1.4 Anwendungen und Beispiele

1.4.1 Lorentz-Transformation des elektromagnetischen Felds

$F^{\mu\nu}$ mit den Komponenten

$$F^{\mu\nu} = \left(\begin{array}{c|ccc} 0 & & -\vec{E}/c & \\ \hline & 0 & -B_z & B_y \\ \vec{E}/c & B_z & 0 & -B_x \\ & -B_y & B_x & 0 \end{array}\right)$$

ist ein Tensor 2. Stufe, also gilt:

wobei

es gilt:

$$
\left(\begin{array}{c|ccc}
0 & & -\vec{E}'/c & \\
\hline
 & 0 & -B_z' & B_y' \\
\vec{E}'/c & B_z' & 0 & -B_x' \\
 & -B_y' & B_x' & 0
\end{array}\right)
$$

$$
= \underbrace{\left(\begin{array}{cc|c}
\gamma & -\beta\gamma & \\
-\beta\gamma & \gamma & 0 \\
\hline
0 & & \mathbb{1}
\end{array}\right)
\left(\begin{array}{c|ccc}
0 & & -\vec{E}/c & \\
\hline
 & 0 & -B_z & B_y \\
\vec{E}/c & +B_z & 0 & -B_x \\
 & -B_y & B_x & 0
\end{array}\right)}
\left(\begin{array}{cc|c}
\gamma & -\beta\gamma & \\
-\beta\gamma & \gamma & 0 \\
\hline
0 & & \mathbb{1}
\end{array}\right)
$$

$$
\left(\begin{array}{c|c|c|c}
-\beta\gamma\frac{1}{c}E_x & \gamma\left(-E_x/c\right) & \gamma\left(-E_y/c\right)-\beta\gamma\left(-B_z\right) & \gamma\left(-E_z/c\right)-\beta\gamma B_y \\
\gamma E_x/c & -\beta\gamma\left(-E_y/c\right) & -\beta\gamma\left(E_y/c\right)+\gamma\left(-B_z\right) & -\beta\gamma\left(-E_z/c\right)+\gamma B_y \\
E_y/c & B_z & 0 & -B_x \\
E_z/c & -B_y & B_x & 0
\end{array}\right)
$$

$$
= \left(\begin{array}{c|c|c|c}
\underbrace{-\beta\gamma^2\frac{E_x}{c}+\beta\gamma^2\frac{E_x}{c}}_{0} & -E_x/c & & \\
\underbrace{\gamma^2\frac{E_x}{c}-\beta^2\gamma^2\frac{E_x}{c}}_{\frac{E_x}{c}} & 0 & & \\
\gamma\frac{E_y}{c}-\beta\gamma B_z & -\beta\gamma\frac{E_y}{c}+\gamma B_z & 0 & \\
\gamma\frac{E_z}{c}+\beta\gamma B_y & -\beta\gamma\frac{E_z}{c}-\gamma B_y & B_x & 0
\end{array}\right)
$$

also:

explizite Formeln für die Transformation der Felder !

Bsp: $\vec{B} = 0 \qquad \vec{v} = (v, 0, 0)$

$$\Rightarrow \vec{B}' = \left(0, \beta\gamma \frac{E_z}{c}, -\beta\gamma \frac{E_y}{c}\right)^T$$

$$= \left(0, \frac{v}{c^2} E_z', -\frac{v}{c^2} E_y'\right)^T$$

$\vec{E} = 0$

$$\Rightarrow \vec{E}' = \left(0, -\beta\gamma c B_z, \beta\gamma c B_y\right)^T$$

