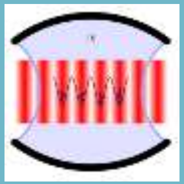
FWF Der Wissenschaftsfonds.

SFB FoQuS Foundations and Applications of Quantum Science

Cavity QED with Ultracold Gases

Ultracold Gases ***Cavity QED***



Optomechanics

Helmut Ritsch

Theoretische Physik
Universität Innsbruck

Quantum Dynamics Workshop

Hamburg, March 2014

People

PD‘s +PhD‘s:

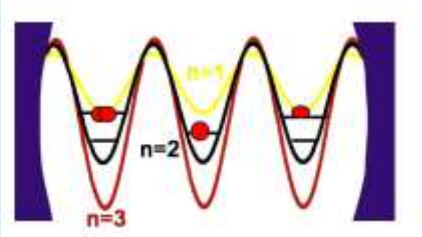
Claudiu Genes
(Wolfgang Niedenzu
Kathrin Sandner)

Tobias Griesser
Laurin Ostermann
Raimar Sandner
Matthias Sonnleitner
Sebastian Krämer
Stefan Ostermann

Master students:
Thomas Maier
Dominik Winterauer

Collaborations (theory):
Peter Domokos, Andras Vukics, Janos Asboth (Budapest)
Giovanna Morigi (Saarbrücken), Aurelian Dantan (Aarhus)
Igor Mekhov (Oxford), Maciej Lewenstein (IFCO)
Hashem Zoubi (Dresden), M. Holland (JILA)
Fancesco Piazza (Munich)

Basic Theme and Physics



Quantum optics:
quantized light modes +
fixed classical matter

Ultracold gases:
quantum particle motion +
fixed classical optical potential

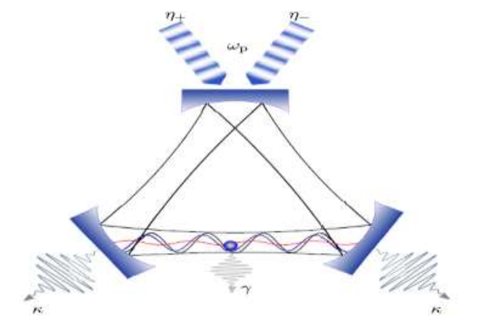
Quantum optics with quantum gases:
full quantum description of light and matter waves

light induces a
dynamic optical lattice with quantized depth

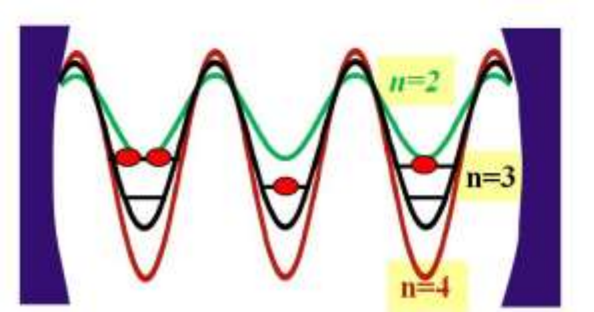
atoms generate a
dynamic refractive index with quantum properties

H. R., P Domokos, F. Brennecke, T. Esslinger, Rev. Mod. Phys., 2013

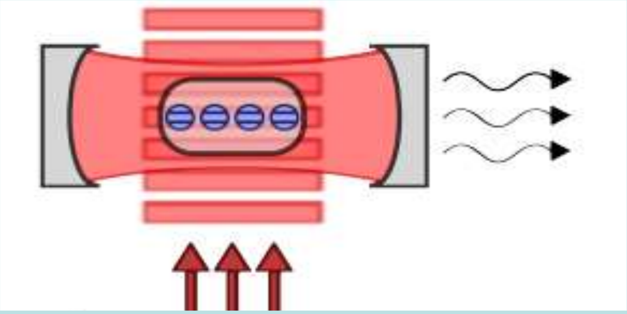
- *Cavity QED and Cavity Cooling*



- *Ground state cooling and atomic quantum statistics in (multimode) resonators*



- *Quantum dynamics of selfordering in multimode cavities*



Light force basics

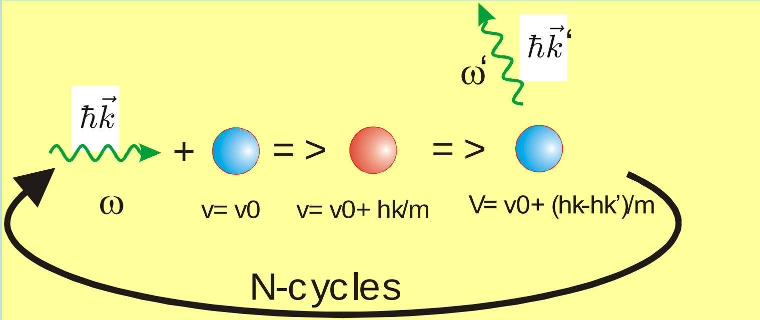
Two classes of forces on point particles *(CCT, JD)* :

Radiation pressure

(absorption + spontaneous re-emission)

Dipole force

(absorption + stimulated emission)



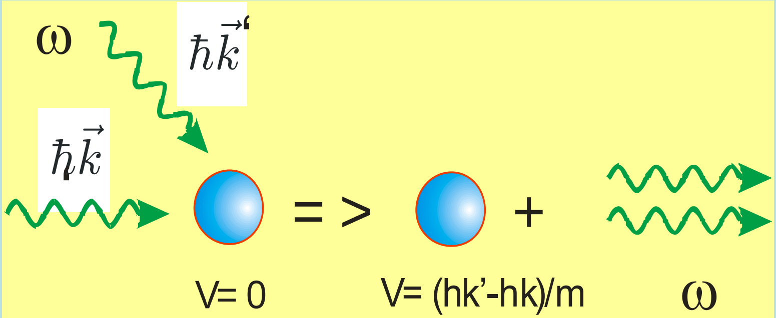
$\hbar\vec k$

in… t rando… ons

$\vec F_{rad} =$ … $+\Gamma^2/4+\frac{1}{2}\Omega^2$

negligible
in dispersive
large detuning
limit !

- **di**… fluct… ce
- **scale**… **1/detuning**…



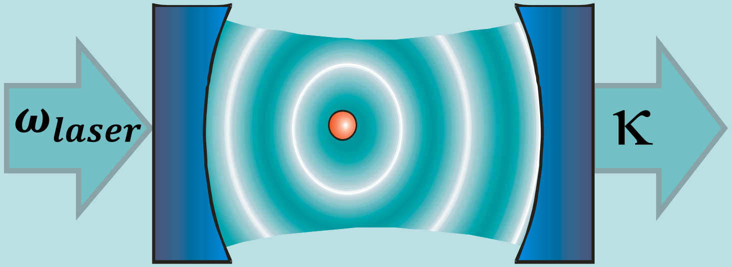
coherent transfer of momentum, which **depends on relative phases** of fields

$$\vec F_{dip} = \hbar(\omega_l-\omega_a)\frac{\frac{1}{2}(\vec\nabla\Omega)^2}{(\omega_l-\omega_a)^2+\Gamma^2/4+\frac{1}{2}\Omega^2}$$

- **conservative optical potential ~ intensity**
- **scales as 1/detuning**
- **optical tweezers,**

(mirror reflection = called “radiation pressure”)

Lightforces on moving polarizable particles in optical resonators



dispersive regime
(= dipol-force) at
large laser-atom detuning:

$$\Delta = \omega_{laser} - \omega_{atom} > \gamma, \kappa$$
$$\omega_{laser} \sim \omega_{cavity}$$

dipole force dominates
spontaneous emission

***Light forces** of resonatorfield determine atomic motion*

↓

interaction

↑

- **self - trapping**
- **friction + diffusion**
- **correlation and entanglement**

***Particle position and motion** changes resonator field dynamics*

Dispersive Cavity QED:

at large atom-cavity detuning Δ ≫ γ

***U(x)* = *optical potential* per photon = cavity *frequency shift* per atom**

$$U(x) := \frac{\Delta_a}{\Delta_a^2 + \gamma_0^2} g_0^2 \cos^2(kx)$$

go … coupling
γ … atomic width
κ ... cavity linewidth

***γ(x)* = *photon loss* per particle = radiation pressure photon**

$$\gamma(x) := \frac{\gamma_0}{\Delta_a^2 + \gamma_0^2} g_0^2 \cos^2(kx)$$

Dispersive limit Δ > γ

$$U >> \gamma$$

⇒ atom-field interaction via **optical potential U**
⇒ **dipole force** dominates radiation pressure

Strong coupling re-defined

$$U >> \kappa >> \gamma$$

⇒ single atom shifts cavity in or out of resonance
⇒ single photon creates an optical trap for an atom

Ultrastrong dispersive coupling

$$U >> \Delta\omega_{mode} >> \kappa$$

⇒ single mode picture breaks down
⇒ nonlinear coupled multi mode model

Point particle in optical resonator

field amplitude:

momentum:

position:

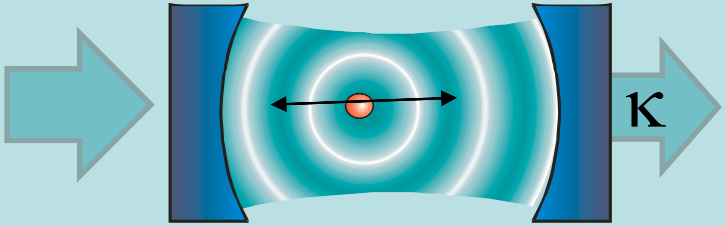
$$\begin{aligned}\dot{E} &= [-\kappa - \boldsymbol{\gamma}(\boldsymbol{x}) + i\Delta_c - i\boldsymbol{U}(\boldsymbol{x})]\,E - \alpha,\\ \dot{p} &= -|E|^2\frac{d}{dx}\boldsymbol{U}(\boldsymbol{x}),\\ \dot{x} &= p/m.\end{aligned}$$

detuning + loss of mode depend on atom position

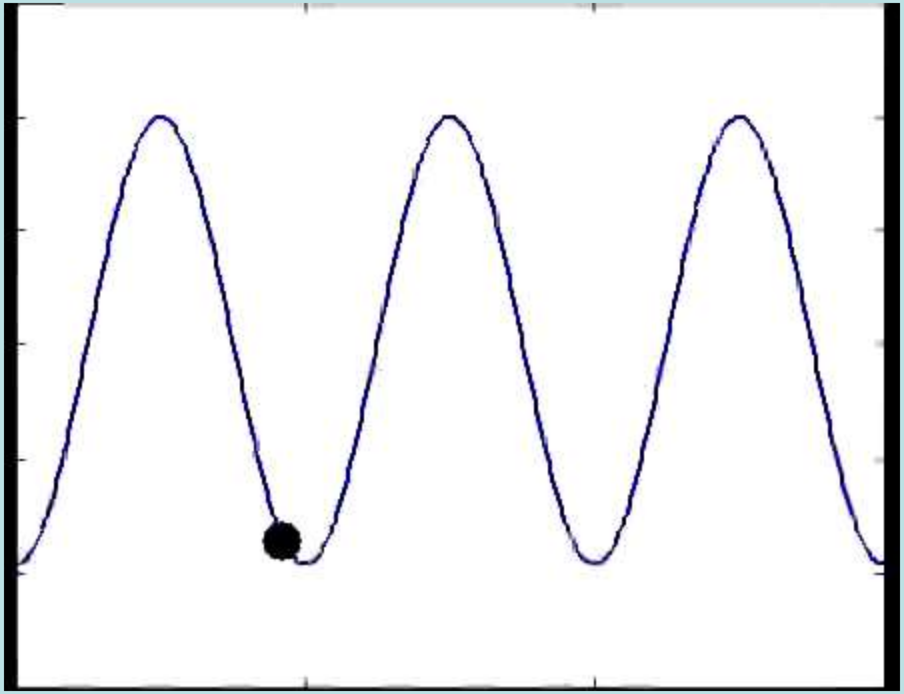
red detuning:

- *atoms drawn to field antinodes*
- *field gets maximal for atom at antinode*

particle moving in optical potential along axis



Lewenstein , PRL 95: ions
Horak, PRL 97: atoms
Vuletic, Chu, PRL 00: atoms
Vitali, PRL 02: mirrors



Semiclassical model:

Master equation for ρ *=> PDE for Wignerfunction =>*
truncate at 2. order => Fokker Planck equation with drift+ diffusion
=> equivalent ***Ito****-stochastic differential equations*

$$dx = \frac{p}{M}dt,$$

$$dp = -\hbar U_0 \left(\alpha_r^2 + \alpha_i^2 - \frac{1}{2}\right) \nabla f^2(x) dt + dP,$$

$$d\alpha_r = -\eta dt + \left(U_0 f^2(x) - \Delta_C\right) \alpha_i dt - \left(\kappa + \Gamma_0 f^2(x)\right) \alpha_r dt + dA_r$$

$$d\alpha_i = -\left(U_0 f^2(x) - \Delta_C\right) \alpha_r dt - \left(\kappa + \Gamma_0 f^2(x)\right) \alpha_i dt + dA_i$$

(quantum)noise forces (dP, dA) => quantum expectation value ⇔ stochastic average

Analytic solution for slow atoms for friction and diffusion

friction

$$\overline{\overline{F}}_1 = -k^2 \frac{\eta^2 U_0^2}{4\kappa^4}$$

diffusion

$$\overline{D} = k^2 \kappa \frac{\eta^2 U_0^2}{8\kappa^4}$$

temperature

$$k_B T = -\frac{\overline{D}}{\overline{F_1}} = \frac{\kappa}{2}$$

Cooling limited by cavity linewidth
$\boldsymbol{\kappa}$ ***!***

Cooling time scale:

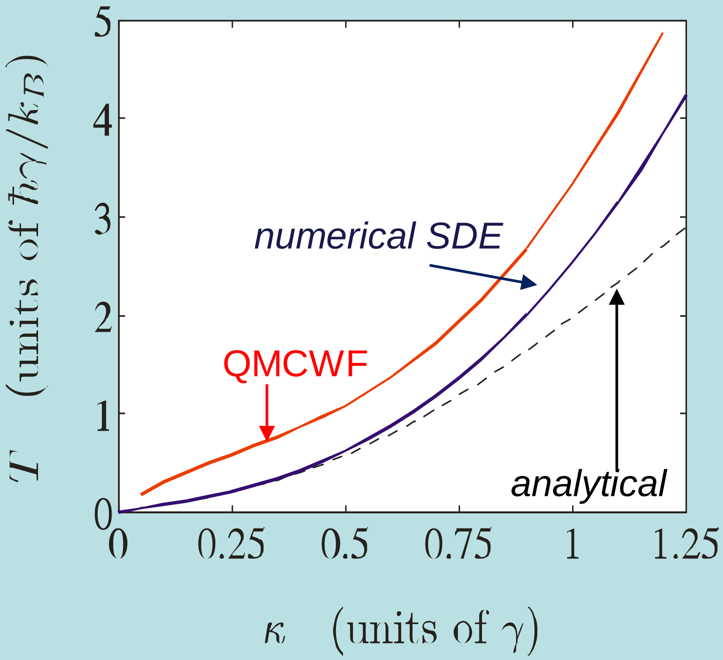
$$\tau_c = \frac{m}{2|\overline{F_1}|} = \frac{\kappa^4}{\eta^2 U_0^2}\omega_R^{-1}$$

First Application: Cavity cooling

Horak, PRL 97

Theoretical simulation checked by analytical limits and QMC-Wavefunction Simulations

