

# Operational Amplifiers

## **Electronics:**

Experimental techniques in the photon sciences

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# Outline

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## Basics

- Complex impedance & the low pass-filter
- The decibel scale and Bode plots

## Introduction

### General characteristics

- Basic operation
- The ideal op-amp

### The Concept of Feedback

- Basic idea of feedback
- The noninverting and inverting amplifier

### Circuit Examples

- Summing and differential amplifier
- Integrator and Differentiator
- Nonlinear applications

### Real op-amps

- Frequency Response and Slew Rate
- Input Offset Voltage and Bias Current

### Grounding problems

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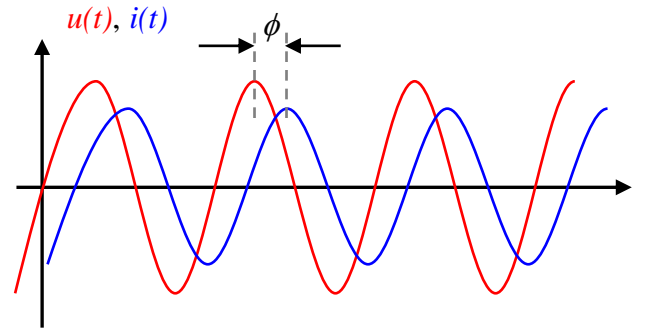
# Complex impedance

## Sinusoidal signals

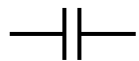
voltage  $u(t) = \hat{u} \cdot \exp(j\omega t)$

current  $i(t) = \hat{i} \cdot \exp(j\omega t + \phi)$

impedance  $Z = \frac{u(t)}{i(t)} = |Z| \cdot \exp(i\phi)$

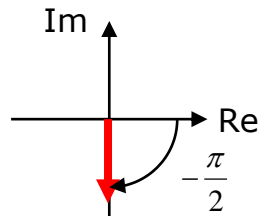


- **Capacities** have to be charged first to have a certain voltage



$$i = C \frac{du}{dt}$$

$$Z(\omega) = \frac{1}{j\omega C}$$

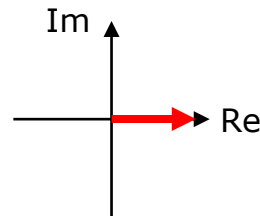


- Voltage drop over ohmic **resistor** is proportional to current



$$u = R \cdot i$$

$$Z(\omega) = R$$

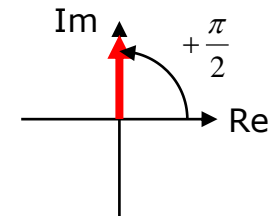


- Current change through **coil** induces voltage drop



$$u = L \frac{di}{dt}$$

$$Z(\omega) = j\omega L$$



# The decibel scale

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- decibels (dB) measure the power (intensity) ratio on a logarithmic scale  
→ transfer functions of consecutive elements can be “added”, e.g. gain of two amplifiers

$$G = 10 \cdot \log_{10} \left( \frac{P_E}{P_A} \right)$$

- In electronics, the power of a signal is usually proportional to the squared field amplitude (e.g.  $P = u^2 / R$  for an ohmic resistor)

$$G = 10 \cdot \log_{10} \left( \frac{u_E^2}{u_A^2} \right) = 20 \cdot \log_{10} \left( \frac{u_E}{u_A} \right)$$

<b>dB</b>	<b>-20</b>	<b>-3</b>	<b>0</b>	<b>3</b>	<b>10</b>	<b>20</b>	<b>30</b>
power	1/100	1/2	1	2	10	100	1000
amplitude	1/10	1/√2	1	√2	√10	10	√1000

- Suffix indicates fixed reference → absolute quantity
  - dBV → relative to 1 V (0 dBV = 1 V, 20 dBV = 10 V)
  - dBm → relative to 1 mW (0 dBm = 1 mW, 30 dBm = 1W)
  - dBc → power ratio to carrier (e.g. for sideband modulation)

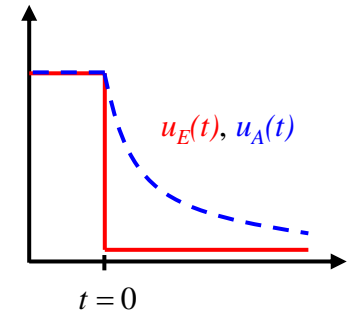
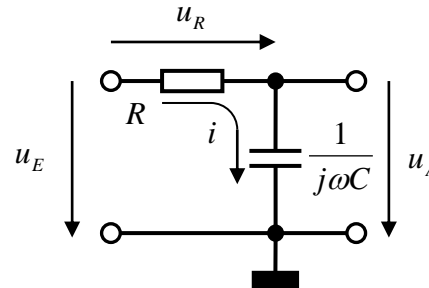
# The low-pass filter

- Many physical systems show low-pass characteristic, i.e. they linearly transmit a signal with a certain time delay (PT1 element in controller theory)
- Exponential step response characteristic

input signal step  $u_E = \begin{cases} u_0 & t < 0 \\ 0 & \text{else} \end{cases}$

$\Rightarrow$  for  $t > 0$  is  $i_C + i_R = 0 \Rightarrow C \frac{du_A}{dt} + \frac{u_A}{R} = 0$

$\Rightarrow u_A(t) = u_0 \cdot \exp(-t/\tau)$  with  $\tau = RC$



- The complex transfer function  $H(j\omega)$  relates the output  $u_A(t)$  to the input  $u_E(t)$  signal via a (magnitude) and a phase shift
- translate linear DE into frequency space via Laplace- / Fourier-transformation, transfer function is a fraction of polynomials
- poles and zero characterize transfer function ( $\rightarrow$  control theory)

ohm's law

$$u_E = i \cdot (Z_R + Z_C)$$

$$u_A = i \cdot Z_C$$

transfer function  $H(j\omega) = \frac{u_A}{u_E} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$

gain

$$G(\omega) = |H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}} \quad \text{with} \quad \omega_0 = \frac{1}{RC}$$

phase

$$\phi(\omega) = \arctan[H(j\omega)] = \arctan\left[-\frac{\omega}{\omega_0}\right]$$

# Bode plots

- Characterize transfer function by plotting magnitude (in dB) and phase on a log. frequency scale
- Polynomials appear as straight lines, poles and zeros can be seen as "kinks"

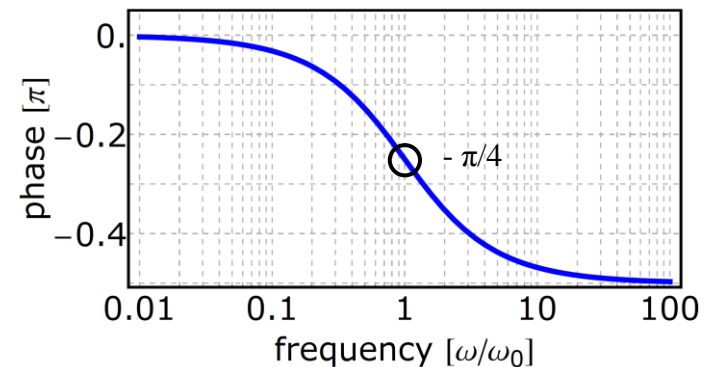
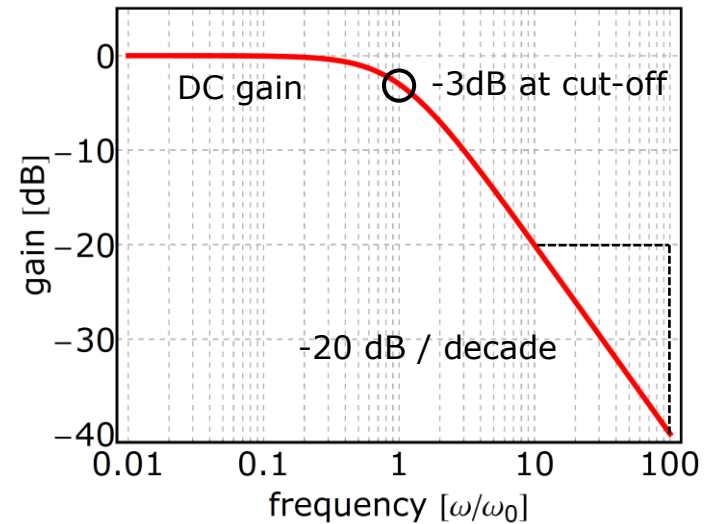
gain  $G(\omega) = \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}$  with  $\omega_0 = \frac{1}{RC}$

phase  $\phi(\omega) = \arctan\left[-\frac{\omega}{\omega_0}\right]$

cut-off  $|Z_R| = |Z_C| \Rightarrow \omega = \frac{1}{RC} = \omega_0$

output  $u_A = \frac{1}{\sqrt{2}} u_E$

phase  $\phi = \arctan(-1) = -\pi/4$



# Pros and Cons of op-amps

## Basic idea:

Use integrated circuit (**black box** – internal realization unknown) as a modular device to manipulate signals, so that behavior of the circuit is purely characterized by external elements.