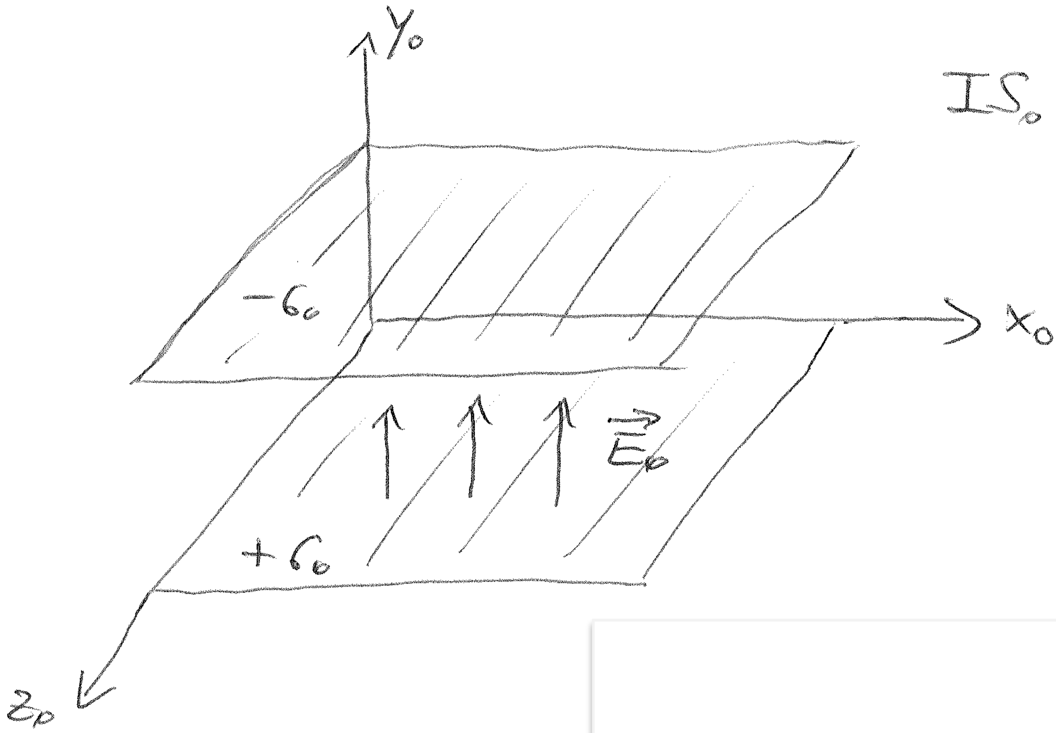
$$= \left(0, -v B_z', v B_y'\right)^T$$

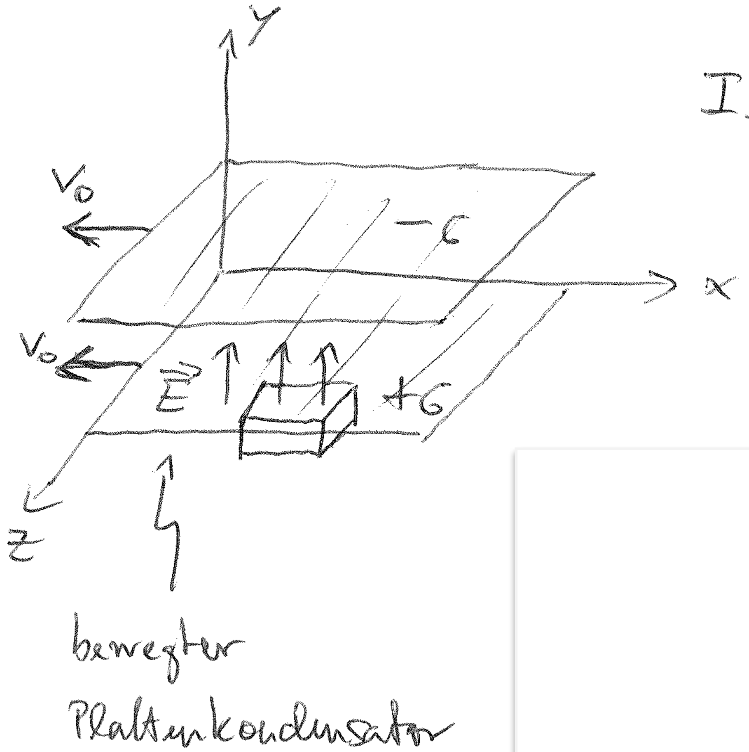
einfache Beziehungen zwischen den Feldern, die gelten, falls $\vec{E}$ bzw. $\vec{B}$ in einem IS verschwinden

1.4.2 Bewegte Kondensatoren und Spulen

anschauliche Verifikation der Transformationsformeln?

in IS_0 ruhender Plattenkondensator:





IS (bewege sich relativ zu IS_0 mit $v_0 = \text{const}$ in x-Richtung.)

es ist: (Ladung ist ein Lorentz-Skalar)

(Längenkontraktion)

$\Rightarrow \quad A = \frac{1}{\gamma_0} A_0 \qquad \gamma_0 = \frac{1}{\sqrt{1 - v_0^2/c^2}}$

also:

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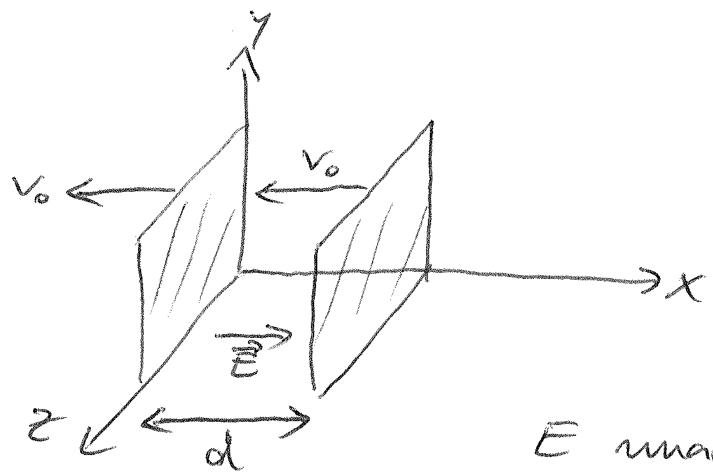
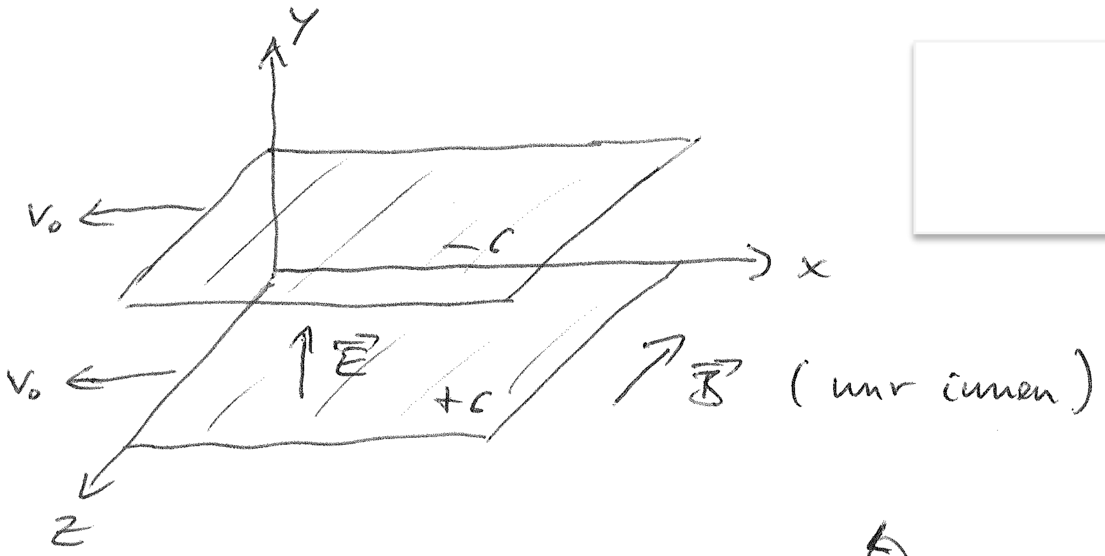
analog:

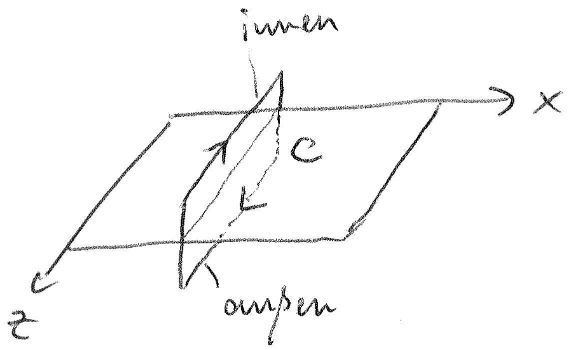


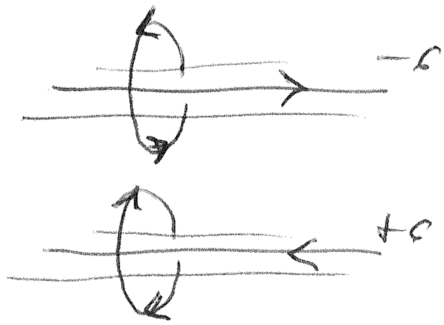
E unabhängig von d!