Temperature limit ~ hκ



applications: **trap + cool atoms, molecules, nanoparticles**

analogous: optomechanic cavity cooling of **mirrors/membranes** …

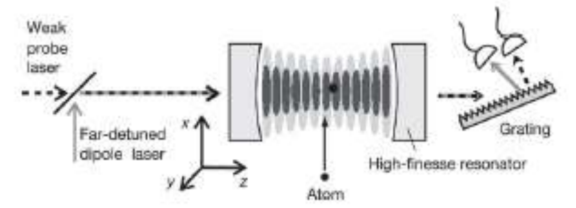
Few particle cavity cooling experiments in classical regime

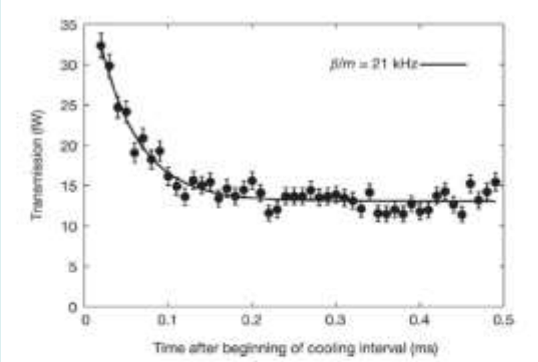
first proof of principle experiment:
MPQ München, Nature 2004

Cavity cooling of a single atom

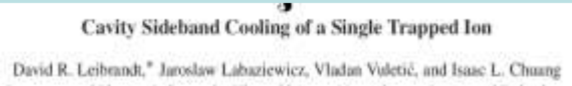
P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse & G. Rempe

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1,
D-85748 Garching, Germany



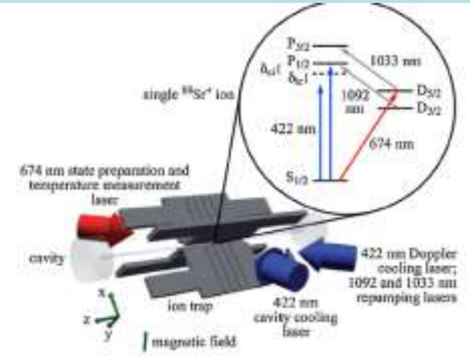


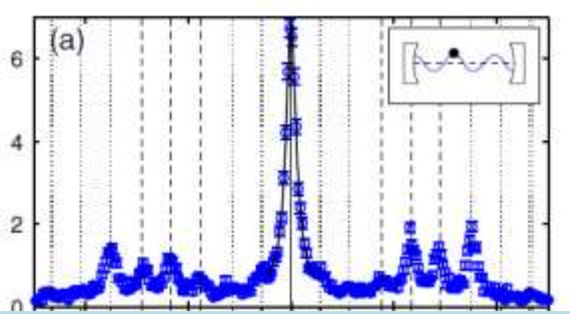
Ions: MIT PRL 2009



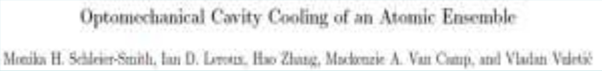
Cavity Sideband Cooling of a Single Trapped Ion

David R. Leibrandt,* Jaroslaw Labaziewicz, Vladan Vuletić, and Isaac L. Chuang





Atomic ensembles:
MIT, Vuletic PRL 2009



Optomechanical Cavity Cooling of an Atomic Ensemble

Monika H. Schleier-Smith, Ian D. Leroux, Hao Zhang, Mackenzie A. Van Camp, and Vladan Vuletić

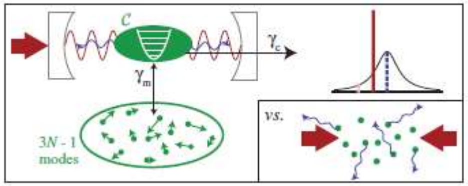


FIG. 1. (Color online) Ensemble cavity cooling. A probe

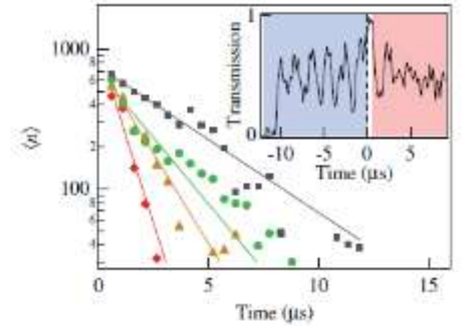
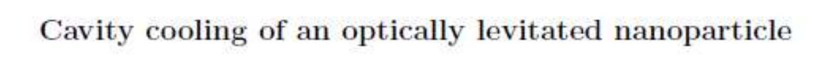


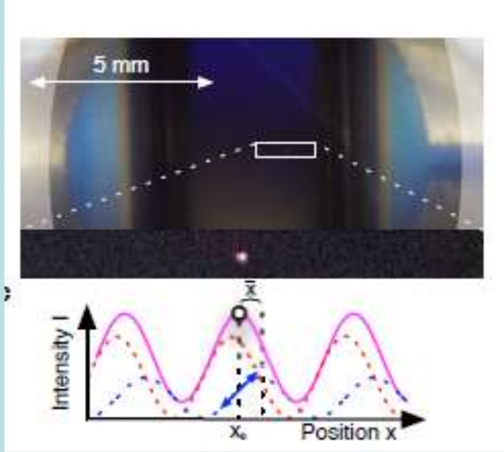
FIG. 2. (Color online) Mean occupation number ⟨n⟩ of mode

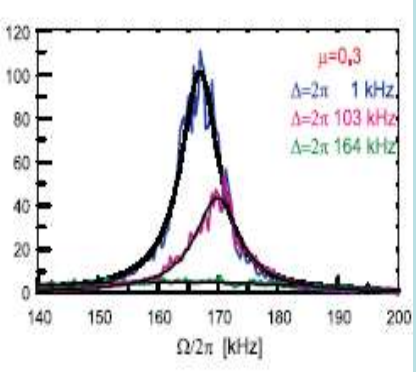
- More experiments by other groups : Meschede (EIT-cavity cooling), Stamper-Kurn, Barret, Baden , Barker, Renzoni ,.???
- temperature und cooling rate agree well with theory
- Ions: multiple vibrational modes addressed (Vuletic 2011), Barrett (2012) ,

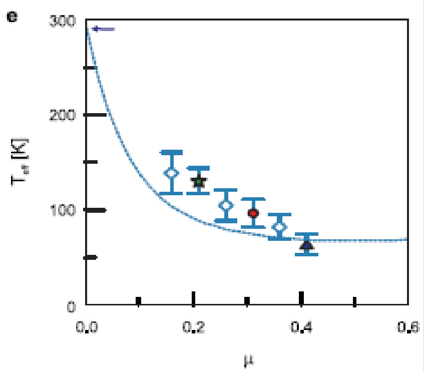
micro beads, Vienna 2013


Cavity cooling of an optically levitated nanoparticle

Kiesel, Blaser, Aspelmeyer, (PNAS 2014) ..

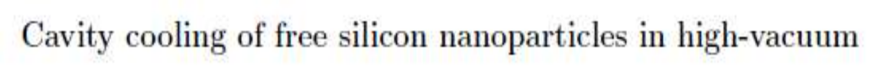






Trapping and efficient axial cooling

micro grain particles,Vienna 2013


Cavity cooling of free silicon nanoparticles in high-vacuum

Asenbaum, Arndt, (2014) ...

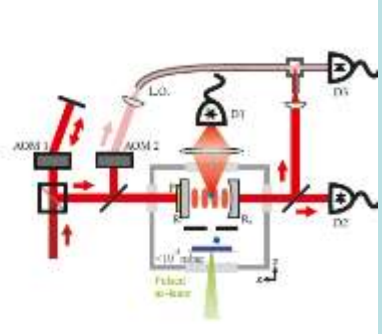
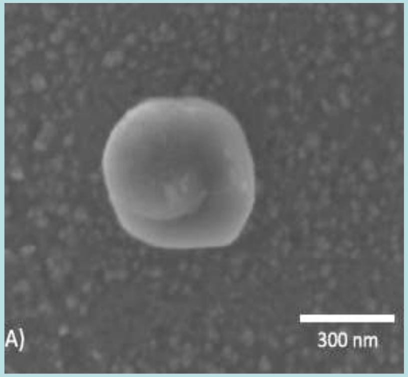
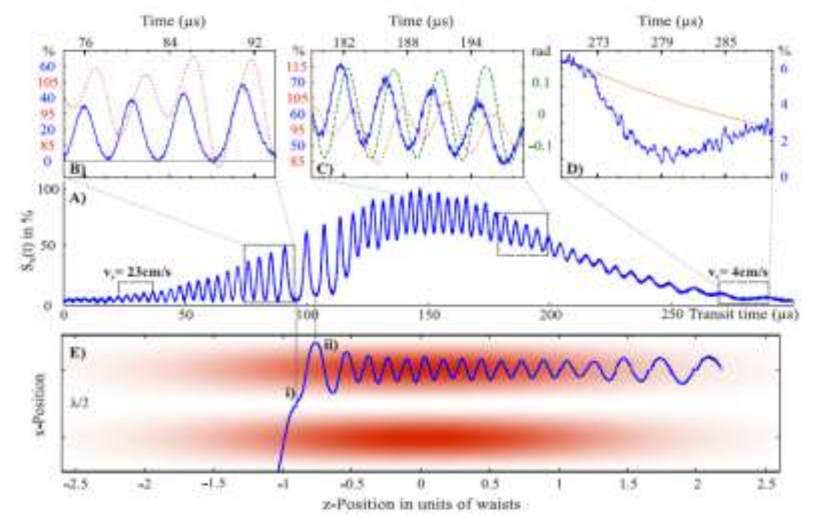


FIG. 1. Cavity cooling of pure silicon nanoparticles in a high-vacuum environment. The fiber laser beam (1550



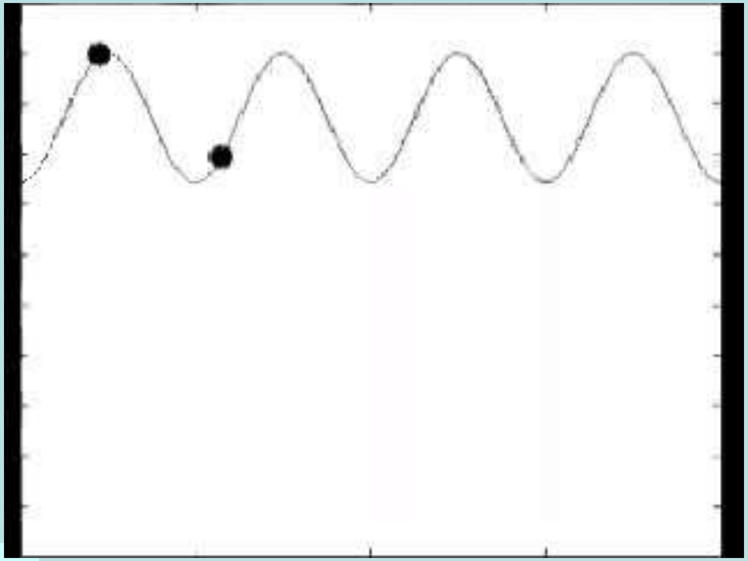


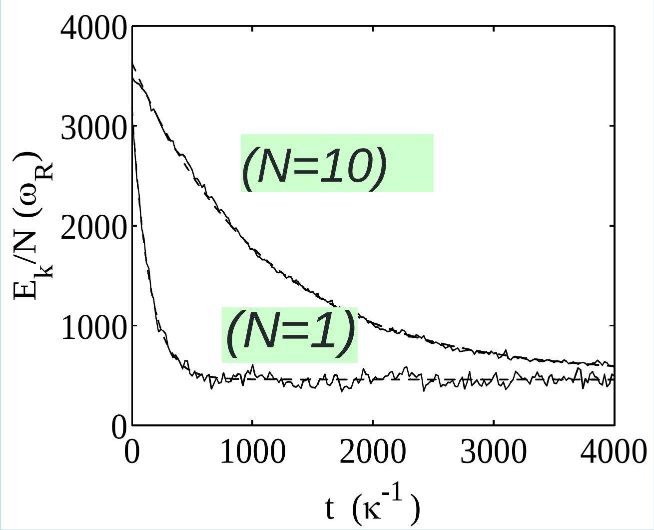
radial and axial cooling in high vacuum

New results by K.Dholakia (St. Andrews), P. Barker (Imperial)

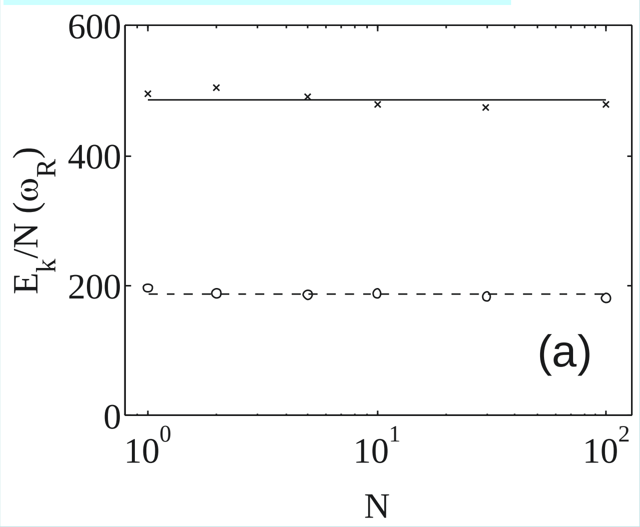
Scaling of resonator induced cooling

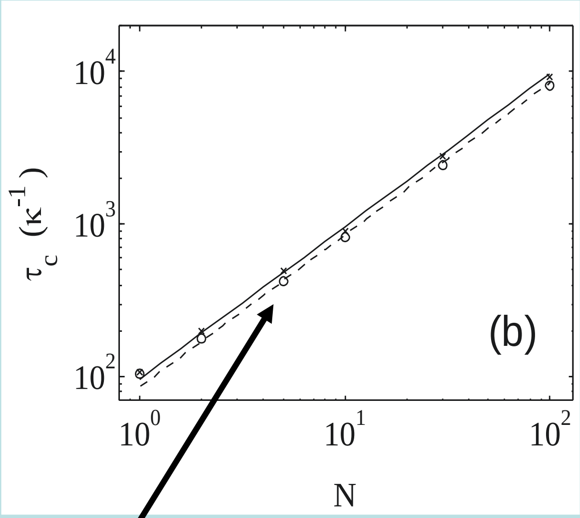
- *Is resonator cooling good for larger ensembles ?*
- *How far can we go in T ?*





temperature unchanged

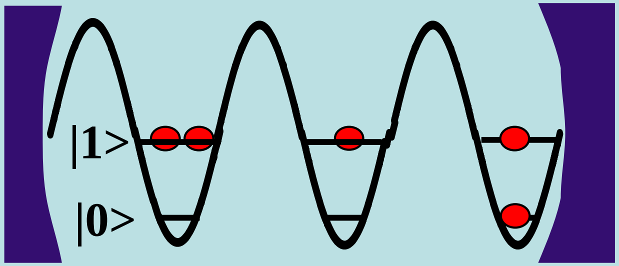




Slow cooling for large ensembles !
(Scaling determined from numerics)

Quantum limit of cavity cooling: atomic motion + field quantized: very good cavity with width smaller than recoil frequency (~hk^2/2m)

$\kappa < \omega_{rec}$



$$H = \frac{p^2}{2m} + \underbrace{\hbar U_0 a^\dagger a \cos^2(kx)}_{\text{quantum potential}} - \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right)$$

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho] + \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right)$$

