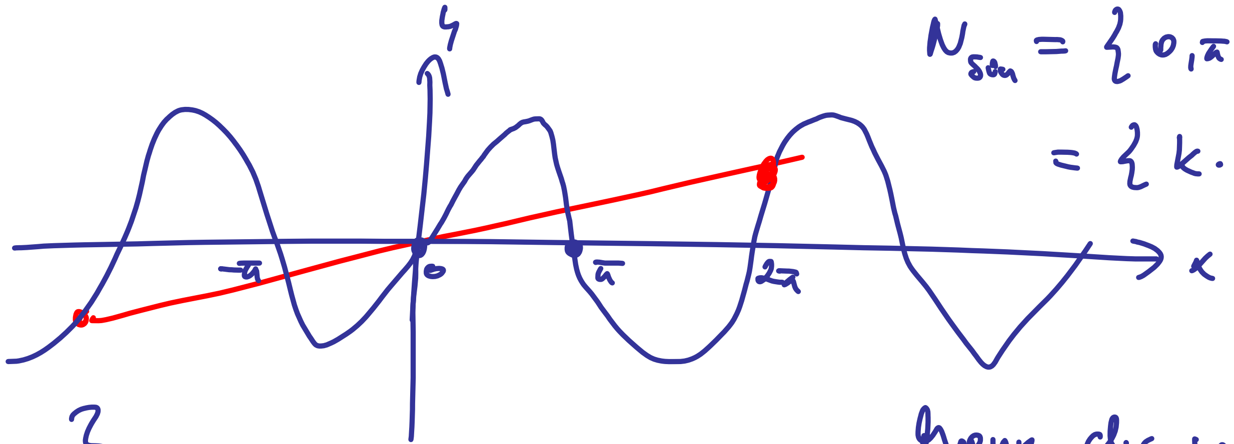



gerade Funktion

$N_{\sin} = \{ 0, \pi, 2\pi, -\pi, \dots \}$
$= \{ k \cdot \pi \mid k \in \mathbb{Z} \}$

Bsp: $f(x) = \sin x$

ungerade Funktion



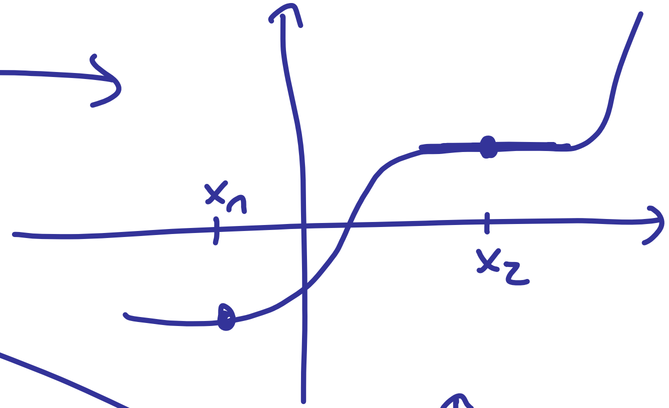

$\mathbb{Z} = \{ 0, \pm 1, \pm 2, \dots \}$ Menge der ganzen Zahlen

## Monotonie von Funktionen

Sei $f : \mathbb{D} \to \mathbb{W}$ eine Funktion. $f$ heißt...