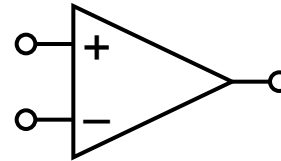
→ Why not use a transistor?

## Advantages of op-amps

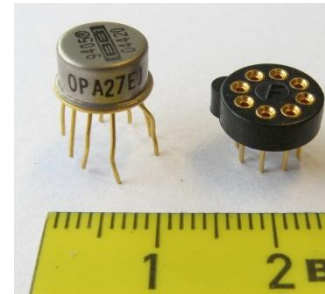
- **Versatility** - operation only determined by external surrounding circuit
- No bias current needed to determine operation point
- High input impedance, low output impedance
- High intrinsic gain
- Very linear and precise amplification over broad voltage and frequency range

## Disadvantages

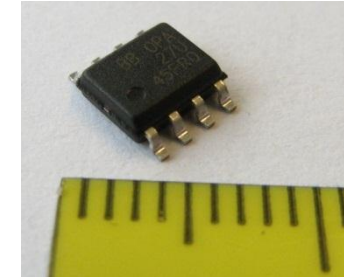
- higher noise due to multiple amplifier stages
- lower cut-off-frequency than single-stage transistor amplifiers
- Strong feedback over multiple amplifier stages causes non-linear distortions



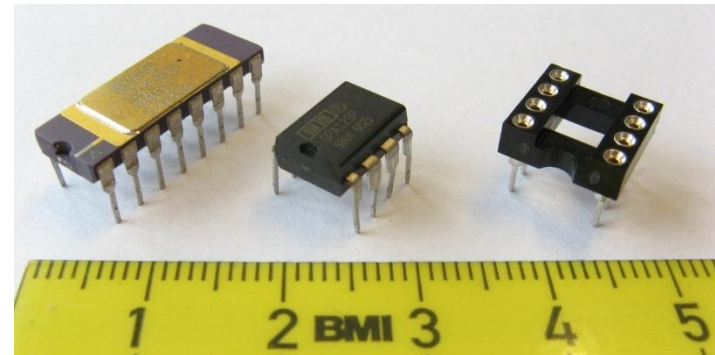
circuit symbol



TO-99 (transistor-single-outline)



SOIC-8 (small-outline-integrated-circuit)

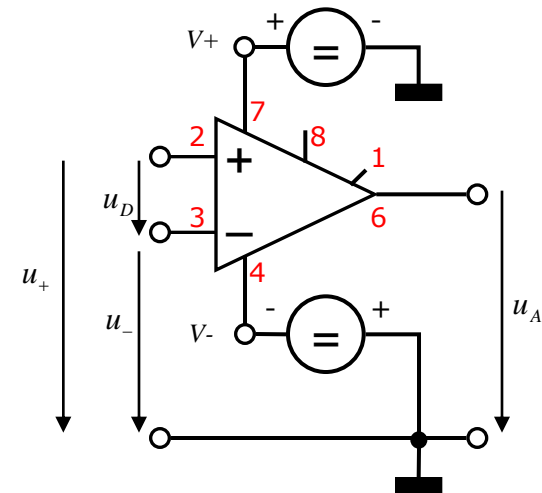
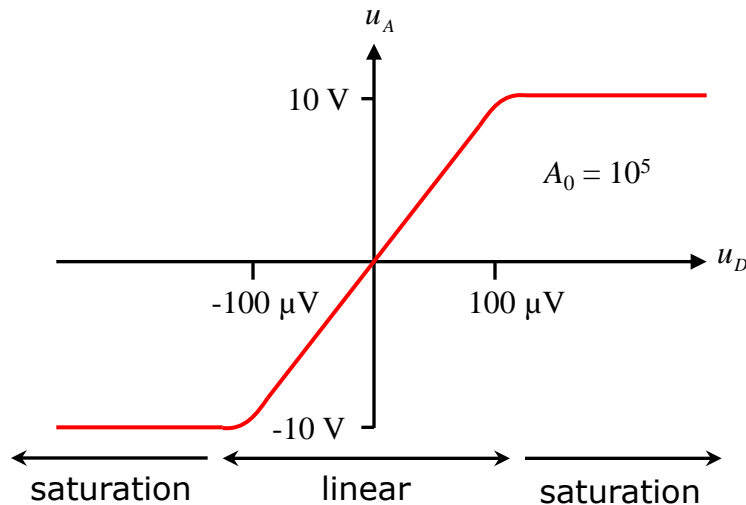
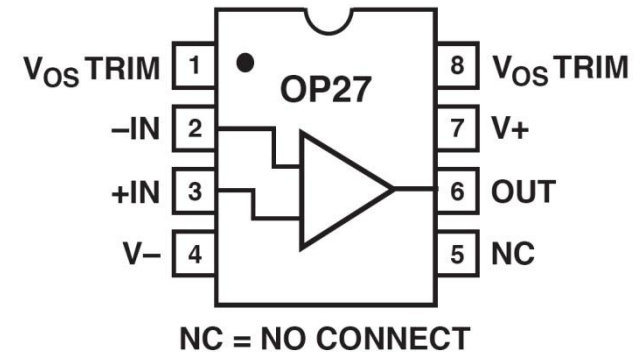


CERDIP / PDIP (dual-in-line-package)

# Basic operation and wiring

## Typical pin assignment

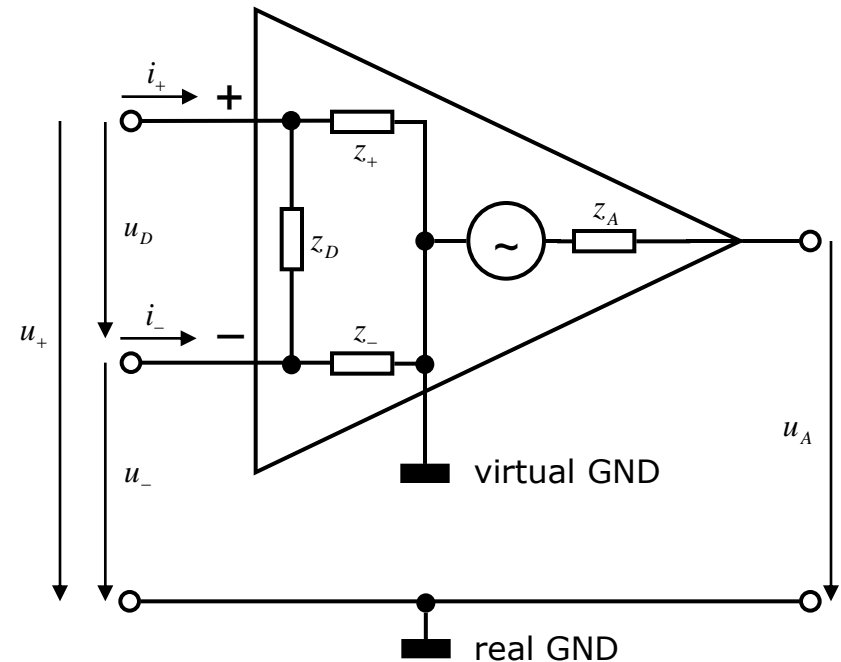
- inverting (-) and noninverting (+) input (2,3)
- voltage difference  $u_D = u_+ - u_-$  between inputs is amplified linearly by factor  $A_0$  (open-loop-gain) to output  $u_A = A_0 \cdot (u_+ - u_-)$  (6)
- every potential is measured relatively to ground potential (usually GND or 0V)
- two connections (4,7) for power supply  $V_{\pm}$  (usually  $\pm 15V$ )
- no separate ground connection, output ground reference by  $u_A = 0$  for  $u_+ - u_- = 0$  (for ideal op-amp)
- usually two extra pins for external offset compensation (8,1)





# The ideal op-amp

Parameter	Symbol	Ideal	Real (OP27)
Input Impedance Differential-Mode	$z_D$	$\infty$	$4 \text{ M}\Omega \parallel \leq 1 \text{ pF}$
Input Impedance Common-Mode	$z_+, z_-$	$\infty$	$2 \text{ G}\Omega \parallel \leq 1 \text{ pF}$
Input Bias Current	$i_+, i_-$	0	$\pm 15 \text{ nA}$
Input Offset Voltage	$u_0$	0	$30 \mu\text{V}$
Output Impedance	$z_A$	0	$70 \Omega$
Unity Gain Bandwidth	$f(A_0 = -3 \text{ dB})$	$\infty$	8 MHz
Open-Loop-Gain	$A_0 = du_A/du_D$	$\infty$	$10^6 = 120 \text{ dB}$
Common-Mode Rejection Ratio	$A_0 / du_A/d(u_+ + u_-)$	$\infty$	$10^6 = 120 \text{ dB}$
Slew rate	$\max(du_A/dt)$	$\infty$	$2.8 \text{ V}/\mu\text{s}$