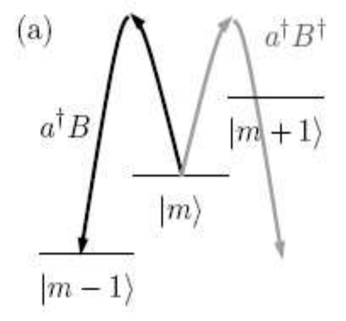
Strong pump:
deep lattice

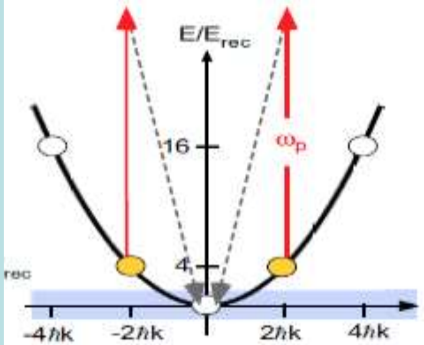
Cavity mode
tuned to

Weak pump:
free motion with eff. mass

„blue“ vibrational sideband
of trapped atoms

higher momentum states
for a free gas





vibrational trap states in cavity field

free space momentum states

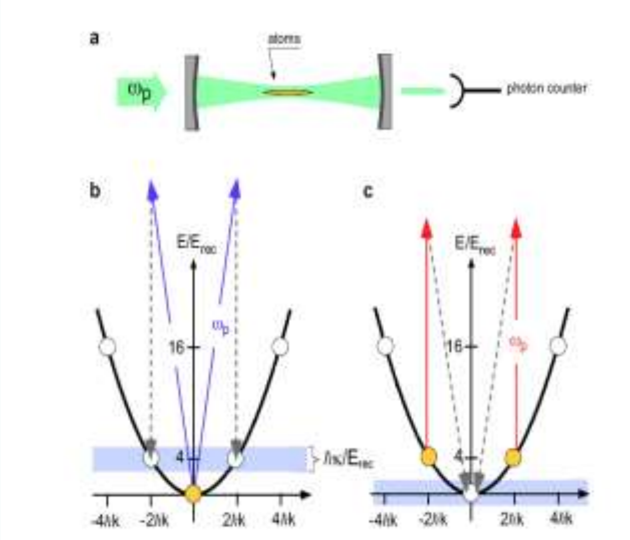
Experiment: cavity ground state cooling of „free“ atoms

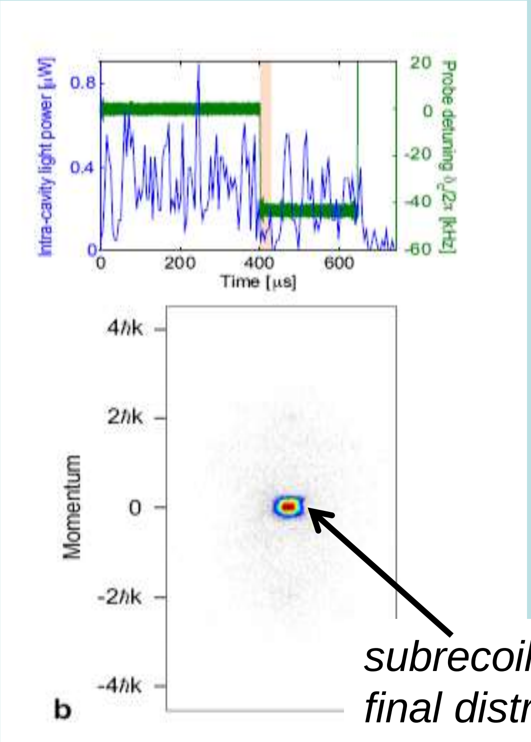
„sub-recoil“ regime:

$\kappa < \omega\_r$

A. Hemmerich, Hamburg (Science 2012)

momentum distribution
smaller than
single photon recoil ?
=> cooling towards degeneracy ?



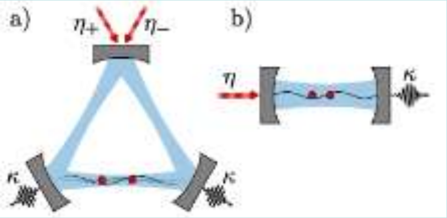


subrecoil width
final distribution !

⇒ Cavity cooling with Bose stimulation to replace evaporation
⇒ BEC formation without particle loss

Cavity cooling dynamics and quantum statistics of two particles

(can we ‚cavity‘-cool down to degeneracy from finite T)



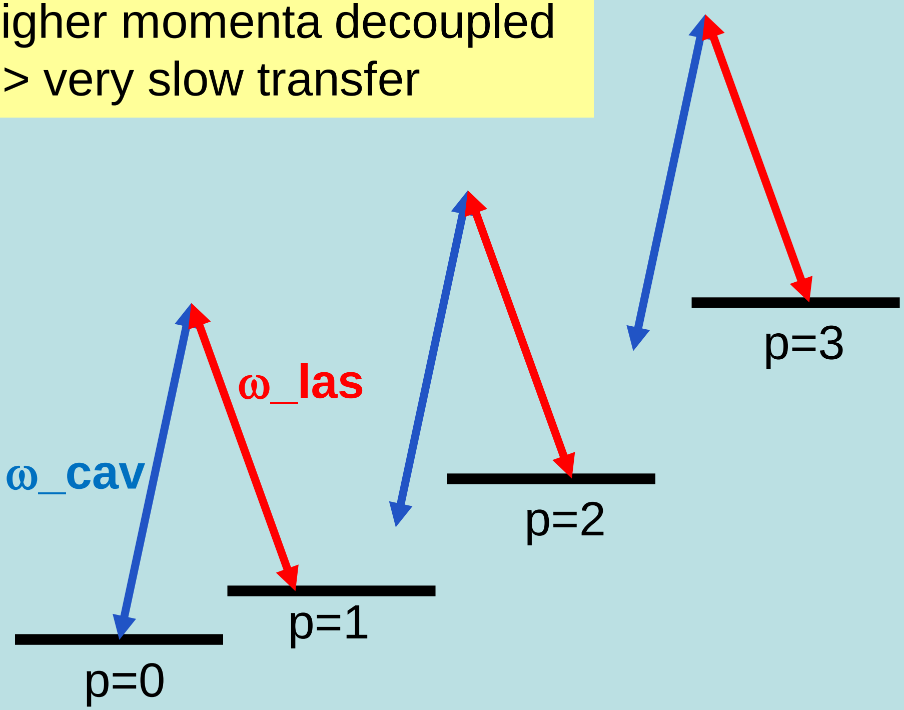
„sub-recoil“ regime :

κ < ω_r

$$H = \sum_{i=1}^{2}\left[\frac{p_i^2}{2m} + \hbar U_0\left(a_c^\dagger a_c \cos^2(kx_i) + a_s^\dagger a_s \sin^2(kx_i)\right)\right] + \frac{\hbar U_0}{2}\left(a_c^\dagger a_s + a_s^\dagger a_c\right)\sum_{i=1}^{2}\sin(2kx_i) - \hbar\Delta_c a_c^\dagger a_c - \hbar\Delta_c a_s^\dagger a_s - i\hbar\eta\left(a_s - a_s^\dagger\right). \quad (1)$$

Groundstate cooling for Δ ~ - 4 ω_r

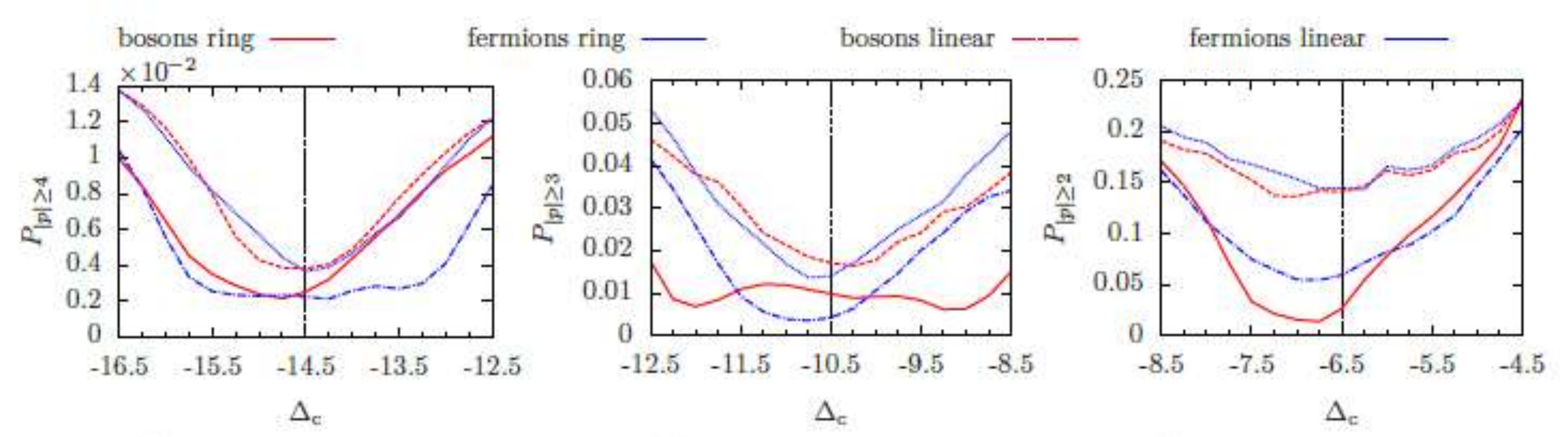
Problem: nonlinear ladder
higher momenta decoupled
=> very slow transfer



=> Compress momentum distribution in a multistep cooling sequence

Simulate last three steps:

(1) Δ < -12 ω_r => **(2)** Δ < -12 ω_r **=> (3)** Δ < -12 ω_r



(a) Stage 1, $\eta = 3\omega_R$ (b) Stage 2, $\eta = 2\omega_R$ (c) Stage 3, $\eta = 0.5\omega_R$

Efficiency of cooling stages depends on detuning
and particle type (symmetric or antisymetric wavefunction)

Example:
time evolution of optimized cooling sequences of two particcles

„bosons“

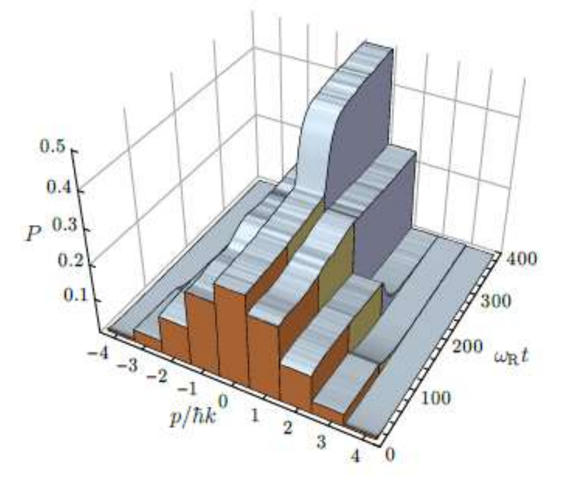


Fig. 4: (Colour on-line) Single-particle momentum distribution for $\Delta_c/\omega_R = -14.75|-12|-7$ (ring cavity bosons).

„fermions“

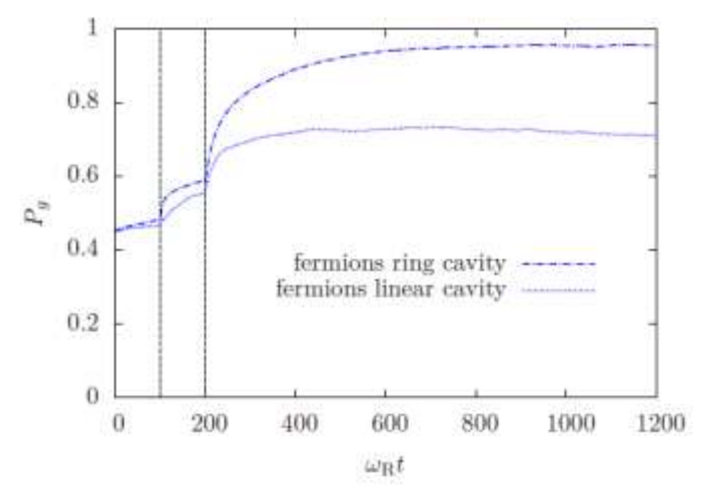
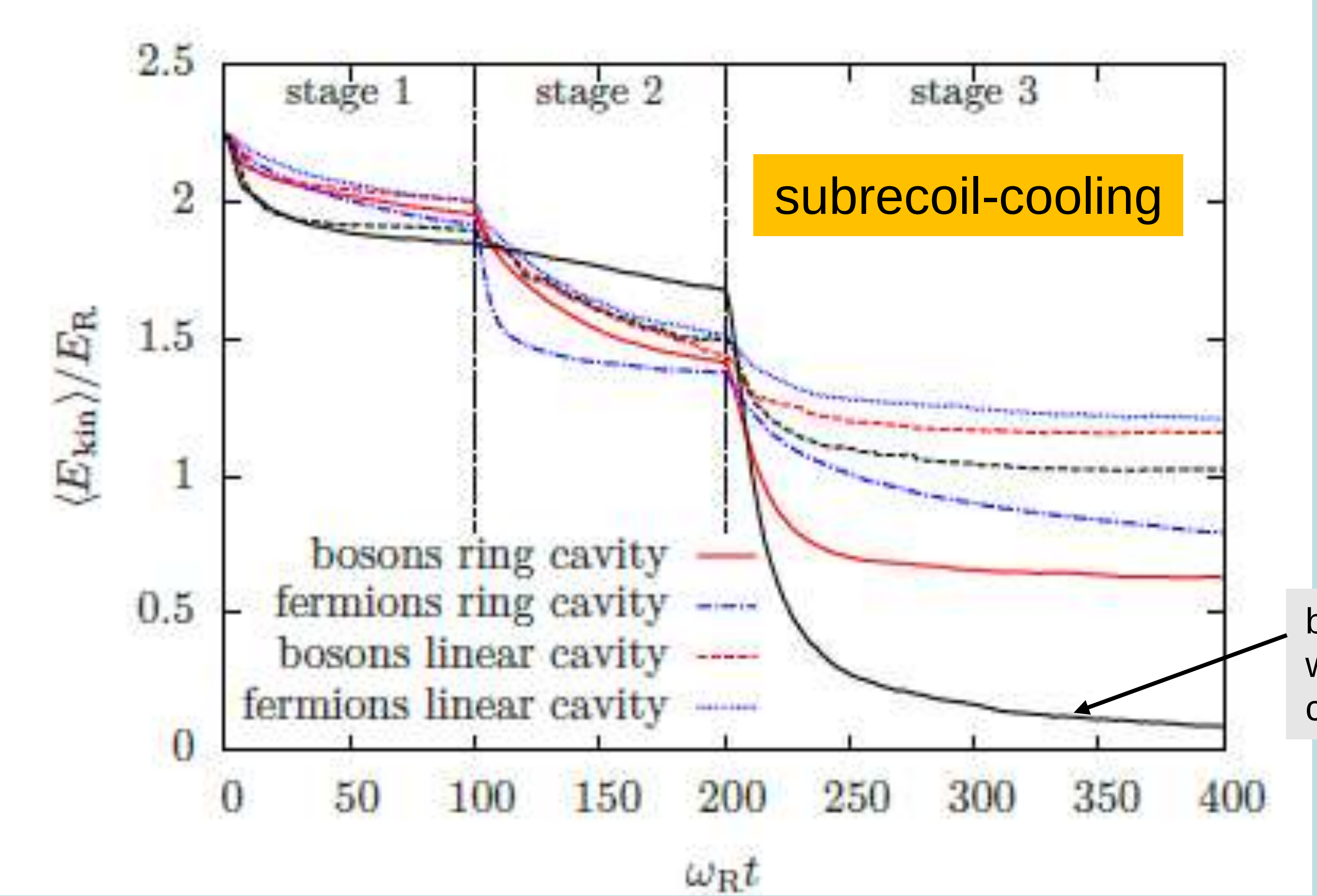


Fig. 8: Projection onto the fermionic ground state P_g.

