Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica E 18 (2003) 337–338
PHYSICA E
www.elsevier.com/locate/physe

Rashba spin splitting in different quantum channels

Wolfgang Häusler

Fakultät für Physik, Universität Freiburg, Hermann-Herder-Str. 3, 79104 Freiburg, Germany

Abstract

Rashba precession of spins moving along a one-subband quantum channel is calculated. The quantitative influence of unoccupied higher subbands depends on the shape of the transversal confinement and can be accounted for perturbatively. Coulomb interactions are included within the Tomonaga–Luttinger model with spin–orbit coupling incorporated. Increasing interaction strength at decreasing carrier density is found to *enhance* spin precession.

Keywords: Rashba spin splitting; Spin transport; Quantum wires; Tomonaga–Luttinger liquid

1. Introduction

Complete understanding of the Datta–Das spin transistor [1] requires to know how different physical parameters influence the precession of spins which is caused by the spin–orbit coupling

$$H^{so} = \alpha(\sigma_x p_z - \sigma_z p_x)\,. \tag{1}$$

H^{so} breaks spin rotation invariance proportional to the momentum of spins moving along the 'active' x–z-plane. For isotropic single particle energy dispersions $\varepsilon(|k|)$ in the plane, ignoring Coulomb interactions, this yields the well-known spin-split bands $\varepsilon_\pm(|k|) = \varepsilon(|k|) \pm \alpha|k|$. The Rashba parameter α is proportional to the intrinsic or by means of gates externally applied electric field perpendicular to the layer [2], here taken as the y-direction. Spin precession occurs then on the length scale $|k_+ - k_-|^{-1}$ of the spin-split momenta $k_\pm$ at the Fermi energy. It is special to the effective mass approximation $\varepsilon(|k|) = k^2/2m$, describing many semiconductors, that the Rashba length $2/|k_+ - k_-| = (m\alpha)^{-1}$ does *not* depend on the Fermi energy nor the carrier density.

Coherent precession of many spins down to the spin selective drain of the transistor requires to diminish the directional spread of the electron momenta by introducing quasi one-dimensional (1D) 'waveguides' [1]. To leading order in α the waveguide simply projects the momenta in $\varepsilon_\pm(|k|)\to\varepsilon_\pm(k_x)$ onto the x-axis, leaving the basic features of the 1D case valid, cf. Fig. 1. In particular, will the linear kinetic energy dispersion in carbon nanotubes or in narrow-gap semiconductors lead to enhanced precession with carrier density.

To higher-order H^{so} will mix different transport channels in each waveguide. Upto $O(\alpha^5)$ this effect can be accounted for by a renormalizing $\alpha\to\alpha^*$ in $\varepsilon_\pm(k)$ and, within the effective mass approximation, by $m\to m^*$. In this latter case, the Rashba length modifies according to $(m\alpha)^{-1}\to(m^*\alpha^*)^{-1}$. A quantitative estimate requires the intra-subband eigenfunctions

$$\psi_{kns}(x,z) = \mathrm{e}^{\mathrm{i}kx}\phi_n(z)(\cos(m\alpha z)|s\rangle + \mathrm{i}\sin(m\alpha z)|-s\rangle), \tag{2}$$

which are plane waves of momentum k along the waveguide and, without inter-subband scattering, slightly modified subband states ϕ_n (subband index n) in z-direction of the spin polarization $s = \pm$ on the axes. For a harmonic confinement (subband energy ω_0), as relevant for example in gated samples [3], a perturbative estimate yields $\alpha^* = \alpha(1-\eta)$ and $m^* = m(1+8\eta^2)$ in the ground subband. Here, the dimensionless parameter $\eta = (mw\alpha/2)^2$ compares the waveguide width $w = 2/\sqrt{m\omega_0}$ with the Rashba length. For a hard wall confinement on the other hand (again of width w), as possibly more relevant for wires fabricated by the

E-mail address: haeusler@physik.uni-frieburg.de (W. Häusler).

1386-9477/03/$ - see front matter © 2003 Elsevier Science B.V. All rights reserved.
doi:10.1016/S1386-9477(02)01077-9

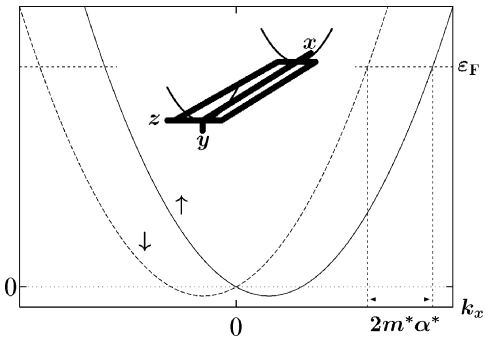

Fig. 1. Energy dispersion in the lowest spin-split subband of a quantum wire. On the wire axis the spins $s=\uparrow,\downarrow$ are polarized in the plane of the heterostructure parallel to the z-direction.

cleaved edge technique [4,5], the renormalizations become $\alpha^* = \alpha(1-(1/6-1/\pi^2)\eta)$ and $m^* = m(1+3(4/3\pi)^6\eta^2)$, i.e. they are significantly reduced compared to the soft wall case since $1/6-1/\pi^2 \approx 0.065$ and $3(4/3\pi)^6/8 \approx 0.002$.