# Basic idea of feedback

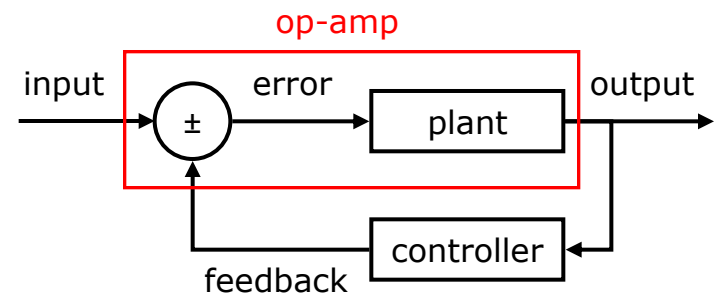
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## Problem

- open-loop-gain  $A_0$  too high, input voltage range of  $\leq 100 \mu\text{V}$  too small
- amplification  $A$  not adjustable
- gain  $A_0$  determined by device (and hence differs between various devices), instabilities

## Solution

- feed factor  $k_F$  of output signal back into input
- signal experiences certain amplification / attenuation and phase shift while passing the loop, effect determined by phase difference at inputs



## Consequences

- reduced gain, but improved **linearity, frequency response, bandwidth and stability**
- more negative feedback results in less dependency on device parameters
- in general, feedback can be frequency-dependant (equalizers and filters) or amplitude-dependant (logarithmic amplifiers or multipliers)

## The “golden rules” concerning op-amps

1. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero (infinite open-loop-gain).
2. The inputs draws no current (infinite input impedance).

# The noninverting amplifier

- realize controller by voltage divider  $R_2$ - $R_1$ , rising output voltage hinders input difference  $u_+ - u_-$ .

negative feedback 
$$u_- = \frac{R_1}{R_1 + R_2} \cdot u_A$$

output signal 
$$u_A = A_0(u_+ - u_-) = A_0\left(u_E - \frac{R_1}{R_1 + R_2} \cdot u_A\right)$$

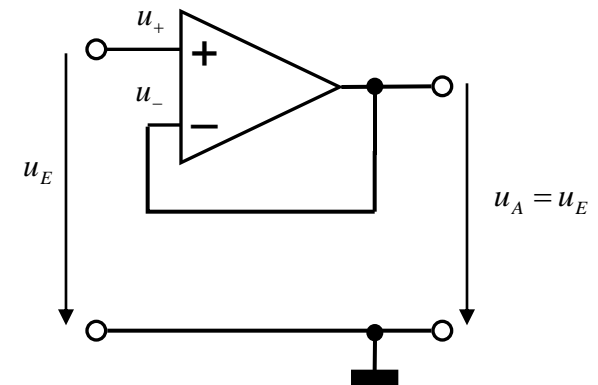
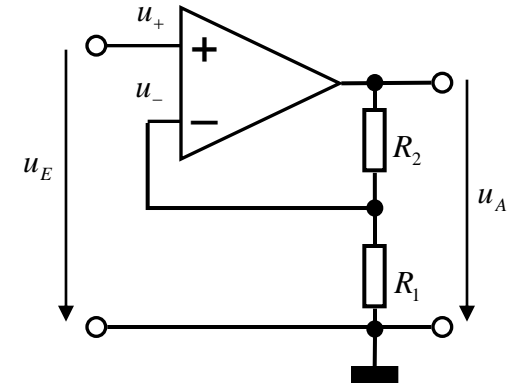
closed-loop-gain 
$$A = \frac{u_A}{u_E} = \left(\frac{1}{A_0} + \frac{R_1}{R_1 + R_2}\right)^{-1}$$

ideal OP-Amp 
$$A \cong \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

from feedback 
$$u_A = A_0(u_+ - u_-) = \frac{R_1 + R_2}{R_1} \cdot u_-$$

virtual short-circuit 
$$u_+ = \underbrace{\left(\frac{1}{A_0} \cdot \frac{R_1 + R_2}{R_1}\right)}_{\approx 0} \cdot u_- + u_- \cong u_-$$

- because generally  $A_0 \gg A$ , the input voltage difference tends to zero (**virtual short-circuit**) (complies with **1st golden rule**)
- high input impedance ( $10^{12}\Omega$ ), low output impedance ( $10^1\Omega$ )
- for  $R_2 \rightarrow 0$  and  $R_1 \rightarrow \infty$  the closed-loop-gain  $A \rightarrow 1$ , op-amp works as buffer (voltage follower)



# The inverting amplifier

- connect input signal  $u_E$  to lower end of feedback voltage divider  $R_2$ - $R_1$
- assume input voltage  $u_E = 1V$ , but no output  $u_A = 0V$ 
  - since noninverting input is connected to GND  $u_+ = 0V$ , op-amp sees high input unbalance
  - this forces the output  $u_A$  to go negative, until both input are on GND  $u_+ = u_- = 0V$

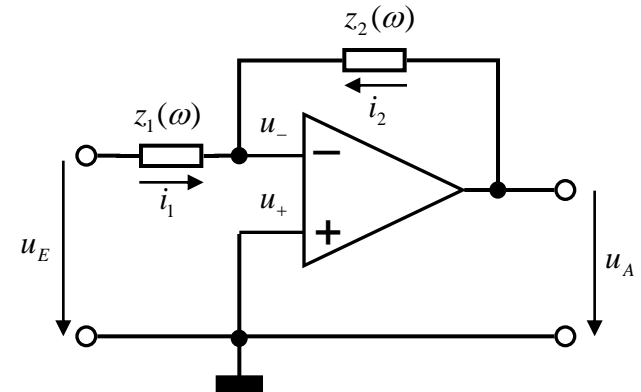
kirchhoff's junction rule  $0 = i_1 + i_2 + i_-$

open loop gain  $u_D = u_A / A_0$

high op-amp impedance  $0 = i_-$

$$\Rightarrow 0 = \frac{u_E - u_D}{z_1(\omega)} + \frac{u_A - u_D}{z_2(\omega)}$$

$$\Rightarrow u_A \cdot z_1 + \frac{u_A \cdot z_2}{A_0} + \frac{u_A \cdot z_1}{A_0} = -u_E \cdot z_2$$



closed loop gain

$$A(\omega) = \frac{u_A}{u_E} = -\frac{z_2}{z_1 + z_2/A_0 + z_1/A_0} \approx -\frac{z_2(\omega)}{z_1(\omega)}$$

input impedance  $z_E(\omega) = \frac{du_E}{di_E} = z_1(\omega)$

- frequency-dependant (but linear) feedback provides active filters, integrators, differentiators
- non-linear feedback allows to build amplitude-dependant devices, i.e. exponential or logarithmic amplifiers

# The summing amplifier

## Problem

- summing currents is easy, but since all potentials are grounded, how to add potentials?

## Solution

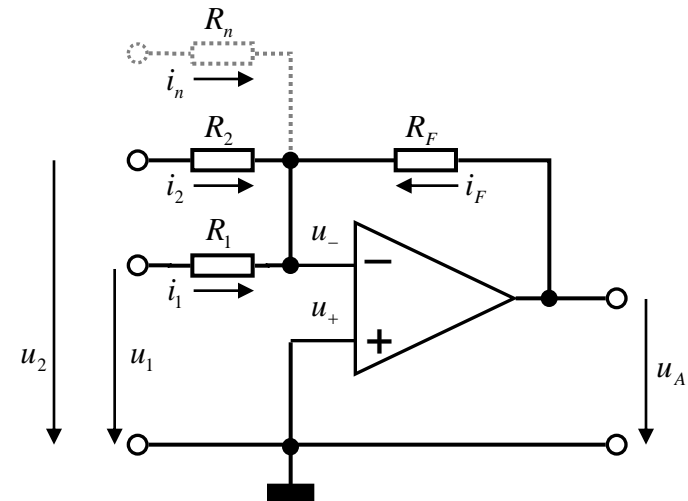
- take inverting amplifier and add currents in feedback loop
- assume that high input impedance  $i_- = 0$  and virtual GND  $u_+ \approx u_- = 0$

kirchhoff's junction rule  $0 = i_1 + i_2 + \dots + i_n + i_F$

$$= \frac{u_1}{R_1} + \frac{u_2}{R_2} + \dots + \frac{u_n}{R_n} + \frac{u_A}{R_F}$$
$$u_A = -R_F \left( \frac{u_1}{R_1} + \frac{u_2}{R_2} + \dots + \frac{u_n}{R_n} \right)$$

assume equal inputs  $R = R_1 = R_2 = \dots = R_n \Rightarrow$

closed-loop-gain  $u_A = -\frac{R_F}{R} (u_1 + u_2 + \dots + u_n)$



# The differential amplifier

## Problem

- measure voltage drop over some impedance, what if none of both connections has GND contact?