Ring cavity much better in final stage

Cooling dynamics at optimal detuning for different particle quantum statistics



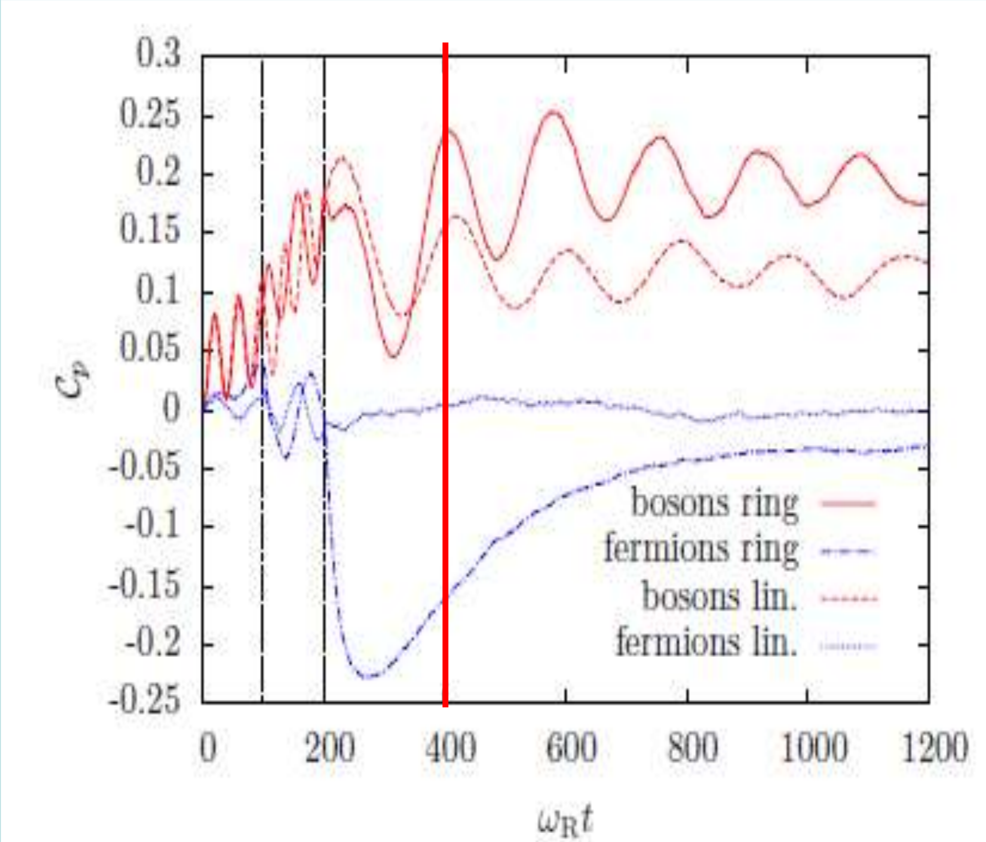
subrecoil-cooling

best results with weak collisions

Ring cavity cools much better in the final stage towards degeneracy

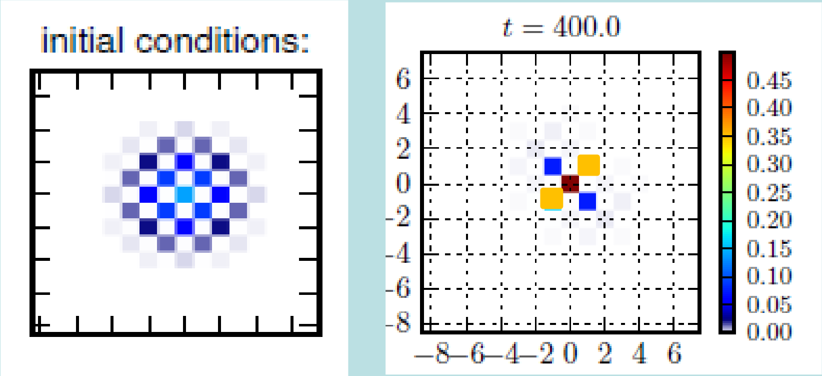
Momentum space pairing and quantum statistics: optimal cooling

momentum pair correlations

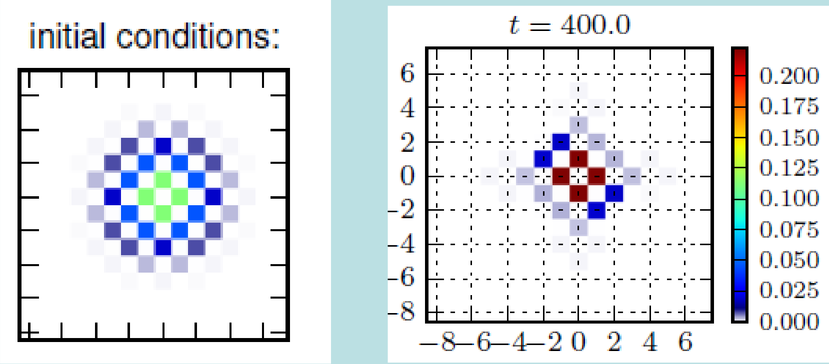


$$C_p := \frac{\mathrm{Cov}(p_1, p_2)}{\Delta p_1 \Delta p_2} = \frac{\langle p_1 p_2 \rangle - \langle p_1 \rangle \langle p_2 \rangle}{\Delta p_1 \Delta p_2}.$$

two **bosons**/distinguishable: correlations



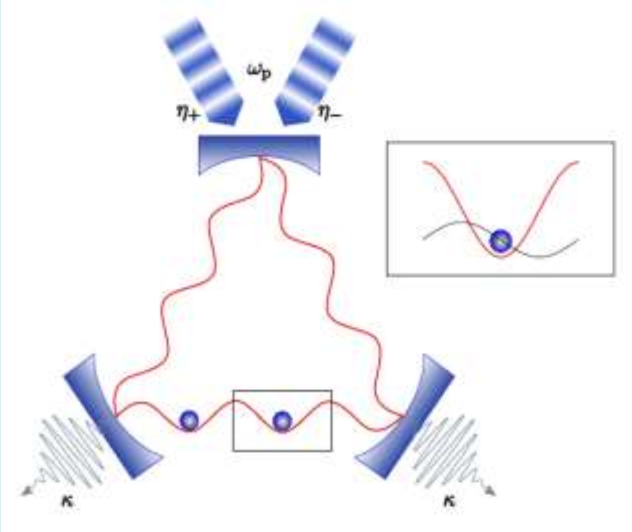
two **fermions**: anticorrelations



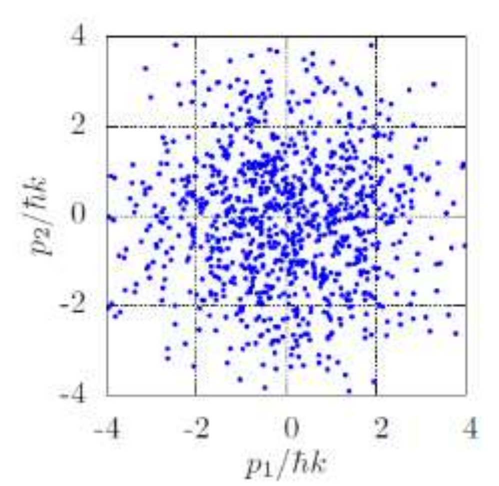
- quantum statistics changes particle momentum correlations
- positive correlations enhanced in ring cavity and for bosons

Two bosonic particles in a ring cavity with dispersive interaction near motional ground state: **momentum pairing**

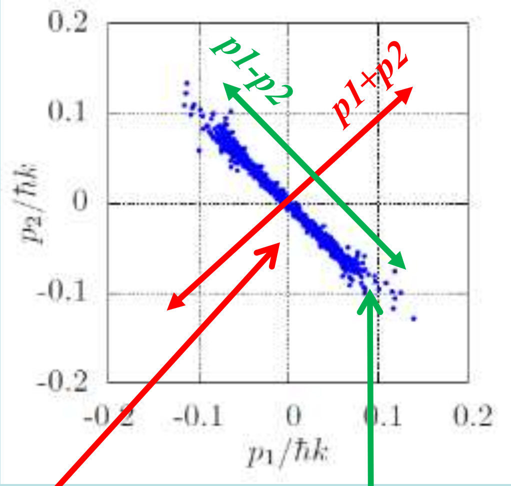
semiclassical classical point particle simulation



start: random momenta



final: correlated momenta



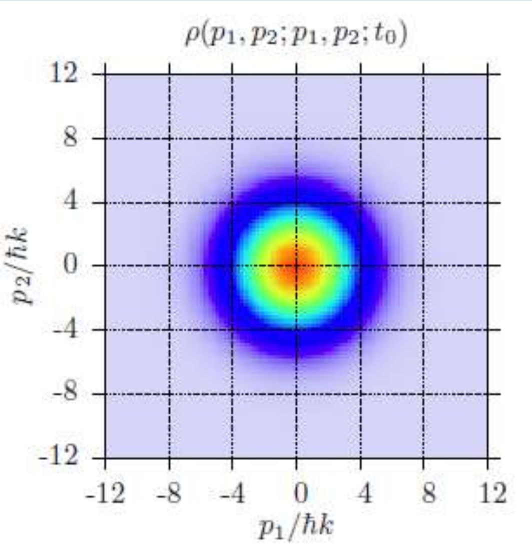
- * Strongly pumped Cos - mode
  => mean field **α**
- * Scattered photons in Sin-mode
  => quantum operator **a**

center of mass
momemtum damps fast
+
anti correlated pairs
decouple from
dissipation

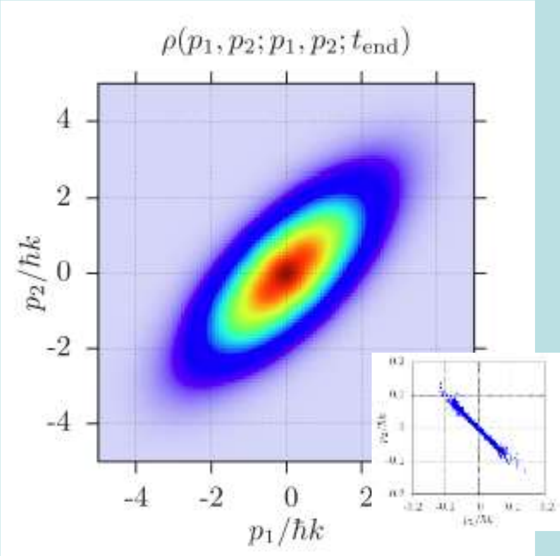
relative motion:
pairing by
momentum
anti-correlation

Quantum trajectory simulations (C++QED): two trapped bosons

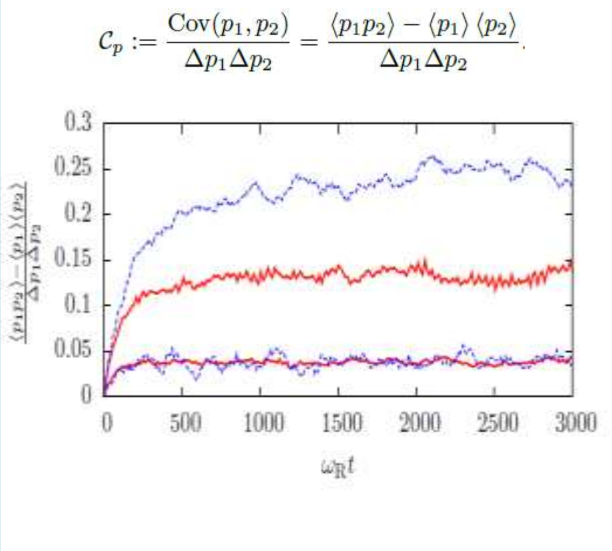
start



final



time evolution



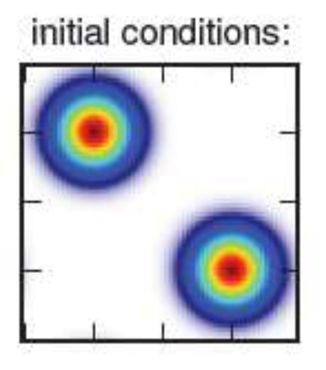
Particles show positive correlations despite zero average center of mass momentum !

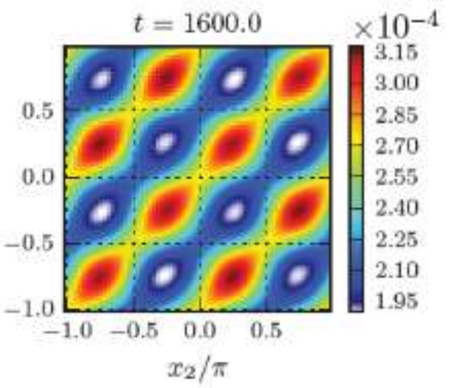
This is allowed by quantum mechanics for a momentum entangled state:

$$|\psi\rangle \propto |p, p\rangle \pm |-p, -p\rangle$$

How is this state forming ??

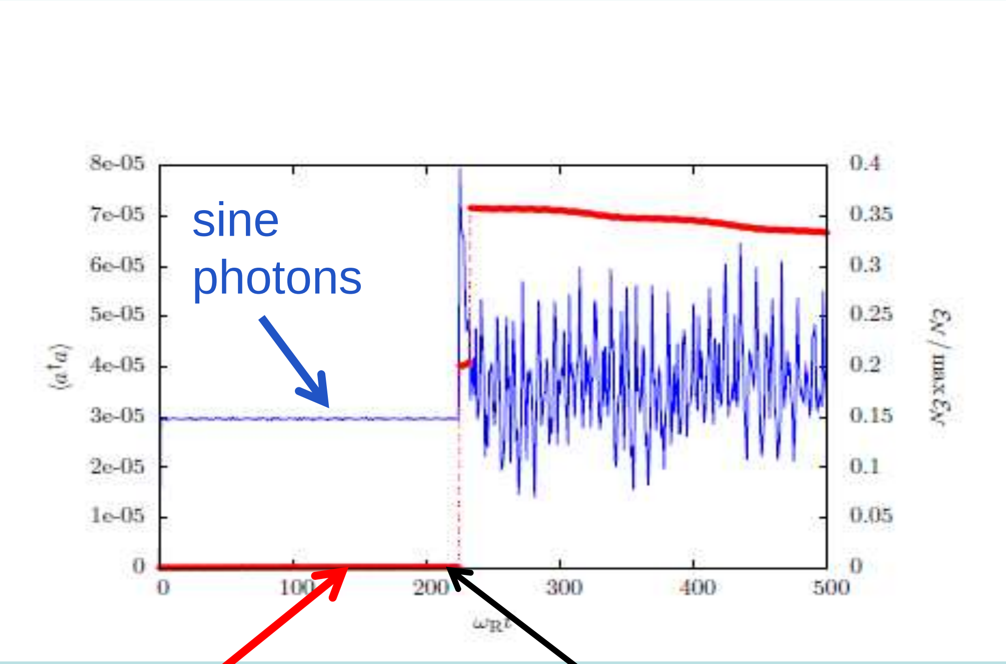
real space distribution: doubled periodicity





Single trajectory analysis shows correlated quantum jumps of particles and field !

single jump:

