Symmetry Groups in Physics: Problems

Problem 16 — Platonic solids
In three-dimensional space, a Platonic solid is a regular polyhedron. It is constructed by congruent regular polygonal faces with the same number of faces meeting at each vertex. There are five solids: The tetrahedron, the cube, the octahedron, the dodecahedron, and the icosahedron with 4, 6, 8, 12, 20 faces, respectively.

Determine the orders of the symmetry groups of the Platonic solids!

Problem 17 — Direct product
Recalling that $G_1 \lhd G$, $G_2 \lhd G$, and $G_1 \cap G_2 = \{e\}$, and $G_1 G_2 = G$ implies that $G = G_1 \times G_2$, prove that:

$$GL(n, \mathbb{R}) = SL(n, \mathbb{R}) \times \mathbb{R}^*$$

for the case that n is odd! Here

$$\mathbb{R}^* = \{\lambda \mathbf{1} \mid \lambda \in \mathbb{R}\,,\ \lambda \neq 0\}\,.$$

Problem 18 — Positive determinant
We define

$$GL_+(n, \mathbb{R}) = \{A \in GL(n, \mathbb{R}) \mid \det A > 0\}\,.$$

Show that this defines a group!

Show that $GL_+(n, \mathbb{R})$ is a normal subgroup of $GL(n, \mathbb{R})$!

Show that $GL(n, \mathbb{R})/GL_+(n, \mathbb{R}) \cong Z_2$!