## Solution

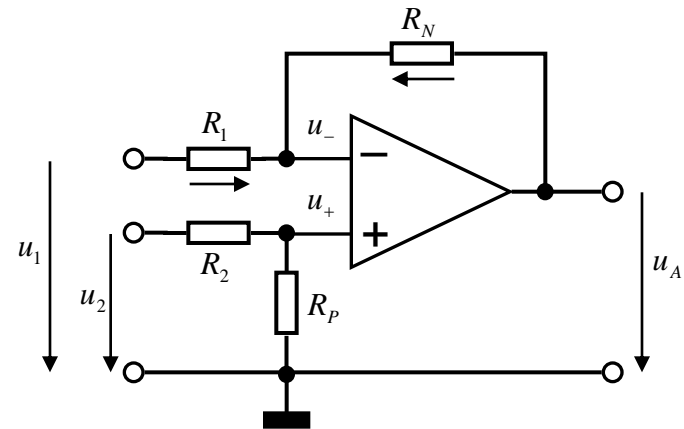
- use features from both inverting and noninverting amplifier

virtual short-circuit  $u_- \cong u_+ = u_2 \cdot \frac{R_p}{R_2 + R_p}$

kirchhoff's junction rule  $0 = i_1 + i_N = \frac{u_1 - u_-}{R_1} + \frac{u_A - u_-}{R_N}$

insert  $u_-$   $u_A = -\frac{R_N}{R_1} \cdot \left[ u_1 - \frac{R_1/R_N + 1}{R_2/R_p + 1} \cdot u_2 \right]$

if  $R_1 = R_2$  and  $R_p = R_N$   $u_A = -\frac{R_N}{R_1} \cdot (u_1 - u_2)$



## Caution

- assume  $R_N = \alpha \cdot R_1$  and thereby the closed-loop-gain  $A = -\alpha$ , resistors have tolerance  $\Delta\alpha$

$$\text{CMRR} = A \left/ \frac{du_A}{d(u_+ + u_-)} \right. \approx (1 + \alpha) \frac{\alpha}{\Delta\alpha} \Rightarrow \text{error, if resistor values differ}$$

- different input impedances require source with low output impedance
- better results with instrumentation amplifier

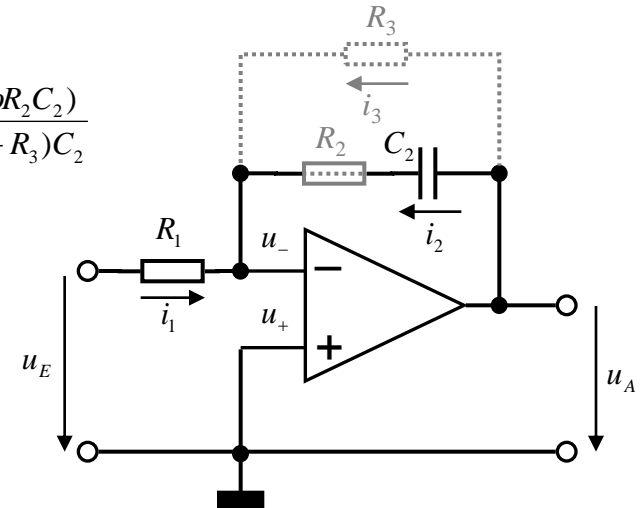
# The integrator – frequency response

- use inverting amplifier with RC-high-pass  $C_2$ - $R_1$  filter in feedback
- strong negative feedback for high frequencies (respectively weak feedback for low frequencies) results in **low-pass-characteristic**

use  $z_1(\omega) = R_1$  and  $z_2(\omega) = (C_2 + R_2) \parallel R_3 = \frac{R_3 \cdot (1 + j\omega R_2 C_2)}{1 + j\omega(R_2 + R_3)C_2}$

closed loop gain  $A(\omega) = -\frac{z_2(\omega)}{z_1(\omega)} = -\frac{R_3}{R_1} \cdot \frac{1 + j\omega R_2 C_2}{1 + j\omega(R_2 + R_3)C_2}$

- missing feedback at  $\omega = 0$  makes circuit instable, insertion of bypass  $R_3$  limits feedback at low frequencies
- attenuation ( $|A| < 1$ ) at high frequencies can be avoided by insertion of  $R_2$

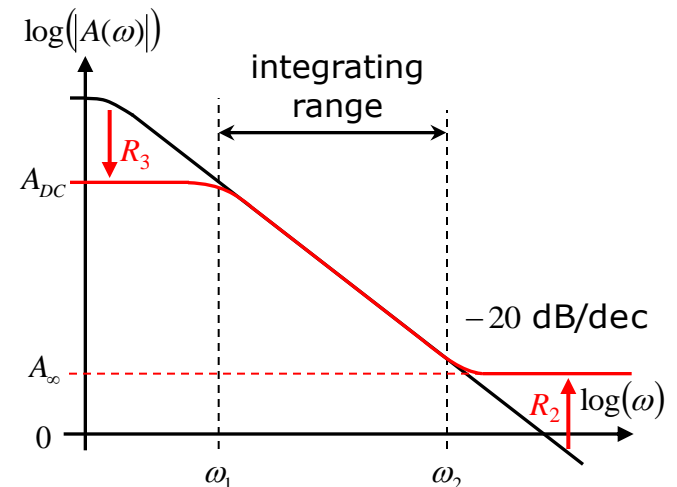


lower limit  $|A(\omega=0)| = A_{DC} = -\frac{R_3}{R_1}$

lower cut-off  $\left(|A(\omega_1)| = \frac{1}{\sqrt{2}} \cdot |A_{DC}|\right) \quad \omega_1 = \frac{1}{\sqrt{(R_3^2 + 2R_3R_2 - R_2^2)} \cdot C_2}$

upper cut-off  $\left(|A(\omega_2)| = \sqrt{2} \cdot |A_{\infty}|\right) \quad \omega_2 = \frac{\sqrt{(R_3^2 + 2R_3R_2 - R_2^2)}}{R_2(R_2 + R_3) \cdot C_2}$

upper limit  $|A(\omega=\infty)| = A_{\infty} = -\frac{R_2 \cdot R_3}{R_1 \cdot (R_2 + R_3)}$



# The integrator – operating method

closed loop gain  $A(\omega) = -\frac{z_2(\omega)}{z_1(\omega)} = -\frac{R_3}{R_1} \cdot \frac{1 + j\omega R_2 C_2}{1 + j\omega(R_2 + R_3)C_2}$

kirchhoff's  $0 = i_1 + i_2 + i_3$   
 junction rule  $= \frac{u_E}{R_1} + C_2 \frac{d(u_A - u_{R_2})}{dt} + \frac{u_A}{R_3}$

## Approximations

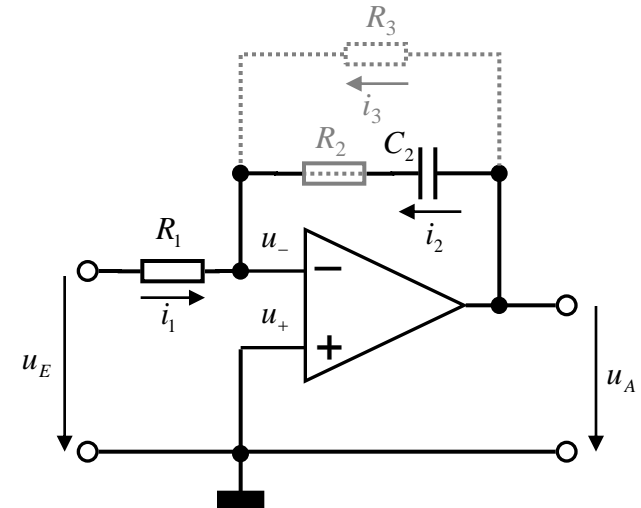
- for frequencies far above **lower limit**  $\omega \gg \omega_1$ , current  $i_3 = u_A / R_3$  can be neglected (shortened by  $C_2$ )
- for frequencies far beneath **upper limit**  $\omega \ll \omega_2$ , impedance of  $C_2$ - $R_2$  is dominated by condenser, so the voltage change  $du_{R_2}/dt$  over  $R_2$  can be neglected

$$0 = \frac{u_E}{R_1} + C_2 \frac{du_A}{dt} \Rightarrow \boxed{u_A(t) = -\frac{1}{R_1 C_2} \cdot \int_0^t u_E(t') dt' + u_A(0)}$$

- integration constant  $u_A(0) = Q_0 / C$  is determined by initial condenser charge and capacity
- input bias current  $i_-$  and input offset voltage  $u_0$  result in additional condenser current  $u_0 / R + i_-$ , which changes output voltage by

$$\frac{du_A}{dt} = \frac{1}{C_2} \cdot \left( \frac{u_0}{R} + i_- \right)$$

- $i_- = 1 \mu A$  results with  $C = 1 \mu F$  in a change of 1 V per second (**offset compensation**)





