Condensed-Matter Theory - Special Topics

Problem 11 — Variance of the free density of states of the $D = \infty$ hypercubic lattice
Compute the D-dependence of the variance of the noninteracting density of states,

$$\Delta(D) = \int d\omega\, \omega^2 \rho_0(\omega) \; - \; \left(\int d\omega\, \omega\, \rho_0(\omega) \right)^2$$

for a (spinless) tight-binding model with nearest-neighbor hopping $-t$ on the D-dimensional hypercubic lattice!

Hint: If $\boldsymbol{t}$ is the hopping matrix,

$$\int d\omega\, \omega^k \rho_0(\omega) = \frac{1}{L} \text{tr}\, \boldsymbol{t}^k \; .$$

Argue that the scaling $t = t^*/\sqrt{D}$ with $t^* =$ const ensures a non-trivial density of states in the limit $D \to \infty$!

Problem 12 — Locality of the Luttinger-Ward functional in the infinite-D limit
Consider the diagrammatic representation of the Luttinger-Ward functional $\Phi[\boldsymbol{G}]$ for the Hubbard model on the D-dimensional hypercubic lattice and show that the nonlocal elements of $\boldsymbol{G}$ do not contribute in the limit $D \to \infty$, i.e., show that $\Phi[\boldsymbol{G}] = \Phi[G_{\text{loc}}]$ is a functional of the local Green's function $G_{\text{loc}}(\omega) = G_{ii\sigma}(\omega)$ only, in this limit!

Derive the scaling

$$\Sigma_{ij\sigma}(\omega) = \mathcal{O}(D^{-3/2})$$

of the self-energy in this way!

Problem 13 — Free Green's function of the Hubbard model on the hypercubic lattice
The free Green's function $G^{(0)}_{ij}(\omega)$ behaves as

$$G^{(0)}_{n}(\omega) = \mathcal{O}(D^{-n/2})$$

in the large-D limit, where n is the Manhattan distance between the sites i and j.

Prove this relation by considering the equation of motion for a *typical* element of $G^{(0)}_n(\omega)$ $(n = 0, 1, 2, ...)$ for large D and by making use of the scaling $t = t^*/\sqrt{D}$!