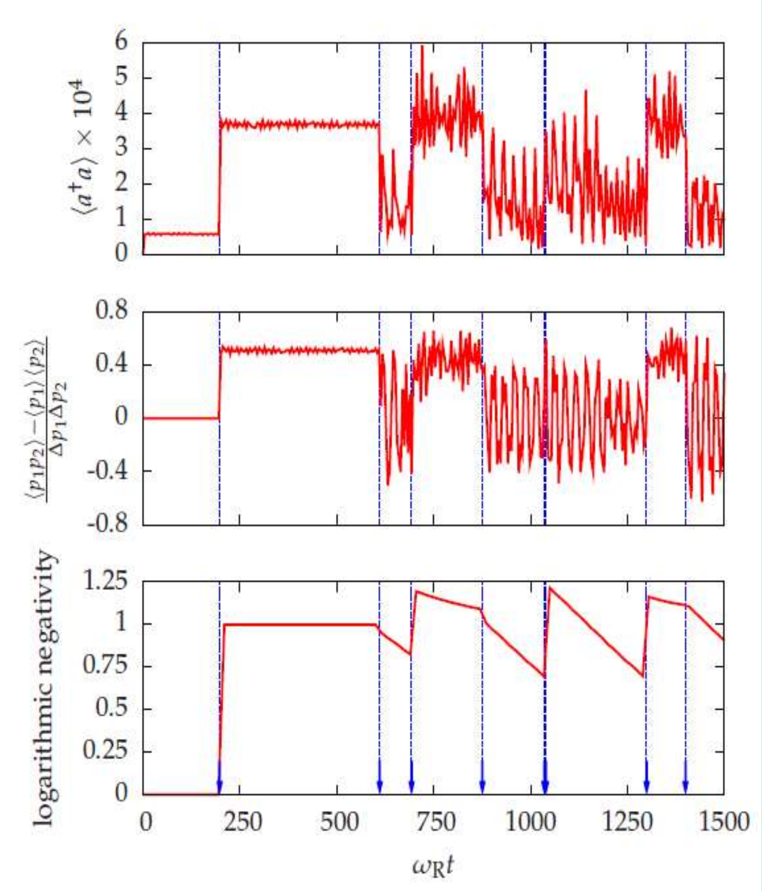


(negativity)

photon click

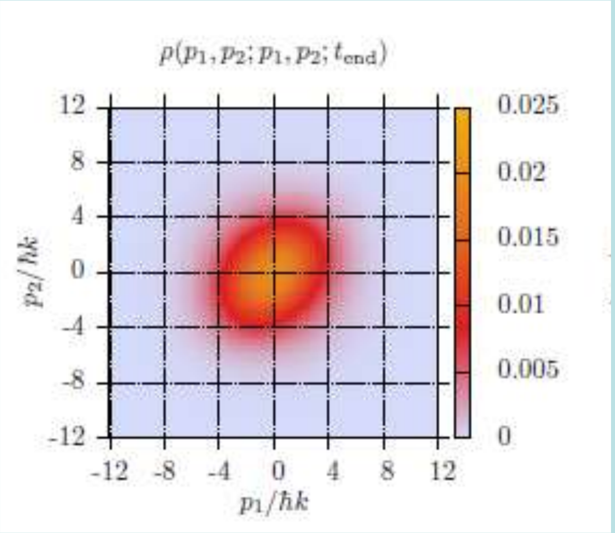
Quantum jumps (= photodetection) increase motional entanglement between the particles
=> strong conditional (heralded) entanglement induced by cavity dissipation / measurement

quantum trajectory simulations

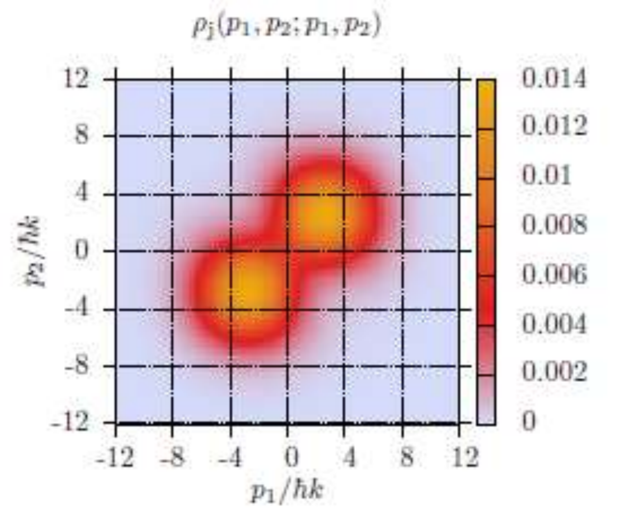


- conditional density matrix exhibitis nonclassical correlations and entanglement
- „squeezing“ of particle (= mirror) distance !

average density matrix



conditional density matrix after click = jump

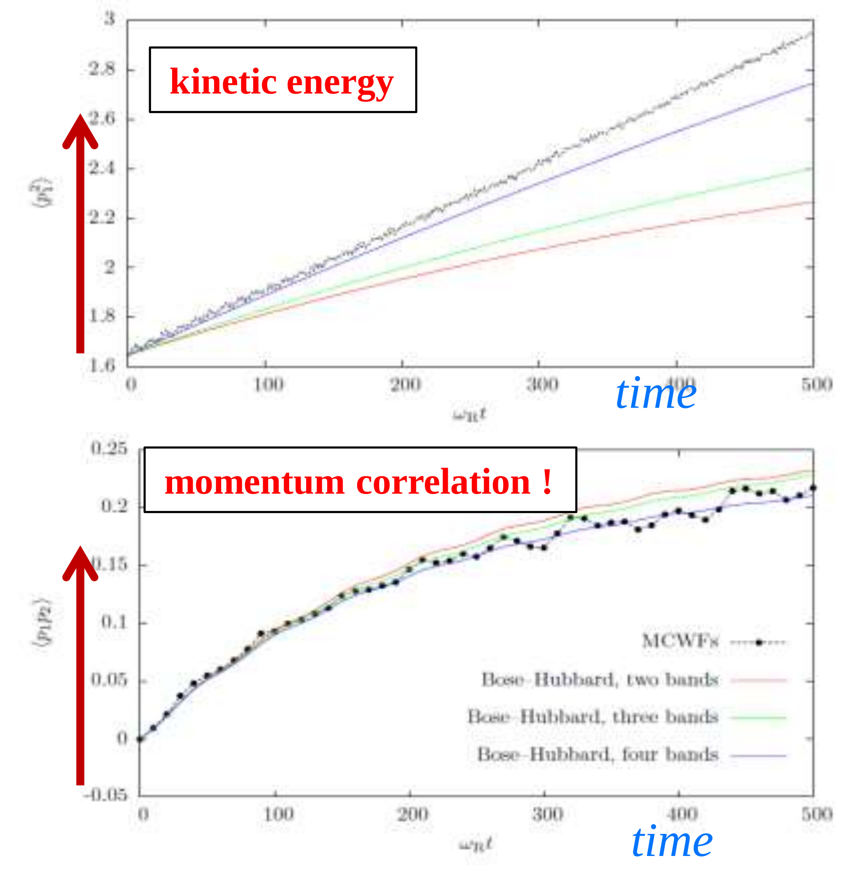


Quantum simulations near T~0

$$H = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \hbar U_0 \alpha^2 \cos^2(kx_i) + \frac{\hbar U_0 \alpha}{2} \sin(2kx_i)(a + a^\dagger) \right) - \hbar \left(\Delta_c - N U_0 \right) a^\dagger a.$$

Particles started independent (= product state) in lattice ground state

diffusion and friction => heating + correlations



kinetic energy

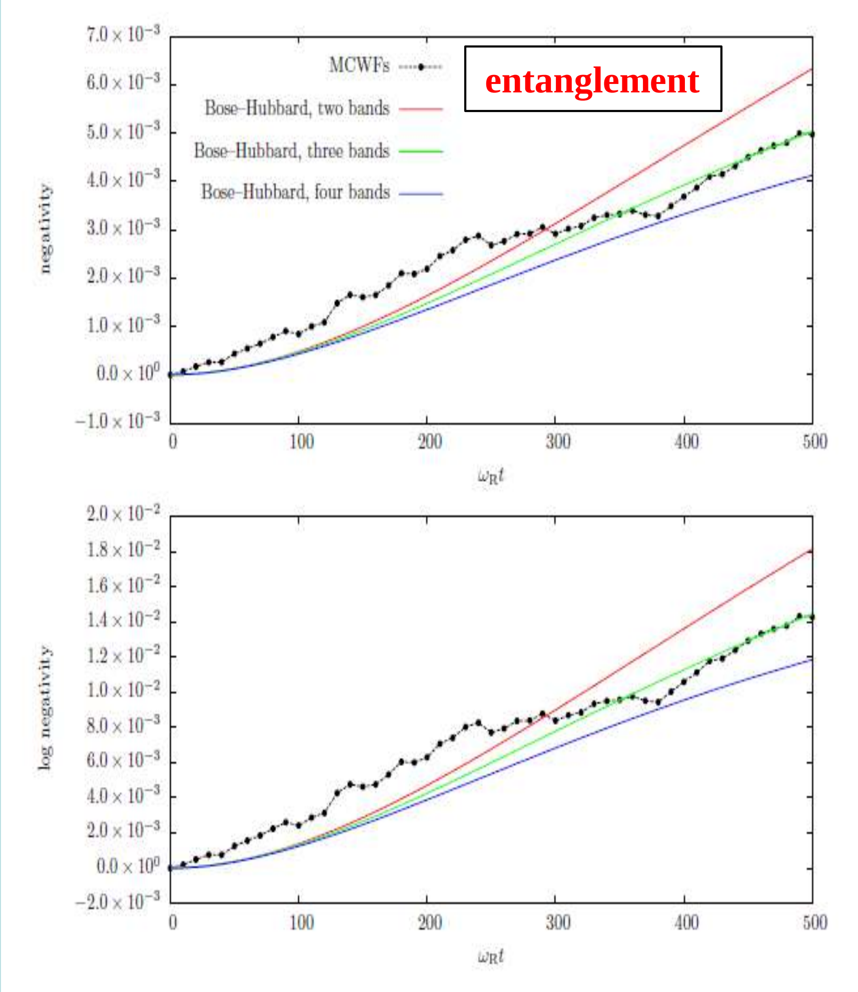
time

momentum correlation !

time

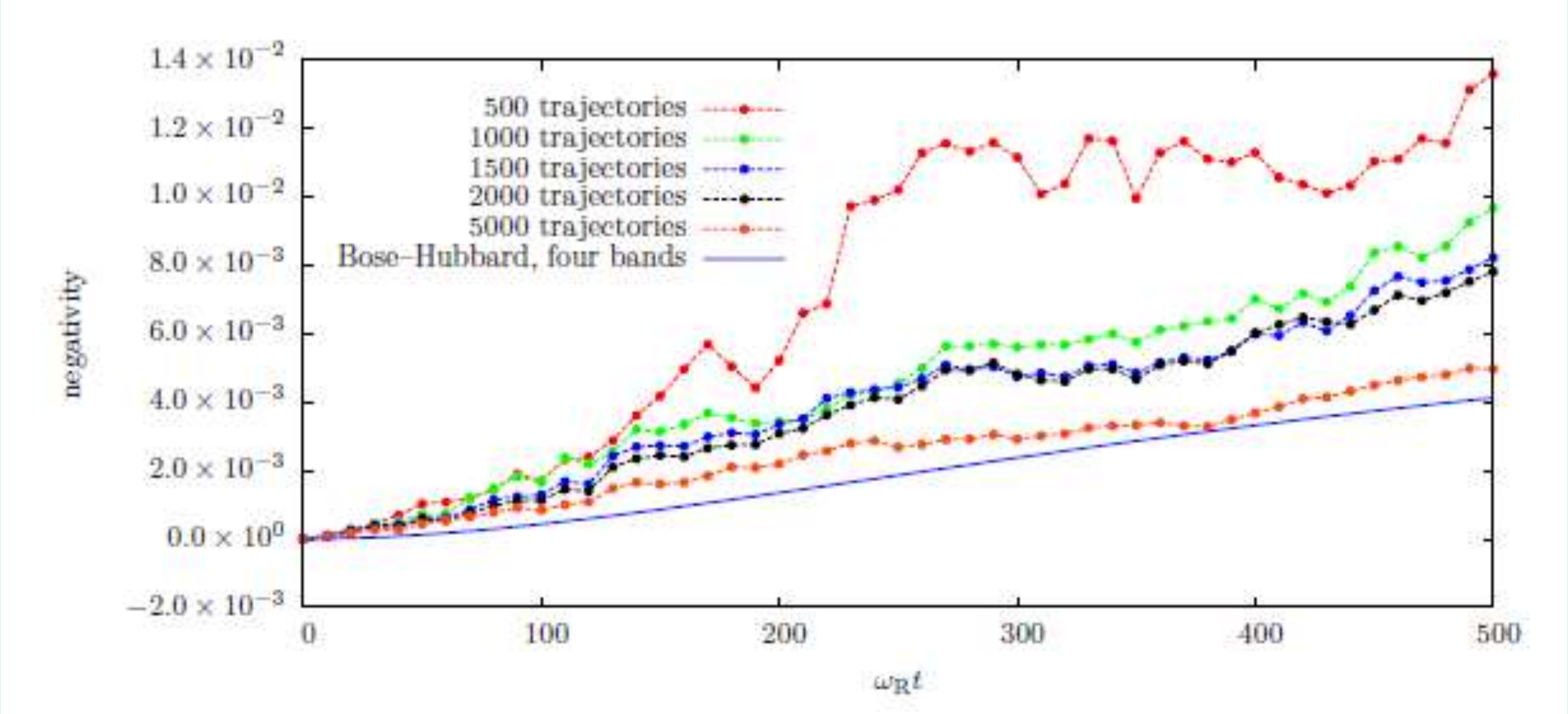
Figure 2: Momentum correlation. The parameters are the same as in figure 1 and max $\langle p_1 p_2 \rangle / \sqrt{\langle p_1^2 \rangle \langle p_2^2 \rangle} \sim 0.08$.

particle-particle entanglement (trace out field)



entanglement

- * Large entanglement in individual trajectories
- * small average steady state entanglement
- * very slow convergence of negativity with sample number



QMCWF-Simulations show slow convergence on correlations and entanglement !

intermediate summary :

- ***Light forces in optical resonators:***

  ** self trapping and cooling of all particles with sufficient polarizability*

  ** subrecoil cooling towards BEC’s and atom lasers*

  ** idealized implementation of optomechanics with ensemble*

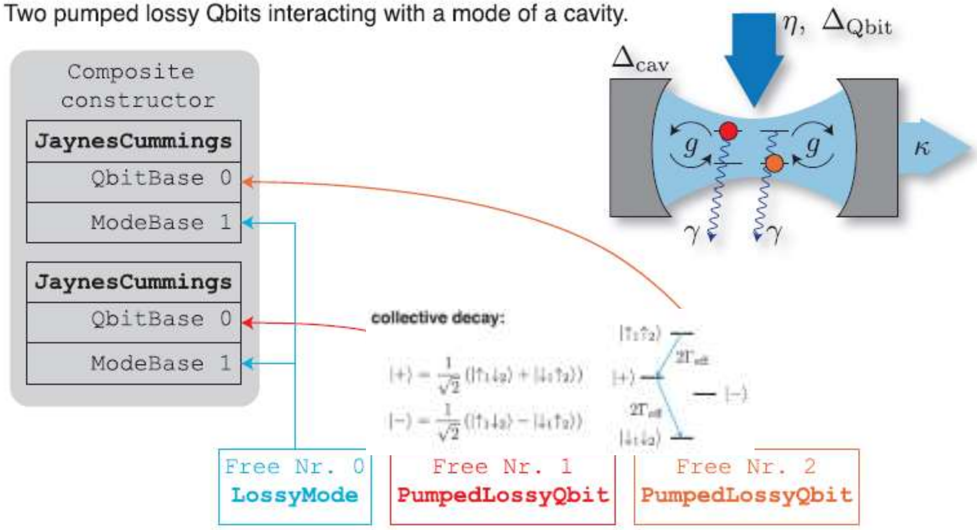
  ** “infinite” range interactions perturb cooling and induce correlations and entanglemnet*

efficient numerical studies by quantum wave-function simulation framework C++QED available via (http://cppqed.sourceforge.net/)

by Andras Vukics (IBK-> Wigner Institute)
Sebastian Krämer, Raimar Sandner (Ibk)
(CCQED ITN-network)