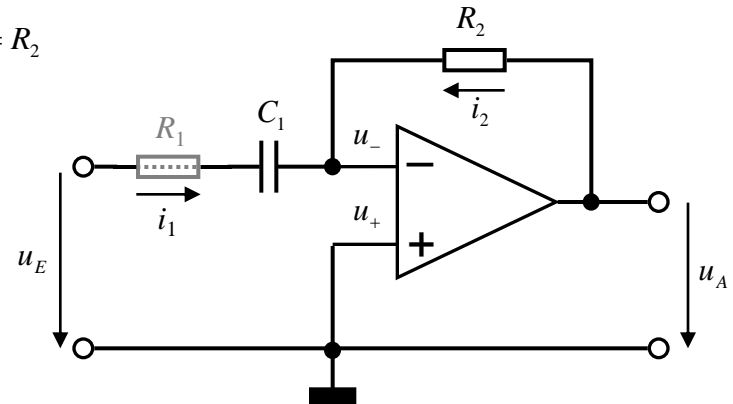
# The differentiator – frequency response

- swap condenser and resistor in integrator
- RC-low-pass filter  $R_2$ - $C_1$  delivers in high negative feedback for low frequencies, resulting in a **high-pass characteristic**

use  $z_1(\omega) = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$  and  $z_2(\omega) = R_2$

closed loop gain  $A(\omega) = -\frac{z_2(\omega)}{z_1(\omega)} = -\frac{j\omega R_2 C_1}{1 + j\omega R_1 C_1}$

- at high frequencies  $\omega$ , missing feedback makes circuit unstable
- HF noise is strongly amplified, circuit tends to oscillate due to phase delay of op-amp and feedback
- gain limiting at high frequencies can be achieved by insertion of  $R_1$

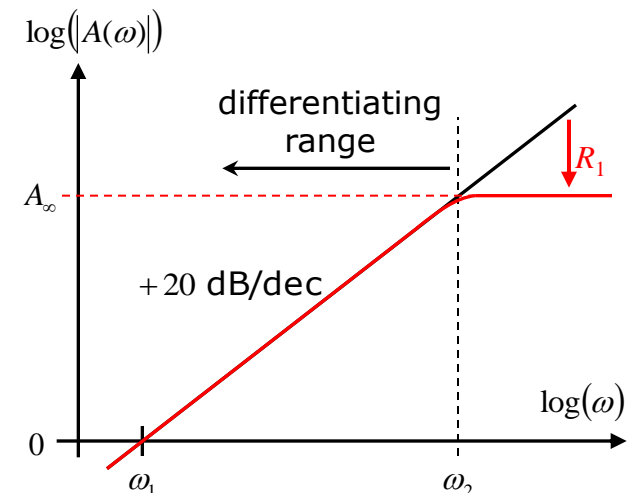


lower limit  $|A(\omega=0)| = A_{DC} = 0$

lower cut-off ( $|A(\omega_1)| = 1$ )  $\omega_1 = \frac{1}{\sqrt{R_2^2 - R_1^2} \cdot C_1}$

upper cut-off ( $|A(\omega_2)| = \frac{1}{\sqrt{2}} \cdot |A_\infty|$ )  $\omega_2 = \frac{1}{\sqrt{R_1 R_2 - R_1^2} \cdot C_1}$

upper limit  $|A(\omega=\infty)| = A_\infty = -\frac{R_2}{R_1}$



# The differentiator – operating method

$$\text{closed loop gain } A(\omega) = -\frac{z_2(\omega)}{z_1(\omega)} = -\frac{j\omega R_2 C_1}{1 + j\omega R_1 C_1}$$

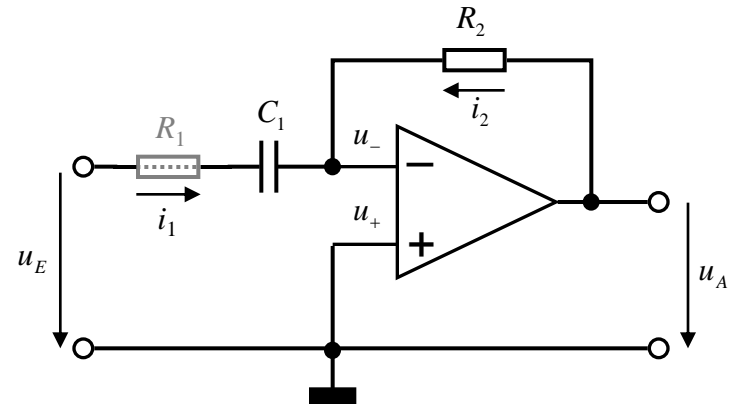
$$\text{kirchhoff's } 0 = i_1 + i_2$$

$$\text{junction rule } = C_1 \frac{d(u_E - u_{R1})}{dt} + \frac{u_A}{R_2}$$

## Approximation

- for frequencies far beneath **upper limit**  $\omega \ll \omega_{2f}$ , impedance of  $R_1$ - $C_1$  is dominated by condenser, so the voltage change  $du_{R1}/dt$  over  $R_1$  can be neglected

$$0 = C_1 \frac{du_E}{dt} + \frac{u_A}{R_2} \Rightarrow \boxed{u_A(t) = -R_2 C_1 \cdot \frac{du_E}{dt}}$$



- differentiator is bias-stable, optional roll-off-condensator in feedback can reduce bandwidth (**bandpass**)
- capacitive input impedance draws current from source, problems possible at high frequencies

# The logarithmic amplifier

## Problem

- measured signal has large dynamic range

## Idea

- instead of linear characteristic curve  $i_R = u_R / R$  of a resistor, use exponential I-U-dependency of semiconductors
- use collector current  $i_C$  of a bipolar transistor

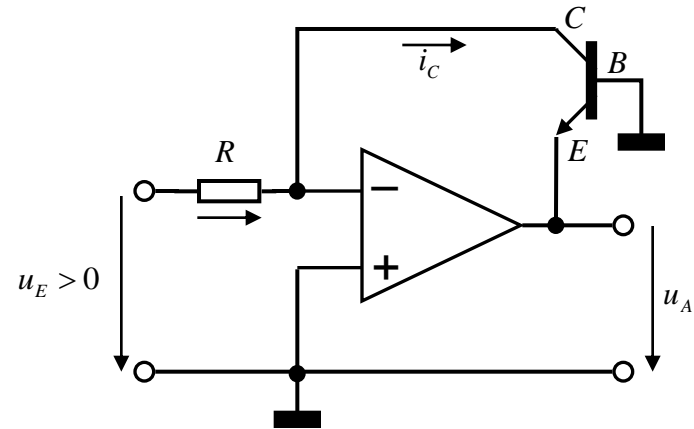
$$i_C = i_{CS}(T, u_{CE}) \cdot \exp(u_{BE}/u_T) \Rightarrow u_{BE} = u_T \cdot \ln(i_C/i_{CS})$$

$i_{CS}$  reverse leakage current

$u_T = kT/e$  thermal voltage

since  $u_A = -u_{BE}$  ,  $i_C = u_E/R$

$$\Rightarrow \boxed{u_A = -u_T \cdot \ln\left(\frac{u_E}{i_{CS}R}\right)} \quad (\text{for } u_E > 0)$$



- no error due to collector-base-current  $i_{CB}$ , since  $u_{CB} = 0$ , but **temperature drift**
- with appropriate transistor and op-amp with low bias-current, usually **nine decades** available
- swap resistor and transistor: **exponential amplifier**
- multiply / divide signals** by taking logarithm, add / subtract, take exponential value

# The comparator

## Problem

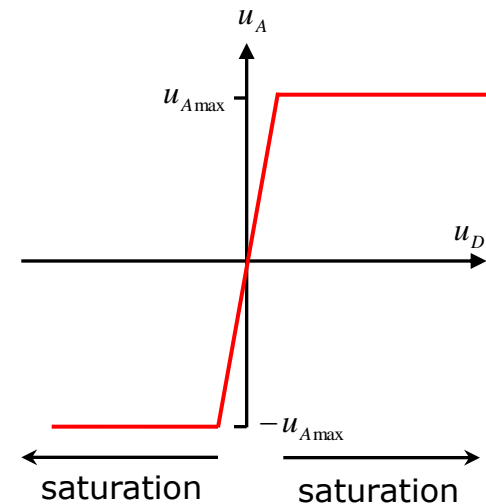
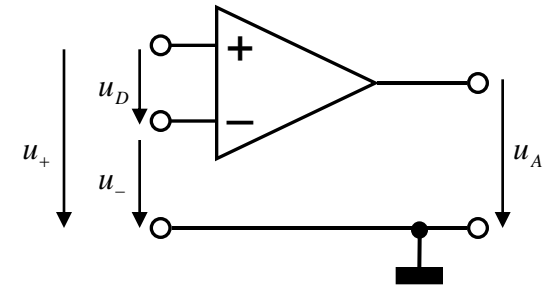
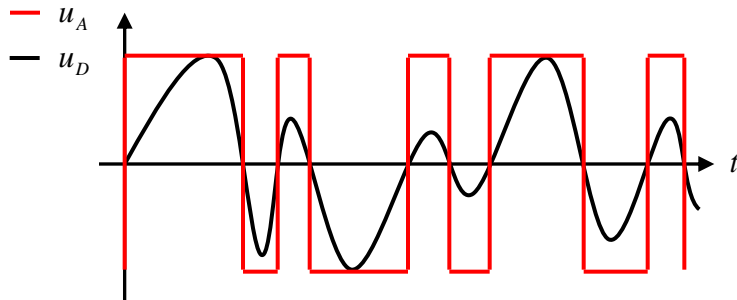
- device for comparison of two signals, which decides if voltage is below / above some threshold

