SoSe 2017

ÜBUNGEN ZUR VORLESUNG

Feynman Path Integral in Solid State Physics

Blatt 4

Fermi Coherent States

1) Let $\hat{a}^{+}$ and $\hat{a}$ are fermionic creation and anihilation operators:

$$\left\{\hat{a}, \hat{a}^{+}\right\} = 1$$

and coherent states:

$$\begin{aligned} |\alpha\rangle &= \exp\left(-\alpha\hat{a}^{+}\right)|0\rangle \\ \langle\alpha| &= \langle 0|\exp\left(-\hat{a}\alpha^{*}\right) \end{aligned}$$

Show that

$$\begin{aligned} \hat{a}\,|\alpha\rangle &= \alpha\,|\alpha\rangle \\ \langle\alpha|\beta\rangle &= \exp\left(\alpha^{*}\beta\right) \end{aligned}$$

and

$$\int d\alpha^{*}d\beta\frac{|\beta\rangle\langle\alpha|}{\langle\alpha|\beta\rangle} = \hat{1}$$

2) Let (α^{*}, α) and (a^{*}, a) be pairs of Grassmann variables,

$f(\alpha^{*})$ and $g(\alpha^{*})$ are 'vector' function of Grassmann variables,

$A(\alpha^{*}, \alpha)$, $B(\alpha^{*}, \alpha)$ and $C(\alpha^{*}, \alpha)$ are 'matrix' function of Grassmann variables

$$\begin{aligned} f(\alpha^{*}) &= f_0 + f_1\alpha^{*} \\ A(\alpha^{*}, \alpha) &= A_{00} + A_{10}\alpha^{*} + A_{01}\alpha + A_{11}\alpha^{*}\alpha \end{aligned}$$

with the scalar product defined as:

$$\langle f|g\rangle = \int d\alpha^{*}d\alpha\exp\left(-\alpha^{*}\alpha\right)\left(f(\alpha^{*})\right)^{*}g(\alpha^{*})$$

Show that:

$$\langle f|g\rangle = f_0^* g_0 + f_1^* g_1$$

Also show that

$$(Af)(a^*) = \int d\alpha^* d\alpha \exp\left(-\alpha^*\alpha\right) A(a^*, \alpha) f(\alpha^*) = g(a^*)$$

is equivalent to

$$\begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \end{pmatrix} = \begin{pmatrix} g_0 \\ g_1 \end{pmatrix}$$

and

$$(AB)(a^*, a) = \int d\alpha^* d\alpha \exp\left(-\alpha^*\alpha\right) A(a^*, \alpha) B(\alpha^*, a) = C(a^*, a)$$

is equivalent to the standard product of 2x2 matrix: $AB = C$

3) Prove the formula for multidimensional Gaussian integral

with a hermitian A over complex Grassmann variable

(using change of variables which diagonalized matrix M):

$$Z = \int \Pi_{i=1}^N d\alpha_i^* d\alpha_i \exp\Big(-\sum_{i,j=1}^N \alpha_i^* M_{ij} \alpha_j\Big) = \det M$$