ÜBUNGEN zur Theorie der kondensierten Materie I
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– Blatt 9 –

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Aufgabe 1) Spin operator

Proof the following representation of Spin operator $\hat{\vec{S}}_i$ on site i in terms of the fermonic creation $\hat{a}_{i\sigma}^\dagger$ and annihilation $\hat{a}_{i\sigma}$ operators:

$$\hat{\vec{S}}_i = \frac{\hbar}{2} \sum_{\sigma\sigma'} \vec{\sigma}_{\sigma\sigma'} \hat{a}_{i\sigma}^\dagger \hat{a}_{i\sigma'}$$

where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are 2×2 Pauli matices

Aufgabe 2) Eigenvalues

Find eigenvalues of bosonic and fermonic occupation operators $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ for the state $|n_i>$

Aufgabe 3) Coulomb Interaction

Using the general representation of the two-particle interaction in the field operators:

$$H_{int} = \frac{1}{2} \int d^3r d^3r' \hat{\Psi}(\vec{r})^\dagger \hat{\Psi}(\vec{r'})^\dagger U(\vec{r} - \vec{r'}) \hat{\Psi}(\vec{r'}) \hat{\Psi}(\vec{r}).$$

show that the Coulomb interaction $U = \frac{1}{(\vec{r} - \vec{r'})}$ (in a.u.) for spinless fermions has the following form in impuls $\vec{k}$-space:

$$H_{\text{int}} = \frac{1}{2V} \sum_{\vec{k}, \vec{k'}, \vec{q}} U_{\vec{q}} \hat{a}_{\vec{k}+\vec{q}}^\dagger \hat{a}_{\vec{k'}-\vec{q}}^\dagger \hat{a}_{\vec{k'}} \hat{a}_{\vec{k}}$$