## Solution

- use op-amp without feedback to compare voltages (**comparator**)
- because of high open-loop-gain, circuit is sensitive to small voltage differences  $u_D$  and switches between saturated values  $-u_{Amax}$  and  $u_{Amax}$
- **in saturation no virtual short-circuit between inputs, i.e.  $u_+ \neq u_-$ .**

$$u_A = \begin{cases} +u_{Amax} & \text{for } u_+ > u_- \\ -u_{Amax} & \text{for } u_+ < u_- \end{cases}$$

- useful for regeneration of digital signals or as trigger
- special devices for fast applications



# The noninverting Schmitt trigger

## Problem

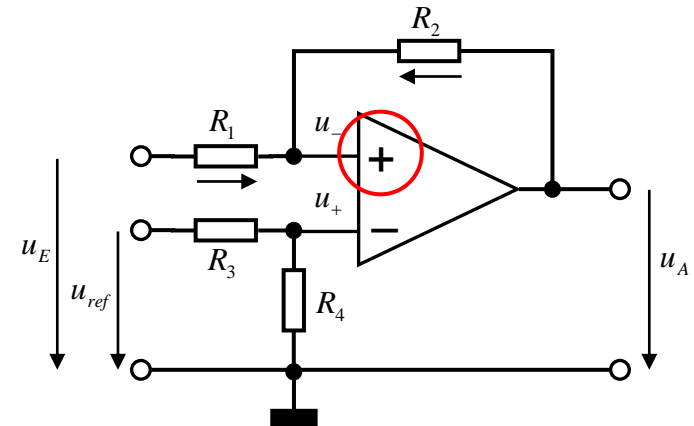
- **comparator** has no well-defined output for small input signals  $u_D \approx 0$
- different switching values for high and low state desired

## Solution

- use positive feedback to create hysteresis for switching

$$\text{input } u_- = \frac{R_4}{R_3 + R_4} \cdot u_{ref} \quad u_+ = \frac{R_2}{R_1 + R_2} \cdot (u_E - u_A)$$

- assume high positive input  $u_{E,I}$  so that output is  $u_{Amax}$
- with falling input voltage  $u_{E,I}$  output  $u_A$  remains unchanged until  $u_+ = u_- = 0$

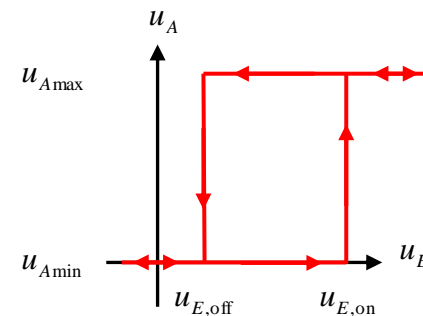
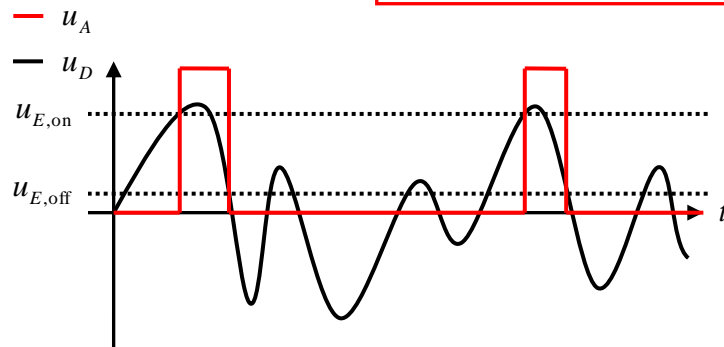


upper switching point

$$u_{E,on} = \frac{R_4 \cdot (R_1 + R_2)}{R_2 \cdot (R_3 + R_4)} \cdot (u_{ref} + u_{Amax})$$

lower switching point

$$u_{E,off} = \frac{R_4 \cdot (R_1 + R_2)}{R_2 \cdot (R_3 + R_4)} \cdot (u_{ref} + u_{Amin})$$



# The real op-amp

- differential **open-loop-gain**  $A_0 = du_A/du_D$  is usually not infinite -> error in approx.  $A_0 = \infty$
- even with shortened inputs  $u_+ - u_- = 0$ , op-amp amplifies common-mode voltage  $u_+ + u_-$ , usually expressed by ratio of differential open-loop-gain  $A_0$  to  $du_A/d(u_+ + u_-)$   
-> **common-mode rejection ratio**
- **finite input impedance** draws current from source, common-mode input impedance (input to GND) usually negligible, effect compensated if adjusted
- **nonzero output impedance** negligible, since decrease of output  $u_A$  by output load is compensated by feedback
- transistors as well as resistors produce **noise**, which is amplified towards the output

Parameter	Symbol	Ideal	Real (OP27)
Input Impedance Differential-Mode	$z_D$	$\infty$	$4 \text{ M}\Omega \parallel \leq 1 \text{ pF}$
Input Impedance Common-Mode	$z_+, z_-$	$\infty$	$2 \text{ G}\Omega \parallel \leq 1 \text{ pF}$
Input Bias Current	$i_+, i_-$	0	$\pm 15 \text{ nA}$
Input Offset Voltage	$u_0$	0	$30 \text{ }\mu\text{V}$
Output Impedance	$z_A$	0	$70 \text{ }\Omega$
Unity Gain Bandwidth	$f(A_0=-3 \text{ dB})$	$\infty$	8 MHz
Open-Loop-Gain	$A_0 = du_A/du_D$	$\infty$	$10^6 = 120 \text{ dB}$
Common-Mode Rejection Ratio	$A_0 / du_A/d(u_+ + u_-)$	$\infty$	$10^6 = 120 \text{ dB}$
Slew rate	$\max(du_A/dt)$	$\infty$	$2.8 \text{ V}/\mu\text{s}$

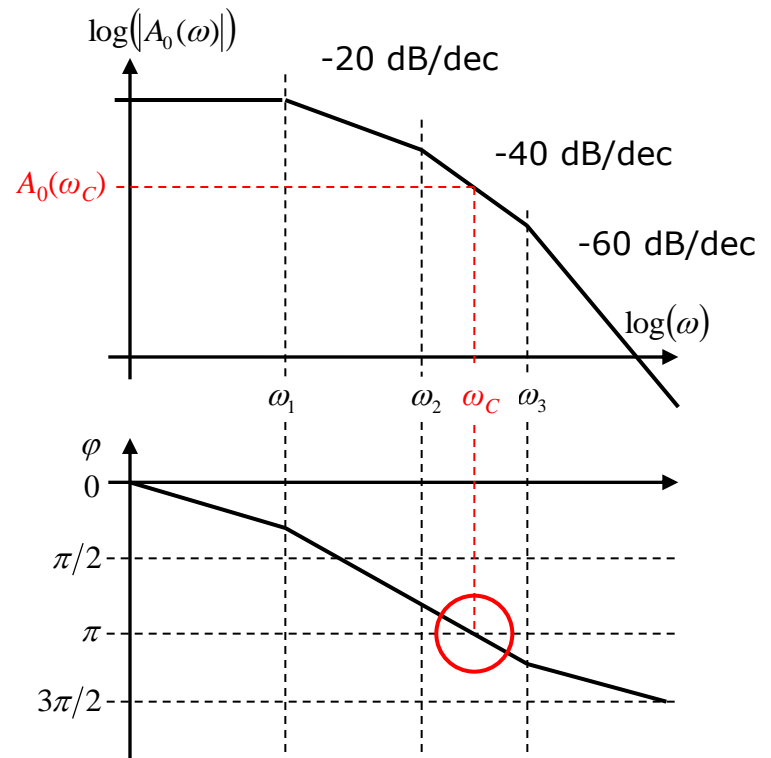
# Real op-amps - frequency response

- real op-amps are multi-stage transistor-amplifiers
- every single stage represents a **low-pass filter** with a particular cut-off-frequency  $\omega_i$  that **reduces the gain by -20 db per decade** and adds a certain **phase shift of maximal  $\varphi = \pi/2$**  (subsequent stages usually have increasing bandwidths)
- if  $A_0(\omega)$  is open-loop-gain, and  $|k_F| \leq 1$  is feedback factor, requirement for oscillation is (negative input adds phase shift  $\varphi = \pi$ )