Example of a composite system

Two pumped lossy Qbits interacting with a mode of a cavity.



$$|+\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\downarrow_2\rangle + |\downarrow_1\uparrow_2\rangle\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle\right)$$

Essential part of the script:

```
qbit::SmartPtr qb(qbit::maker(ppqb,qmp));
mode::SmartPtr m(mode::maker(plm,qmp));
JaynesCummings<> jc(qb,m,pjc);

StateVector<3> psi(mode::init(plm)
                   *(qbit::init(qb1init)*qbit::init(qb2init)
                     +s*qbit::init(qb2init)*qbit::init(qb1init)));
psi.renorm();

evolve(psi, makeComposite(Act<1,0>(jc), Act<2,0>(jc)), pe);
```

only one object!

initial state definition

Goal: simulation of composite open quantum systems

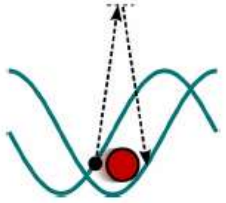
Methods of time evolution

- full master equation simulation
- Monte Carlo wave-function simulation (MCWF, evolution with a non-hermitian Hamiltonian and random quantum jumps)
- ensemble average of MCWF trajectories

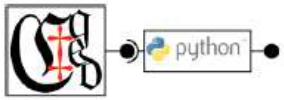
Design objectives

- **Flexibility**: definition of composite systems using elementary free systems and interactions
- **Performance**: e.g. maximal use of compile-time algorithms, adaptive stepsize, interaction picture
- **Extensibility**: maximally reusable code, new elements can build on class hierarchy

C++QED code base



Optimized ring cavity interaction



Python interface

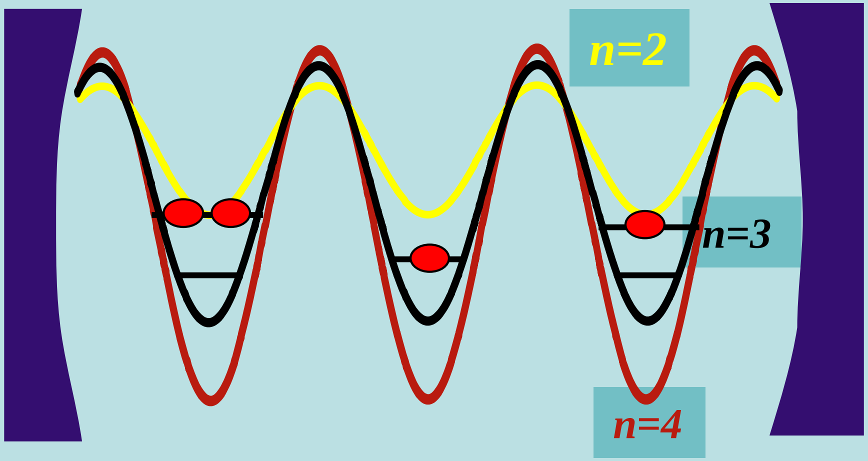
C++QED infrastructure

- New cross-platform build system
- Improved support for Mac OS X / Windows
- Binary packages
- Python tool for convenient computing cluster integration

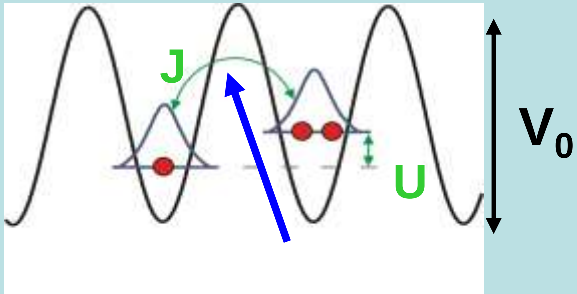
New: Python wrapper …

Part II: Atomic dynamics in cavities beyond 'mean field'

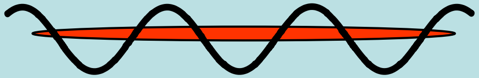
Ultracold gas near T= 0
in a ***quantum*** *optical lattice potential*



Reminder: Bose Hubbard model in fixed optical lattices



- **Superfluid Phase** J>>U



delocalized atoms

- **Mott-Insulator Phase:** J<<U

regular filling

Theory:

Fisher *et al*. (1989),
Jaksch *et al*. (1998)
Zwerger et. al.(2003)

Experiment:

Greiner et. al. (2002)
+ many more .

Effective many body - Hamiltonian

$$H = -J \sum_{\langle n,m \rangle} b_n^\dagger b_m + \frac{U}{2} \sum_n b_n^\dagger b_n \left(b_n^\dagger b_n - 1 \right) + \sum_i (\varepsilon_n - \mu) b_n^\dagger b_n$$

Generalized Bose-Hubbard model in cavity generated fields

$$H = \sum_{l=0,1} \hbar\omega_l a_l^\dagger a_l + \frac{1}{2}\frac{4\pi a_s\hbar^2}{m}\int \mathrm{d}^3\mathbf{r}\Psi^\dagger(\mathbf{r})\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})\Psi(\mathbf{r}) + \int \mathrm{d}^3\mathbf{r}\Psi^\dagger(\mathbf{r})H_0\Psi(\mathbf{r})$$

quantized light modes

Nonlinear atom-atom interaction

Adiabatically eliminated single-particle Hamiltonian

$$H_0 = \frac{\mathbf{p}^2}{2m_a} + V_{\mathrm{cl}}(\mathbf{r}) + \hbar g^2 \sum_{l,m=0,1} \frac{u_l^*(\mathbf{r})u_m(\mathbf{r})a_l^\dagger a_m}{\Delta_{ma}}$$

One-dimensional optical lattice: $\mathbf{r}_m = x_m\mathbf{e}_x = md\mathbf{e}_x \quad \text{for } m = 1, 2, \ldots, M$

$$\left.\begin{array}{ll}\text{Travelling wave cavities:} & u_{0,1}(\mathbf{r}_m) = \exp[i(mk_{0,1}d + \phi)] \\ \text{Standing wave cavities:} & u_{0,1}(\mathbf{r}_m) = \cos(mk_{0,1}d + \phi)\end{array}\right\} k_{0,1} = \mathbf{k}_{0,1}\cos\theta_{0,1}$$

Bose Hubbard model for a single standing wave mode resonator

effective single atom Hamiltonian

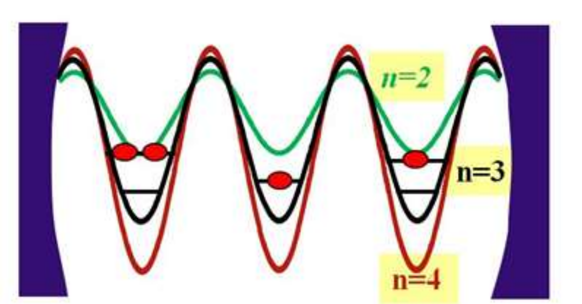
quantized light potential

extra classical potential

$$\begin{aligned} H_{\mathrm{eff}} &= \frac{p^2}{2m} + \cos^2(kx)\left(\hbar U_0 a^\dagger a + V_{cl}\right) - \hbar\Delta_c a^\dagger a \\ &- i\hbar\eta\left(a - a^\dagger\right) + \hbar\eta_{\mathrm{eff}} h(y)\cos(kx)\left(a + a^\dagger\right) \end{aligned}$$

transverse pump laser

Hubbard model for a quantized single mode



$$\begin{aligned} H &= E_0\hat{N} + E\hat{B} + \left(\hbar U_0 a^\dagger a + V_{\mathrm{cl}}\right)\left(J_0\hat{N} + J\hat{B}\right) \\ &- \hbar\Delta_c a^\dagger a - i\hbar\eta\left(a - a^\dagger\right) + \frac{U}{2}\hat{C}. \end{aligned}$$

$$\hat{N} = \sum_k \hat{n_k} = \sum_k b_k^\dagger b_k$$

$$\hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c.\right)$$

Looks similar to standard Bose Hubbard model

but

parameters for lattice dynamics contain field operators

(local and infinite range interactions)

single field mode as observable for atomic quantum statistics

Heisenberg equation for field amplitude operator ***a*** *:*

$$\dot{a} = \left\{ i \left[\Delta_c - U_0 \left(J_0 \hat{N} + J \hat{B} \right) \right] - \kappa \right\} a + \eta$$

$$\hat{N} = \sum_k \hat{n}_k = \sum_k b_k^\dagger b_k$$

$$\hat{B} = \sum_k \left(b_{k+1}^\dagger b_k + h.c. \right)$$

atom number in cavity

local atom-atom coherence

field amplitude depends of quantum statistics and gets entangled with atomic distribution

Dynamical effects of a quantum potential:

bad cavity limit :
effective Hamiltonian with eliminated field

$$a_0^\dagger a_0 = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$

$$\begin{aligned} H &= \left[E + J\left(V_{cl} - \hbar U_0 \eta^2 \frac{\kappa^2 - {\Delta'_c}^2}{\left(\kappa^2 + {\Delta'_c}^2\right)^2}\right)\right]\hat{B} \qquad (13) \\ &+ 3\hbar U_0^2 \eta^2 \Delta'_c \frac{3\kappa^2 - {\Delta'_c}^2}{\left(\kappa^2 + {\Delta'_c}^2\right)^4} J^2 \hat{B}^2 + \frac{U}{2} \sum_k \hat{n}_k \left(\hat{n}_k - 1\right) \end{aligned}$$

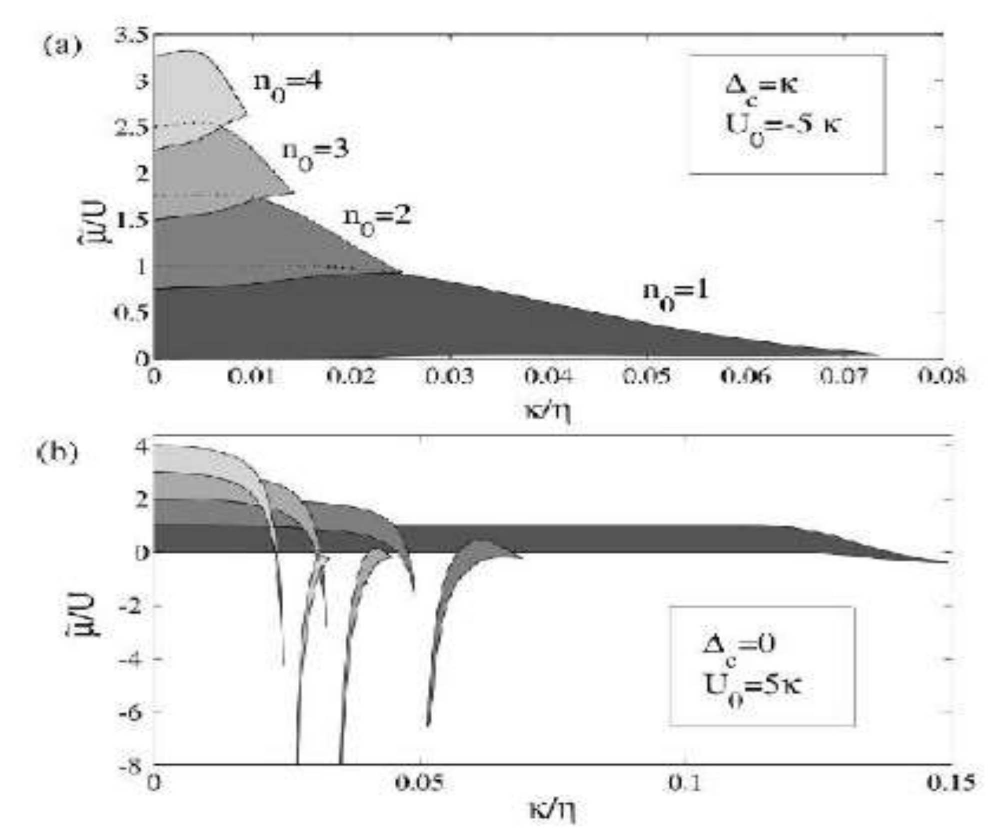
rescaled hopping terms
(sign change possible)

Nonlocal atom-atom interaction
via nonlocal correlated hopping

Cavity parameters can be used to effectively tune size and type of interactions !

Thermodynamic limit and phases of cavity generated lattices

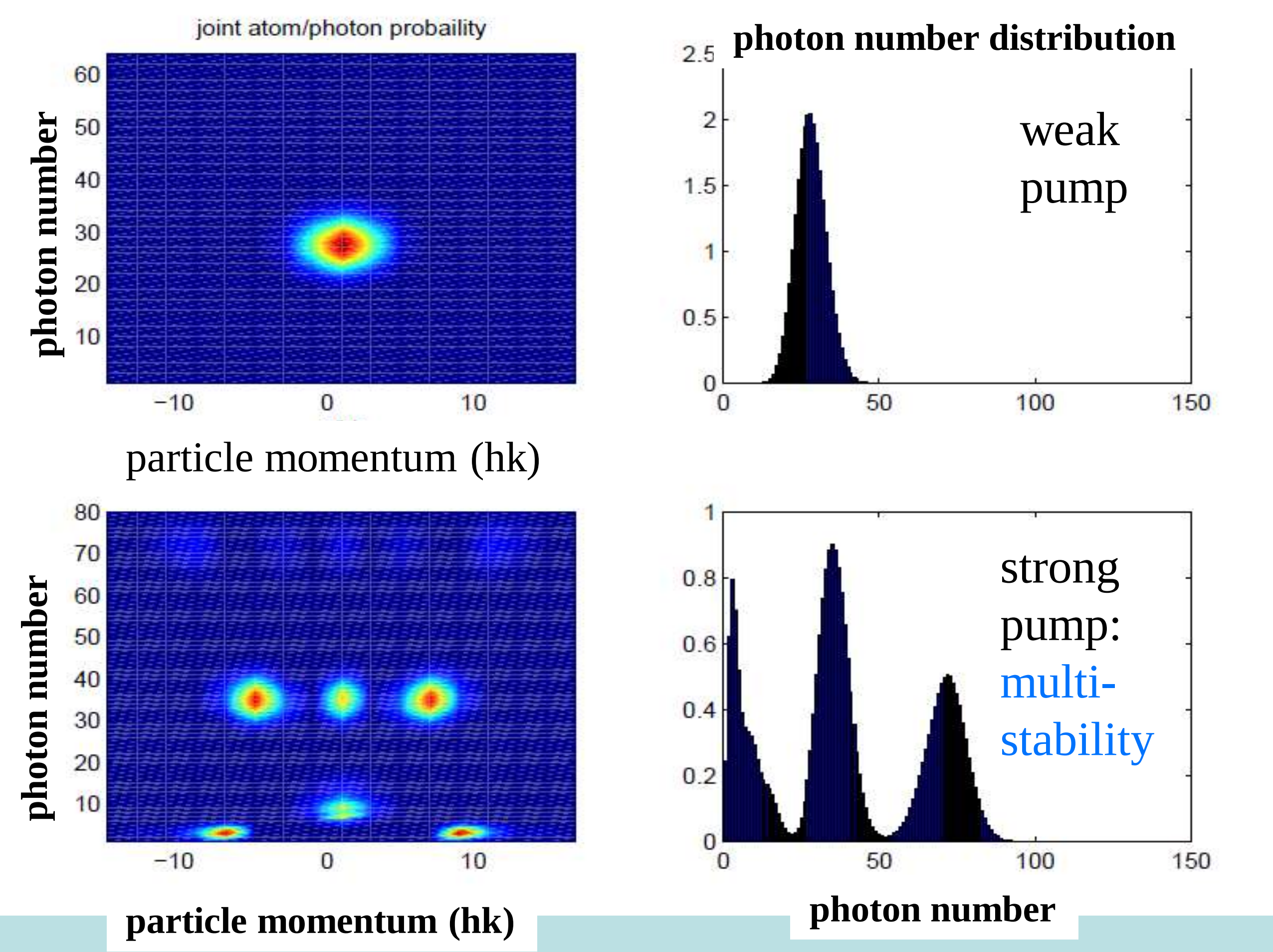
Cavity creates extra effective attraction or repulsion: bistable phases
=> phase superpositions of Mott + Superfluid in principle possible !?