Changing the gate voltage of the transistor when intending to vary the electrical field and therewith α changes at the same time the carrier density. Without interactions and when $\varepsilon(k) = k^2/2m^*$ this would be unimportant. Interactions on the other hand, depend sensitively on the carrier density. Regarding interaction effects the Tomonaga–Luttinger (TL) model [6] provides the most precise low-energy description of 1D metals. Though some of its characteristic power laws can affect spin properties [7] we rather focus here on the question how interactions influence the length over which coherent spin rotation takes place.

As a second striking property quantum wires exhibit spin–charge separation which, interestingly and contrary to statements in the literature [7,8], is *not* spoiled unless spin–orbit coupling is not exceedingly strong $\eta \sim 1$ if the effective mass description applies. On the other hand, for non-quadratic dispersion relations, spin–charge separation is in general destroyed, similar as is the case with Zeeman splitting [9]. An example are carbon nanotubes where $v_\pm = v_F \pm \alpha$ with α originating in this case from the curved surface [10] instead of the Rashba mechanism.

How to include the Rashba term in the TL-model? In previous work [7,8], the Fermi velocities v_+ and v_- have been set to different values, which for $\varepsilon(k) = k^2/2m^*$ does not describe H^{so} as can be seen also in Fig. 1. Rather *both* velocities change slightly but obey $v_+ = v_-$ leading immediately to charge–spin separation. Thus, in effective mass systems H^{so} acts solely in the topological spin sector of the corresponding TL low-energy model (of length L),

$$\frac{\pi}{4L}(v_N N_\sigma^2 + v_J J_\sigma^2) - m^*\alpha^* v_F J_\sigma. \tag{3}$$

N_σ and J_σ denote the usual currents of velocities $v_{N/J}$ where the latter both differ from v_F to account for the Coulomb repulsion [6]. In strictly spin isotropic systems $v_N = v_J$. Since we expect this isotropy being broken only weakly, both of these velocities should be similar in magnitude and also similar to the spin velocity v_σ. This latter quantity has been determined recently by extensive quantum Monte-Carlo simulations [11]. With increasing interaction strength, equivalent to a decreasing carrier density, v_σ/v_F was found to decrease. With parameters for existing quantum wires [4] v_σ/v_F can drop below 0.5.

Many quantities of interest can be calculated exactly using Eq. (3). In particular, it can be shown [12] that spins polarized in x-direction along the wire precess over a length scale

$$m^*\alpha^*\,\frac{v_{\sigma J}}{v_F}.$$

With $v_J = v_\sigma$ we see that this length decreases with increasing interaction strength or decreasing particle density. A similar conclusion has been drawn for two-dimensional electrons after treating the interactions perturbatively [13]. This trend is opposite to what is expected for linear single electron dispersions but agrees with experimental observations [14].

References

[1] B. Datta, S. Das, Appl. Phys. Lett. 56 (1990) 665.
[2] Ė.I. Rashba, Fiz. Tverd. Tela 2 (1960) 1224 [Sov. Phys. Solid State 2 (1960) 1109].
[3] S. Tarucha, T. Honda, T. Saku, Solid State Commun. 94 (1995) 413.
[4] A. Yacoby, et al., Phys. Rev. Lett. 77 (1996) 4612.
[5] M. Rother, et al., Physica E 6 (2000) 551.
[6] F.D.M. Haldane, J. Phys. C 14 (1981) 2585.
[7] A. De Martino, R. Egger, Europhys. Lett. 85 (2000) 3464.
[8] A.V. Moroz, K.V. Samokhin, C.H.W. Barnes, Phys. Rev. Lett. 84 (2000) 4164.
[9] H. Frahm, V.E. Korepin, Phys. Rev. B 42 (1990) 10553; T. Kimura, K. Kuroki, H. Aoki, Phys. Rev. B 53 (1996) 9572.
[10] L.I. Magarill, D.A. Romanov, A.V. Chaplik, JETP 86 (1998) 771.
[11] C.E. Creffield, W. Häusler, A.H. MacDonald, Europhys. Lett. 53 (2001) 221.
[12] W. Häusler, Phys. Rev. B 63 (2001) 121310(R).
[13] G.-H. Chen, M.E. Raikh, Phys. Rev. B 60 (1999) 4826.
[14] J. Nitta, et al., Phys. Rev. Lett. 78 (1997) 1335; G. Engels, et al., Phys. Rev. B 55 (1997) 1958(R); C.-M. Hu, et al., Phys. Rev. B 60 (1999) 7736.