**SoSe 2017**

# ÜBUNGEN ZUR VORLESUNG

*Solid State Physics with Feynman Path Integral*

**Blatt 2**

## 1. Quantum Harmonic Oscillator

Starting with the Feynman path integral, show that the propagator for the one-dimensional quantum Harmonic oscillator:

$$\widehat{H} = \frac{\widehat{p}^2}{2m} + \frac{m\omega\widehat{q}^2}{2}$$

takes the following form:

$$\left\langle q_f \left| e^{-i\widehat{H}t/\hbar} \right| q_i \right\rangle = \left(\frac{m\omega}{2\pi i\hbar \sin\omega t}\right)^{1/2} \exp\left[\frac{i}{2\hbar} m\omega\left(\left[q_i^2 + q_f^2\right]\right)\cot\omega t - \frac{2q_i q_f}{\sin\omega t}\right].$$

Suggest why the propagator varies periodically on the time interval t, and explain the origin of the singularities at t = n$\pi/\omega$, n = 1, 2, . . .. Taking the frequency $\omega \to 0$, show that the propagator for the free particle is recovered.

## 2. Particle in a Periodic Potential

A quantum mechanical particle moves in a periodic lattice potential V with periodicity $a$. Use a definition of imaginary time propagator:

$$G(a, \pm a, \tau) = \left\langle \pm a \left| e^{-\tau\widehat{H}/\hbar} \right| a \right\rangle$$

(a) Taking the Euclidean action for the instanton connecting two neighbouring minima to be $S_{inst}$, express the Euclidean time propagator $G(ma, \pm na, \tau)$, with $m$ and $n$ integer, as a sum over instanton and anti-instanton field configurations.

(b) Making use of the identity $\delta_{qq'} = \int_0^{2\pi} e^{i(q-q')\theta} d\theta/2\pi$ show that

$$G(ma, \pm na, \tau) \backsim e^{\omega\tau/2} \int_0^{2\pi} \frac{d\theta}{2\pi} e^{-i(n-m)\theta} \exp\left[\frac{\Delta\varepsilon\tau}{\hbar} 2\cos\theta\right]$$

where $\Delta\varepsilon$ is the tunnel splitting of energy levels.

(c) Keeping in mind that, in the periodic system, the eigenfunctions are Bloch states $\psi_{p\alpha}(q) = e^{ipq}u_{p\alpha}(q)$ where $u_{p\alpha}(q + ma) = u_{p\alpha}(q)$ denotes the periodic part of the Bloch function, show that the propagator is compatible with a spectrum of the lowest band $\alpha = 0$, $\varepsilon_p = \hbar\omega/2 - 2\Delta\varepsilon\cos(pa)$.