M. Lewenstein, G. Morigi et. al. (PRL 2007, 2008)
Phase diagram in thermodynamic limit

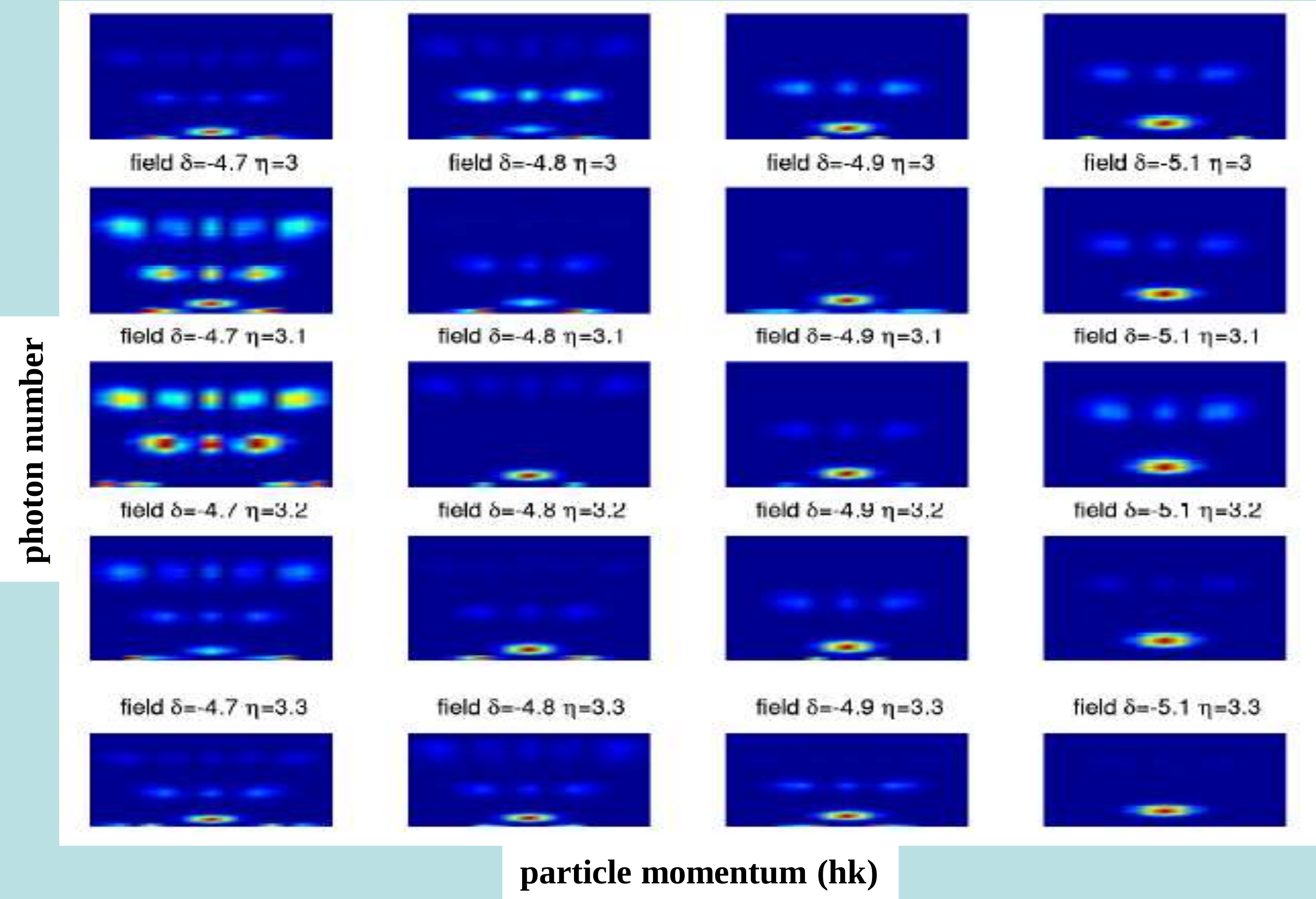
Generalization to fermions, Morigi PRA 2008

full quantum dynamics beyond tight binding approximation: *numerical solution for strong field and few particles by QMC - wavefunction simulations*



efficient numerical studies of full system by quantum wave-function simulation framework C++QED available via (http://cppqed.sourceforge.net/) (Andras Vukics)

stationary solution zoo (two particles)

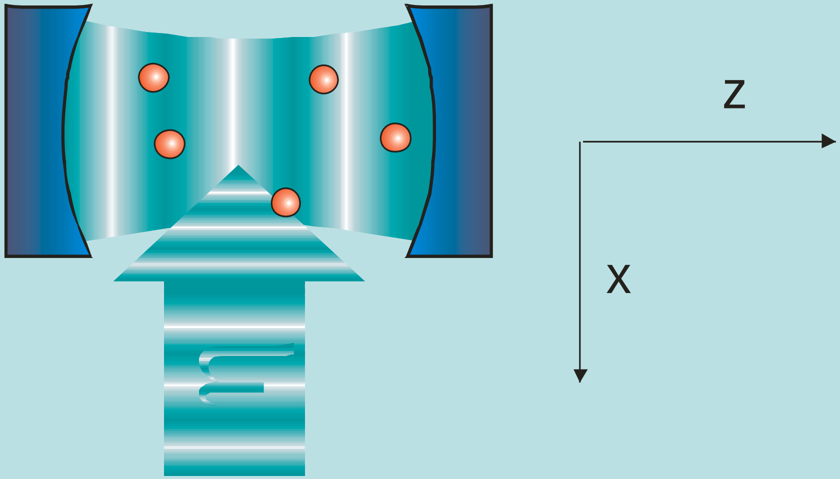


- *atom field correlation and stationary entanglement !*
- *strong atom –atom correlations*
- *very strange states can form at least for few particles*

Part III ***Part III***

Quantum dynamics of self-ordering in cavities

***Modified-geometry:* transverse pump: direct excitation of atoms from side !**



phase of excitation light depends on x - position

$$\dot{\sigma}_i = (i\Delta_A - \gamma)\sigma_i - g(z_i)a + \eta_x + \xi_A$$

$$\dot{a} = (i\Delta_C - \kappa)a + \underbrace{\sum_{i=1}^{N} g^*(z_i)\sigma_i + \xi_i}_{} .$$

collective pump strength ***R***

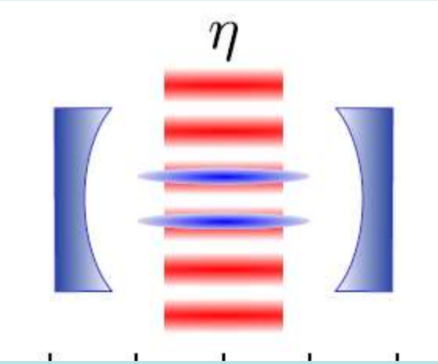
Field in cavity generated only by atoms

R *= 0 for random atomic distribution*

R ~ Ng *for regular lattice (Bragg)*

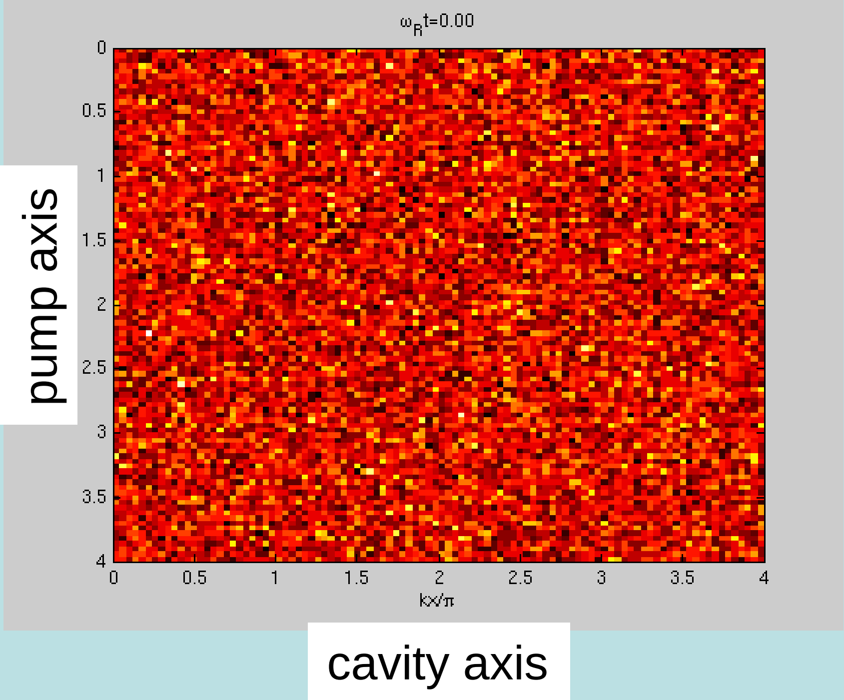
see also: Vuletic, PRL 2001

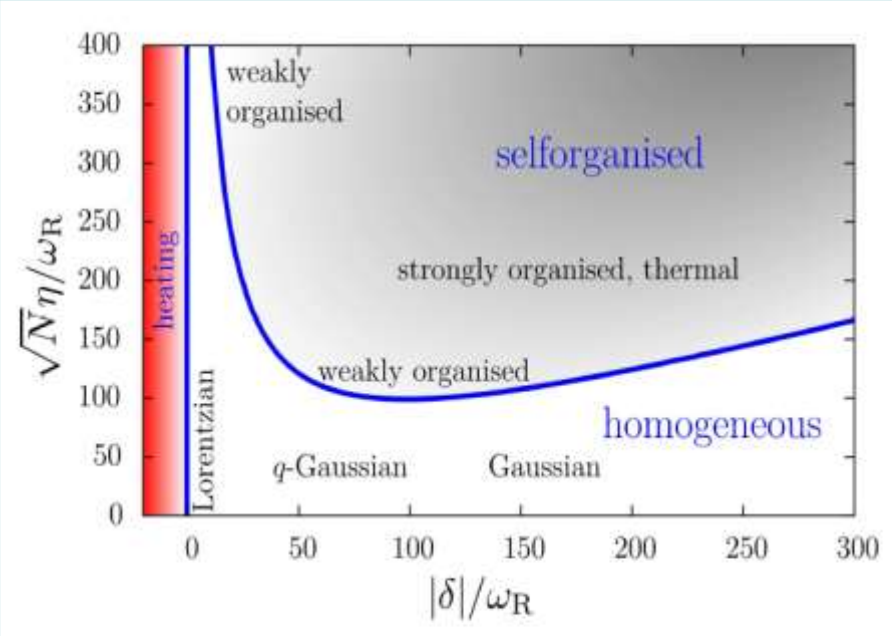
Numerical simulations of coupled dynamics including atomic motion (classical point start with random distribution)



phase diagram from stability analysis including diffusion

G – General audience





particles spontaneously form crystalline order

(Niedenzu, EPL 2011)

Atom-field dynamics for very large particle number : => Vlasov equation for particle distribution

Continuous density approximation for cold cloud: single particle distribution function

$$f_s(x,p,t) := \frac{1}{N_s}\left\langle \sum_{j_s=1}^{N_s} \delta(x - x_{j_s}(t))\delta(p - p_{j_s}(t)) \right\rangle$$

$$\Phi_s(x,\alpha) = \hbar U_{0,s}|\alpha|^2 \sin^2(kx) + \hbar\eta_s(\alpha+\alpha^*)\sin(kx)$$

Vlasov + field equation

$$\frac{\partial f_s}{\partial t} + \frac{p}{m_s}\frac{\partial f_s}{\partial x} - \frac{\partial \Phi_s(x,\langle\alpha\rangle)}{\partial x}\frac{\partial f_s}{\partial p} = 0$$

$$\dot{\alpha} = (i\Delta_c - \kappa)\,\alpha - i\sum_s \int \left(\alpha\, U_{0,s}\sin^2(kx) + \eta_s \sin(kx)\right) f_s\, dx\, dp$$

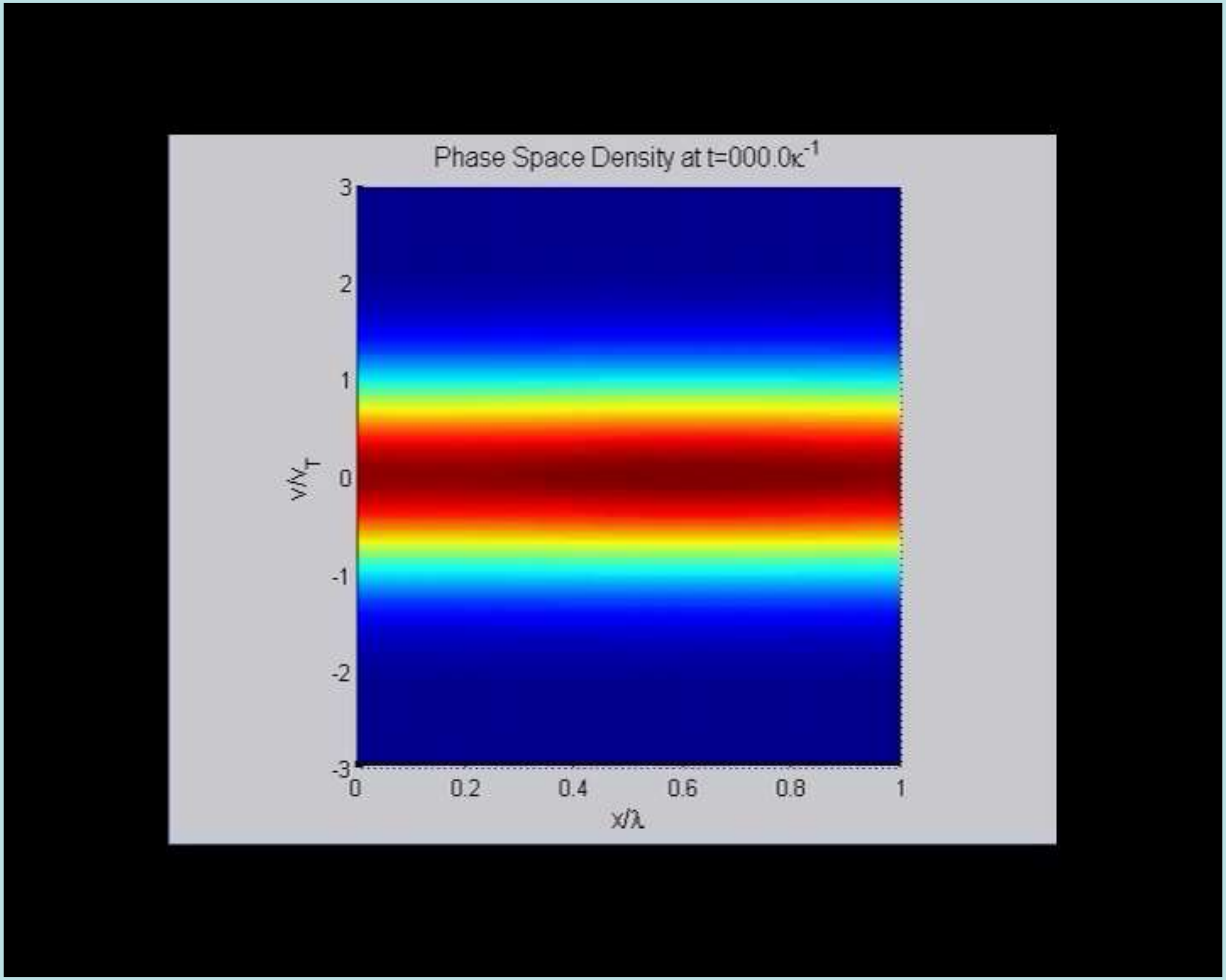
stability threshold of homogeneous distribution:

$$\frac{N\eta^2}{k_B T}\,\mathrm{vp}\int_{-\infty}^{\infty}\frac{g'(\xi)}{-2\xi}\mathrm{d}\xi < \frac{\delta^2+\kappa^2}{\hbar|\delta|}$$

threshold at thermal equilibrium

$$\underbrace{U_0}_{\text{frequency shift of cavity}} N \underbrace{V_{opt}}_{\text{pump laser opt. potential}} > \underbrace{\kappa^2}_{\text{cavity damping}}$$

