

Lehrerfortbildung Elementarteilchen

Physikzentrum Bad Honnef, 23. – 27. Juni 2014

Teilchenbeschleuniger...

... für die Hochenergiephysik

Wolfgang Hillert

Elektronen-Stretcher-Anlage



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Teilchenbeschleuniger...

... für die Hochenergiephysik

Inhalte:

- **Einführung:** Beschleunigung: warum und wie
- **Kreisbeschleuniger:** Magnete, Magnete, Magnete, ...
- **Strahldynamik:** Erwünschtes und Unerwünschtes
- **Luminosität:** Raumladung, Strahl-Strahl, ...
- **Elektronen > 100 GeV:** Synchrotronstrahlung, LINACs
- **Zukunftsträume:** CLIC, FCC, PWA, ...

Warum Beschleuniger?

Fragen der Elementarteilchenphysik:

a. Was sind die elementaren Bausteine der Materie?

→ Auflösung kleinster Strukturen:

Photonen: $E = \hbar\omega = hc/\lambda$

Faustformel: $1\text{\AA} \approx 12345\text{ eV} \leftrightarrow$

$$\Delta x \geq \lambda$$
$$\Delta x[\text{fm}] \approx \frac{1,2}{E[\text{GeV}]}$$

b. Wie wechselwirken sie miteinander?

Eichbosonen und Higgs:

$$W \approx 80,4\text{ GeV}$$

$$Z \approx 91,2\text{ GeV}$$

$$H \approx 125\text{ GeV}$$

Schwere Quarks:

$$c \approx 1,3\text{ GeV}$$

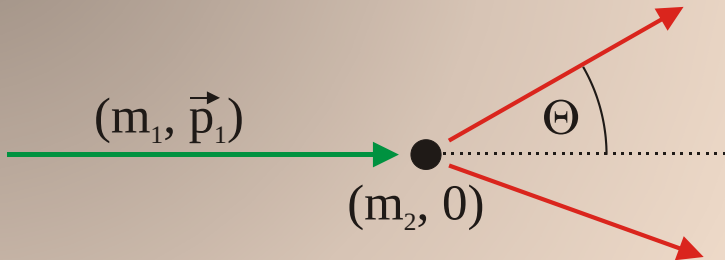
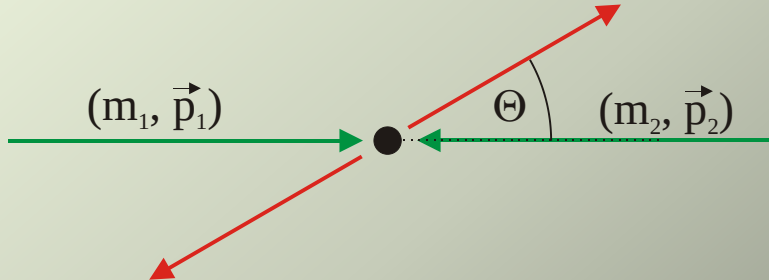
$$b \approx 4,2\text{ GeV}$$

$$t \approx 173,5\text{ GeV}$$

$$E > 1\text{ TeV} \leftrightarrow \text{Terascale} \leftrightarrow \text{„sub“-Attoscale}$$

Collider

(Hochenergie-Einheiten: $\hbar = c = 1$)

Fixed-Target-Experiment	Kollidierende Strahlen
 $S = P_1 + P_2 = (E_1 + m_2, \vec{p}_1 + \vec{p}_2)$ $\vec{\beta}_{CMS} = \frac{\vec{p}_1}{E_1 + m_2}$ $M_{inv}^2 = S^2 = 2 E_1 m_2 + m_1^2 + m_2^2$	 $S = P_1 + P_2 = (E_1 + E_2, \vec{p}_1 + \vec{p}_2)$ $\vec{\beta}_{CMS} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2} = 0$ $M_{inv}^2 = S^2 = (E_1 + E_2)^2$

**Beispiel: p-p Kollisionen, wollen $S = 1 \text{ TeV}$
benötigen:**

$E = 500 \text{ TeV}$

$E = 0,5 \text{ TeV}$

Beschleunigung:

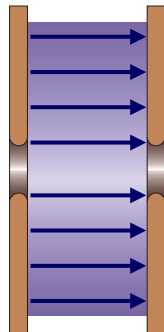
Beeinflussung geladener Teilchen durch die Lorentz-Kraft:

$$\vec{F} = e \cdot \vec{E} + e \cdot (\vec{v} \times \vec{B})$$

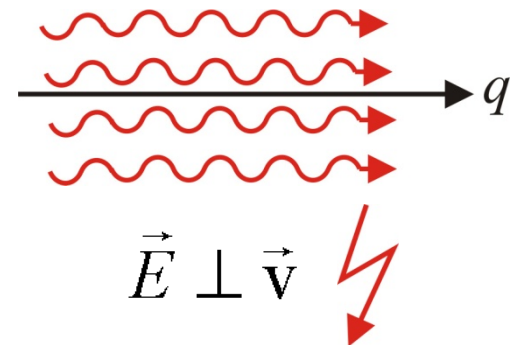
Energiegewinn: $\Delta W_{kin} = \int \vec{F} \cdot d\vec{s} = e \cdot \int E_{\parallel} \cdot ds = e \cdot U$

→ brauchen ein longitudinales elektrisches Feld $E_{\parallel}$!

Kondensator:



Lichtstrahl:
(TEM)

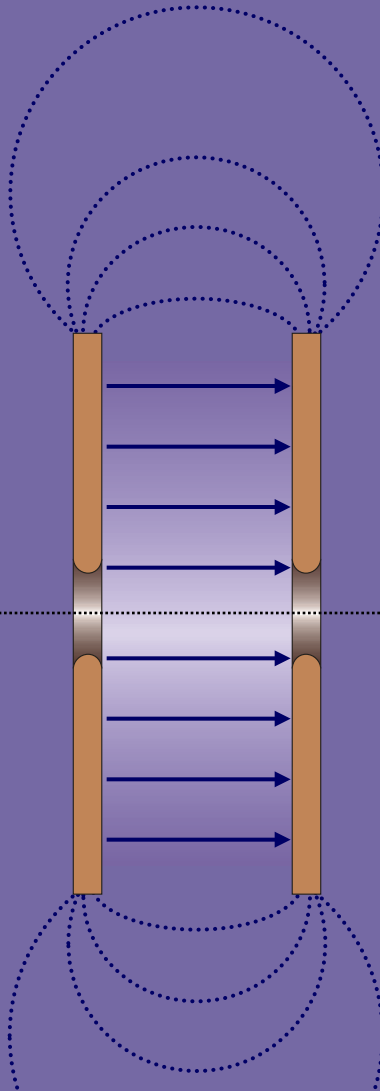


Statische Felder

$$\Delta W = e \cdot U$$

Einheit eV

Wir brauchen
 $E \geq 1 \text{ TeV} = 10^{12} \text{ eV}$



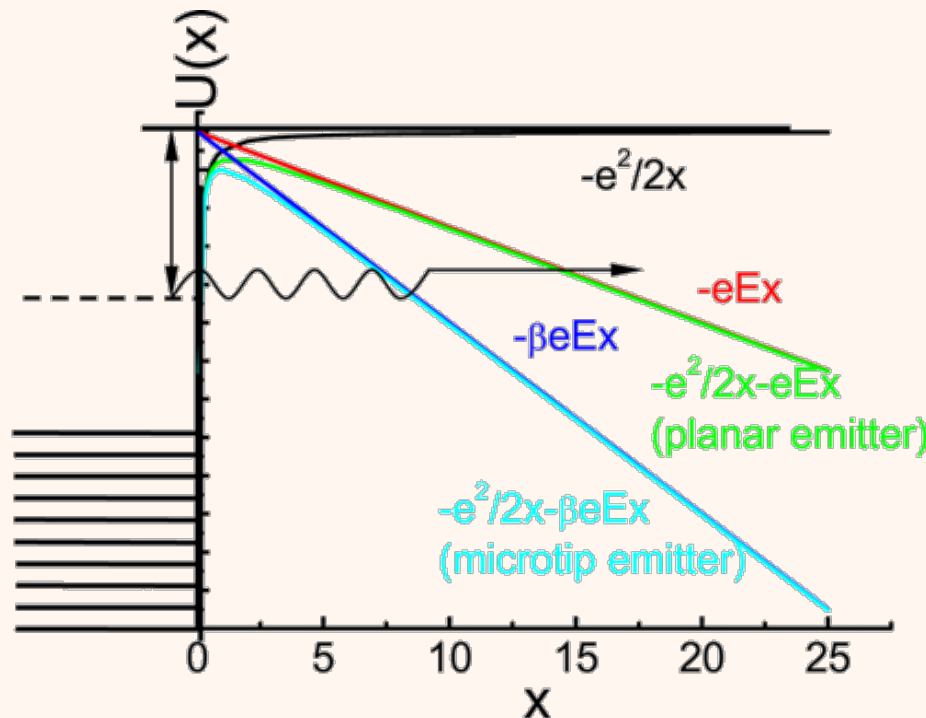
Feldemission:

Coulomb-Potential:

$$U = \int \vec{F} \cdot d\vec{s} = \frac{q^2}{4\pi\epsilon_0} \int \frac{ds}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{-q^2}{2r}$$

homogenes E-Feld:

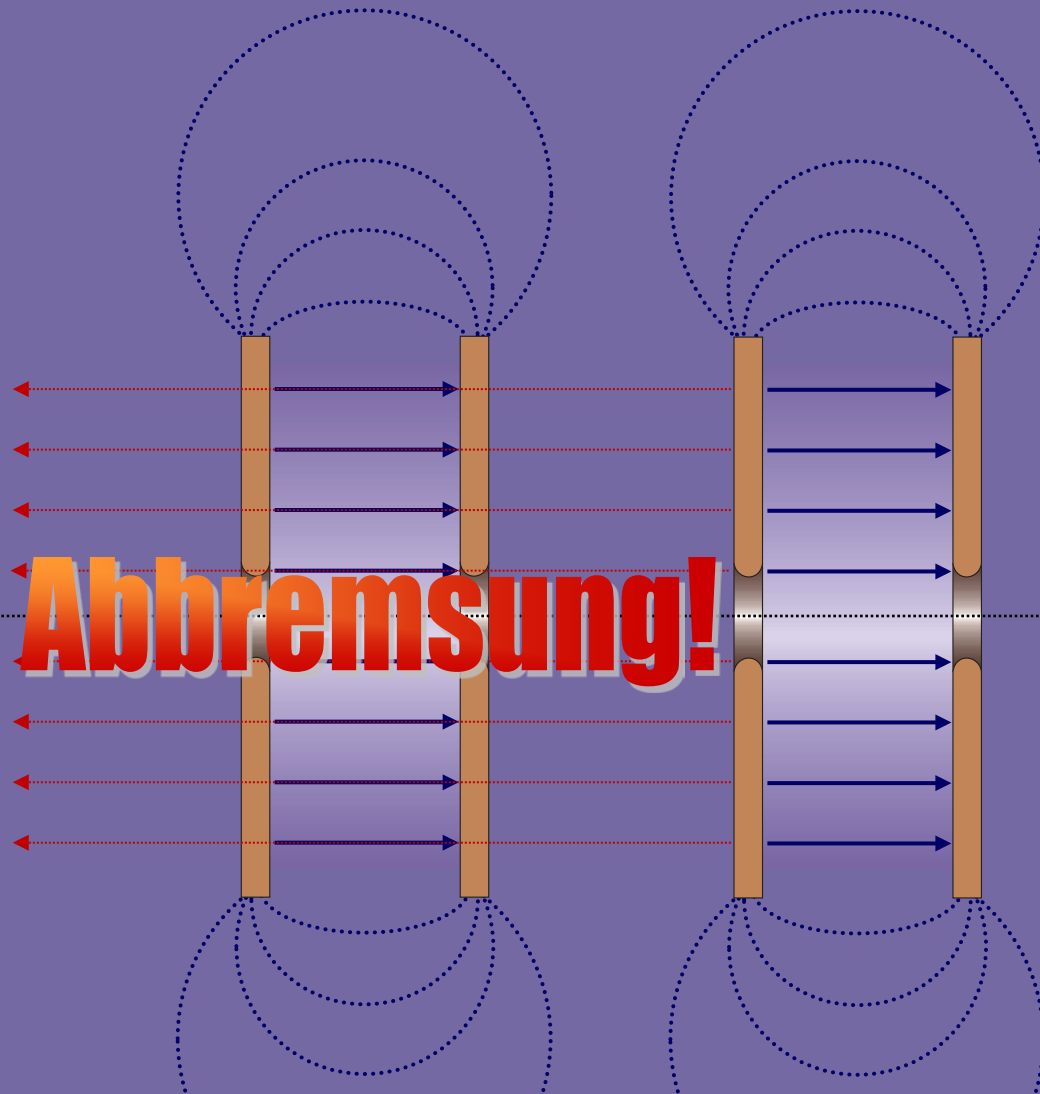
$$U = \int \vec{F} \cdot d\vec{s} = \int q\vec{E} \cdot d\vec{s} = -qEr$$



Tunneleffekt!
Überhöhungsfaktor β !

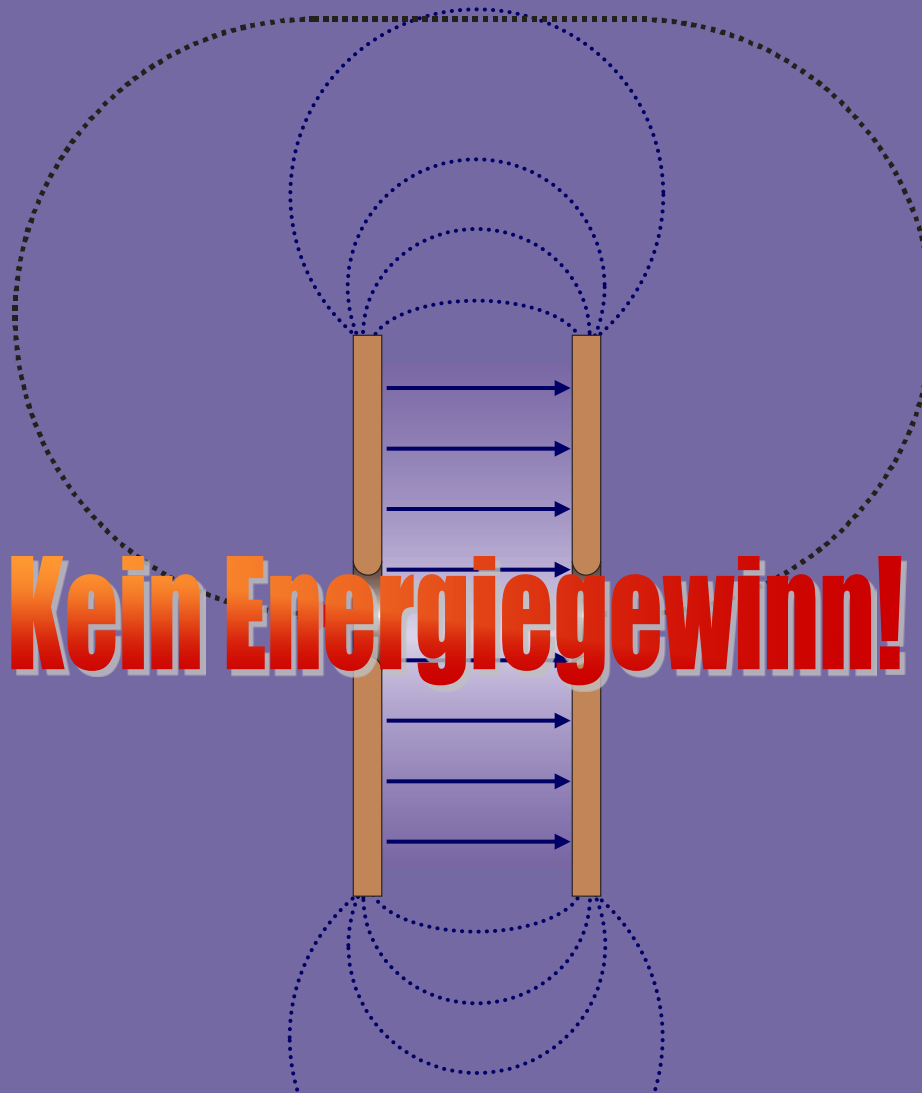
$$E \leq 10 - 100 \text{ MV/m}$$

Statische Felder



Abbremsung!

Statische Felder

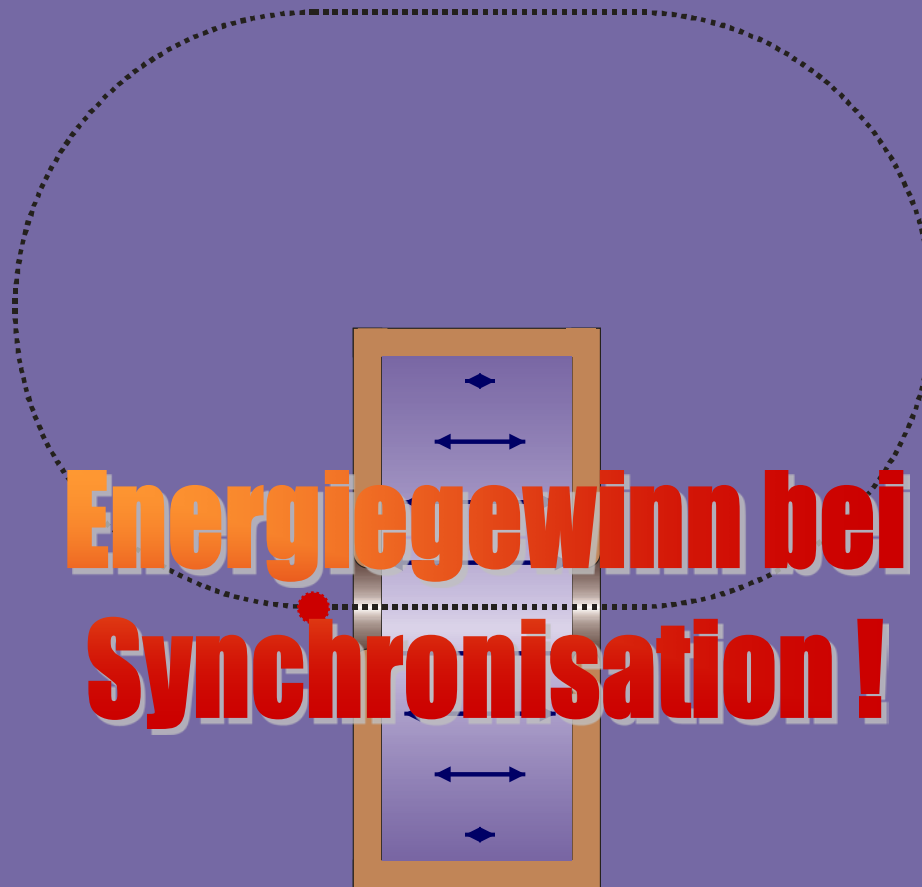


Kein Energiegewinn!



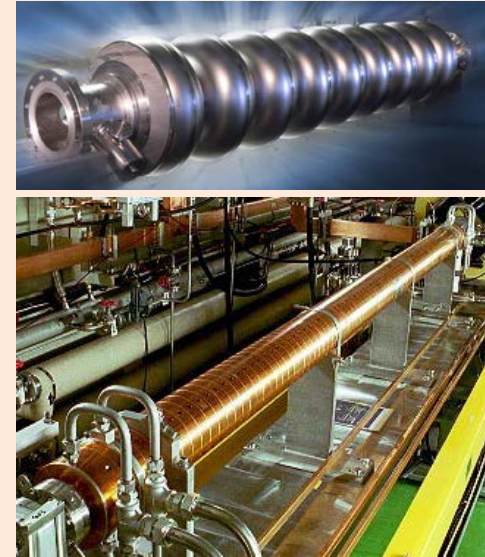
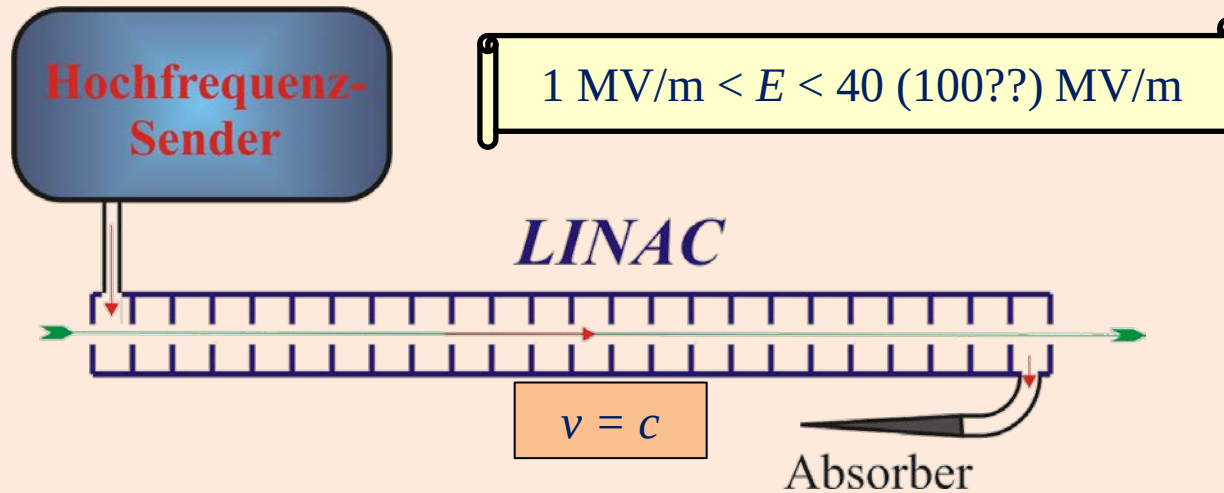
$$\oint \vec{E} \cdot d\vec{s} = 0$$

HF – Wechselfelder



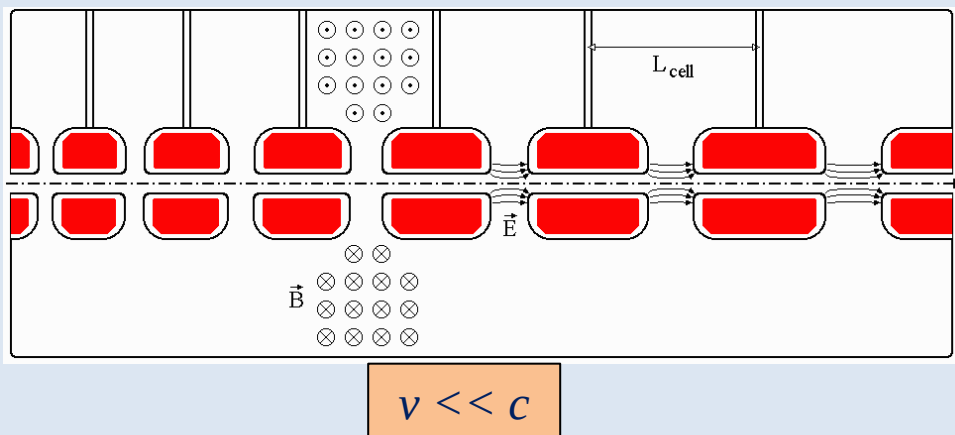
Geradeaus?