also:

(vergleiche allgemeine Formel für verschwindendes Magnetfeld) jetzt Situation mit $\vec{B} \neq 0$, betrachte IS



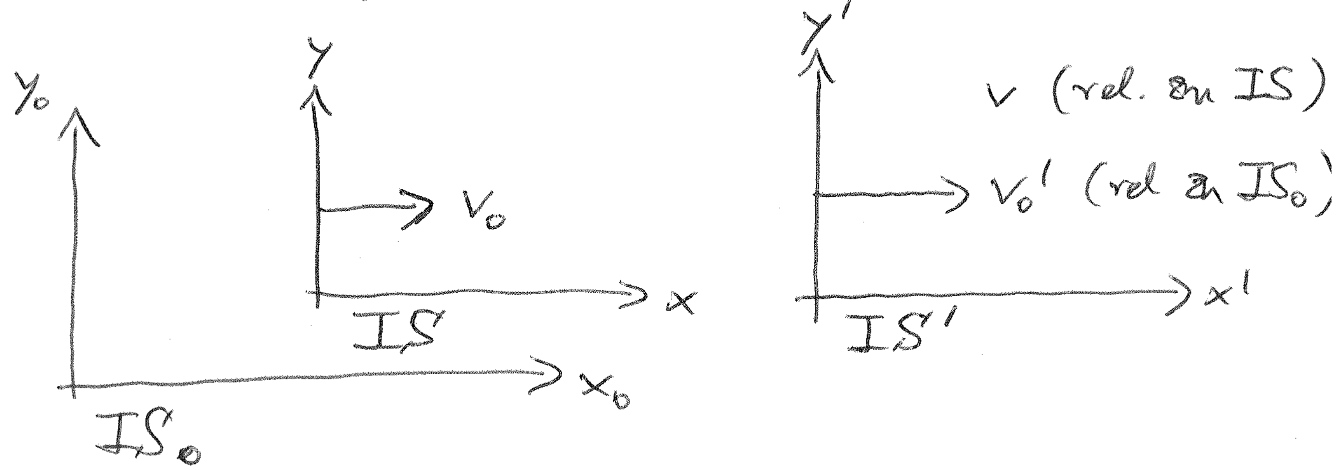




$$\mu_0 I = \oint_C \vec{B} \, d\vec{l}$$

$\Rightarrow$

IS' bewege sich relativ zu IS mit Geschwindigkeit v in x-Richtung



es gilt:

<u>in IS:</u>

$$E_y = \frac{\sigma}{\varepsilon_0}$$

$$B_z = -\mu_0 \sigma v_0$$

$$E_y = -\frac{c^2}{v_0} B_z$$

<u>in IS':</u>

$$E_y' = \frac{\sigma'}{\varepsilon_0}$$

$$B_z' = -\mu_0 \sigma' v_0'$$

$$E_y' = -\frac{c^2}{v_0'} B_z'$$

ausdrücken durch Größen in IS_0:

$\left(\gamma_0 = \frac{1}{\sqrt{1 - v_0^2/c^2}} \right)$ $\left(\gamma_0' = \frac{1}{\sqrt{1 - v_0'^2/c^2}} \right)$

mit $v_0'/v_0 = \left(1 + \frac{v v_0}{c^2}\right)\gamma$ $\qquad \left(\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}\right)$

folgt:

$$E_y' = E_y \left(1 + \frac{v v_0}{c^2}\right)\gamma$$

$$= \gamma E_y + \gamma \frac{v v_0}{c^2} E_y$$

$$B_z' = B_z \frac{v_0'}{v_0}\left(1 + \frac{v v_0}{c^2}\right)\gamma$$

$$= B_z \frac{v + v_0}{1 + \frac{1}{c^2} v v_0} \cdot \frac{1}{v_0} \cdot \gamma \left(1 + \frac{v v_0}{c^2}\right)$$

