SoSe 2017

ÜBUNGEN ZUR VORLESUNG

Feynman Path Integral in Solid State Physics

Blatt 3

Bose Coherent States

1) Let $\widehat{a}^+$ and $\widehat{a}$ are bosonic creation and anihilation operators:

$$\left[\widehat{a},\widehat{a}^+\right] = 1$$

The Boson eigenfunctions:

$$|n\rangle = \frac{(\widehat{a}^+)^n}{\sqrt{n!}}|0\rangle$$

and coherent states:

$$\begin{aligned} |\alpha\rangle &= \exp\left(\alpha\widehat{a}^+\right)|0\rangle \\ \widehat{a}|\alpha\rangle &= \alpha|\alpha\rangle \end{aligned}$$

Show that

$$\langle\alpha|n\rangle = \frac{(\alpha^*)^n}{\sqrt{n!}}$$

2) Let $\widehat{A}$ be the normal ordered operator

$$\widehat{A} = \sum_{n,m=0}^{\infty} A_{nm}\left(\widehat{a}^+\right)^n(\widehat{a})^m$$

Prove that

$$\left\langle\alpha|\widehat{A}|\beta\right\rangle = A(\alpha^*,\beta)\langle\alpha|\beta\rangle$$

where $A(\alpha^*,\beta)$ is obtained from $\widehat{A}$ by means of substitutions:

$$\widehat{a}^+ \to \alpha^* \quad ; \quad \widehat{a} \to \beta$$

3) Prove formula for multidimensional Gaussian integral with a positive defined matrix A over real variable:

$$\int d\overrightarrow{x}\exp(-\overrightarrow{x}A\overrightarrow{x}) = (\det A)^{-\frac{1}{2}}$$

where

$$\int d\overrightarrow{x}... = \int_{\infty}^{\infty}\Pi_i\frac{dx_i}{\sqrt{\pi}}$$