Elektronen:

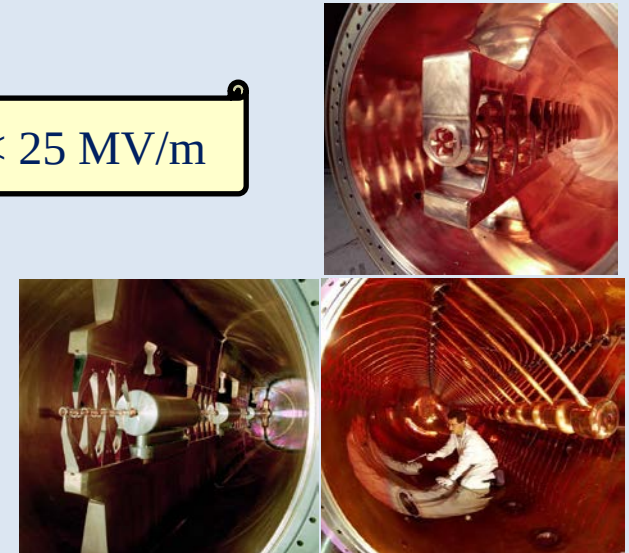


$1 \text{ TeV} \leftrightarrow L > 50 \text{ km}$

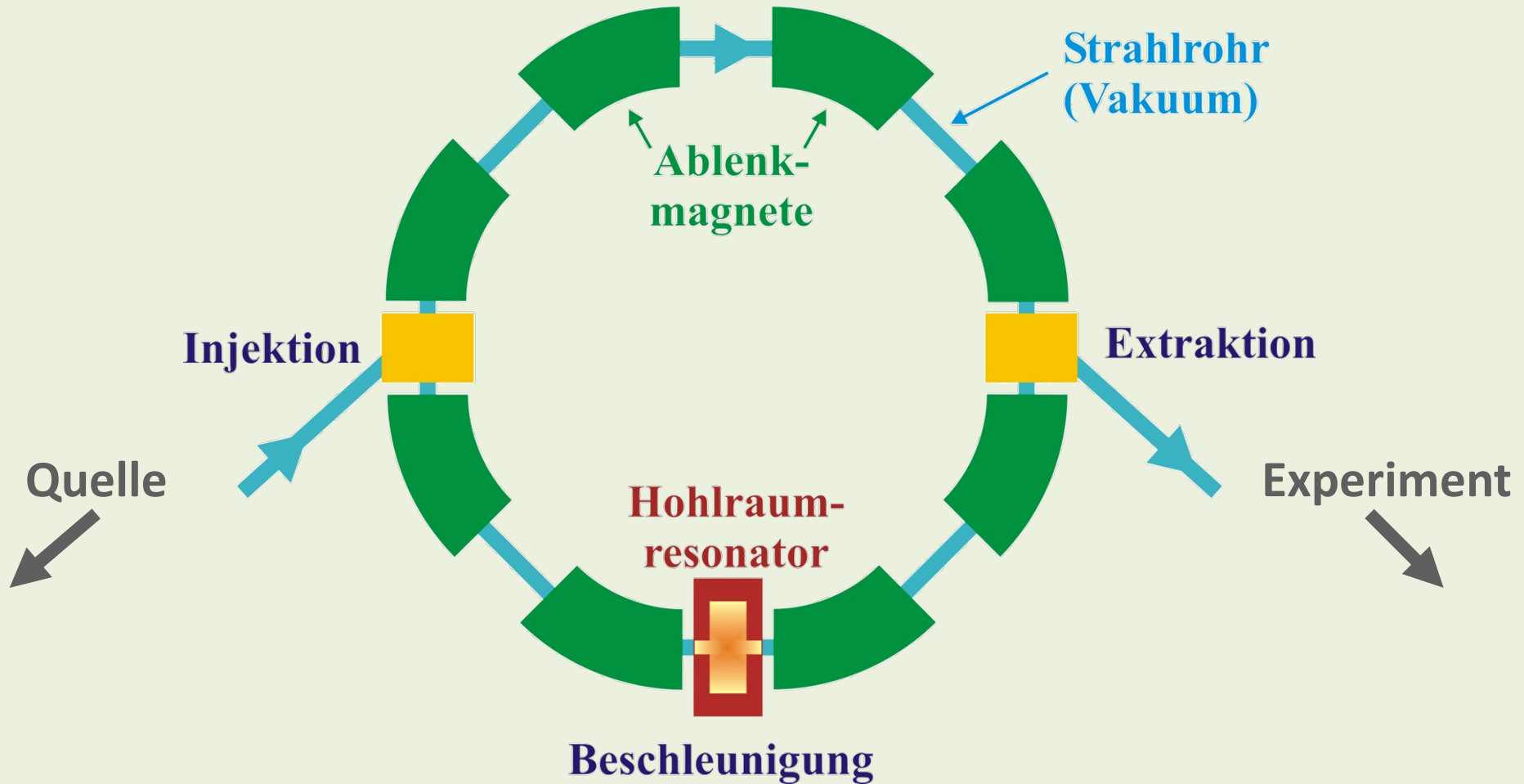
Protonen / Ionen:



$E_{\text{avg}} < 25 \text{ MV/m}$



Kreisbeschleuniger

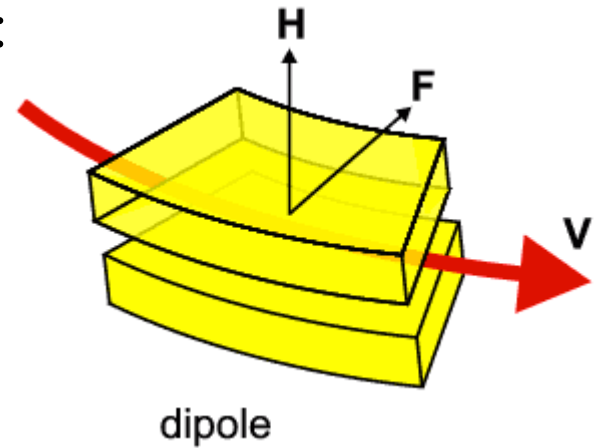


Strahlablenkung

Lorentzkraft: $\vec{F} = e \cdot (\vec{E} + \underline{\underline{\vec{v}}} \times \vec{B}) \rightarrow$ **Magnetfelder**

Kreisbahn $\leftrightarrow$ homogenes Magnetfeld:

$$\frac{mv^2}{R} = e \cdot v \cdot B_z \quad \Rightarrow \quad \frac{1}{R} = \frac{e}{p} \cdot B_z$$

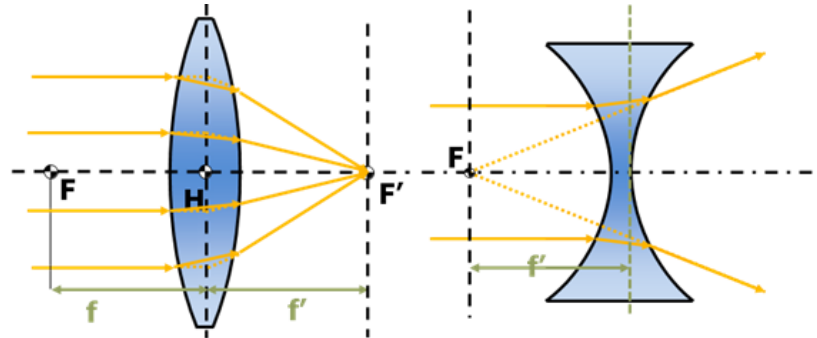


Wichtige Größen:

- Dipolstärke: $\kappa = 1/R$, $[\kappa] = \text{m}^{-1}$ (Krümmung)
- Magn. Steifigkeit: $BR = p/e \approx E/c$ (ultrarelativistisch!)
- Strahlenergie: $p = \frac{e}{2\pi} \cdot \oint B_z \cdot dl$

Strahlfokussierung

Linse (Lichtoptik):

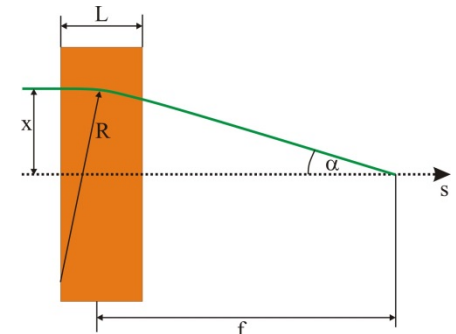


Ablenkung nimmt linear mit Abstand von der optischen Achse zu!

$$\vec{F} = e \cdot \vec{v} \times \vec{B} \Rightarrow B_z = g \cdot x, \quad B_x = g \cdot z, \quad g = \frac{\partial B_z}{\partial x}$$

Wichtige Größen:

- Quadrupolstärke: $k = e/p \cdot g$, $[k] = \text{m}^{-2}$
- Brennweite: $1/f \approx k \cdot L$



$$\frac{x}{f} = \tan \alpha = \frac{L}{R} = L \cdot \frac{e}{p} B_z = \frac{e}{p} g x L = x k L$$



Höhere Ordnungen:

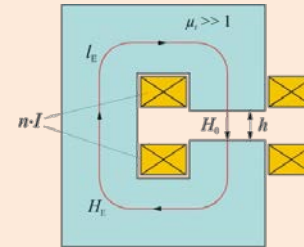


Taylorentwicklung des Magnetfeldes:

$$B_z(x) = \underbrace{B_0}_{\text{Dipol}} + \underbrace{\frac{\partial B_z}{\partial x}}_{\text{Quadrupol}} \cdot x + \frac{1}{2} \underbrace{\frac{\partial^2 B_z}{\partial x^2}}_{\text{Sextupol}} \cdot x^2 + \frac{1}{6} \underbrace{\frac{\partial^3 B_z}{\partial x^3}}_{\text{Oktupol}} \cdot x^3 + \dots$$

Definition eines skalaren magnetischen Potentials:

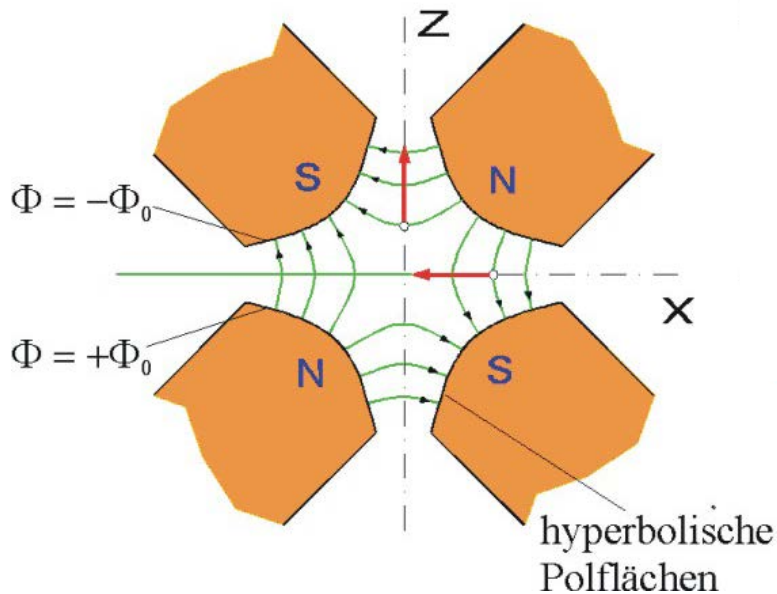
$$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{j} = 0 \quad \Rightarrow \quad \vec{B} = \vec{\nabla} \Phi$$



- Dipol: $-\frac{e}{p} \cdot \Phi_1 = \kappa_z x - \kappa_x z$
- Quadrupol: $-\frac{e}{p} \cdot \Phi_2 = -1/2 \underline{k} (x^2 - z^2) + k x z$
- Sextupol: $-\frac{e}{p} \cdot \Phi_3 = -1/6 \underline{m} (x^3 - 3 x z^2) + 1/6 m (3 x^2 z - z^3)$
- ...

Strahlfokussierung

Quadrupolmagnet:



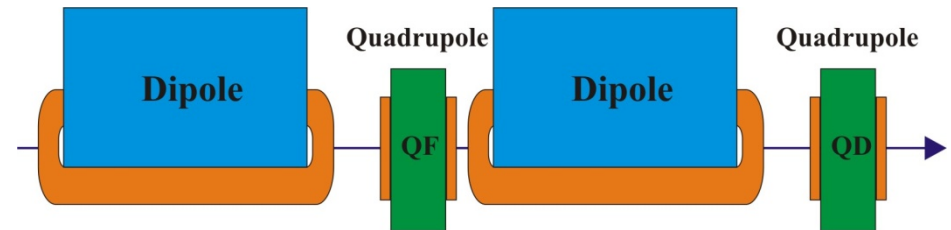
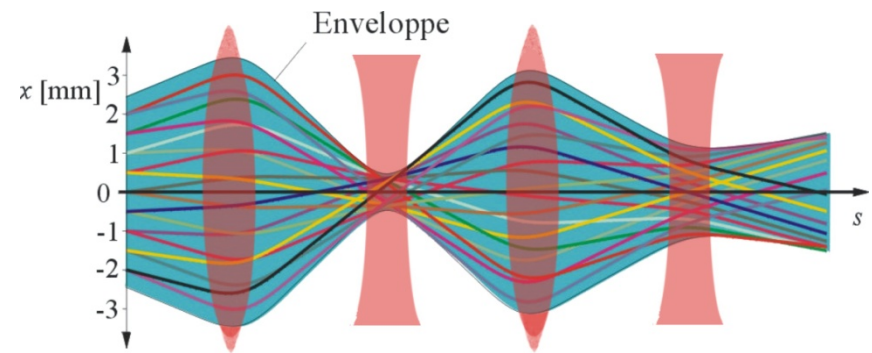
$$\Phi = -g \cdot x \cdot z$$

$$\vec{B} = g \cdot (x \hat{e}_z + z \hat{e}_x)$$

$$\vec{F} = e \cdot (\vec{v} \times \vec{B}) = evg \cdot (x \hat{e}_x - z \hat{e}_z)$$

Starke Fokussierung:

(Courant, Livingston und Snyder, 1952)

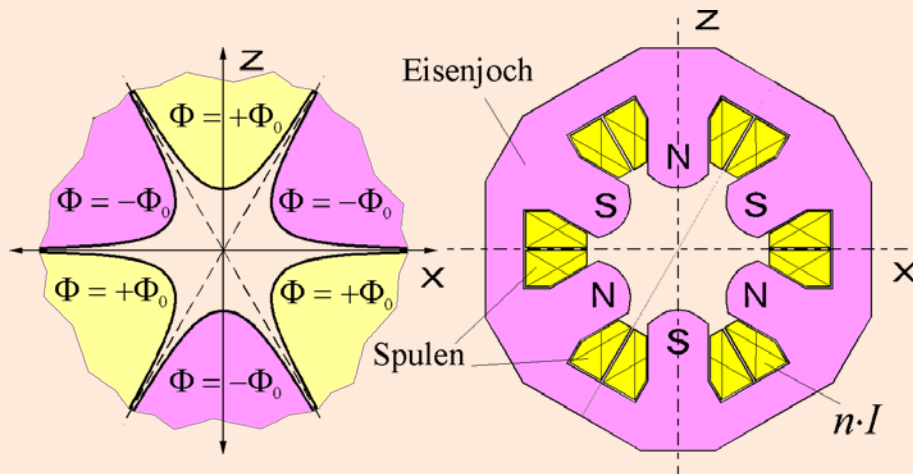


Einfachste Anordnung: FODO

Farbkorrektur



Sextupolmagnet:

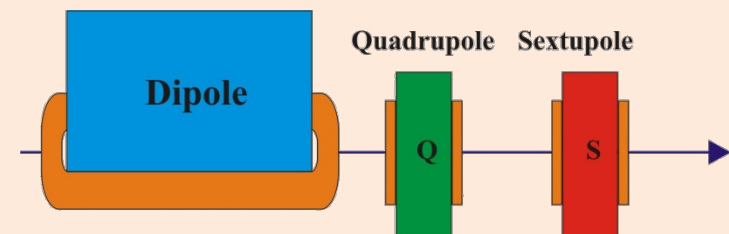
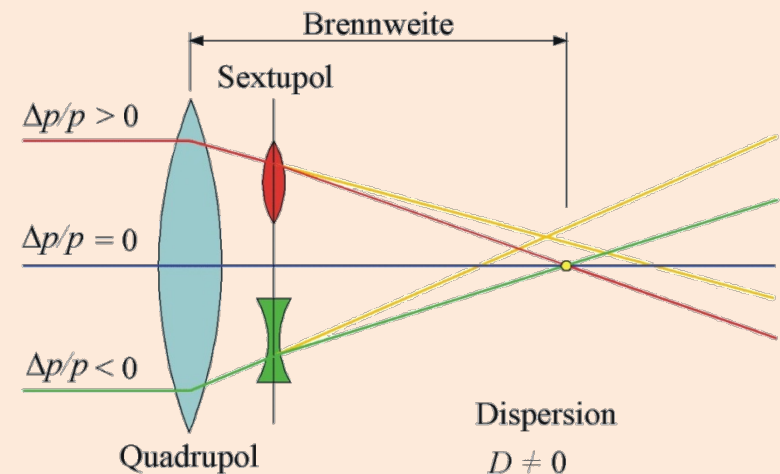


$$\Phi(x, z) = -\frac{1}{6} g' (z^3 - 3x^2 z)$$

$$\vec{B} = g' \left\{ xz \cdot \hat{e}_x + \frac{1}{2} (x^2 - z^2) \cdot \hat{e}_z \right\}$$

$$\vec{F} = evg' \left\{ \frac{1}{2} (x^2 - z^2) \cdot \hat{e}_x - xz \cdot \hat{e}_z \right\}$$

Brennweitenkorrektur:



Eisen-Magnete

Ablenkung

Dipolmagnete

Fokussierung

Quadrupolmagnete

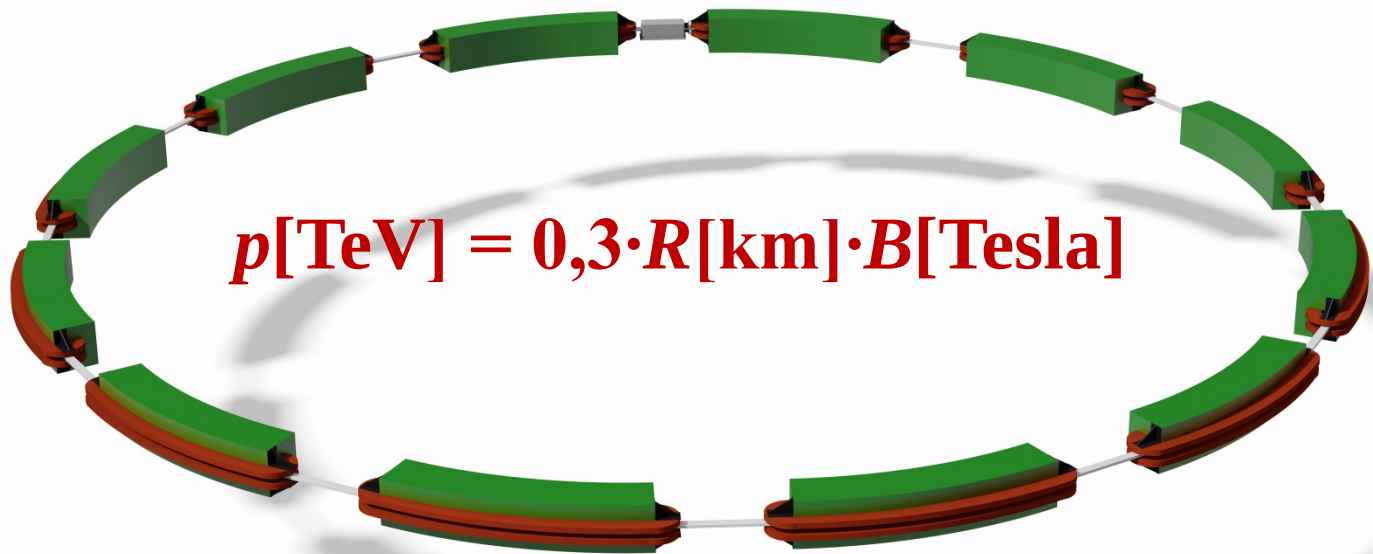
Farbkorrektur

Sextupolmagnete



Eigenschaften definiert durch Polschuhe!
Maximal erreichbare Felder: $B < 2,5$ Tesla

Protonen-Beschleuniger

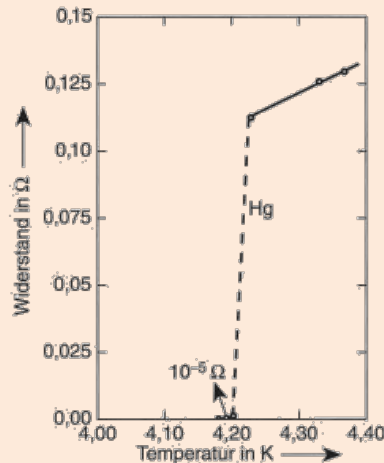


$$p[\text{TeV}] = 0,3 \cdot R[\text{km}] \cdot B[\text{Tesla}]$$

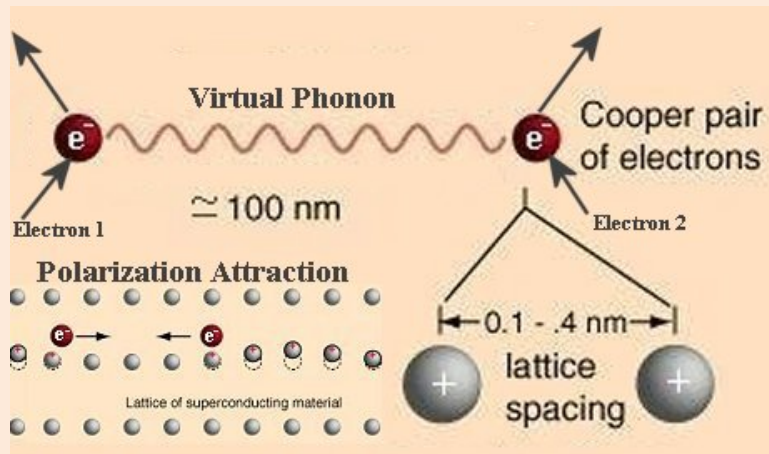
7 TeV @ 2 Tesla $\rightarrow$ $R = 11,6$ km $\leftrightarrow$ $L > 80$ km

Supraleitung

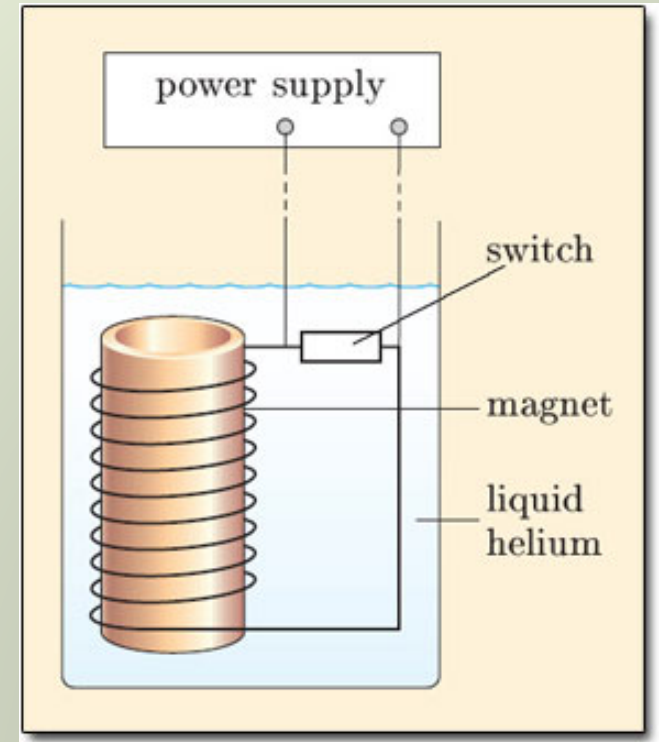
8. April 1911: Entdeckung der Supraleitung durch Heike Kamerlingh Onnes