$$= B_z \left(\frac{v}{v_0} + 1\right)\gamma = \gamma B_z + \gamma \frac{v}{v_0} B_z$$

$$= \gamma B_z - \gamma v \frac{1}{c^2} E_y$$

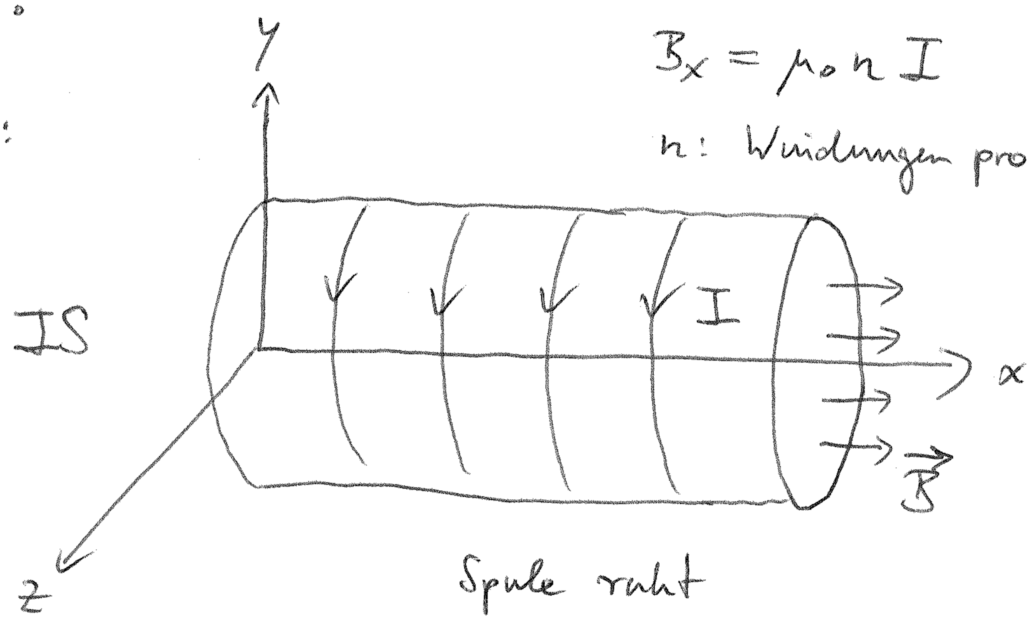
analog (Kondensator in x-y-Ebene):

$B_x' = ?$

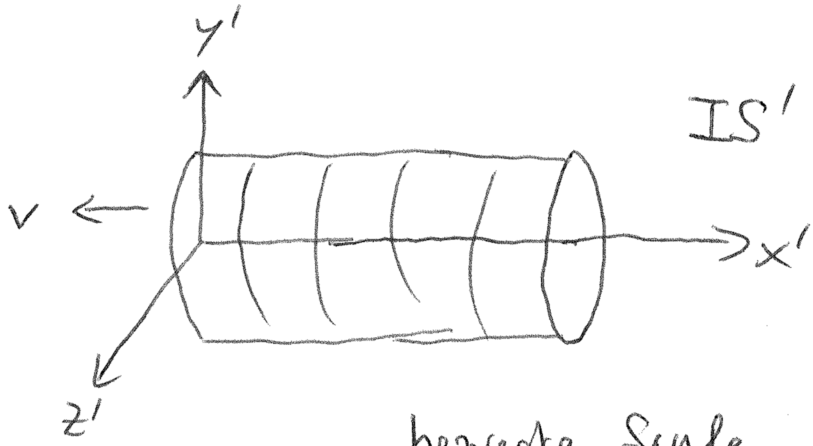
Spule:

$B_x = \mu_0 n I$

n: Windungen pro Länge



Spule ruht



IS' (bewege sich relativ zu IS mit v in x-Richtung)

bewegte Spule

Längenkontraktion:

Zeitdilatation:

also:

$\Rightarrow$