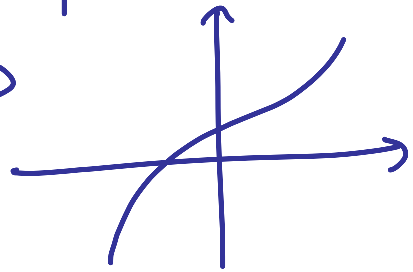
(i) ... monoton steigend, falls

$\forall x_1, x_2 \in \mathbb{D}$ mit $x_1 < x_2$ : $f(x_1) \leq f(x_2)$




(ii) ... streng monoton steigend, falls

$\forall x_1, x_2 \in \mathbb{D}$ mit $x_1 < x_2$ : $f(x_1) < f(x_2)$

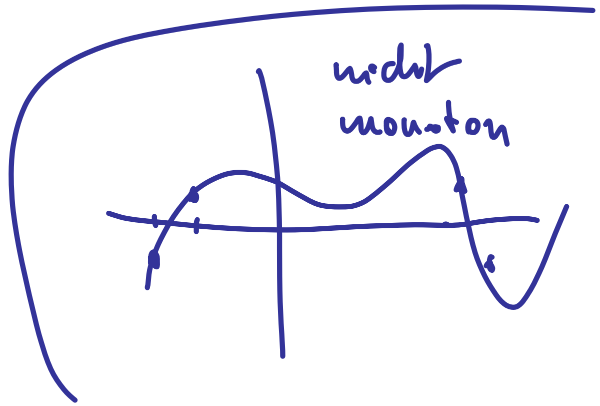


(iii) ... monoton fallend, falls

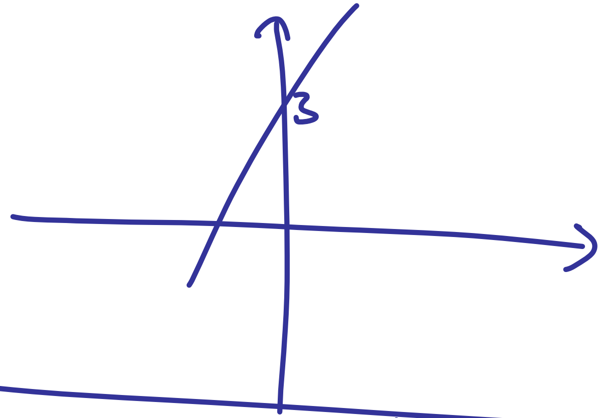
$\forall x_1, x_2 \in \mathbb{D}$ mit $x_1 < x_2$ : $f(x_1) \geq f(x_2)$

(iv) ... streng monoton fallend, falls

$\forall x_1, x_2 \in \mathbb{D}$ mit $x_1 < x_2$ : $f(x_1) > f(x_2)$

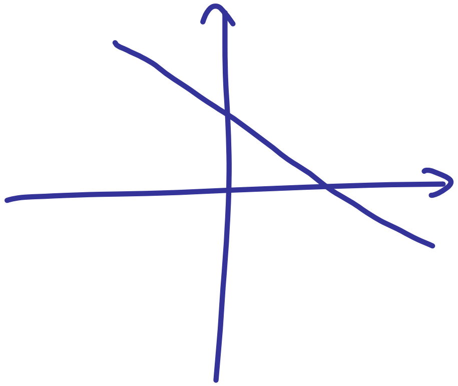


**Bsp:** $f(x) = 2x+3$



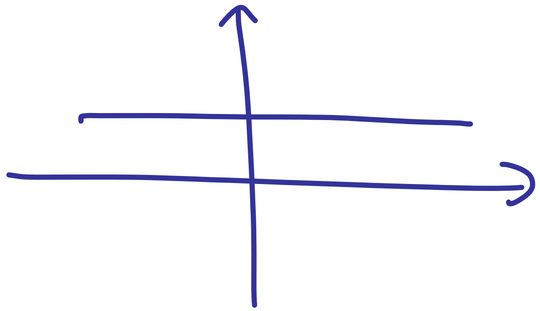

streng monoton steigend

**Bsp:** $f(x) = -x+1$



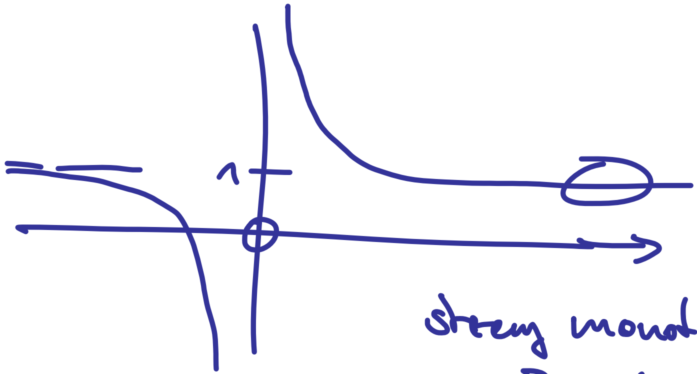
streng monoton fallend

**Bsp:** $f(x) = 1$



monoton steigend
monoton fallend
nicht streng monoton steigend / fallend

**Bsp:** $f(x) = \frac{1}{x} + 1$ $\quad D = \mathbb{R} \setminus \{0\} = \mathbb{R}^*$




streng monoton fallend
im Bereich $x > 0$

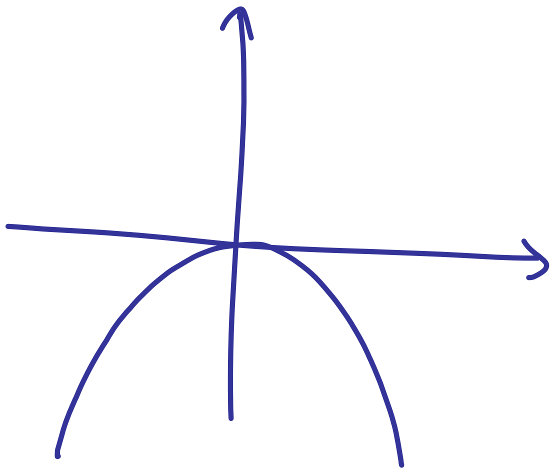
# Beschränktheit

unten

**Def:** Eine Funktion $f: \mathbb{D} \to \mathbb{W}$ heißt nach oben beschränkt, falls es ein $B \in \mathbb{R}$ mit $B \geq f(x)$ für alle $x \in \mathbb{D}$