Unterhalb von T_C kein DC-Widerstand
Grund: Bildung von Cooper-Paaren:



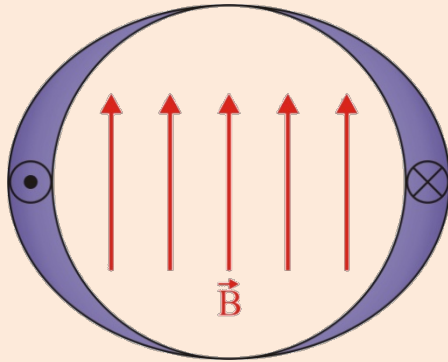
→ „Verlustfreie“ Magnete:



Supraleitende Magnete

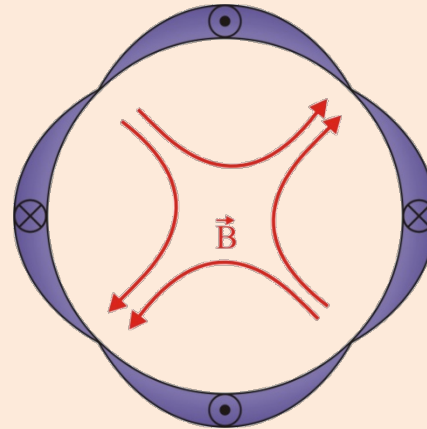


Dipol



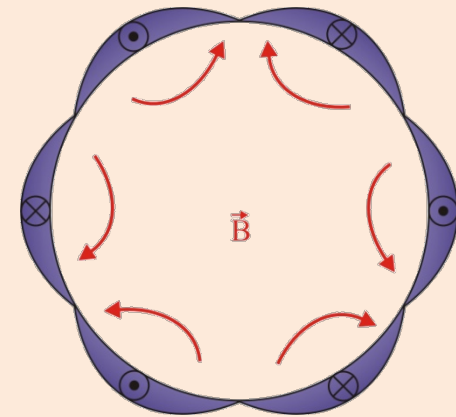
$$I(\varphi) = I_0 \cdot \cos \varphi$$

Quadrupol



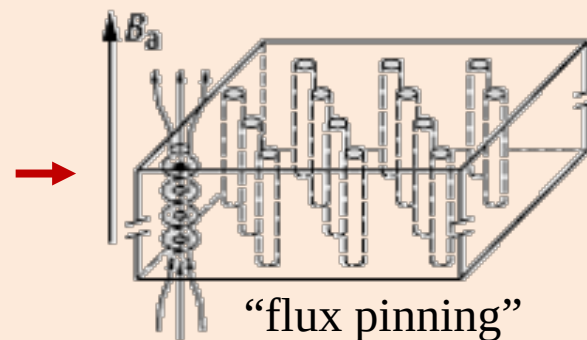
$$I(\varphi) = I_0 \cdot \cos 2\varphi$$

Sextupol

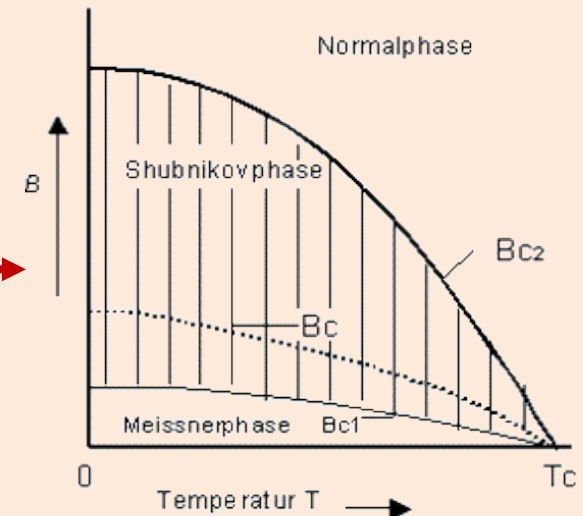


$$I(\varphi) = I_0 \cdot \cos 3\varphi$$

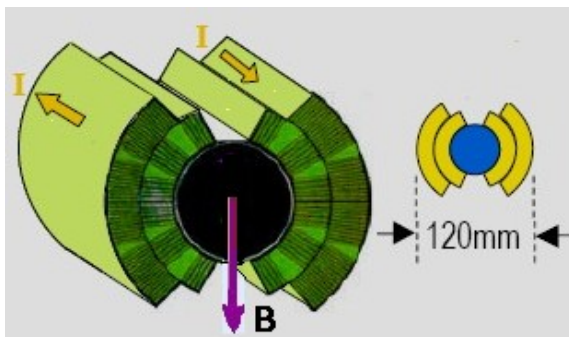
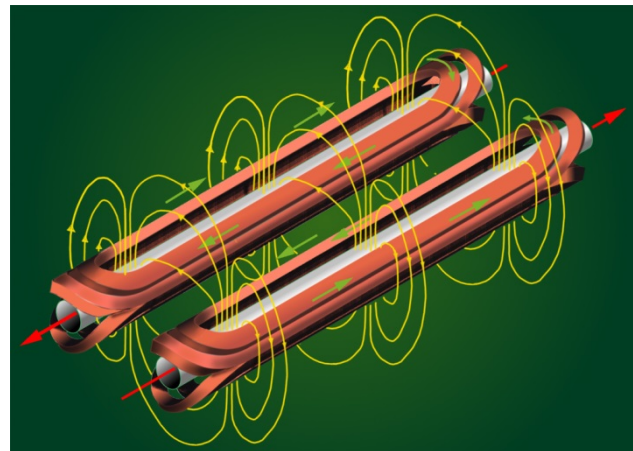
**Typ II
Supraleiter
e.g. Nb₃Sn**



“flux pinning”

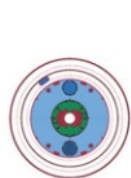
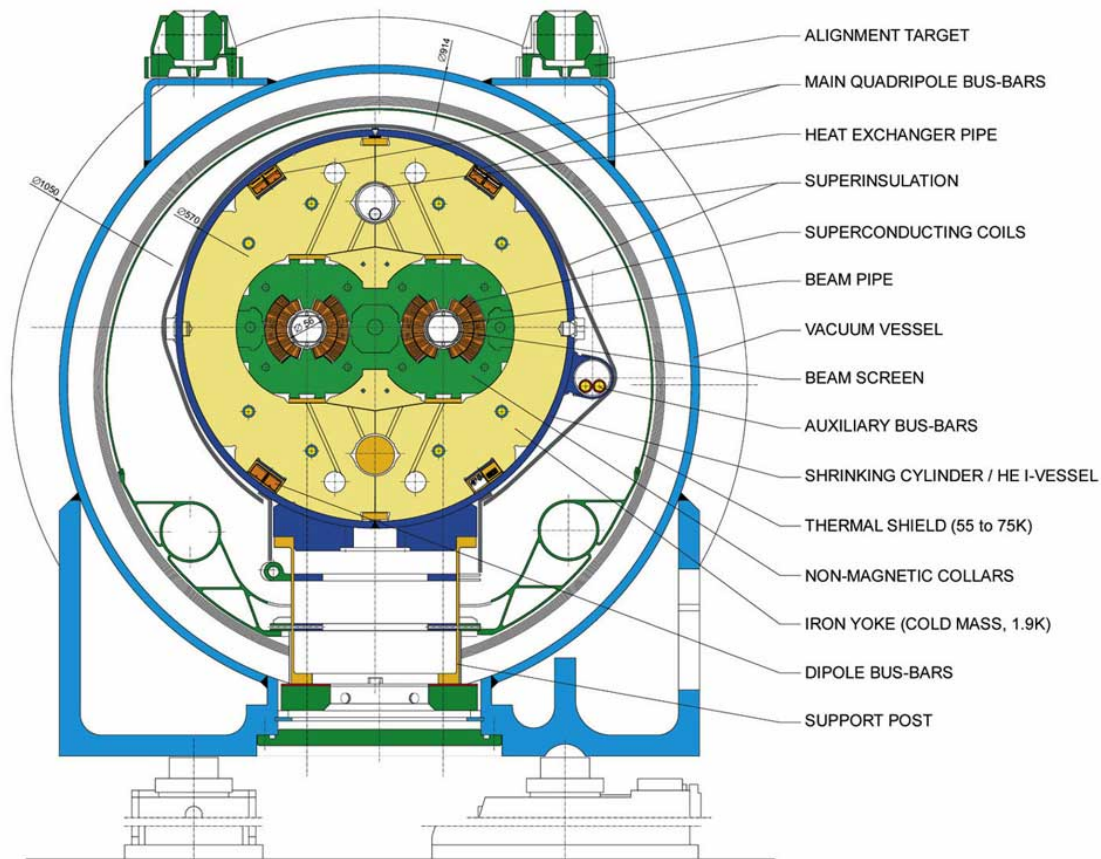


SC Dipolmagnete

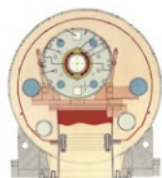


LHC DIPOLE : STANDARD CROSS-SECTION

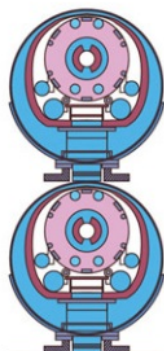
CERN AC/DI/MM - HE107 - 30 04 1999



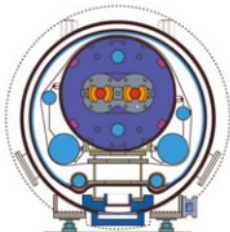
HERA



RHIC



SSC



LHC



TEVATRON

LHC Dipolemagnete:

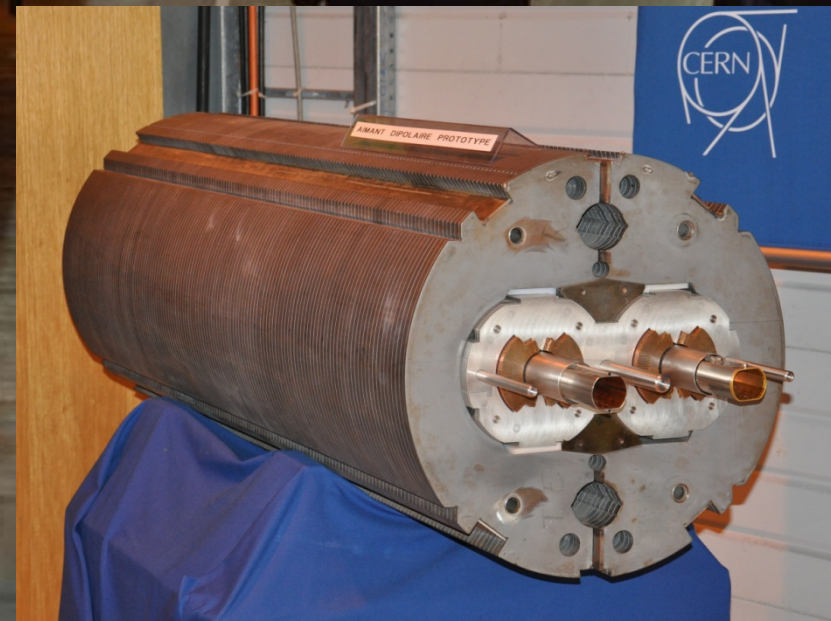
$E_{\max} = 7 \text{ TeV} \leftrightarrow B_{\max} = 8.33 \text{ T}$

He-Temperatur: $T = 1.9 \text{ K}$

nom. Strom: $I = 11850 \text{ A @ } 8.33 \text{ T}$

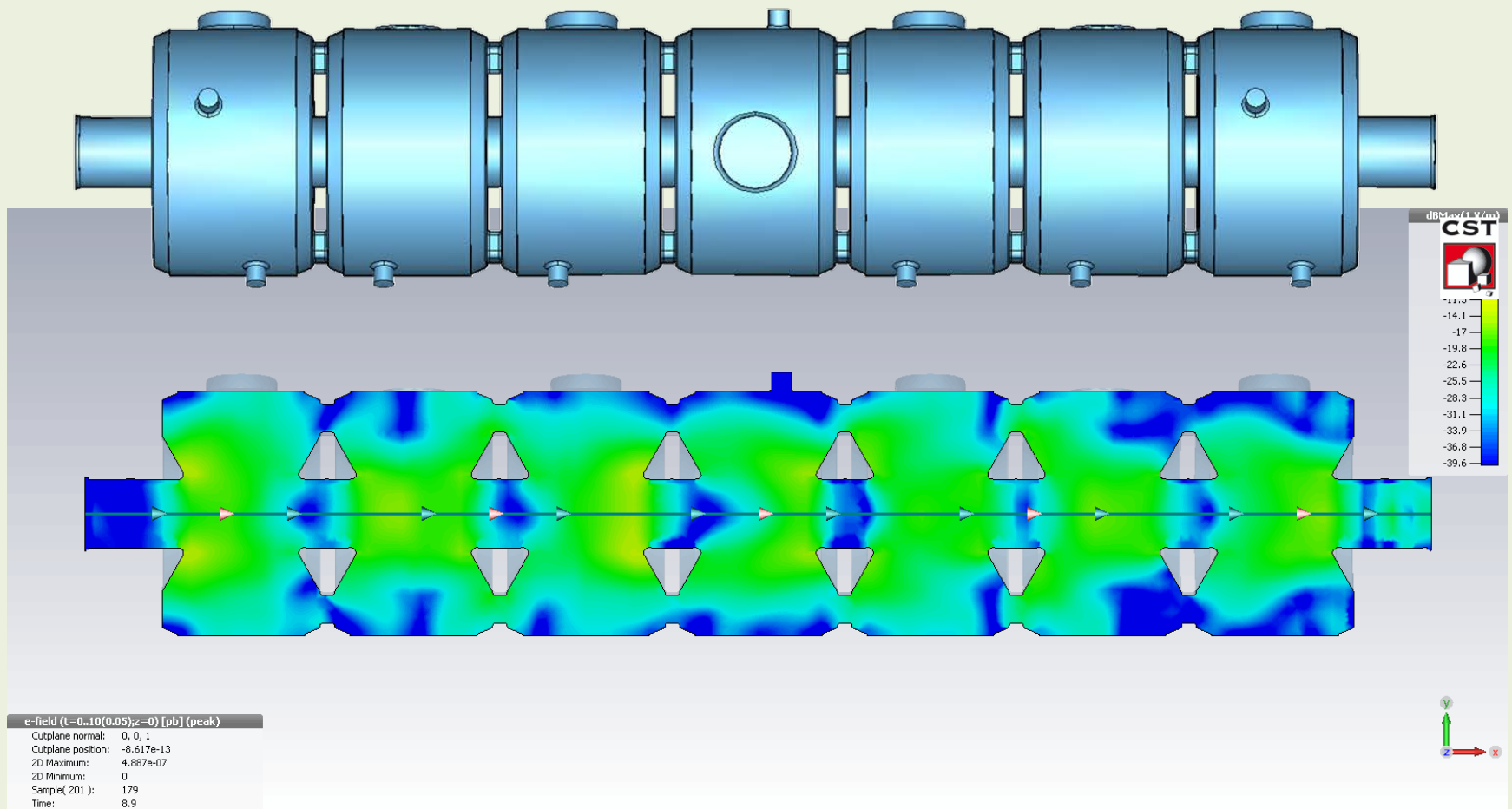
krit. Strom: $I = 13750 \text{ A @ } 1.9 \text{ K}$

gespeicherte Energie: $W = 11 \text{ GJ}$

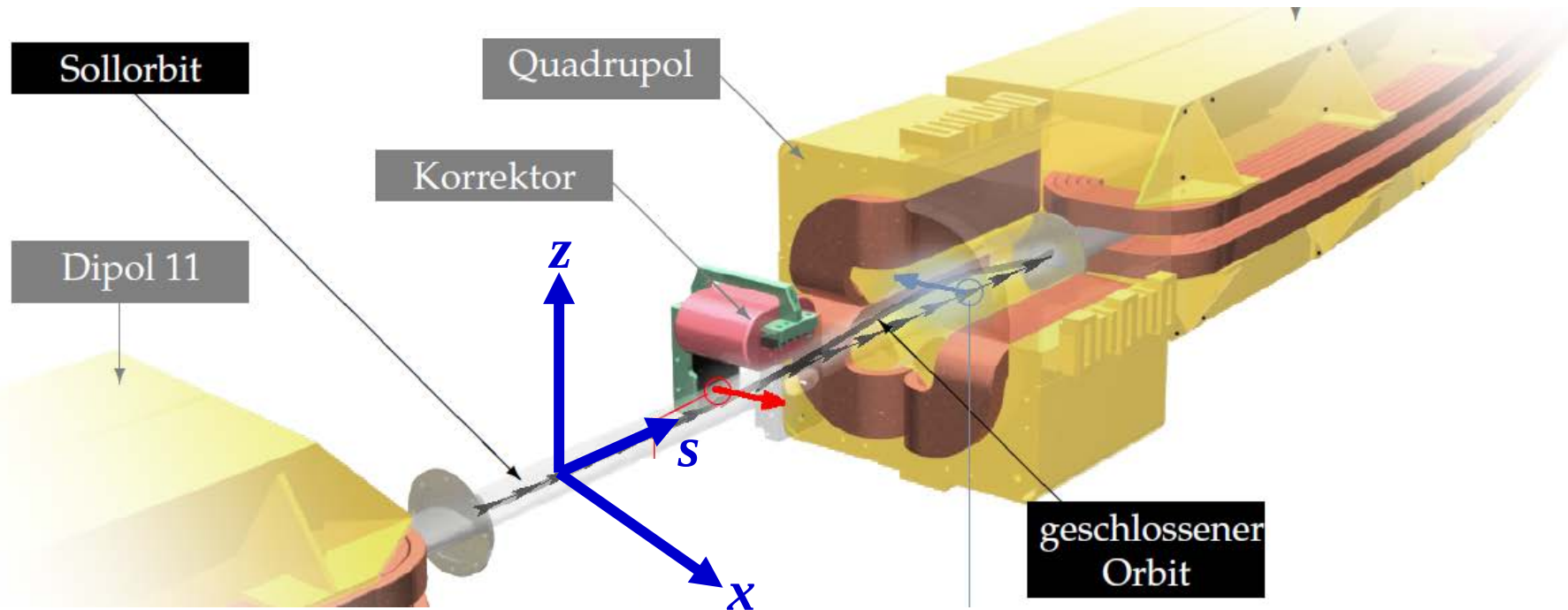


Strahldynamik

Bsp.: Teilchenpaket im PETRA Beschleunigungsresonator:



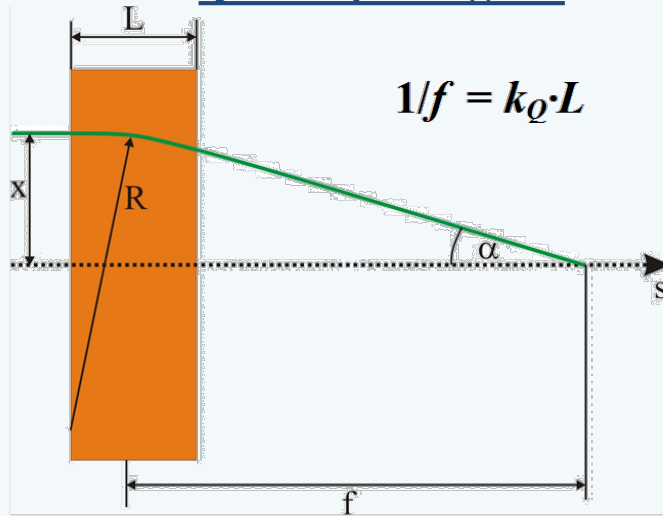
Teilchenbahnen



**Mitbewegtes Koordinatensystem,
an Sollteilchen angeheftet**

Bewegungsgleichung

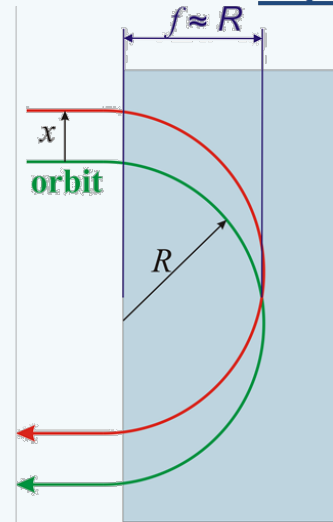
Quadrupolmagnet:



$$1/f = k_Q \cdot L$$

$$\vec{F}_{mag} = m \cdot (\beta c)^2 \cdot k \cdot (x \hat{e}_x - z \hat{e}_z)$$

Dipolmagnet:



Fokussierung
analog zum
Quadrupol:

$$1/f = k_D \cdot L$$

$$k_D = 1/R^2$$

$$\vec{F}_{mag} = m \cdot (\beta c)^2 \cdot \frac{1}{R^2} x \hat{e}_x$$

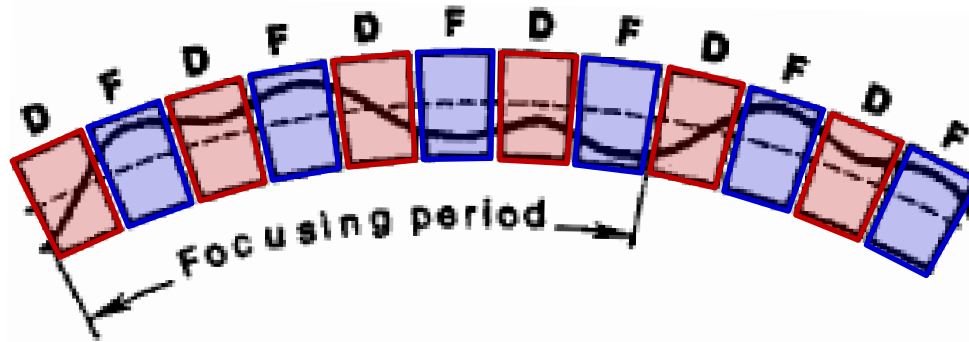
$$m \cdot \ddot{\vec{x}}(t) = m \cdot (\beta c)^2 \cdot \vec{x}''(s) = \vec{F}_{mag}$$