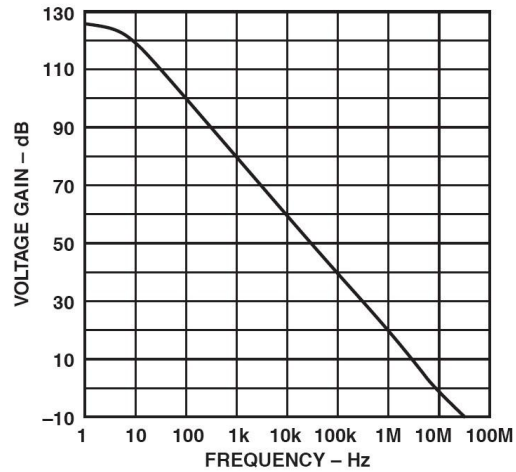
$$k_F(\omega) \cdot A_0(\omega) = 1 \quad \Rightarrow \quad \begin{cases} |k_F| \cdot |A_0| = 1 \\ \varphi = \arg(k_F A_0) = 0, 2\pi, \dots \end{cases}$$

- to prevent oscillation at  $\omega_C$ , feedback factor has to be maximal  $k_F \leq 1 / A_0(\omega_C)$  (signal gain in one loop passage must be smaller than one)

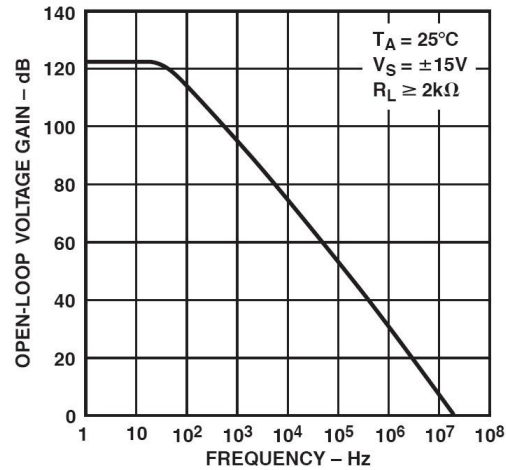
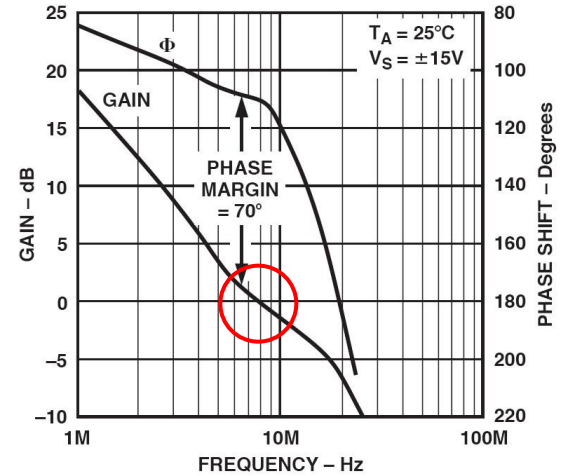
If an op-amp is **not internally frequency-compensated** (and by that **not unity-gain stable**), external frequency compensation or minimal closed-loop-gain is necessary.



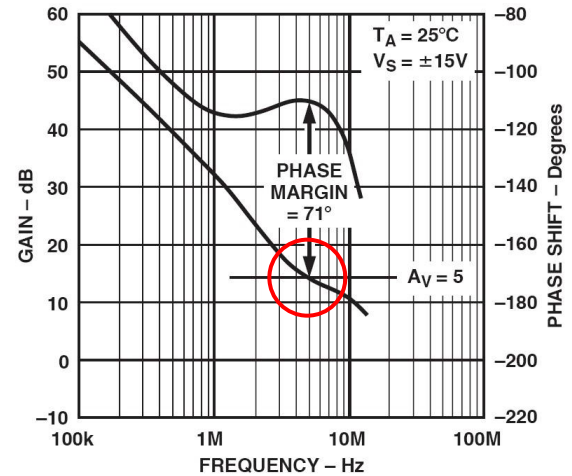
# Comparison OP27 – OP37



**OP27  
(unity-gain stable)**



**OP37 (not  
unity-gain stable)**





# Bandwidth

- frequency-compensated op amps can be regarded as 1<sup>st</sup> order low-pass

$$A_0(\omega) = \frac{A_{DC}}{1 + i \cdot (\omega / \omega_1)} \Rightarrow \omega_1 = R_i C_i$$

- for frequencies  $\omega \gg \omega_1$  above cut-off, open-loop gain is approximately

$$A_0(\omega) \cong -i(\omega_1 \cdot A_{DC} / \omega)$$

$\omega_1 \cdot A_{DC}$  ... gain-bandwidth-product  
(unity-gainbandwidth)

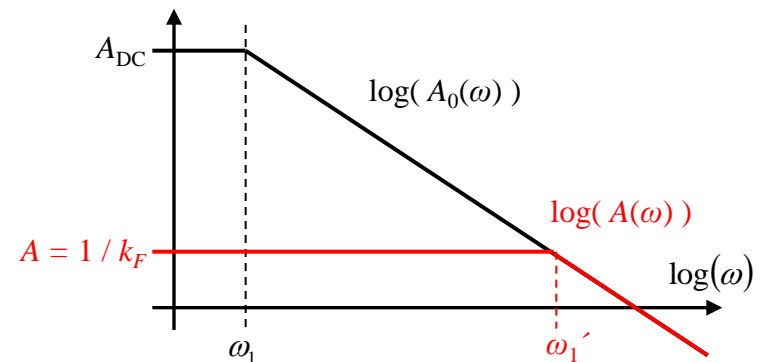
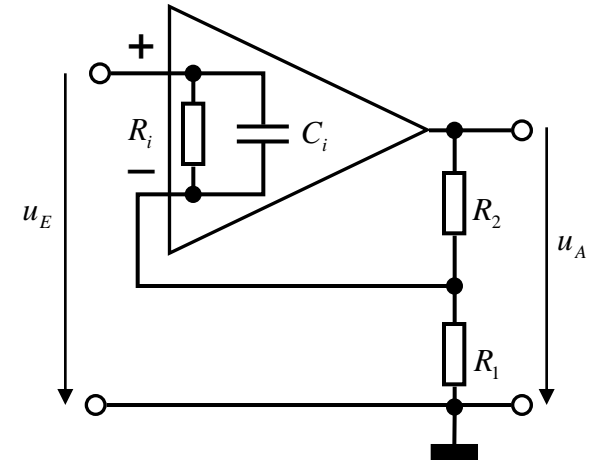
- usually  $A_{DC}$  is in the range of 120 dB, but cut-off frequency very low  $\omega_1 / 2\pi = 10$  Hz
- use closed-loop gain from noninverting amplifier with feedback factor  $k_F = R_1 / (R_1 + R_2)$  to calculate new frequency response

$$A(\omega) = \frac{u_A}{u_E} = \left( \frac{1}{A_0(\omega)} + \frac{R_1}{R_1 + R_2} \right)^{-1} = \frac{A_0(\omega)}{1 + k_F A_0(\omega)} \quad \text{insert } A_0(\omega)$$

$$A(\omega) = \frac{A}{1 + i \cdot (\omega / \omega'_1)} = \begin{cases} A & \text{for } \omega \ll \omega'_1 \\ -i(\omega'_1 / \omega) \cdot A & \text{for } \omega \gg \omega'_1 \end{cases}$$

$A = \frac{1}{k_F} = \frac{R_1}{R_1 + R_2}$  (DC) closed-loop gain

$\omega'_1 = k_F \omega_1 A_{DC}$  closed-loop cut-off frequency



# Slew Rate

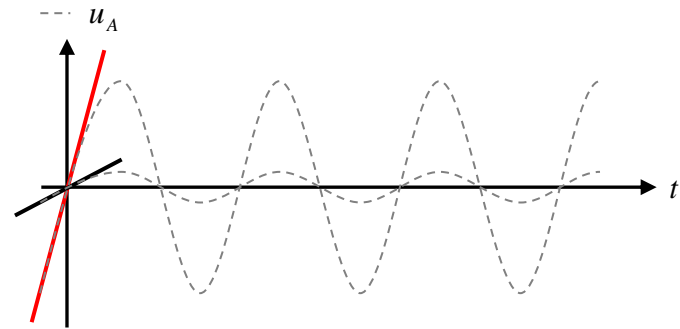
- internal capacities and frequency compensation act as a **low-pass**, whose output for a unit-step input is a (more or less fast) exponential function
- output transistors and capacities need driving current from (previous) driving stage, whose **output current is limited**
- output change rate is limited to the **slew rate**  $SR = \max(du_A/dt)$
- image sine-wave input signal

$$u_E = u_{E\max} \cdot \sin(\omega \cdot t) \Rightarrow \max\left(\frac{du_A}{dt}\right) = A(\omega) \cdot \omega \cdot u_{E\max}$$