$\leq$

**Bsp:** $f(x) = -x^2$

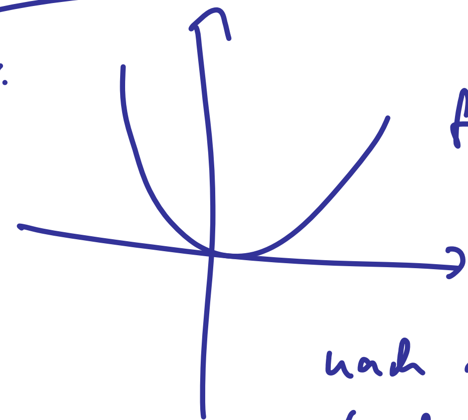


wähle $B = 0$

oder $B = 1$

nach oben beschränkt

**Bsp:** $f(x) = x^2$

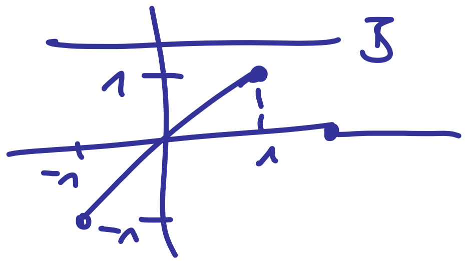


nach unten beschränkt

**Bsp:** $f(x) = x$

$\mathbb{D} = [-1, 1]$

$\mathbb{W} = \mathbb{R}$



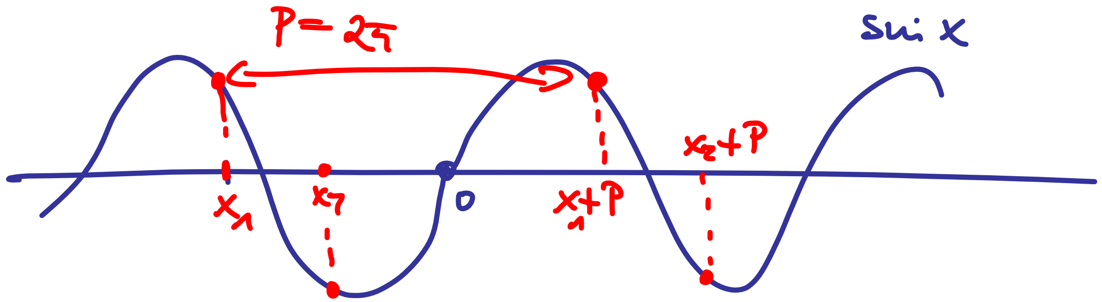
## Periodizität

Eine Funktion $f: D \to W$ heißt periodisch (mit Periode $P$), falls

$\forall x \in D$:
$$f(x+P) = f(x) \qquad (P \neq 0)$$

Bsp: $f(x) = \sin x$

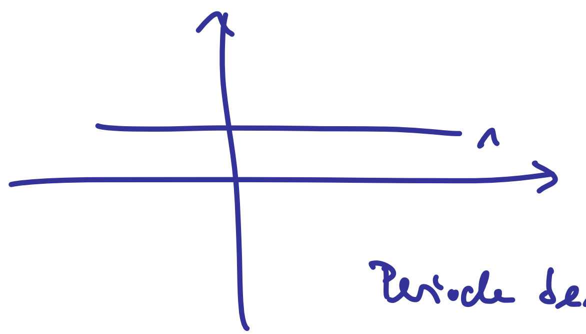
$\forall x \quad \sin(x+2\pi) = \sin x$




Bsp: $f(x) = \cos x$

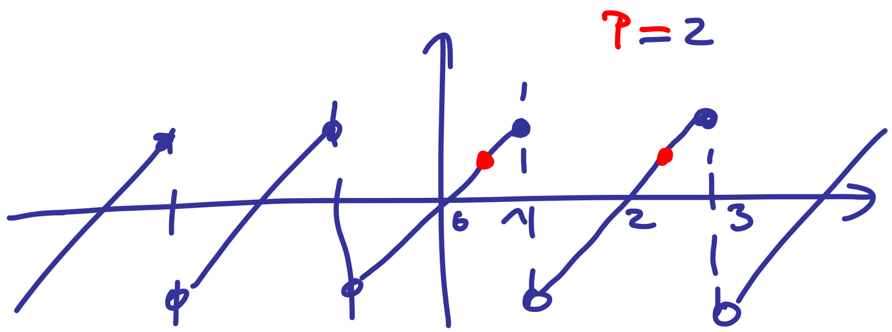
periodisch mit Periode $P = 2\pi$

Bsp:




Periode beliebig

Bsp:




$$f(x) = \begin{cases} x & \text{für } x \in ]-1,1] \\ x-2 & \text{für } x \in ]1,3] \end{cases} \qquad \text{etc...}$$