**Hillsche
Differentialgleichung**

$$x''(s) + \left(\frac{1}{R^2(s)} - k(s) \right) \cdot x(s) = \frac{1}{R(s)} \frac{\Delta p}{p}$$

$$z''(s) + k(s) \cdot z(s) = 0$$

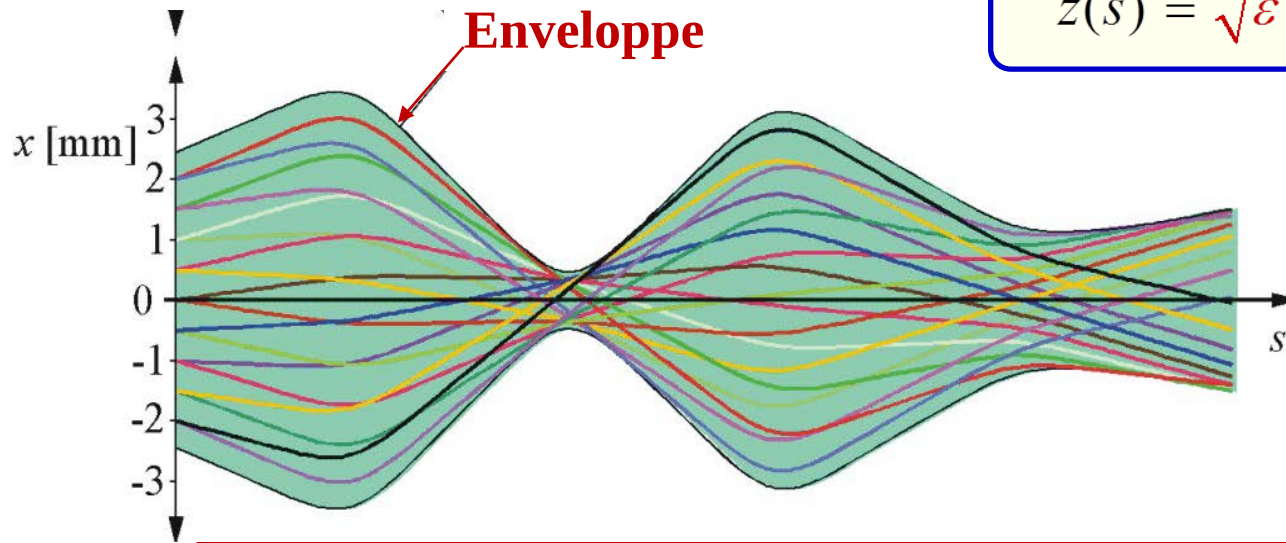
Betatron-Oszillationen



$$z''(s) + k(s) \cdot z(s) = 0$$



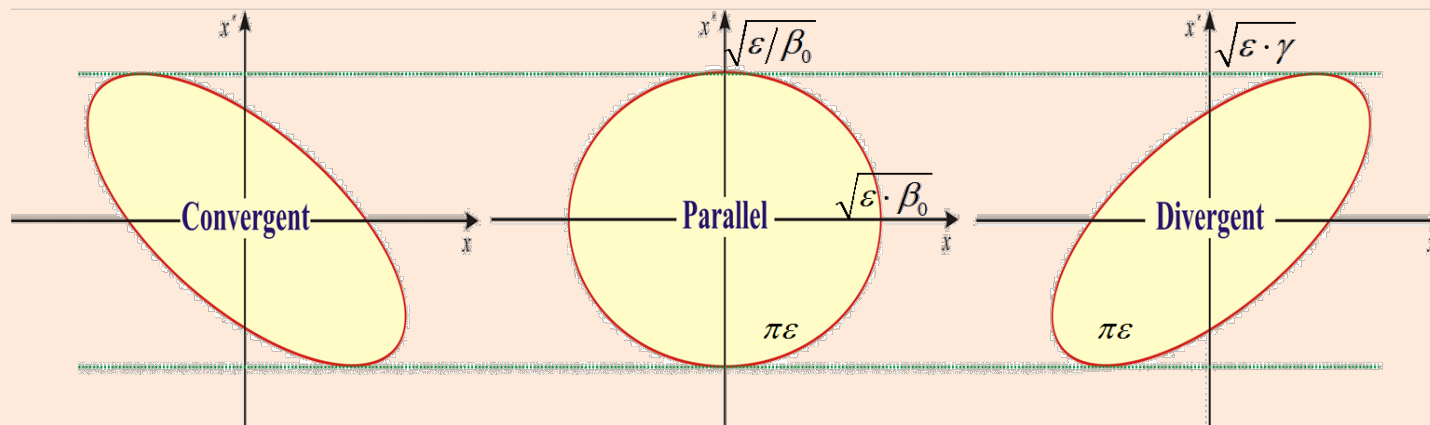
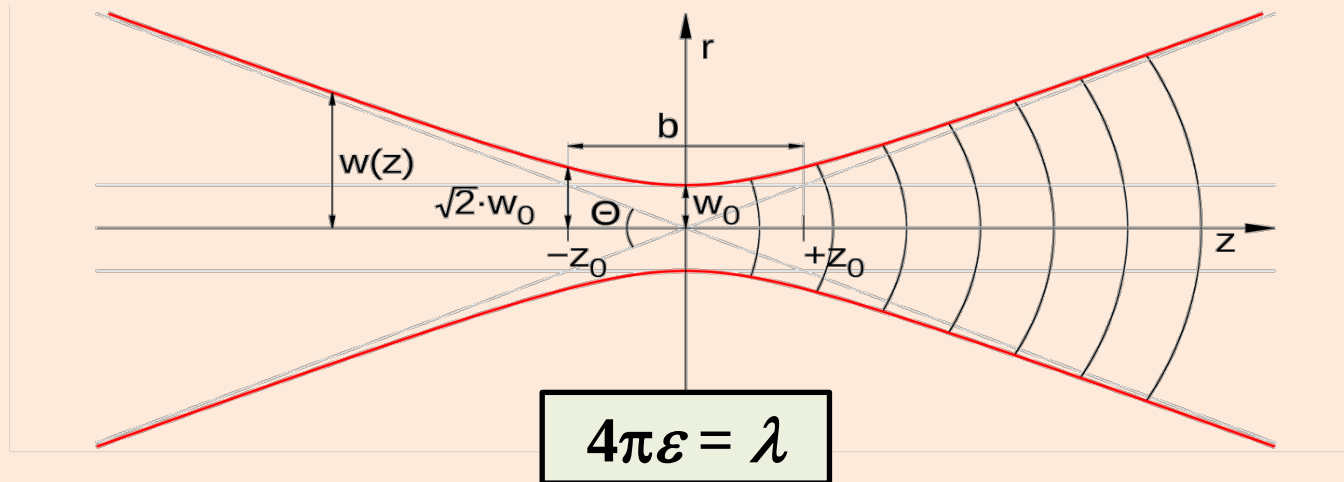
$$z(s) = \sqrt{\varepsilon \cdot \beta_z(s)} \cdot \cos(\phi(s) + \varphi_0)$$



Schwingungen pro Umlauf: Arbeitspunkt $Q_{x,z} = \frac{1}{2\pi} \oint \frac{ds}{\beta_{x,z}(s)}$

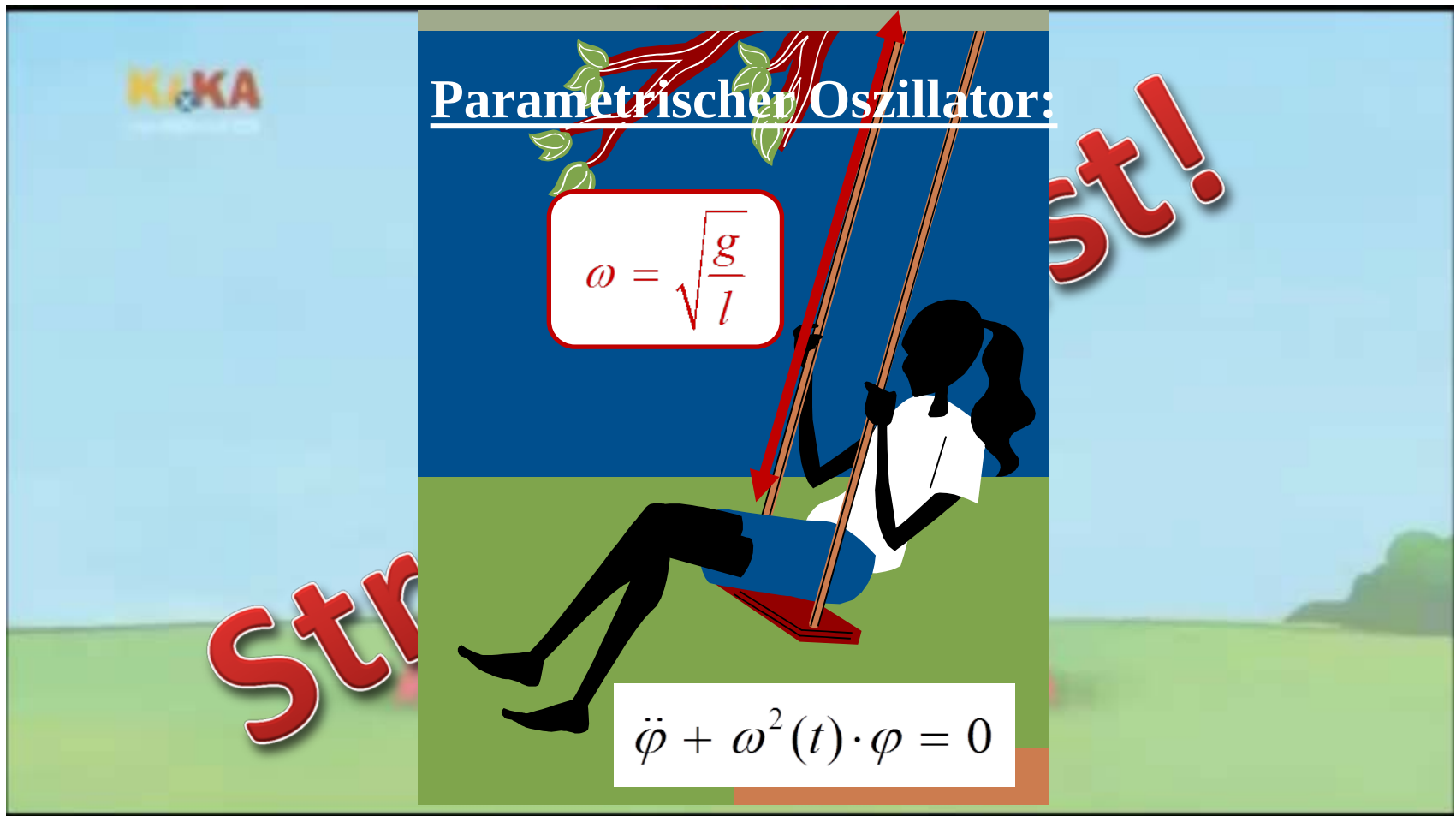


TWISS und Emittanz



$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \varepsilon, \quad \text{wobei} \quad \alpha = -\frac{\beta'}{2} \quad \text{und} \quad \gamma = \frac{1 + \alpha^2}{\beta}$$

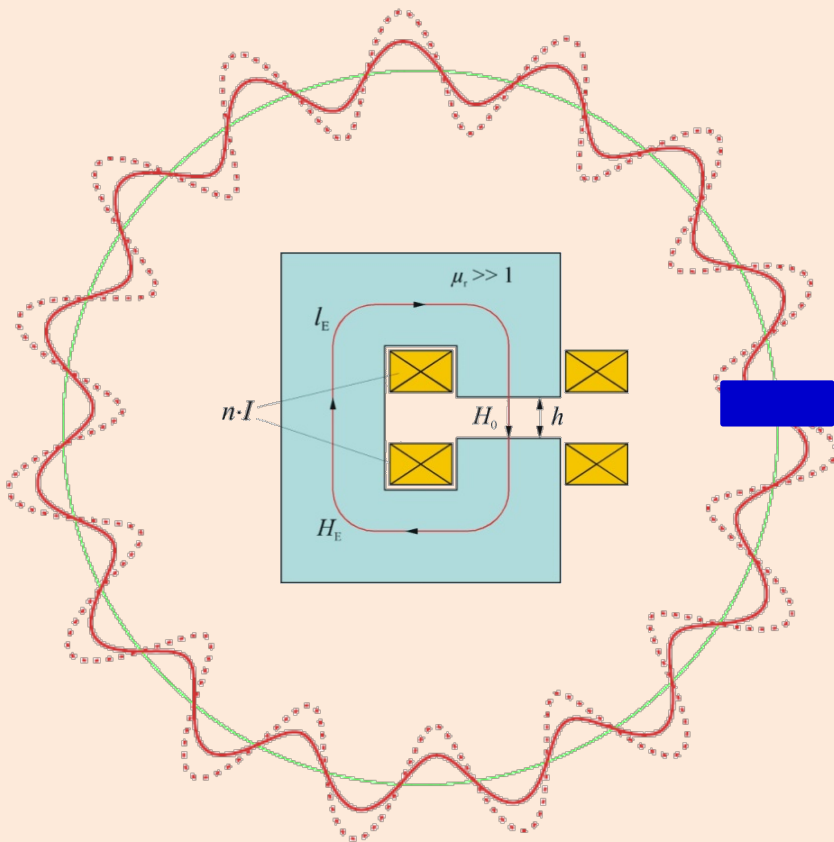
Lösen der Hillschen DGL



Feldfehler

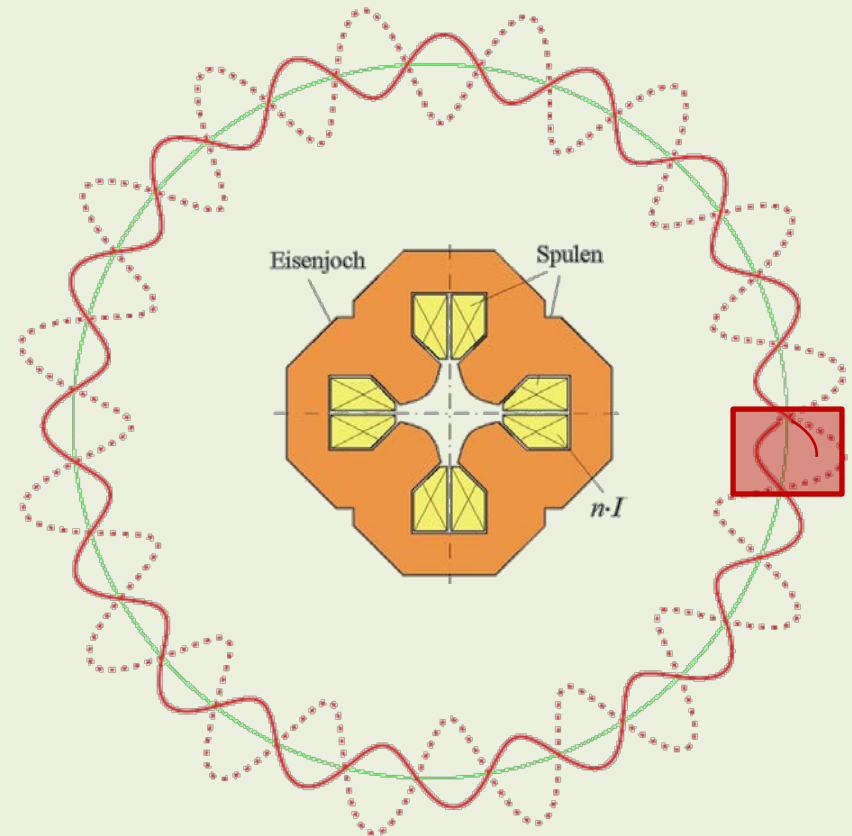
Dipolmagnet

$$Q = n$$

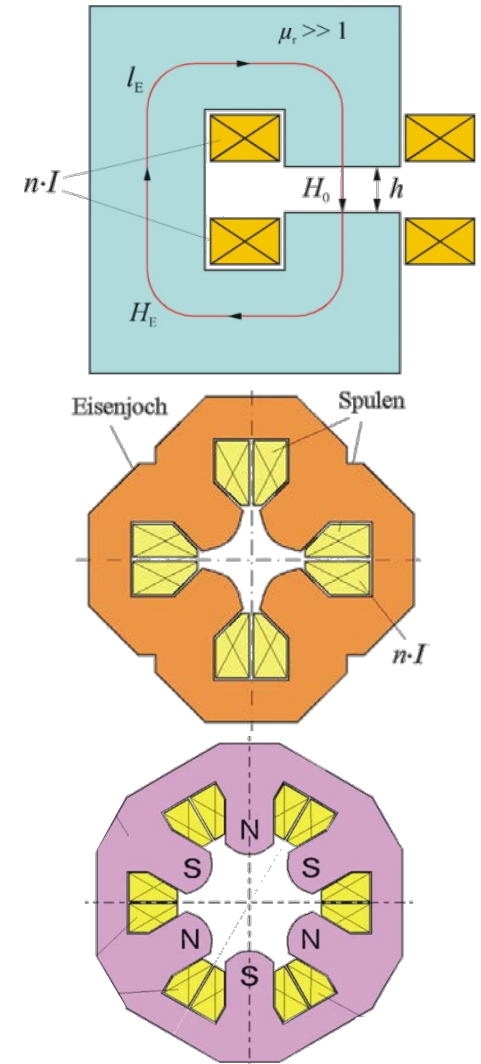
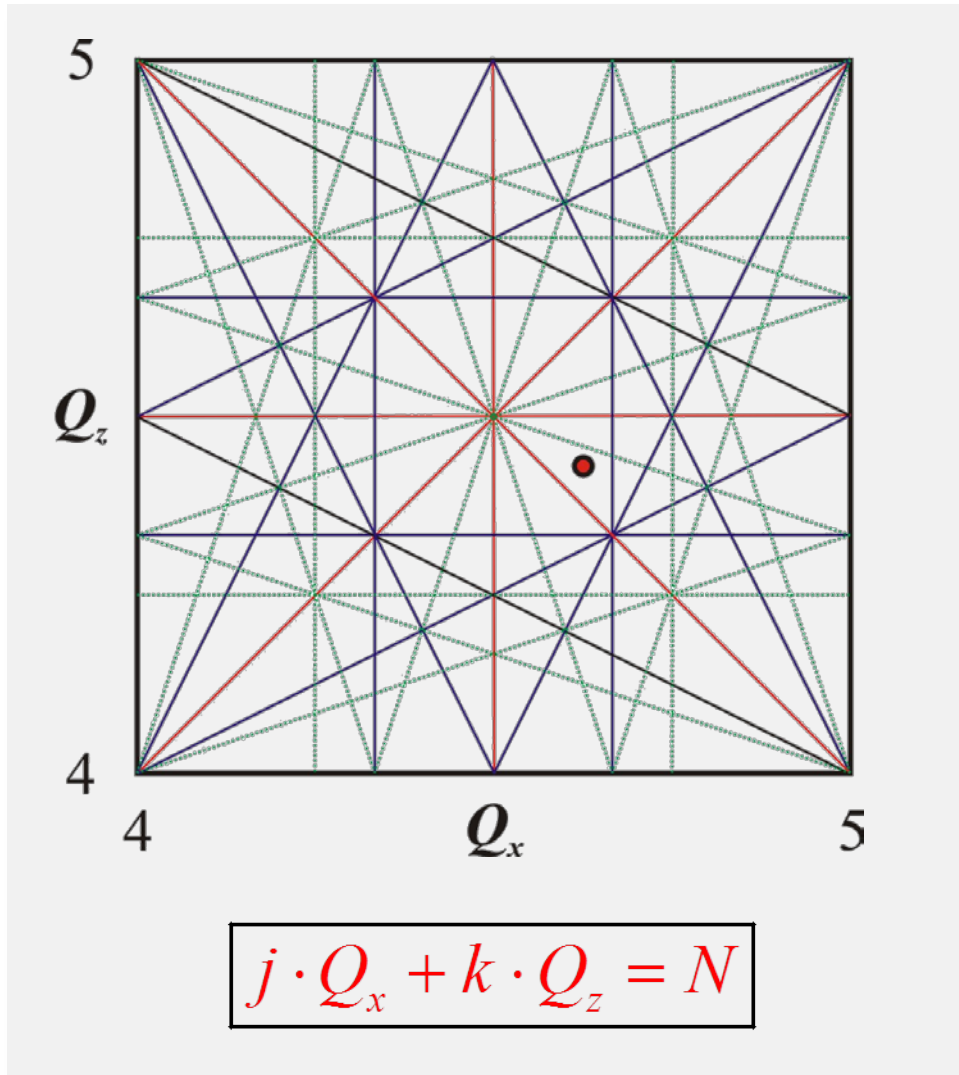


Quadrupolmagnet

$$Q = m/2$$

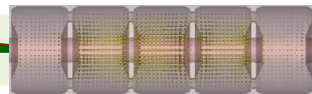
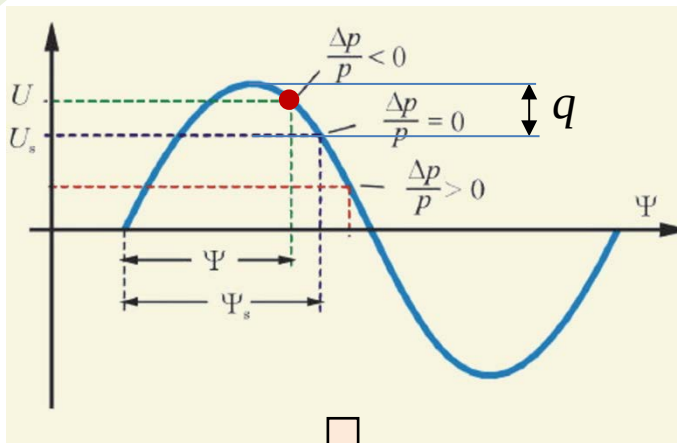


Optische Resonanzen

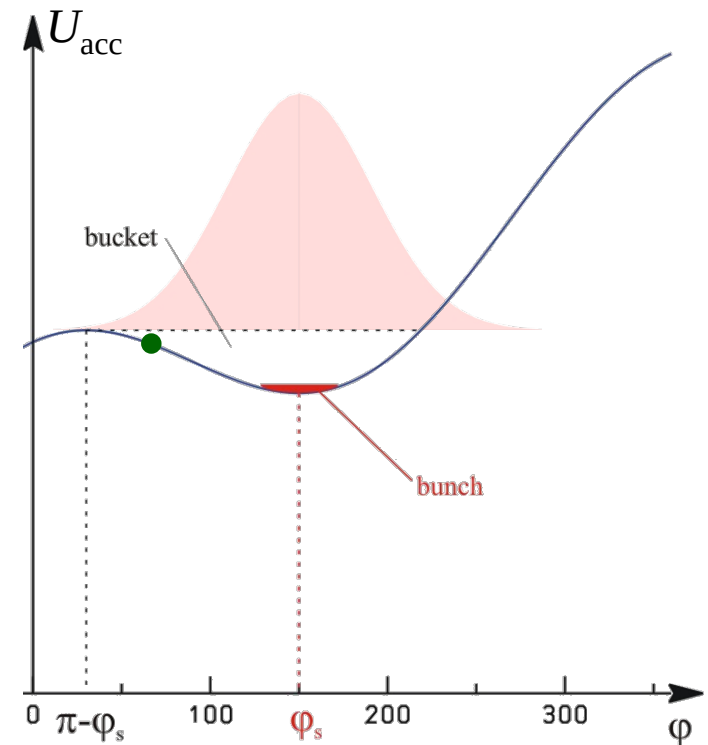


Longitudinale Fokussierung

Phasenschwingung:



Energieschwingung:



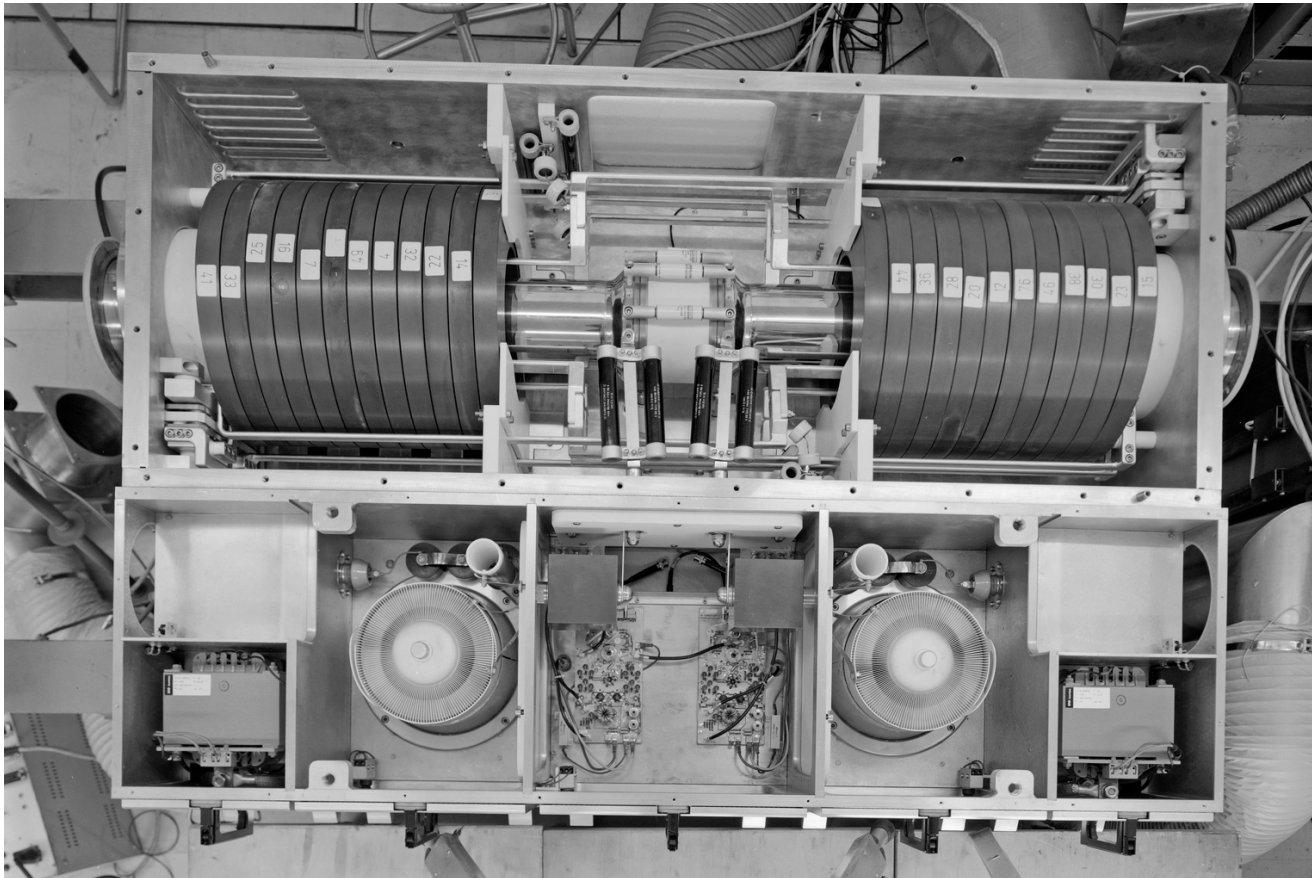
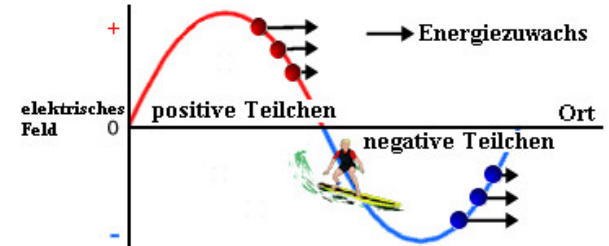
Überspannungsfaktor: $q = U_0/U_s$

Synchronisation

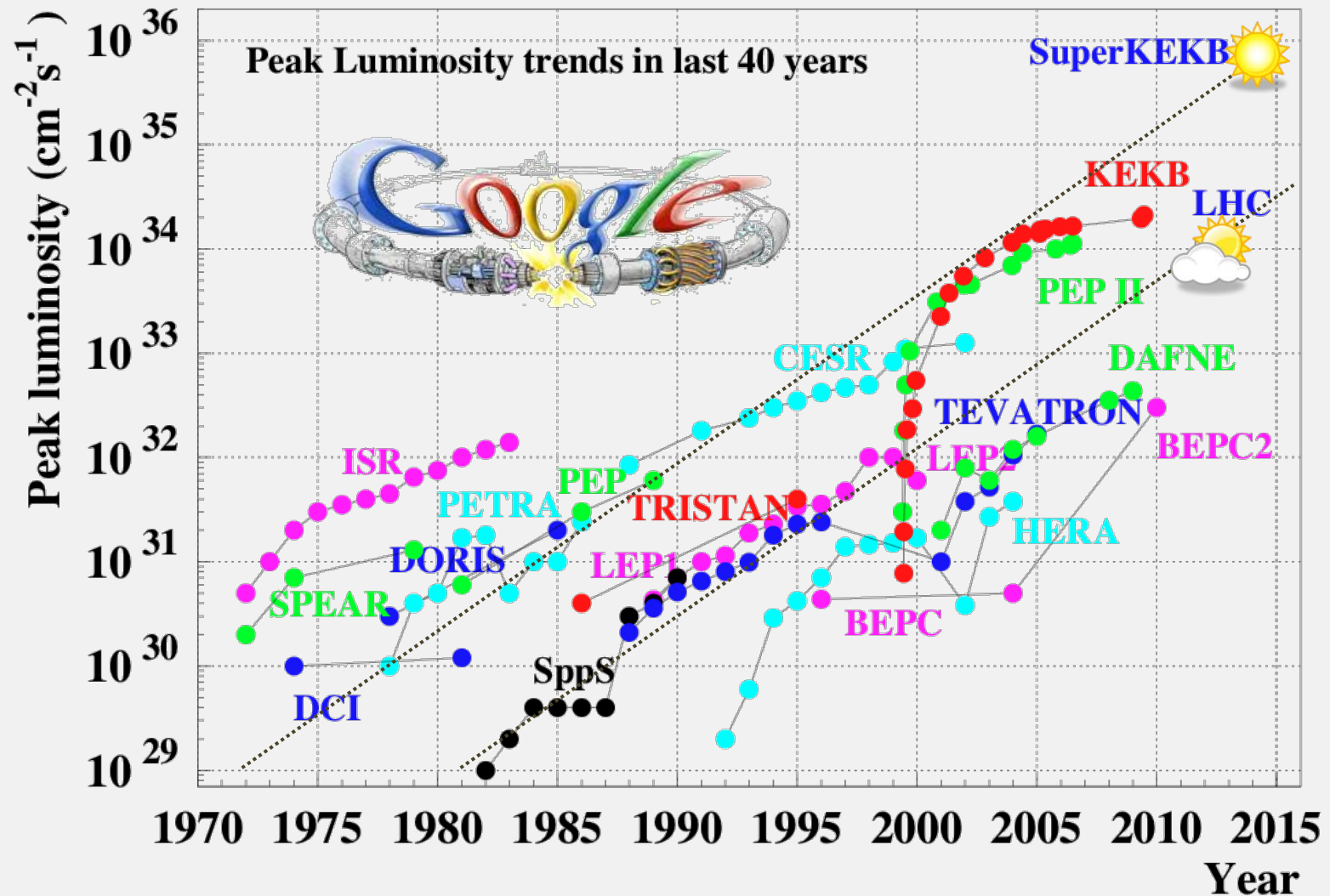
$$B \leftrightarrow \omega_{\text{HF}}$$

Teilchen werden schneller: $h \cdot \omega_0 = \omega_{\text{HF}}$

Bahnlänge bleibt konstant: $L = h \cdot \beta \cdot \lambda_{\text{HF}}$



Luminosität



Luminosität

... das unbekannte Wesen ...

Was interessiert den Teilchenphysiker an Beschleunigern?

- Luminosität**

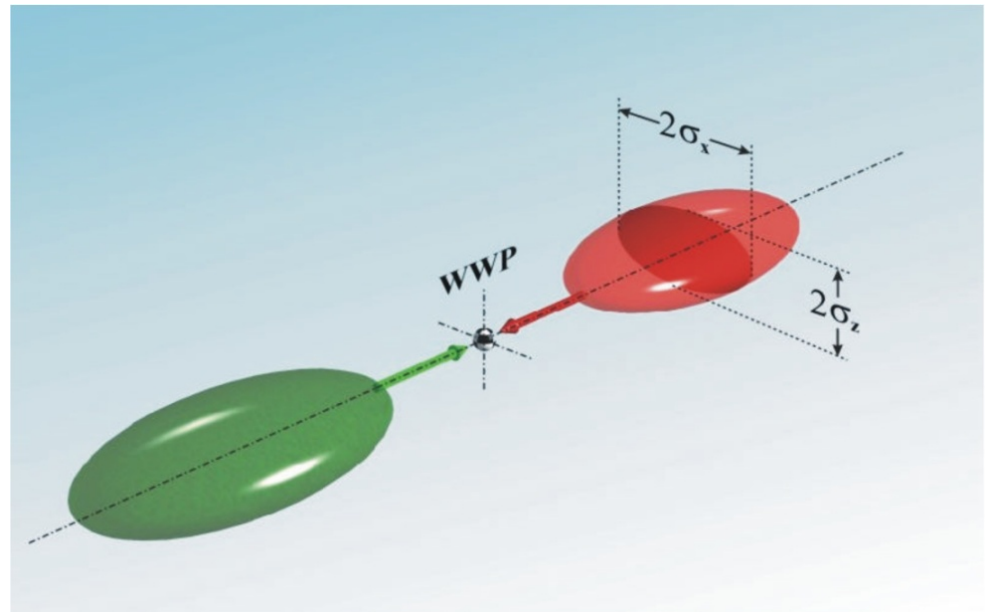
$$\dot{N} = \sigma \cdot \mathcal{L}$$

- Integrierte Luminosität:** $N = \sigma \cdot \int_{\text{Messzeit}} \mathcal{L} \cdot dt = \sigma \cdot \mathcal{I}$

e^+e^- , p-p Collider:

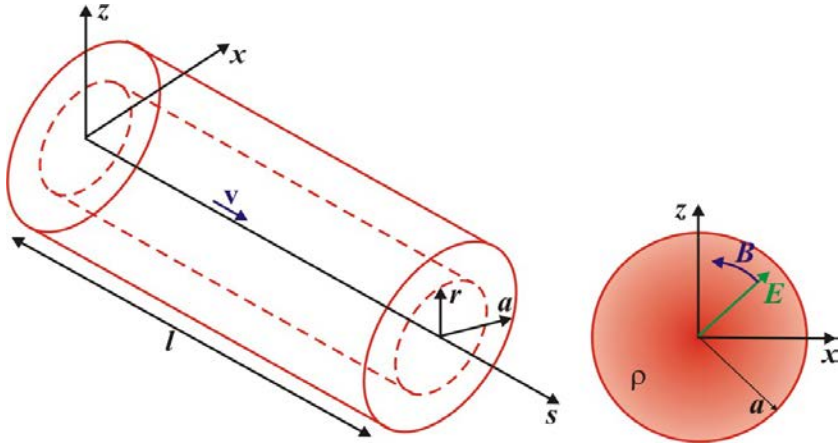
$$\mathcal{L} = \frac{1}{2 e^2 \omega_0 n} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$

$$1 \text{ ab}^{-1} / \text{Jahr} \leftrightarrow \mathcal{L} = 3 \cdot 10^{35} \frac{1}{\text{cm}^2 \text{s}}$$



Direkte Raumladung

Abstoßung im Feld der Nachbarpartikel:



$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot d^3r$$

$$\Rightarrow E_r(r) = \frac{\rho}{2\epsilon_0} \cdot r$$

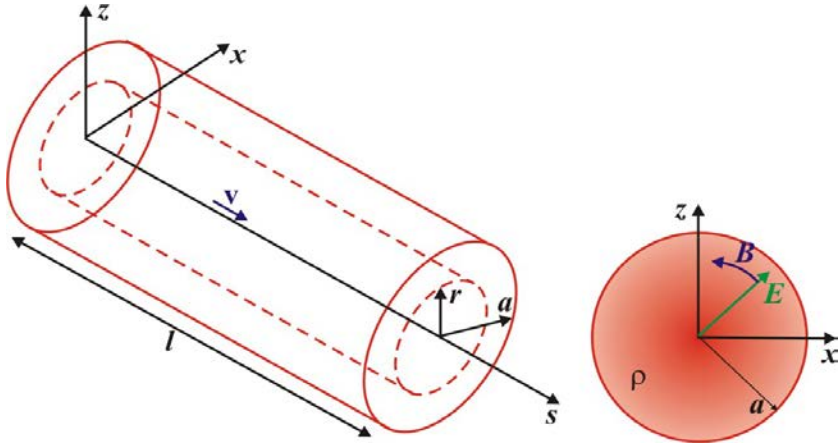
$$F^*(r) = \frac{e\rho}{2\epsilon_0} \cdot r$$

Transformation ins Laborsystem:

- Lorentzkontraktion $\rightarrow$ Faktor γ

Direkte Raumladung

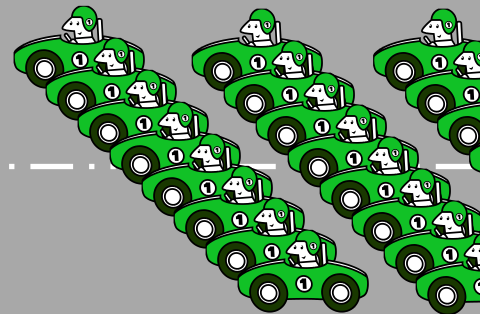
Abstoßung im Feld der Nachbarpartikel:



$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot d^3r$$

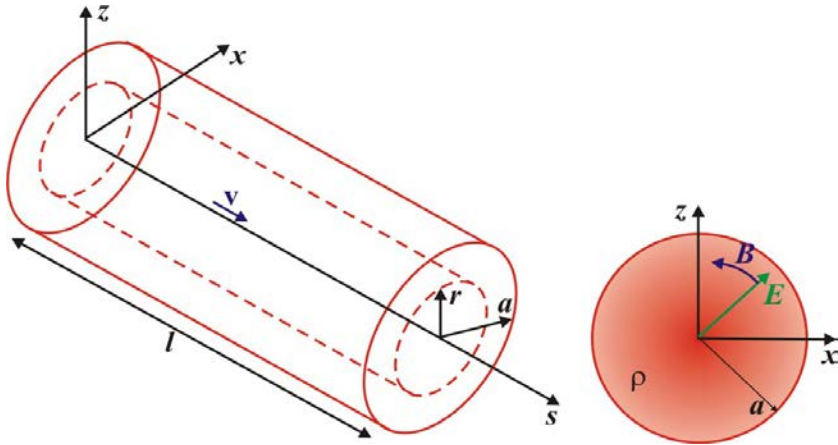
$$\Rightarrow E_r(r) = \frac{\rho}{2\epsilon_0} \cdot r$$

$$F^*(r) = \frac{e\rho}{2\epsilon_0} \cdot r$$



Direkte Raumladung

Abstoßung im Feld der Nachbarpartikel:



$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \iiint_V \rho \cdot d^3r$$

$$\Rightarrow E_r(r) = \frac{\rho}{2\epsilon_0} \cdot r$$

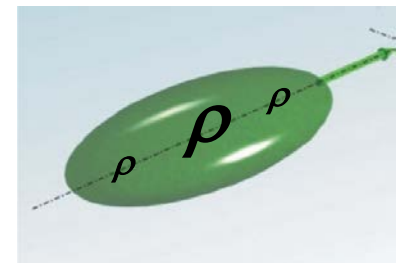
$$F^*(r) = \frac{e\rho}{2\epsilon_0} \cdot r$$

Transformation ins Laborsystem:

- Lorentzkontraktion $\rightarrow$ Faktor γ
- Ladungsdichteverdünnung $\rightarrow$ Faktor γ

$$\gamma m_0 \ddot{x} = F_x = \frac{F_x^*}{\gamma^2}$$

$$\Delta Q_{x,z} = - \frac{e}{8\pi^2 \epsilon_0 m_0 (\beta c)^3 \gamma^3} \cdot \frac{I}{\epsilon_{x,z}} \cdot L$$



Beschleunigerkette @ CERN

Protonen:

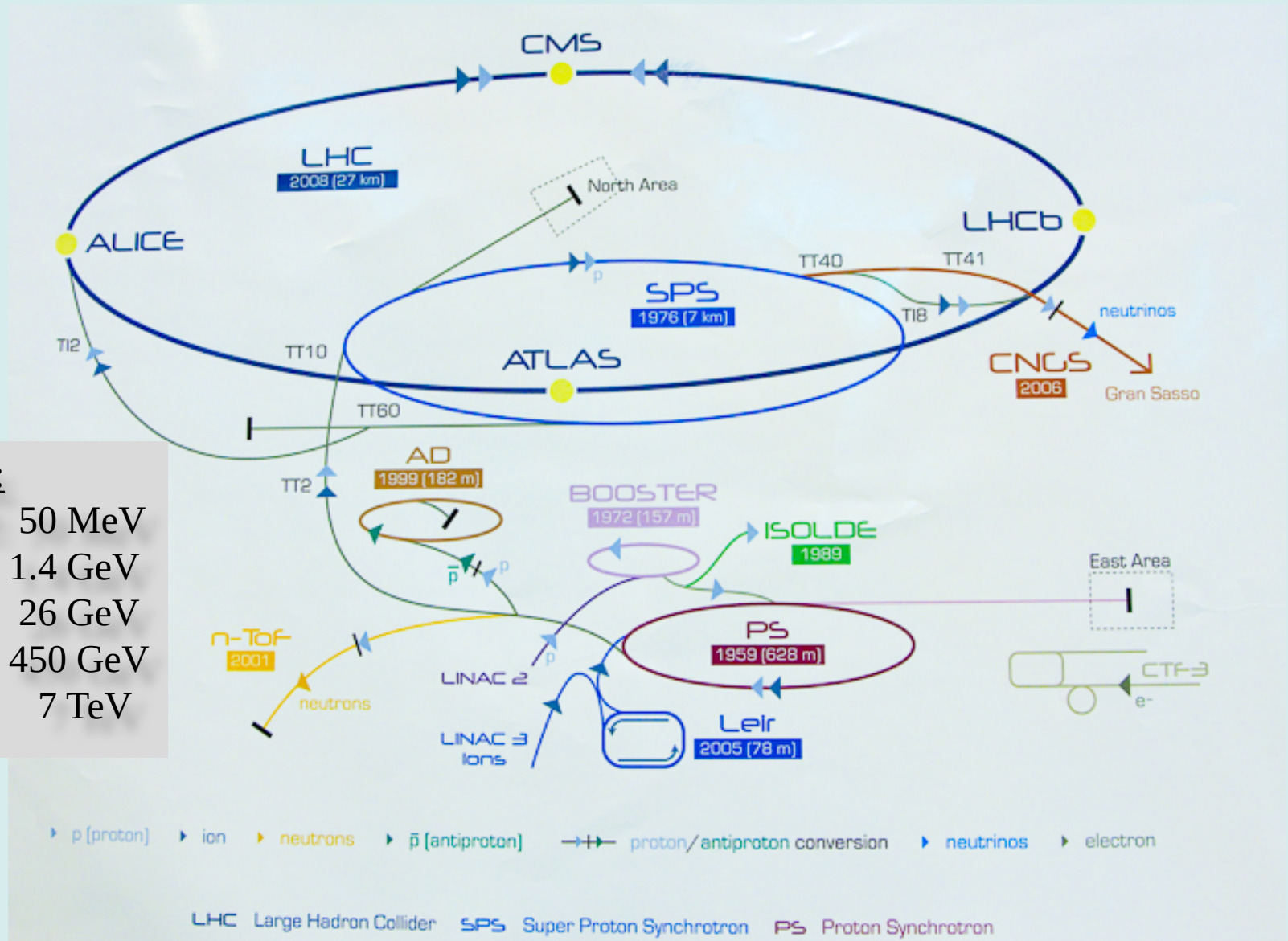
LINAC2: 50 MeV

PSB: 1.4 GeV

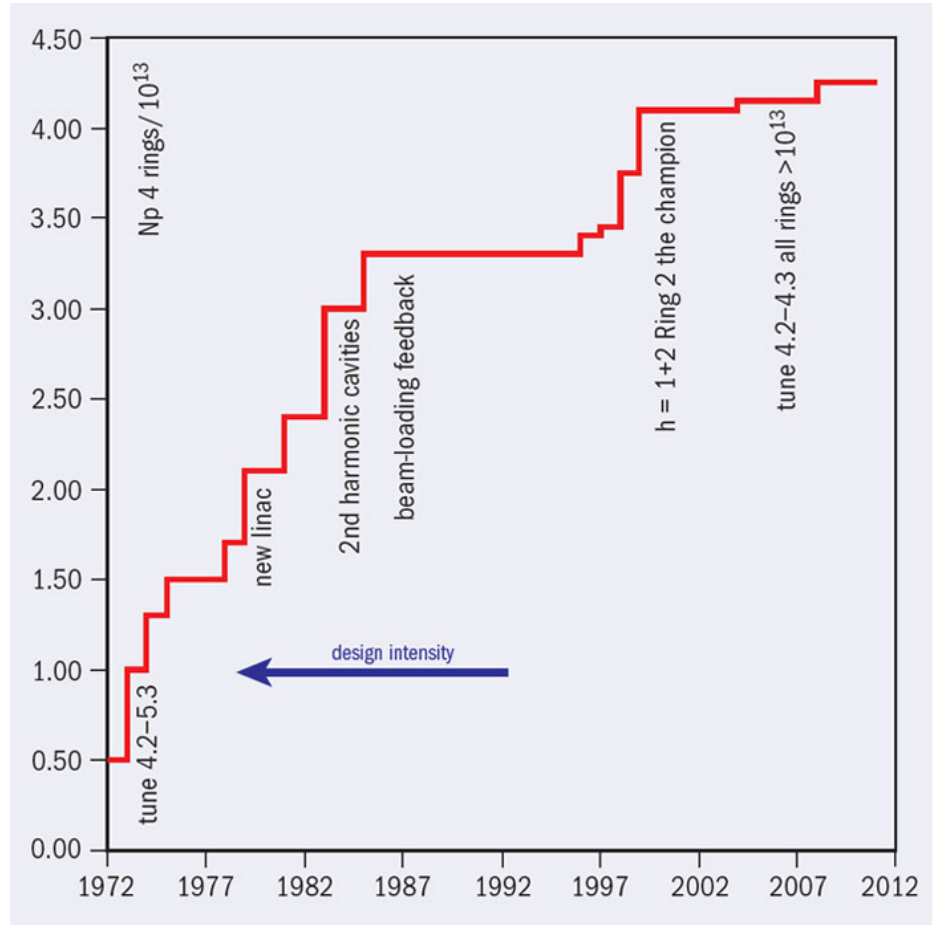
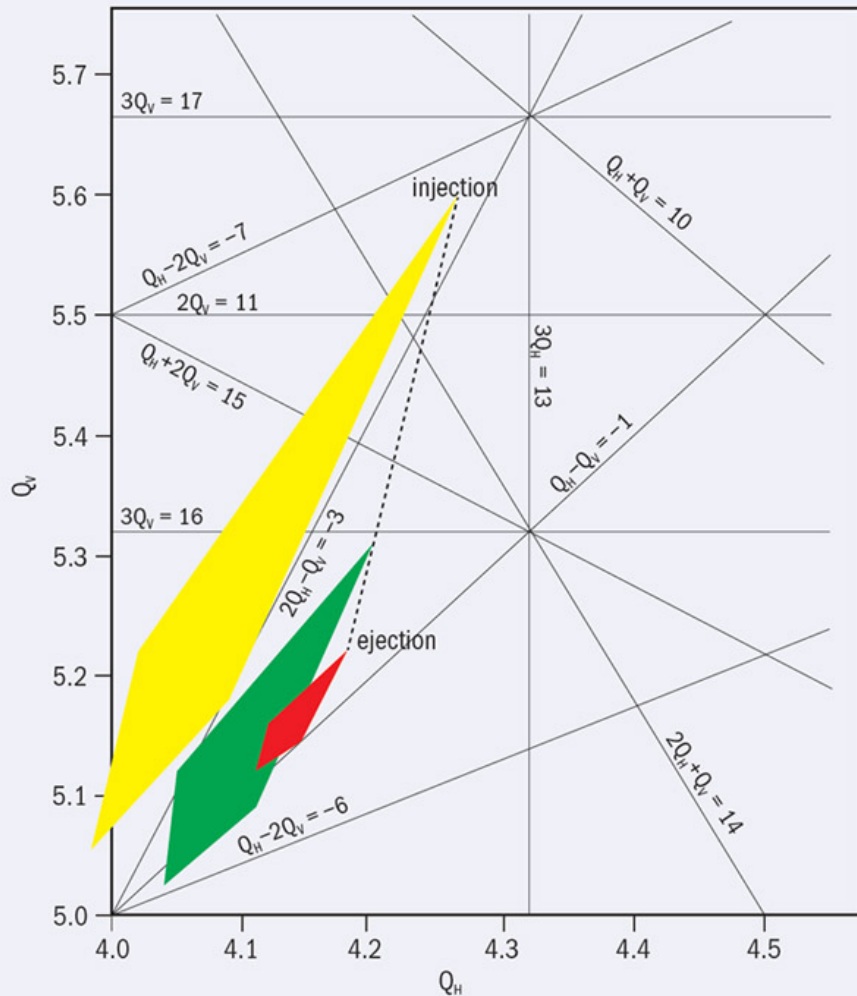
PS: 26 GeV