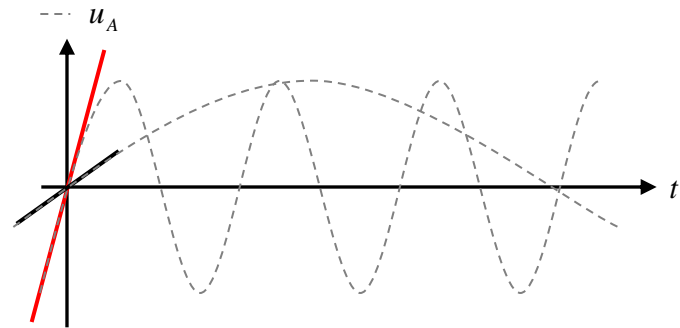
- **slew rate limits amplitude of undistorted sine-wave output swing above some critical frequency (power bandwidth)**

$$u_{E\max} = \frac{SR}{\omega}$$

amplitude-dependant distortion



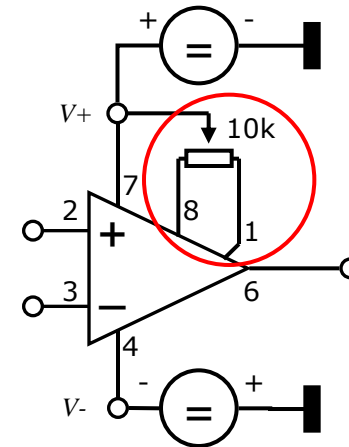
frequency-dependant distortion



# Input offset voltage and bias current

## Offset voltage

- output signal  $u_A \neq 0$  even for no input signal  $u_+ = u_- = 0$ , due to unsymmetrical differential-amplifier at input (usually several  $\mu\text{V}$ )
- **amplified** towards output
- **can be compensated** by potentiometer at extra pins
- problem is **temperature drift** of usually  $1 \mu\text{V}/^\circ\text{C}$



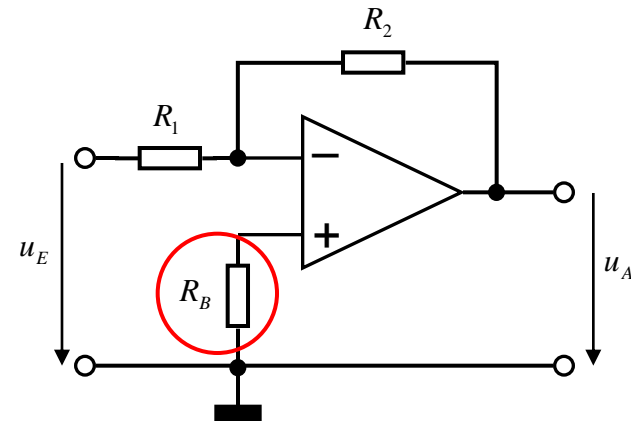
## Bias current

- input transistors need **constant base- or gate-current** for operation, which is delivered by power supply

Bipolar	100 nA
Darlington	1 nA
FET	1 pA

- **can be compensated** by additional bias resistor  $R_B$

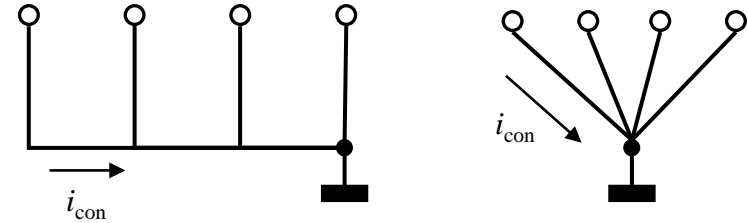
$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$



# Grounding problems

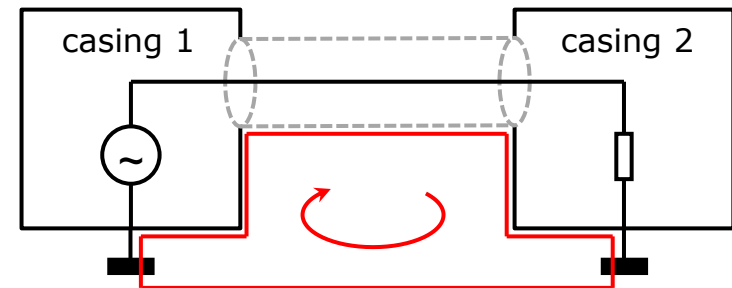
## Ground connections

- Conductive tracks have **nonzero resistance**, so flowing currents produce a voltage drop (ground shift) -> **separate signal ground from consumer ground**
- If possible, connect every device/component **starlike** to one common ground reference



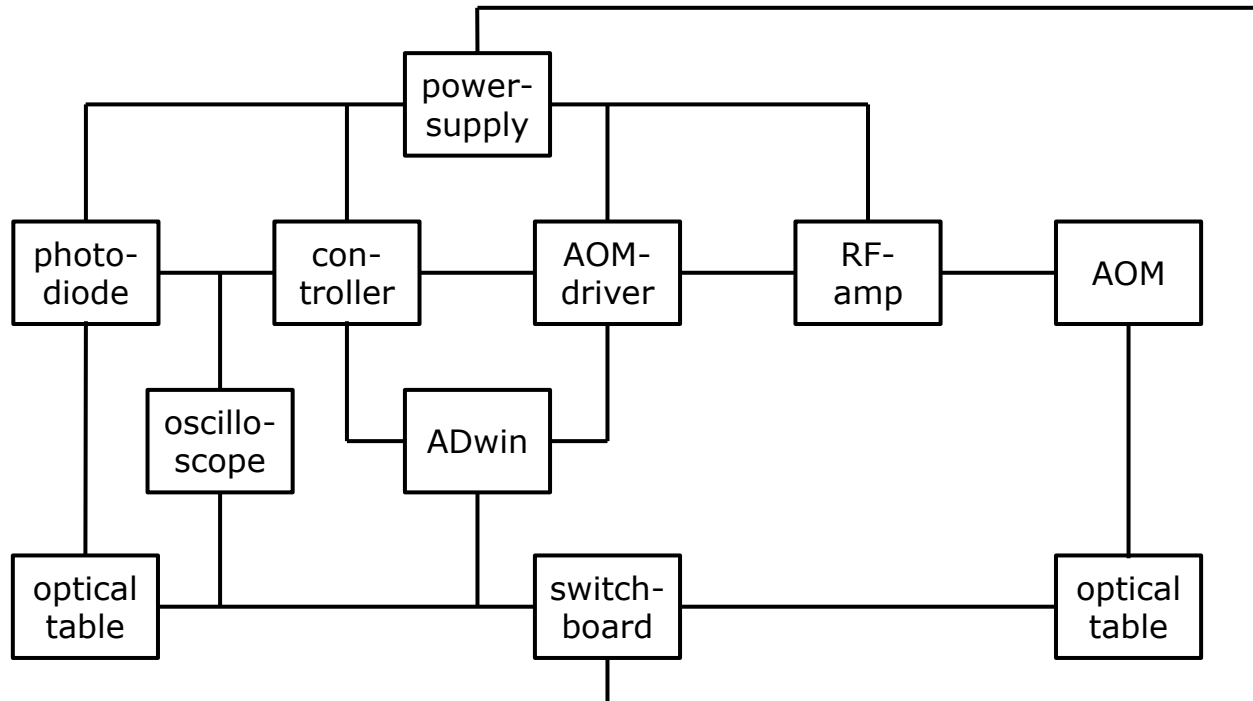
## Ground loops

- RF-signal need shielding to avoid crosstalk  
→ use coaxial cables as waveguides
- **Problem:** DC-signals need two wires, i.e. a clear voltage reference (usually GND)  
→ shielding provides conductor loop between two grounded casings
- Time-varying stray fields from **transformers** induce voltages / currents in loop  
→ a 10  $\mu\text{T}$  stray field of a 50 Hz transformer induces  **$\sim 100 \text{ mV}$**  in a 10  $\text{m}^2$  conductor loop



# Avoiding ground loops

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1. Don't use shielding as conductor, i.e. signal path
2. Isolate all casings from metal surfaces
3. Use separate power supplies
4. Power supplies don't need a ground reference
5. Use digital / analog isolators (e.g. ADUM524x and AD210)
6. Measure signals differentially
7. No oscilloscopes at unbuffered / source signals

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