SPS: 450 GeV

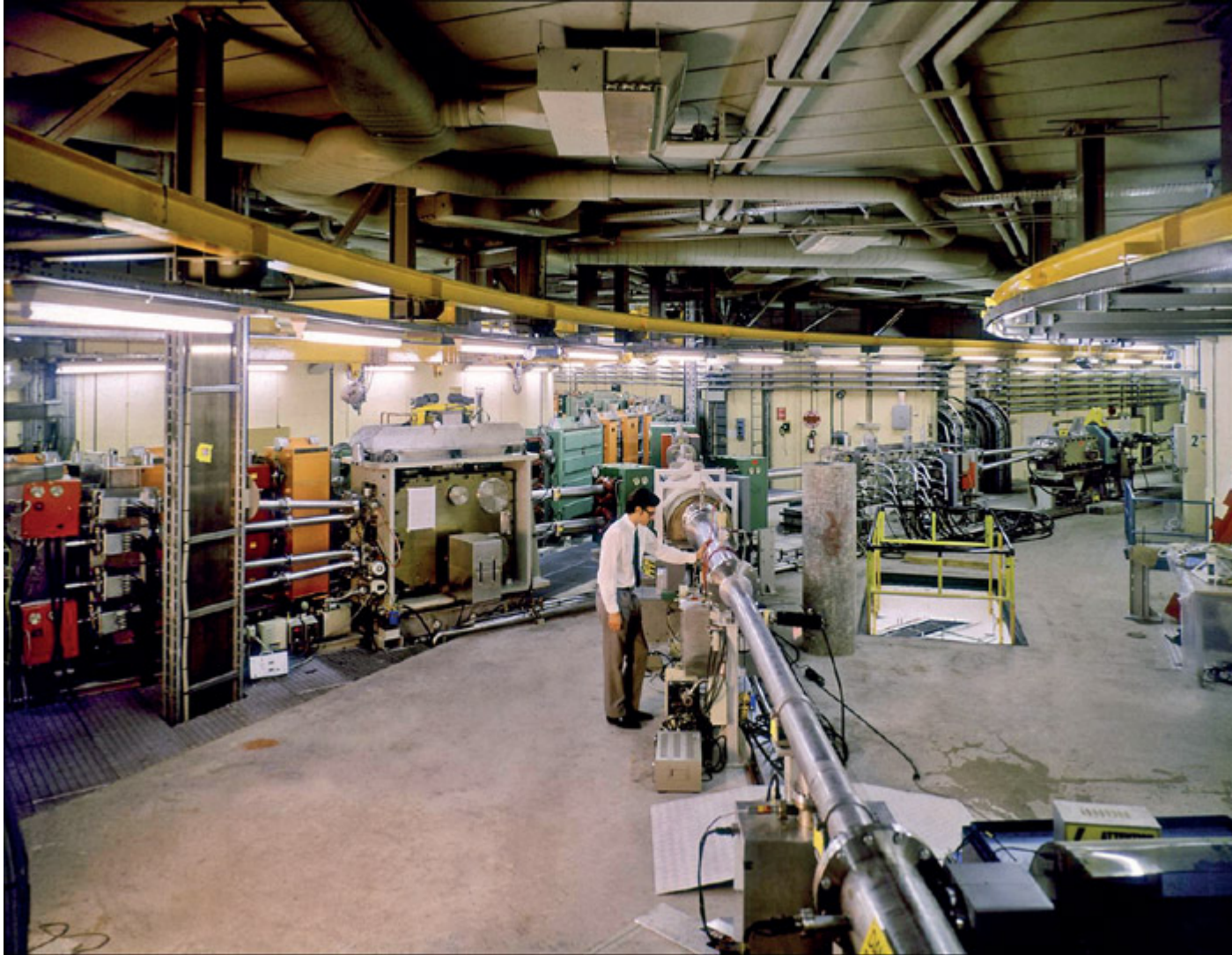
LHC: 7 TeV



PS Booster @ CERN

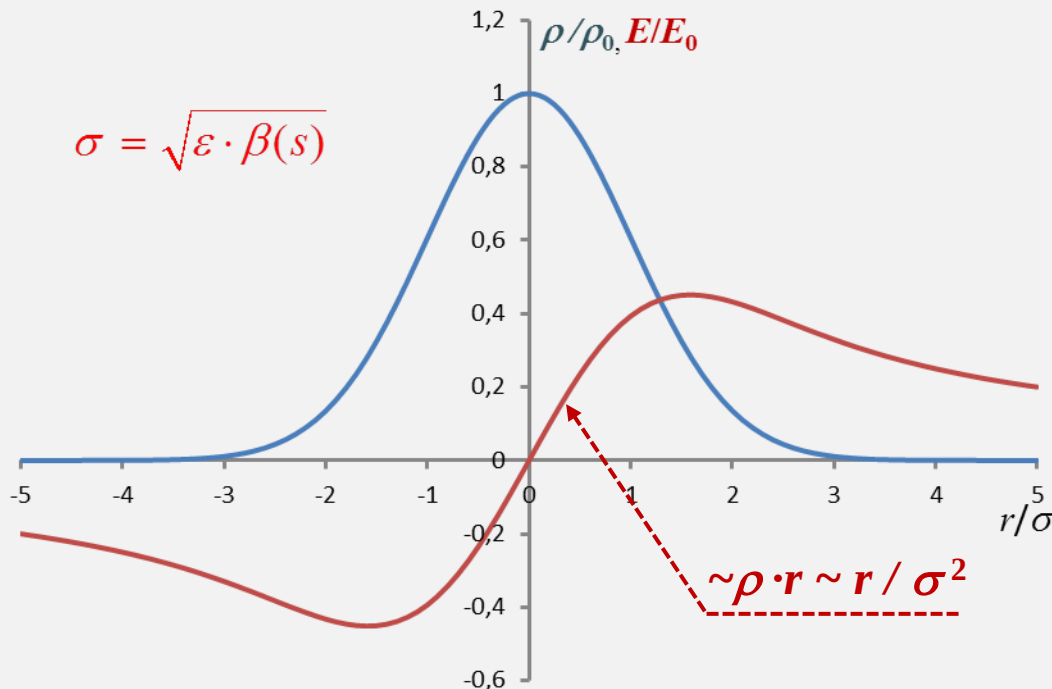
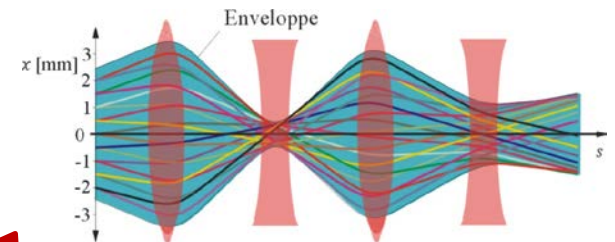
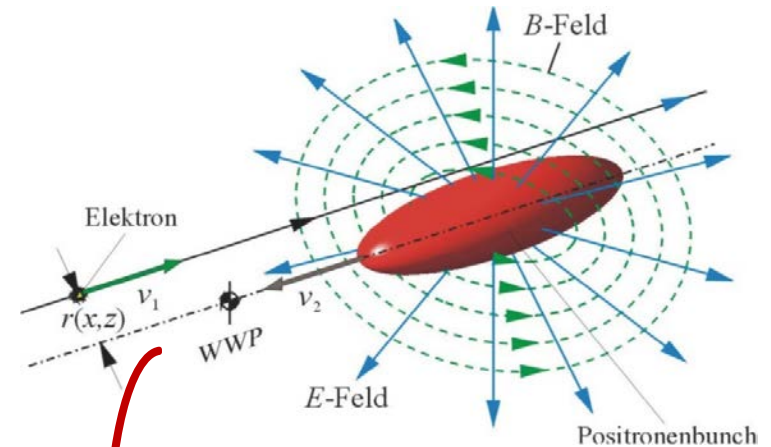


PS Booster @ CERN



Strahl-Strahl-WW

**Zusätzliche Fokussierung
bzw. Defokussierung
bei der Strahlkreuzung**



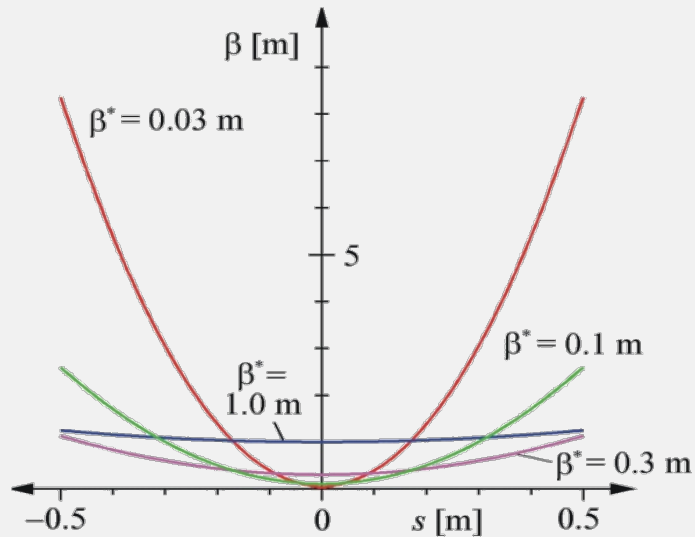
Strahl-Strahl-Parameter:

$$\Delta Q_x = -\frac{N r_e}{2\pi} \cdot \frac{1}{\gamma} \cdot \frac{\beta_x^*}{(\sigma_x + \sigma_z) \sigma_x}$$

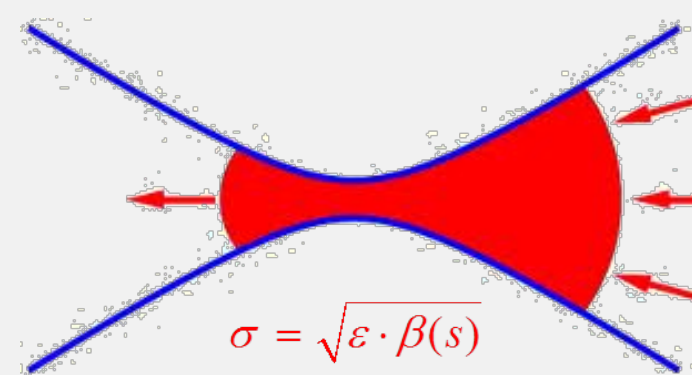
$$\Delta Q_z = -\frac{N r_e}{2\pi} \cdot \frac{1}{\gamma} \cdot \frac{\beta_z^*}{(\sigma_x + \sigma_z) \sigma_z}$$

Strahlbreitenreduktion

Strahlaufweitung:



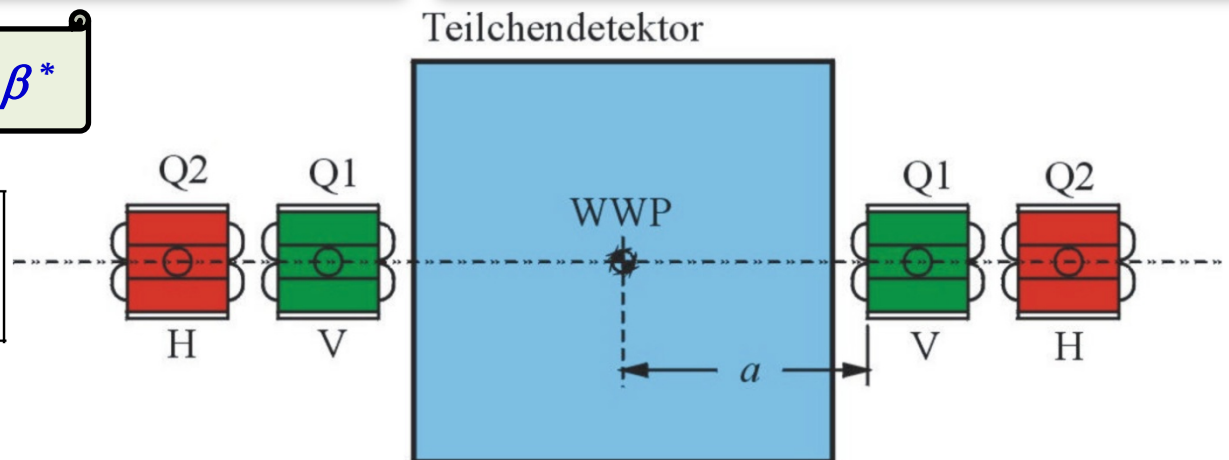
Hourglass-Effekt:



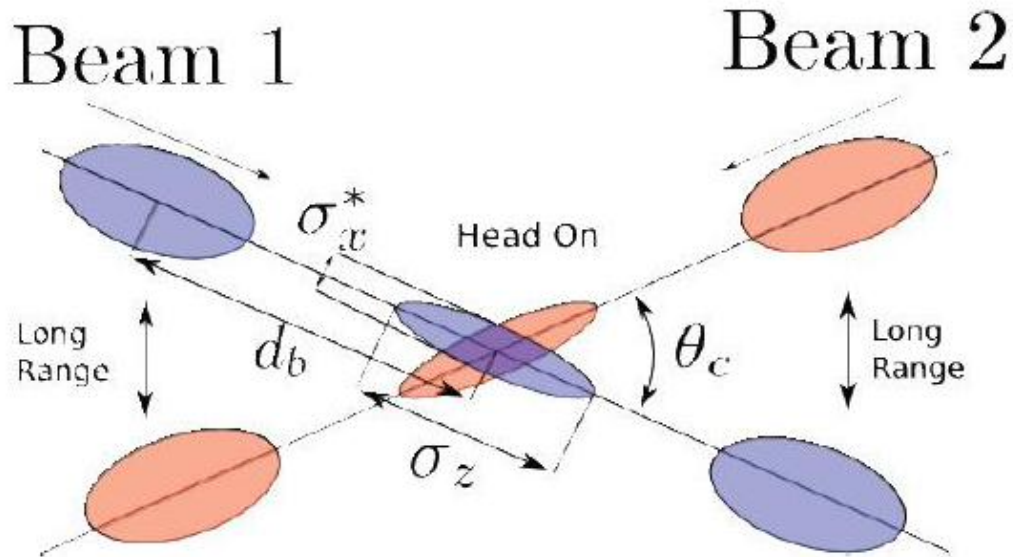
kleine Paketlänge!

$$\sigma < 1,7 \cdot \beta^*$$

$$\mathcal{L} = \frac{1}{2 e^2 \omega_0 n} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$



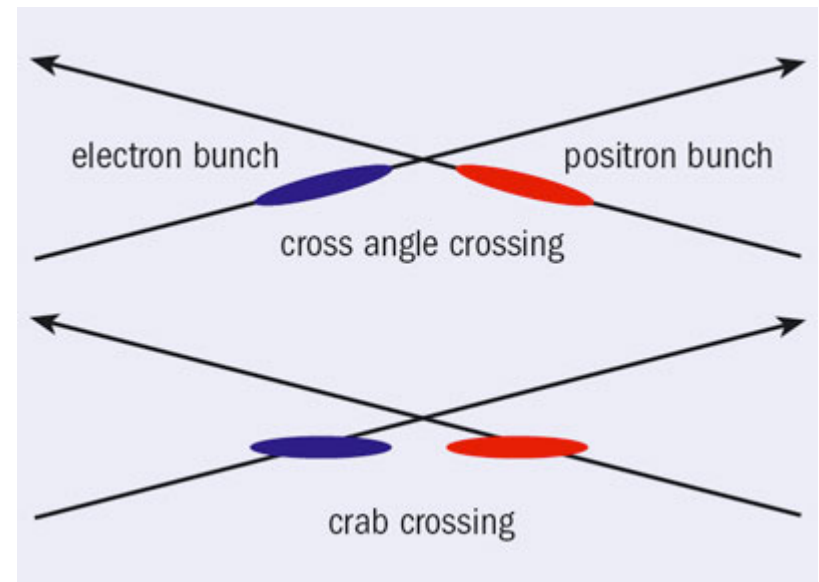
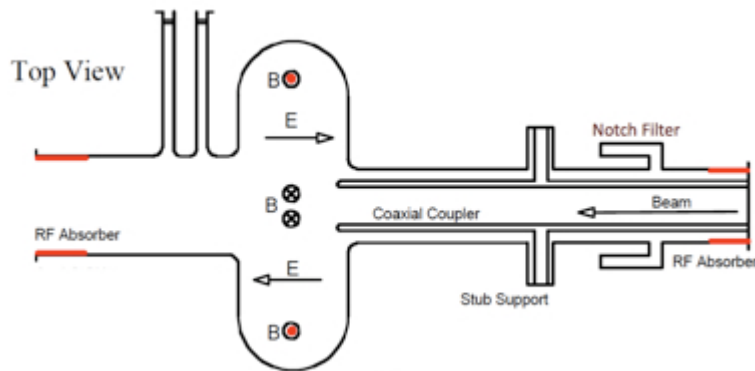
Strahlkreuzung



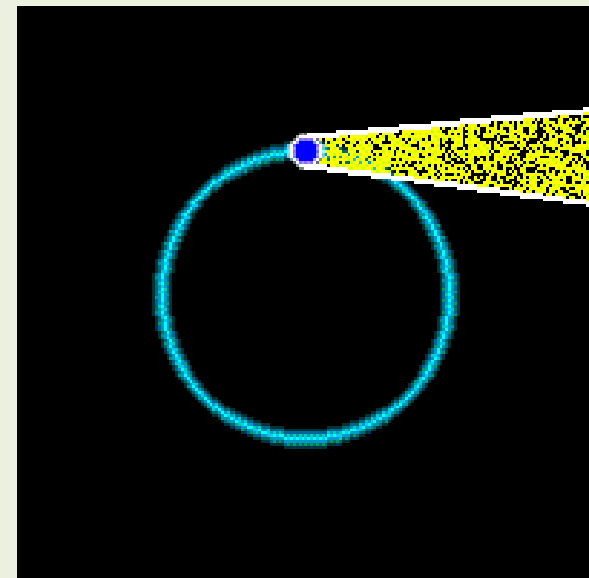
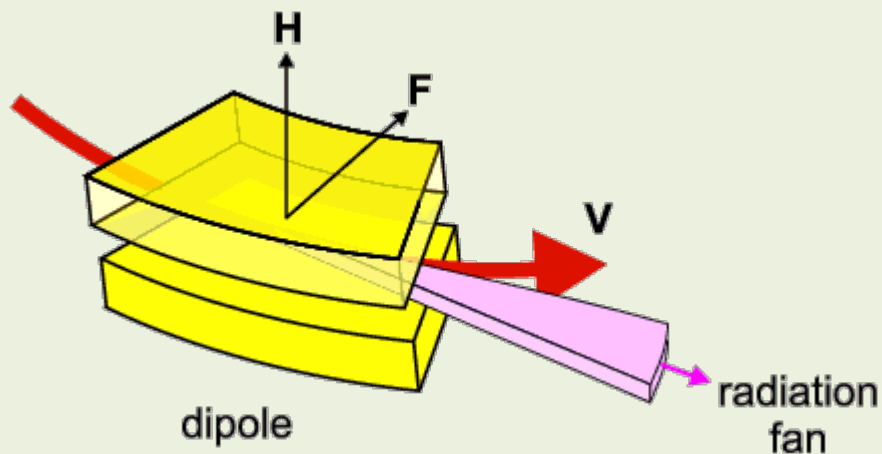
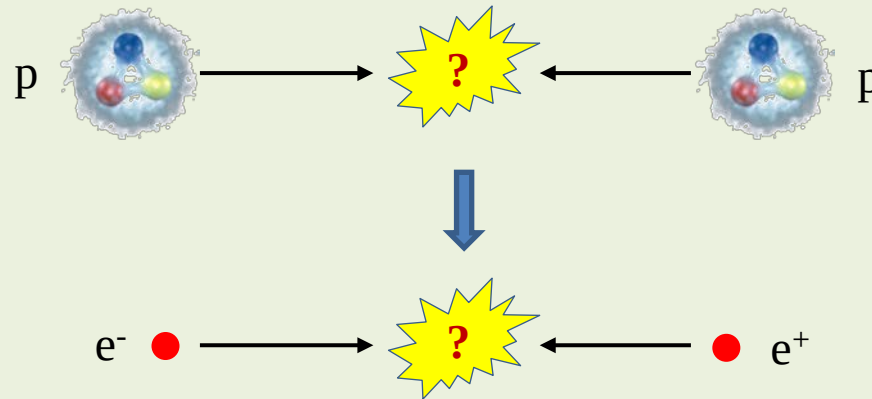
- ausreichende Strahlseparation
- kleiner Kreuzungswinkel
- hinreichende Paketabstände
- ...

$$\mathcal{L} = \frac{1}{2 e^2 \omega_0 n} \cdot \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_z}$$

Crab-Cavities:

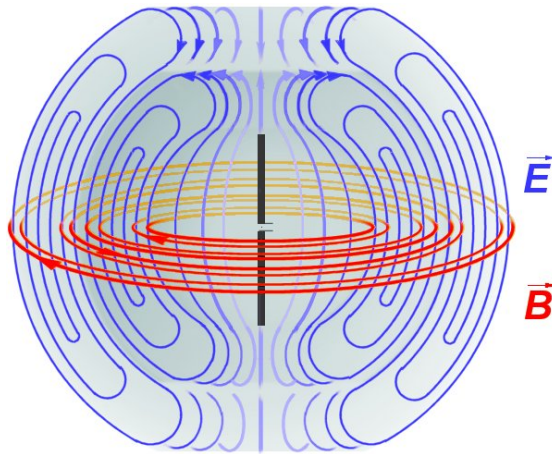


Elektronen / Positronen



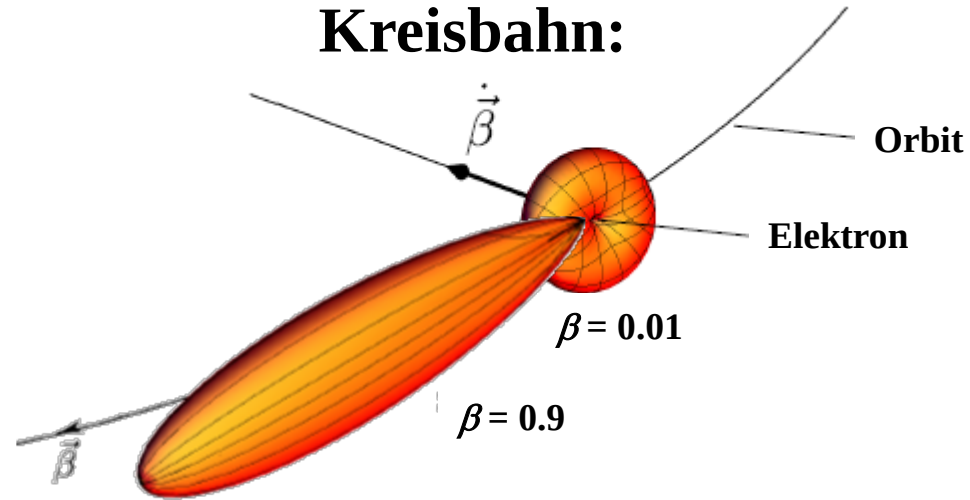
Synchrotronstrahlung

Hertzscher Dipol:



$$P = \frac{e^2}{12\pi\epsilon_0 c^3} \cdot \omega^4 d^2$$

Kreisbahn:



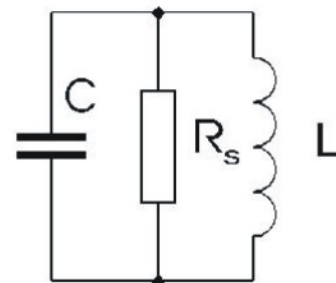
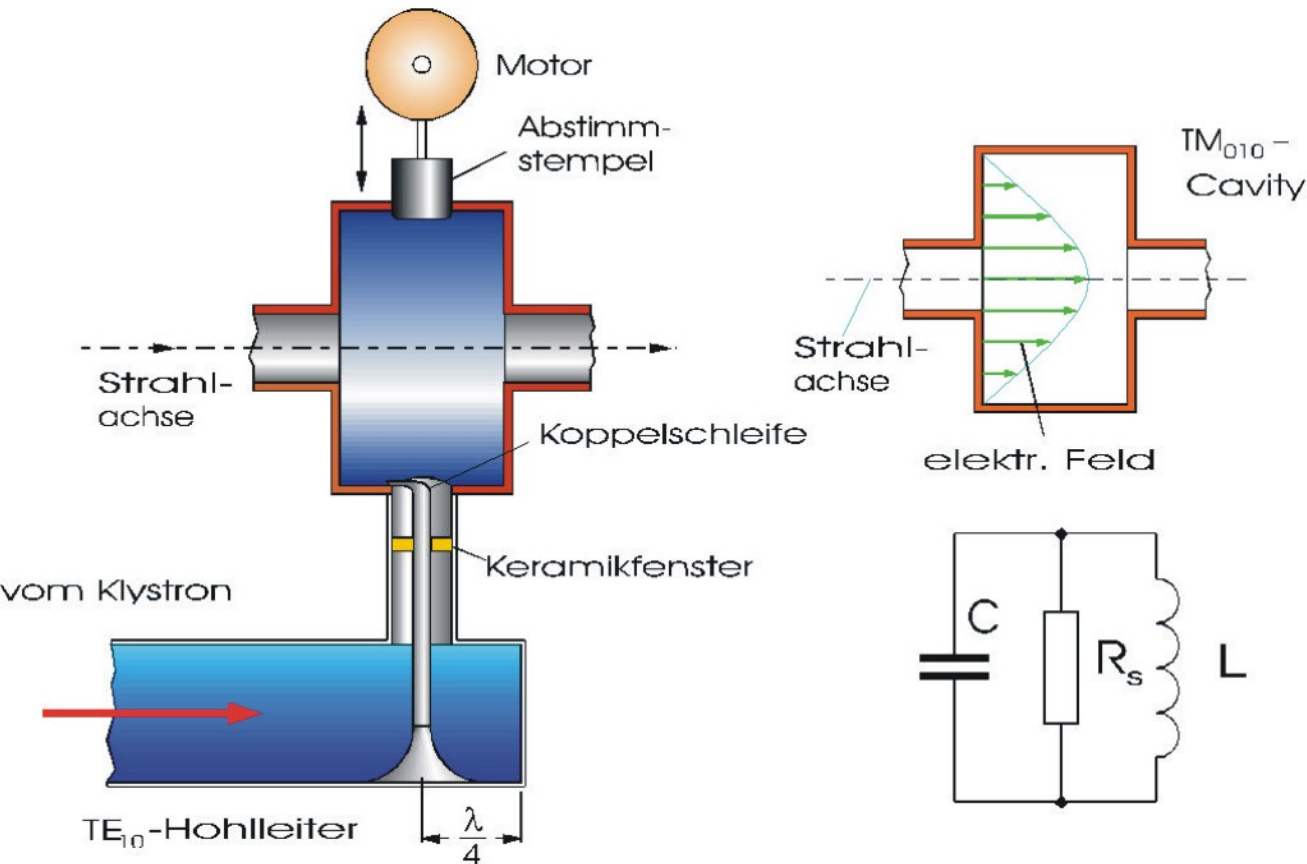
$$P = \frac{e^2 c}{6\pi\epsilon_0} \cdot \frac{\gamma^4}{R^2}$$

$$U[\text{kV}] = 88,5 \cdot \frac{E^4 [\text{GeV}^4]}{R[\text{m}]} \Rightarrow U|_{E=104,5 \text{ GeV}} = \underline{\underline{3,4 \text{ GeV}}}$$

$$P_{\text{rad}}(6 \text{ mA @ } 104,5 \text{ GeV}) = 18 \text{ MW}$$



Resonatoren I



Kenngroößen:

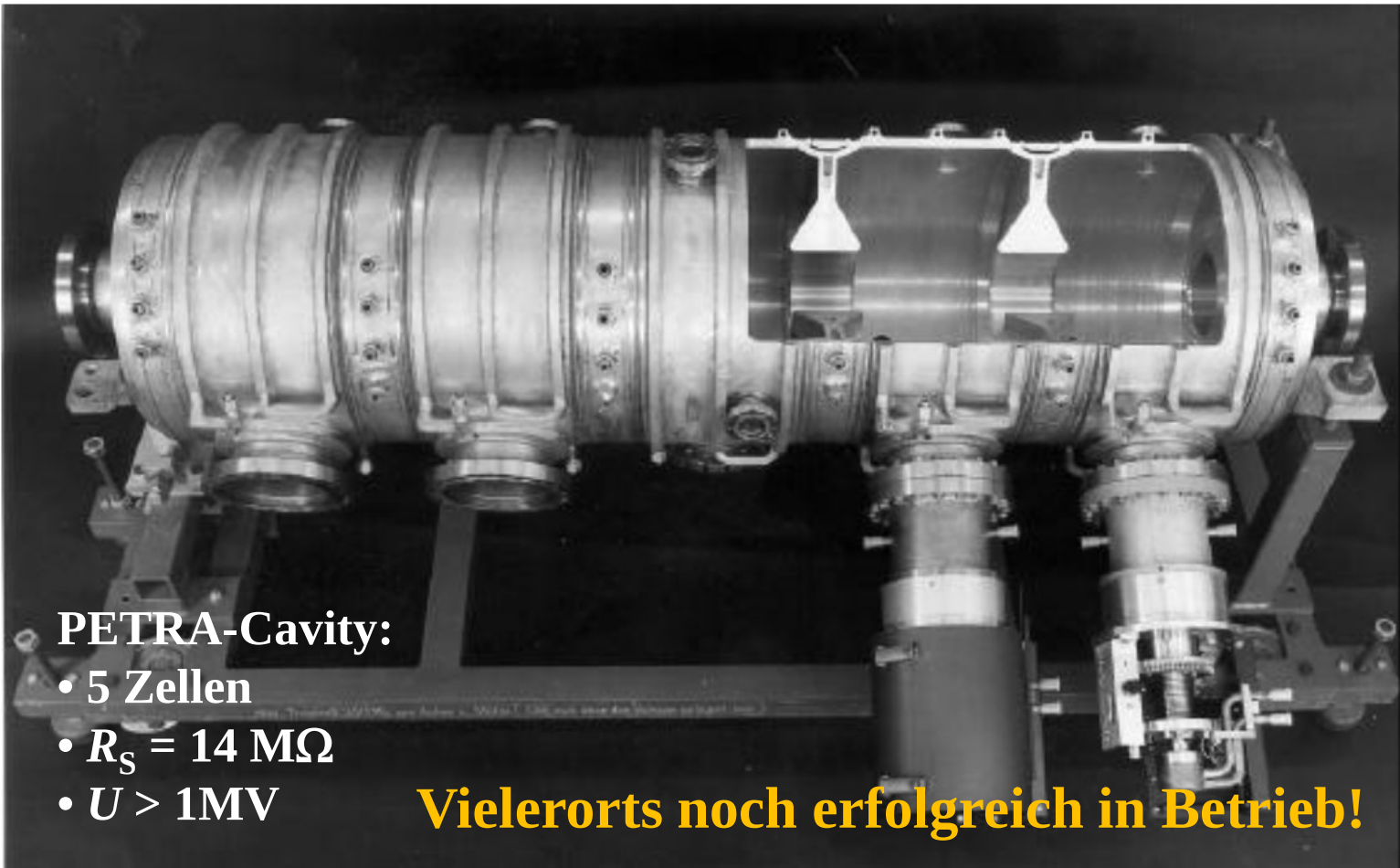
- Shuntimpedanz R_s
nc: $R_s \approx \text{M}\Omega$
- Kreisgüte Q
nc: $Q \approx 10^4$
- Geometriefaktor R_s/Q

Für Spannung U
benötigte HF-Leistung:

$$P = \frac{U^2}{2R_s}$$

$$U = 1\text{MV} \leftrightarrow P = 100\text{ kW}$$

Immerhin:

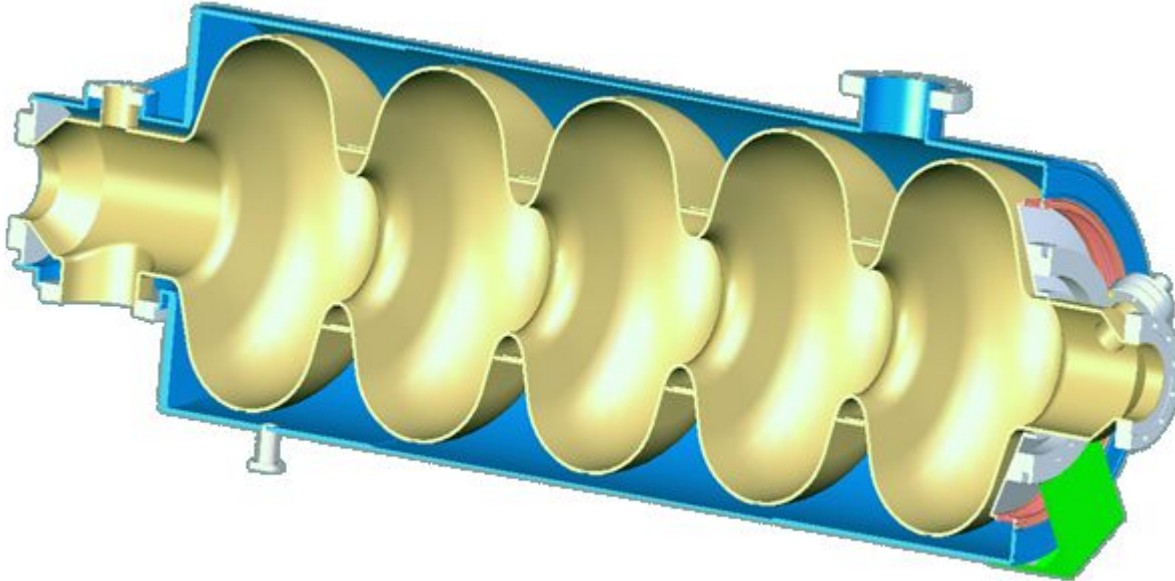


PETRA-Cavity:

- 5 Zellen
- $R_s = 14 \text{ M}\Omega$
- $U > 1 \text{ MV}$

Vielerorts noch erfolgreich in Betrieb!

Resonatoren II



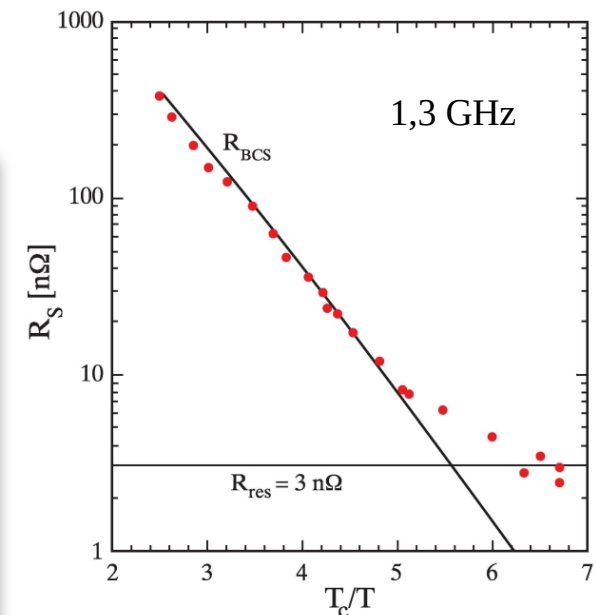
Frequenz begrenzt
durch ansteigenden
Oberflächenwiderstand

Supraleitende Resonatoren aus Niob:

→ **Reduktion der Verluste um Faktor $> 10^5$!**

... hervorgerufen durch nc Elektronen ...

aber: $E_{\max} < 50 \text{ MV/m}$



Large Electron Positron Collider

LEP-Daten:

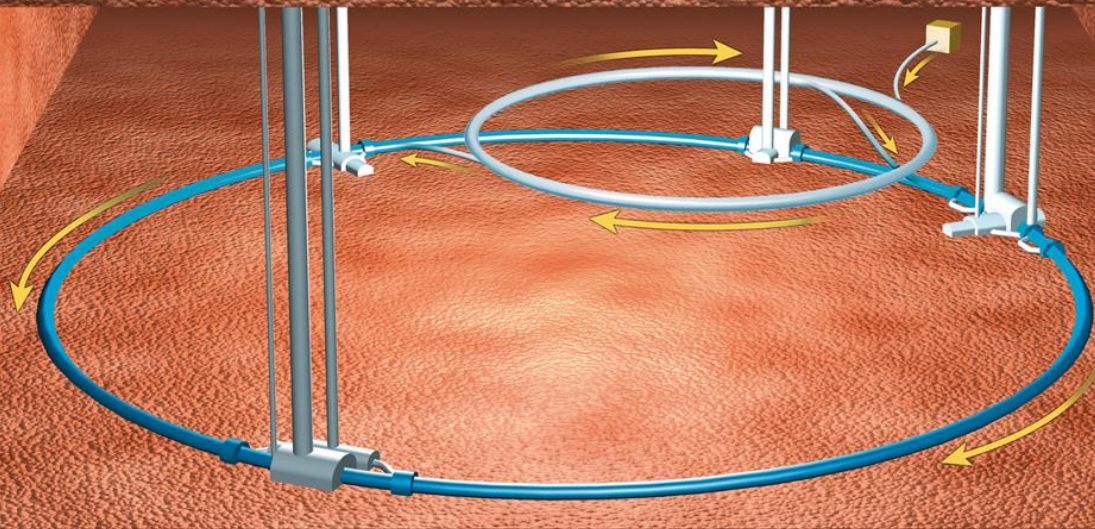
$E \leq 104,5 \text{ GeV}$

$R \approx 3,1 \text{ km}$

$B \approx 0,12 \text{ Tesla}$

$$P, U \sim E^4$$

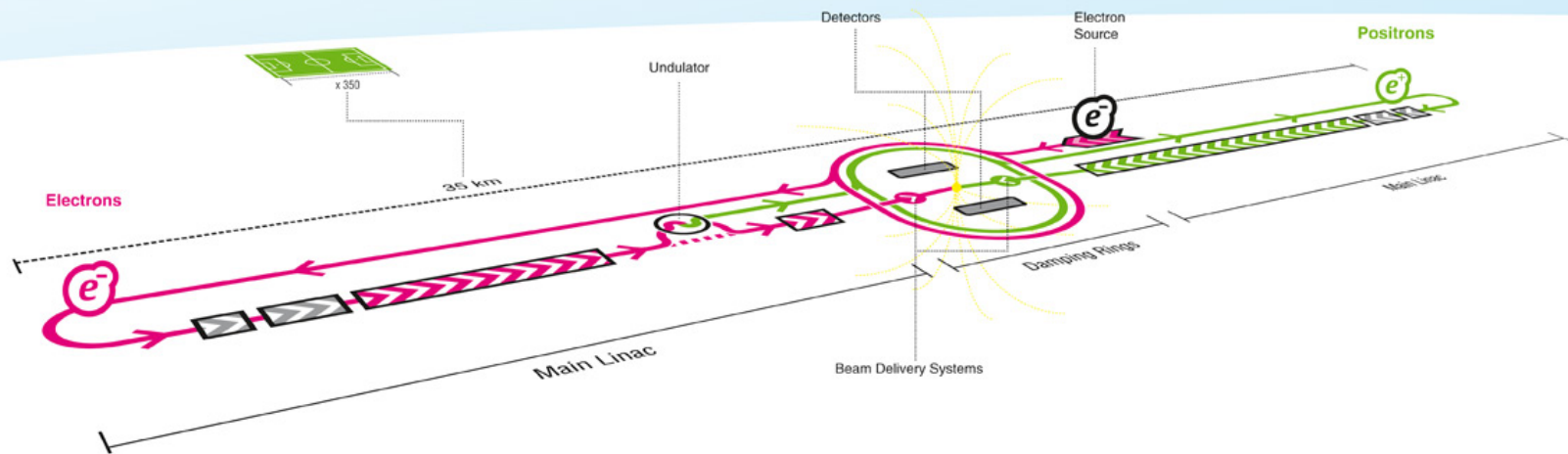
limitiert durch
Synchrotronstrahlung



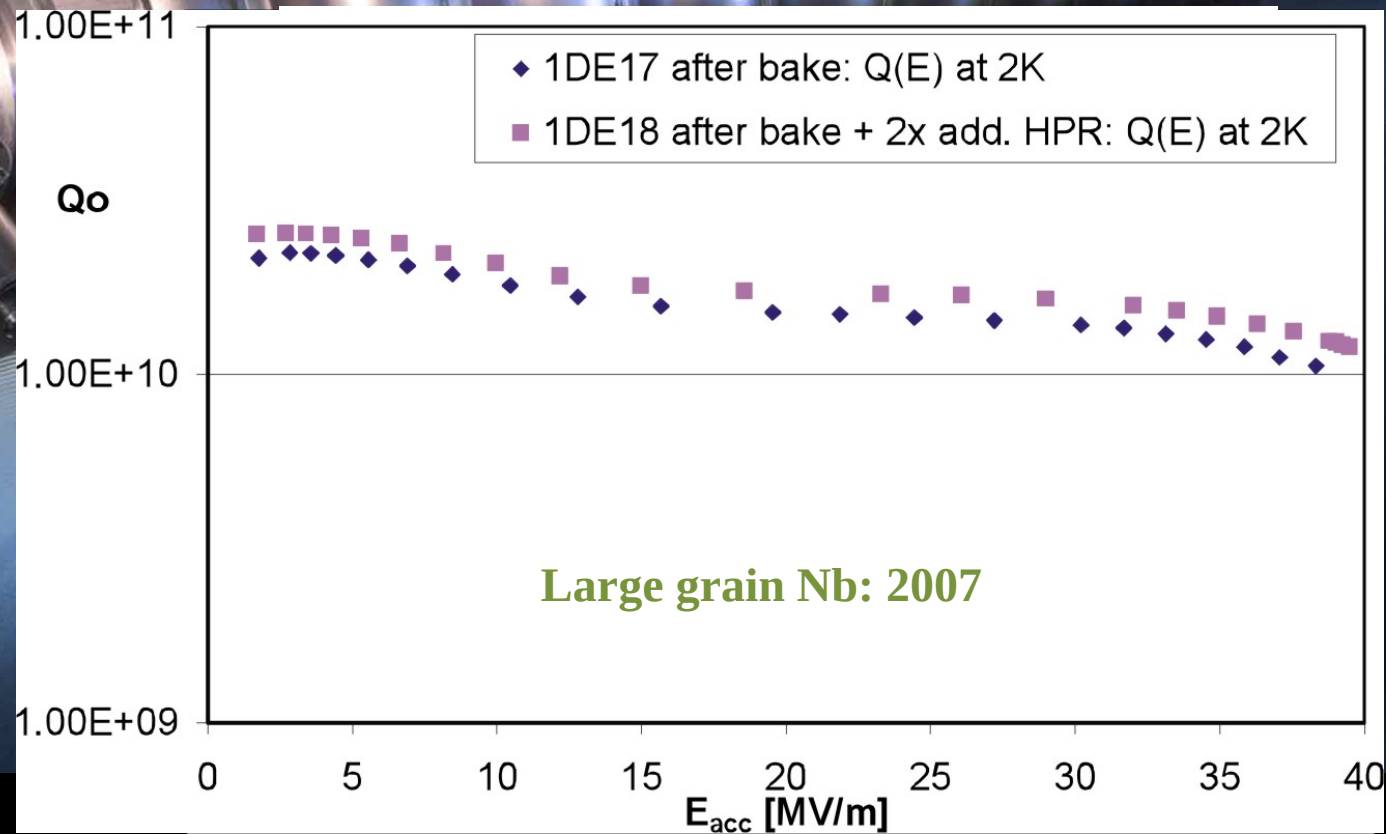
$e^- / e^+ @ 500 \text{ GeV}$

*ILC:
International
Linear
Collider*

*sc linear accelerator
 $\nu = 1.3 \text{ GHz}$
 $E \geq 25 \text{ MV/m}$*



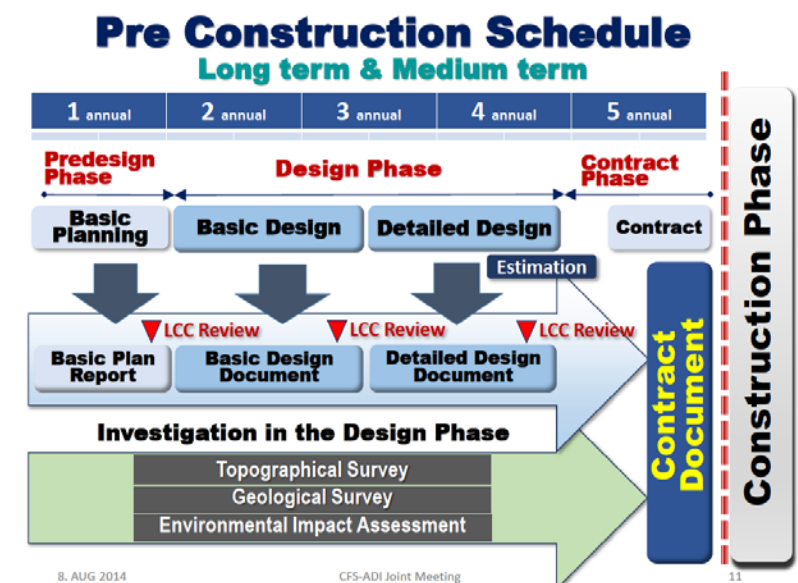
Erreichen hoher Gradienten:





Preparation Steps:

- 80 μ m chemical etching (BCP)
- 800°C firing
- 100 μ m electropolishing (EP)
- high pressure water rinsing (HPR)
- baking (under Ar) atmosphere
- additional HPR
- assembly in cleanroom



... es geht voran ...

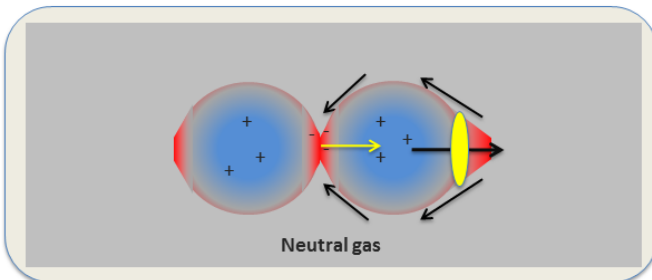
Inbetriebnahme 2030?

Zukunftsträume

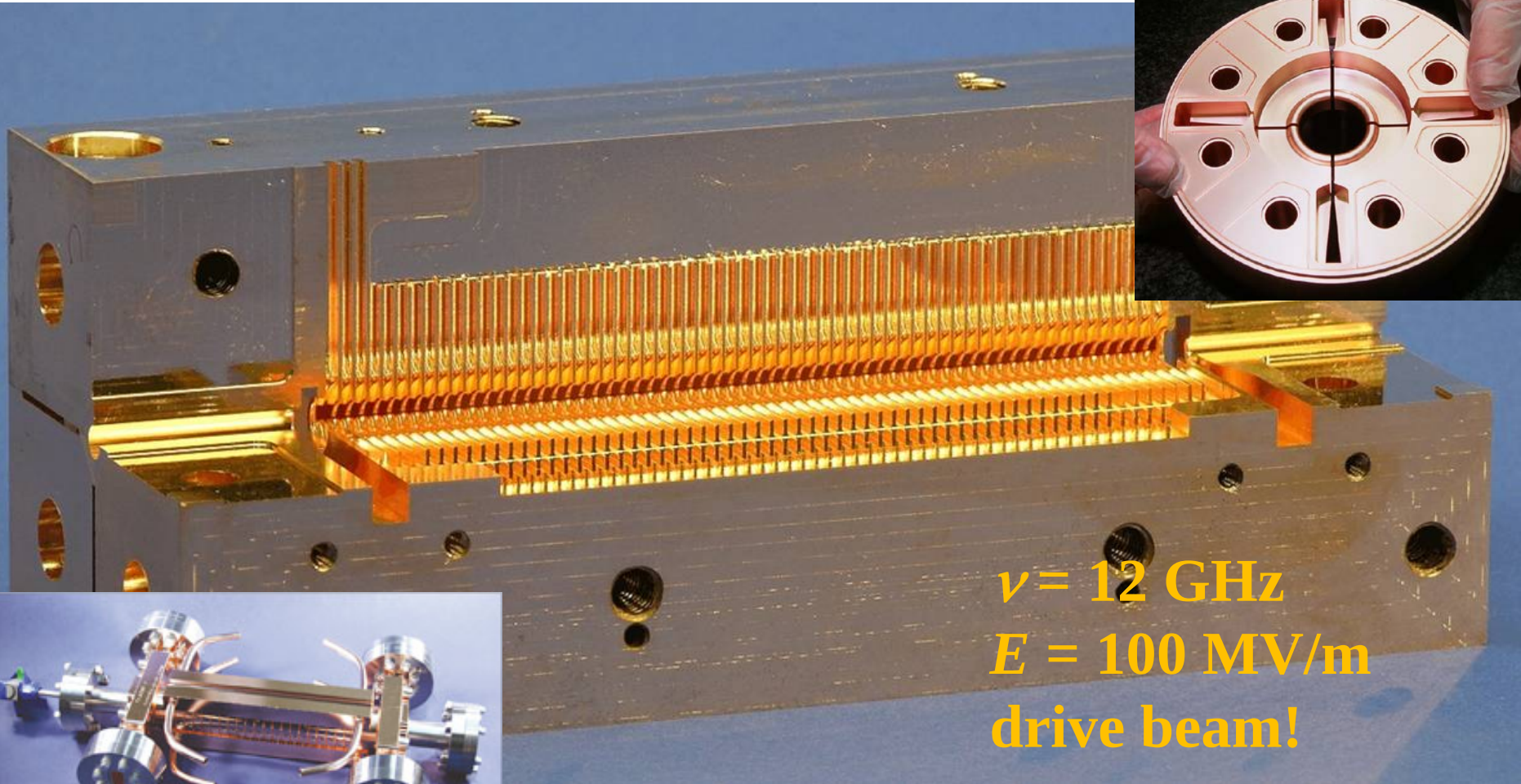


Plasma Wakefield Acceleration

Novel particle acceleration technique



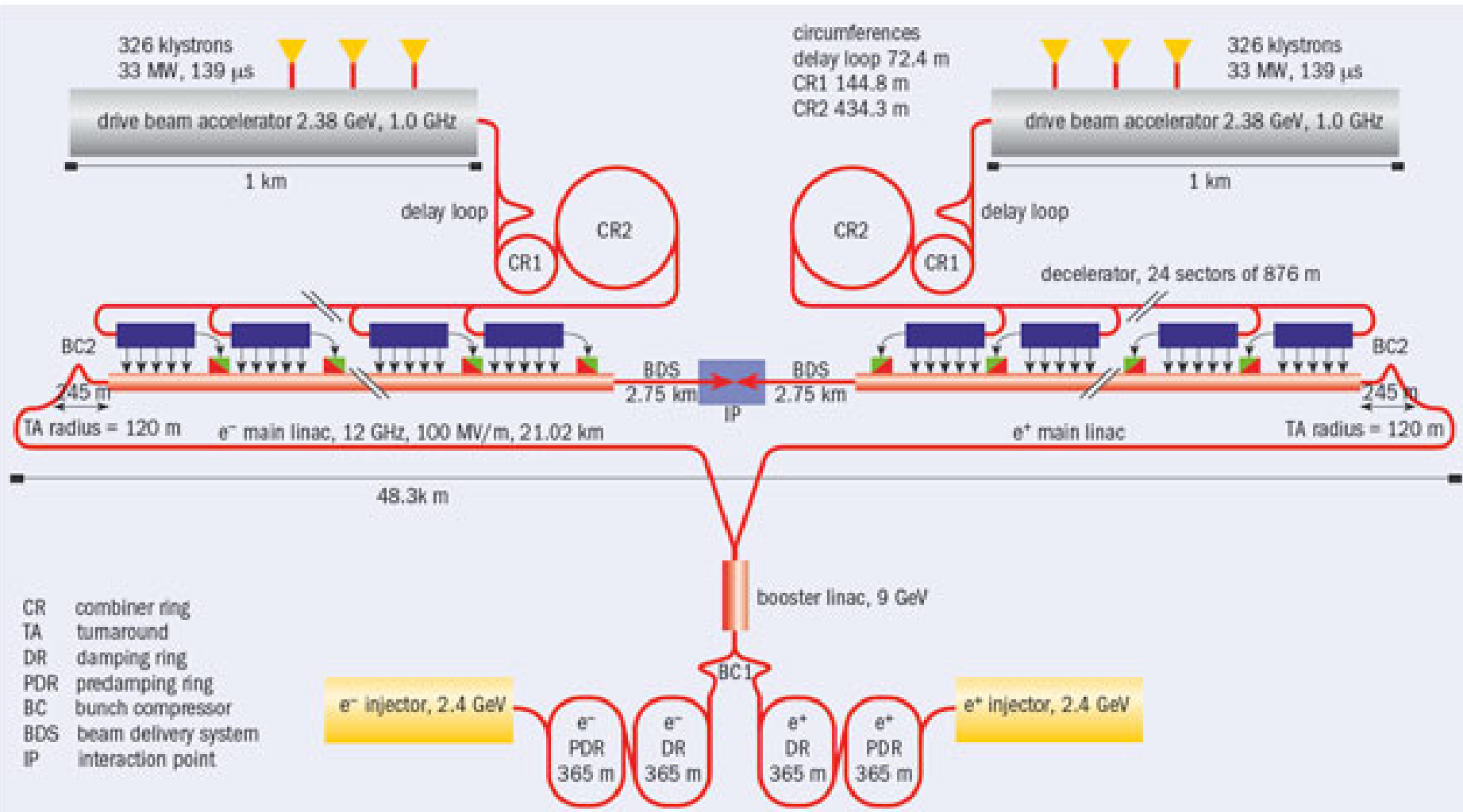
Compact Linear Collider (CLIC)



$\nu = 12 \text{ GHz}$
 $E = 100 \text{ MV/m}$
drive beam!

A promising normal conducting concept!

CLIC set-up



Legend

— CERN existing LHC

Potential underground siting :

●●● CLIC 500 GeV

●●● CLIC 1.5 TeV

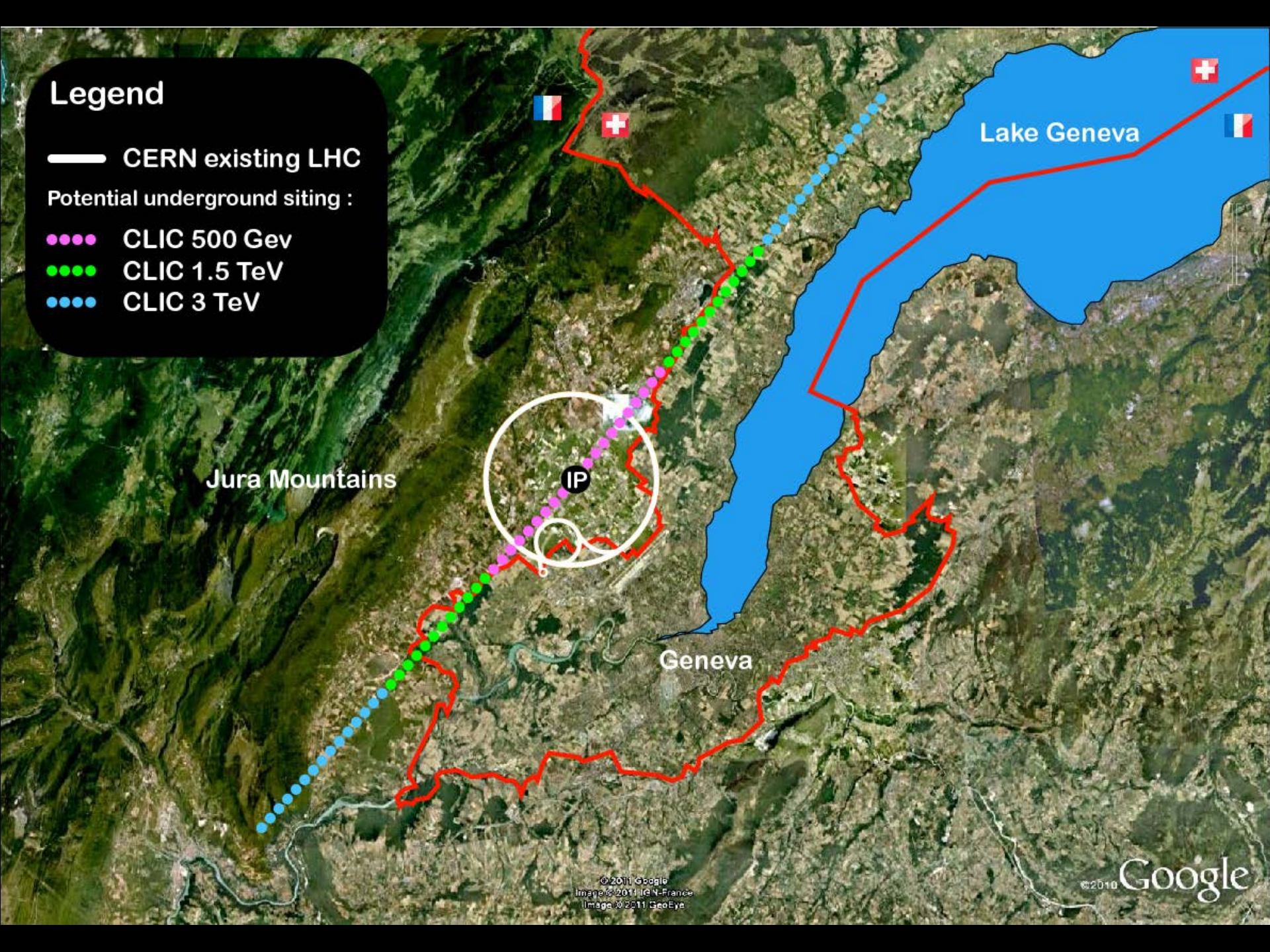
●●● CLIC 3 TeV

Jura Mountains

IP

Geneva

Lake Geneva



FCC Study (Future Circular Colliders)

CDR and cost review for the next ESU (2018)

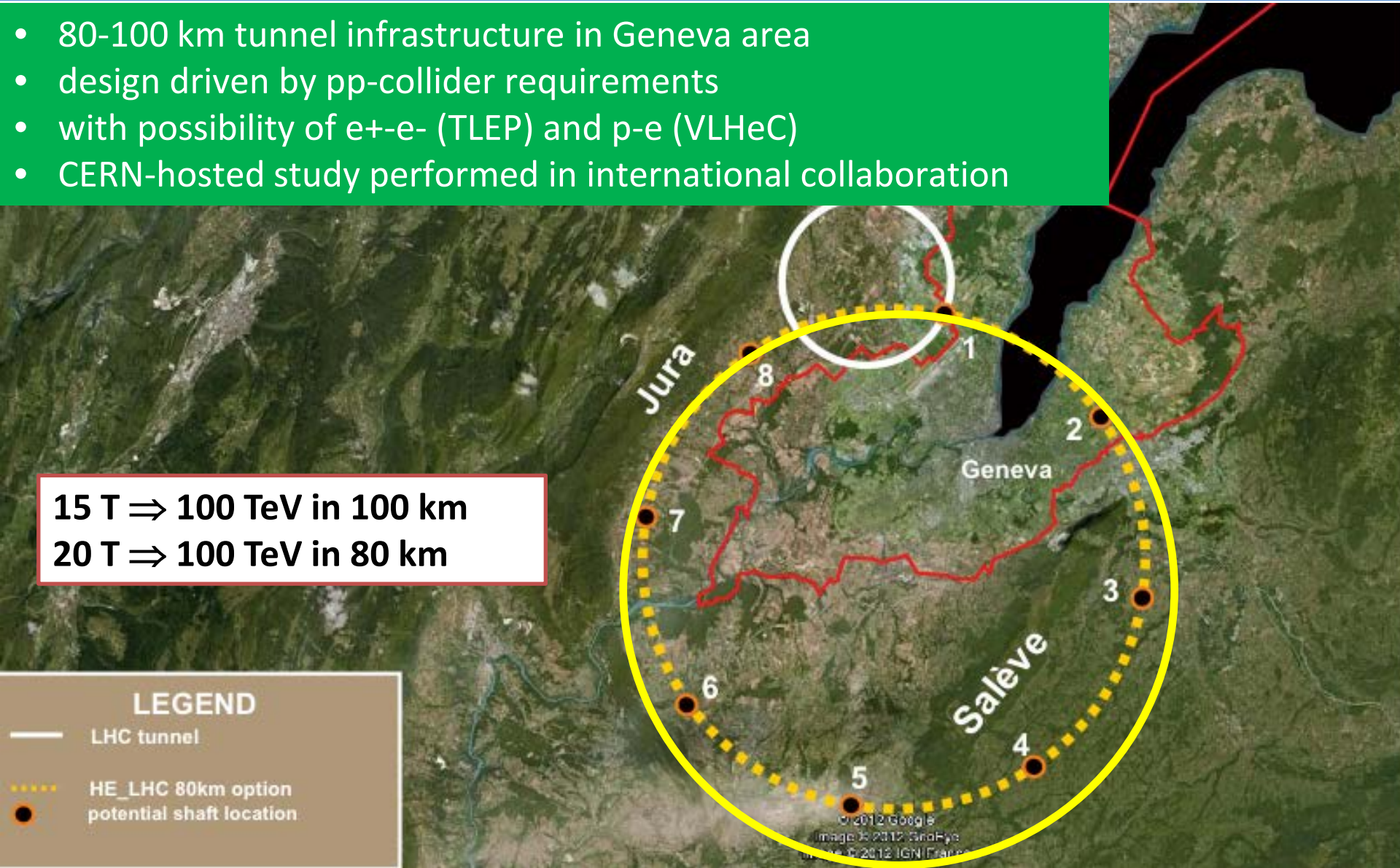
- 80-100 km tunnel infrastructure in Geneva area
- design driven by pp-collider requirements
- with possibility of e^+e^- (TLEP) and p-e (VLHeC)
- CERN-hosted study performed in international collaboration

15 T $\Rightarrow$ 100 TeV in 100 km

20 T $\Rightarrow$ 100 TeV in 80 km

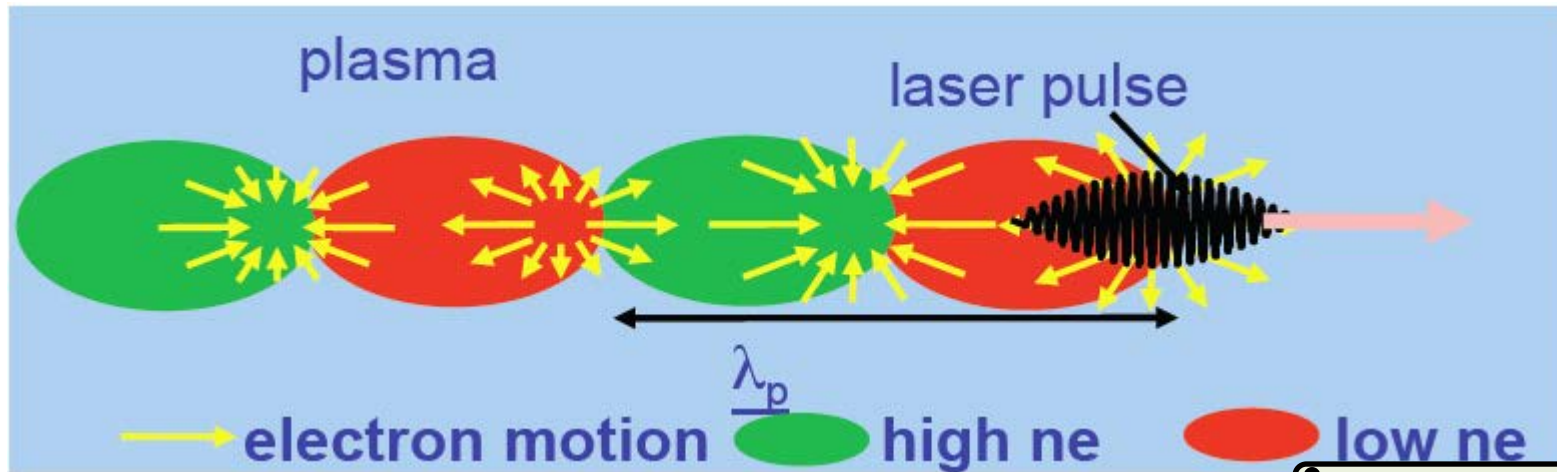
LEGEND

- LHC tunnel
- HE_LHC 80km option
- potential shaft location

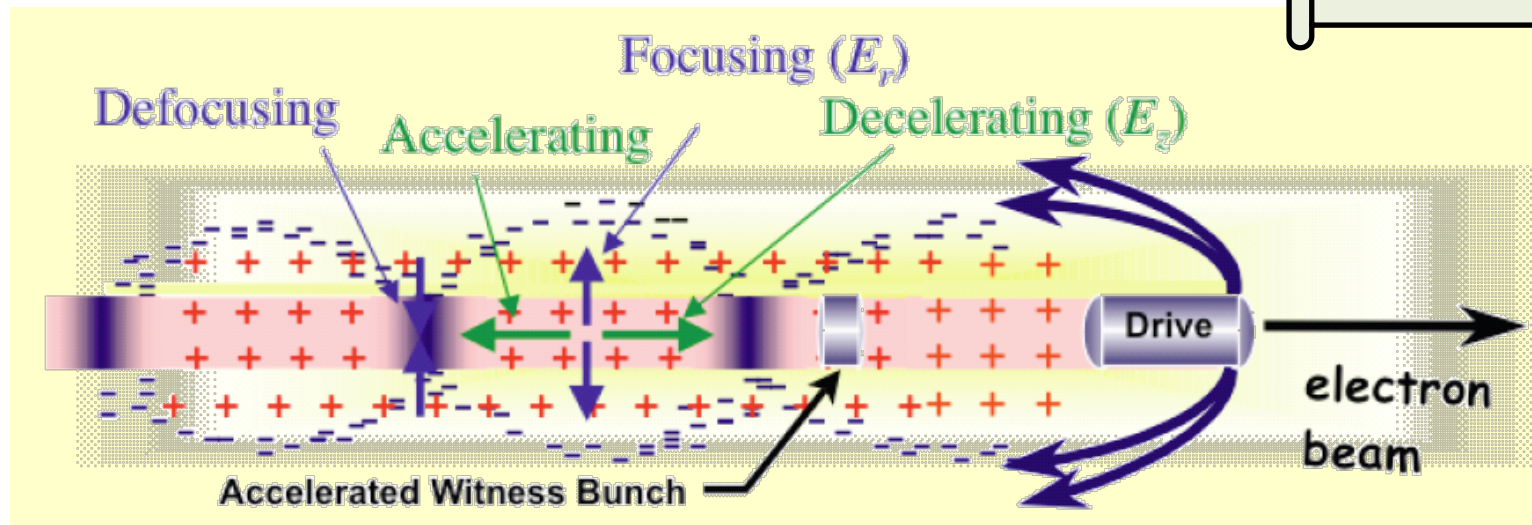


Plasma Wakefield Acceleration

Laser-getrieben:



Strahl-getrieben:



Lehrerfortbildung Elementarteilchen

Physikzentrum Bad Honnef, 23. – 27. Juni 2014

Resümee:

**Wunsch nach höchsten Energien führt
zu unterschiedlichen Konzepten für Collider:**

- **Hadronen:** limitiert durch Ablenkung:
Magnetfelder $B < 10$ Tesla...
- **Elektronen:** limitiert durch Beschleunigung:
AC-E-Felder ≤ 40 MV/m

**Luminositäten $\geq 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ bilden eine
zusätzliche Herausforderung!**