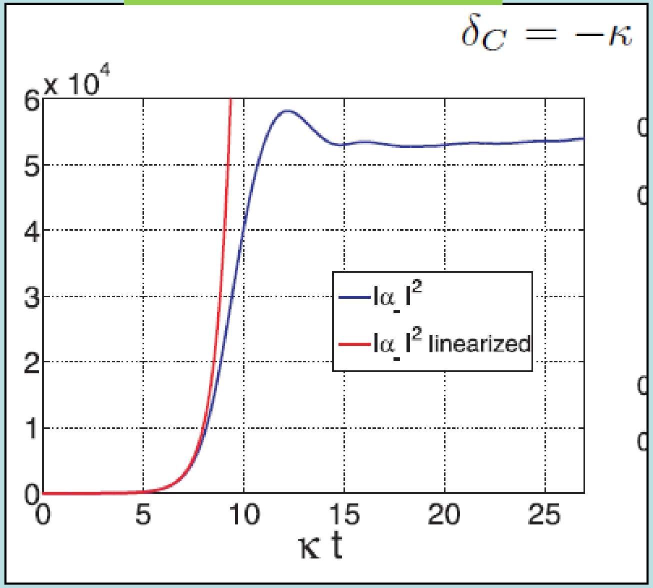
Numerical simulation of Vlasov equation: cooling limit



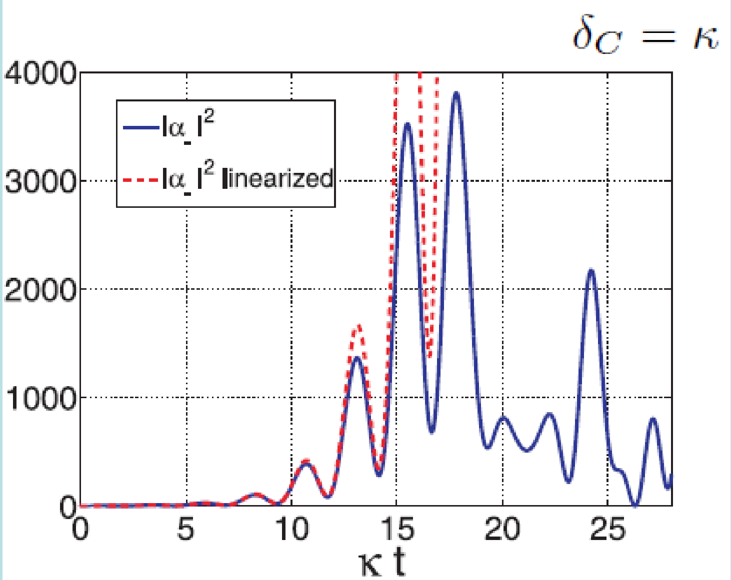
periodic boundary conditions – 1 wavelength

time evolution of field intensity above threshold (~ δ_c^2)

negative detuning



positive detuning



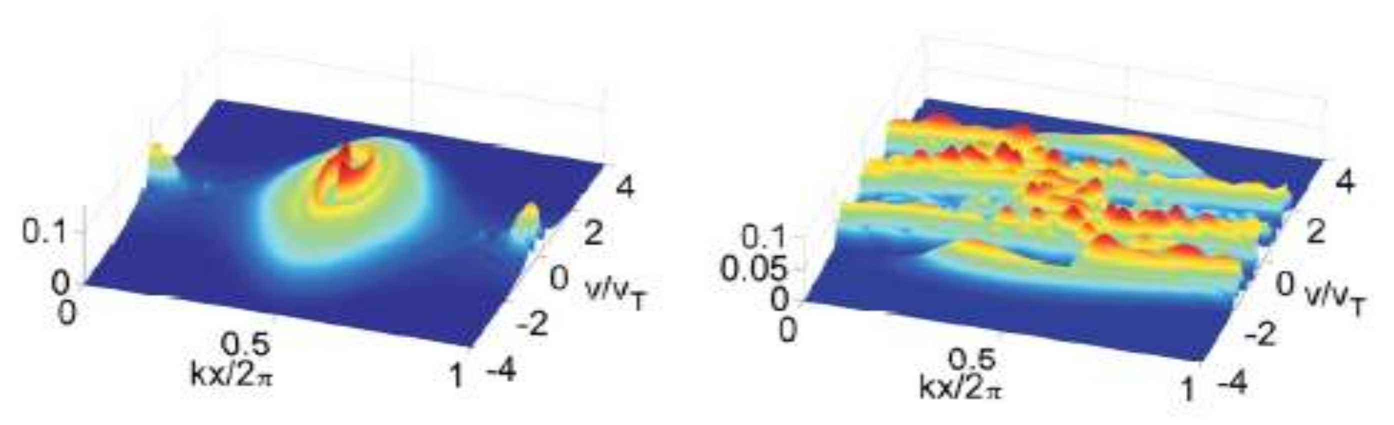


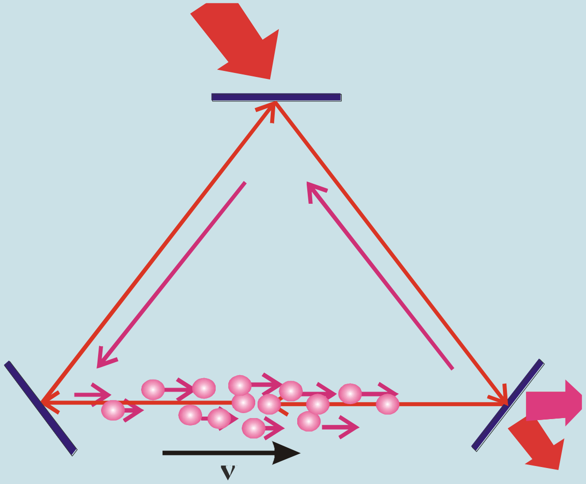
FIG. 24 Phase space densities of the particles for negative detuning $\delta_C = -\kappa$ (left) and positive detuning $\delta_C = \kappa$ (right)

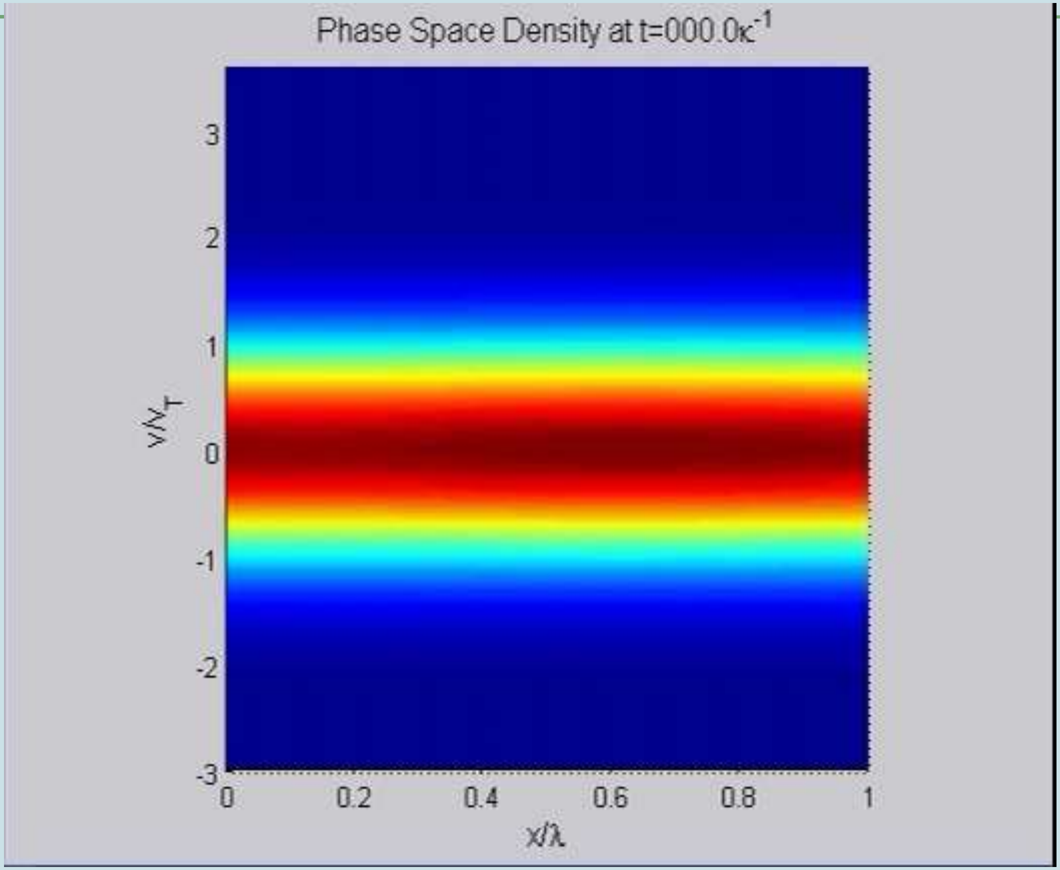
instability confirmed but selfordering for negative detuning only !

special case here at CFEL: single side pumped ring (CARL /FEL) Experiment : Zimmermann (Tübingen)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \phi}{\partial \mathbf{x}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\int f(\mathbf{x}, \mathbf{v}, 0) d^3x d^3v = 1$$

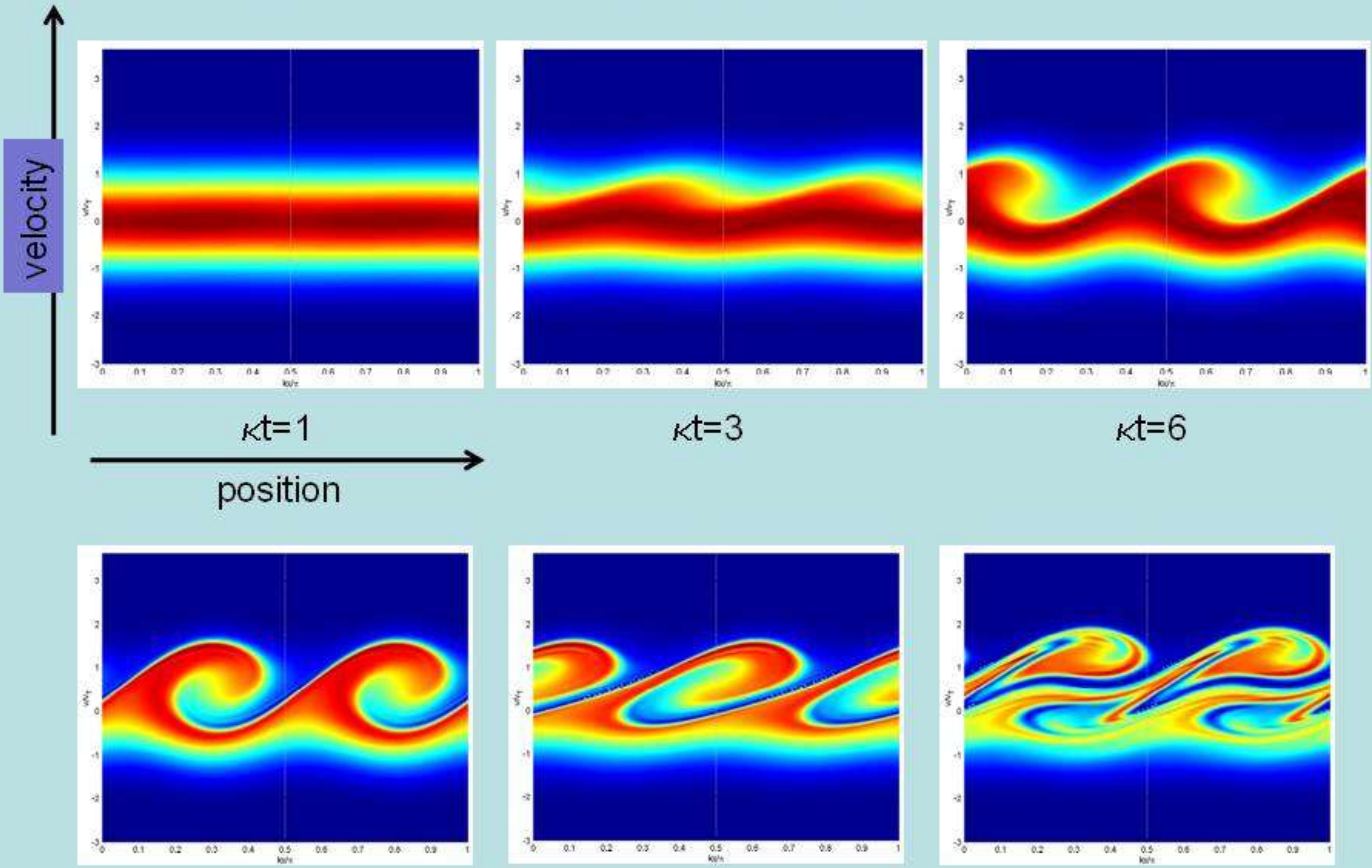




- ***density fluctuations backscatter light and get amplified***

=> selfconsistent accelerated field

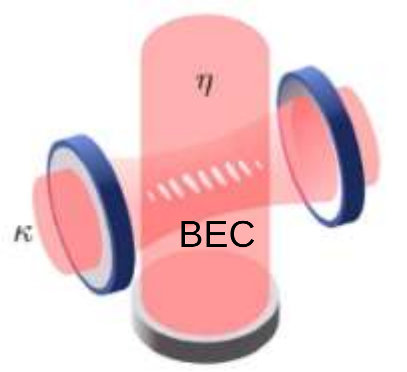
Simplest case: Instability in unidirectional cavity



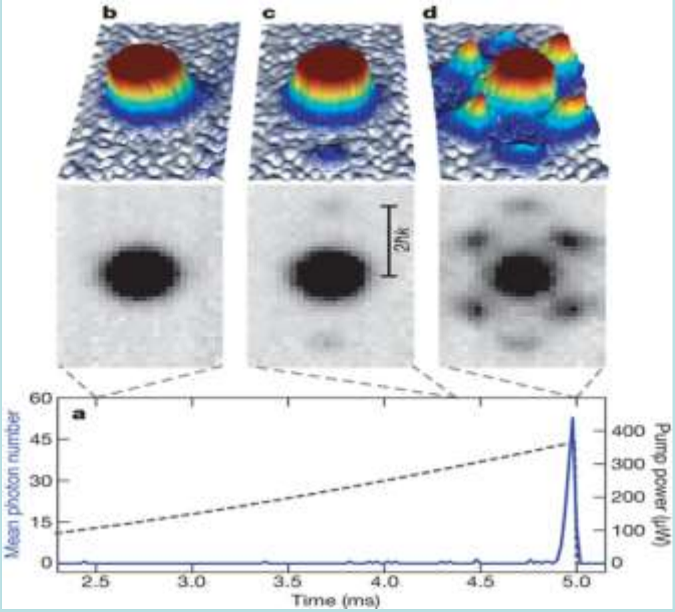
$\kappa t=1$ $\kappa t=3$ $\kappa t=6$

$\kappa t=11$ $\kappa t=15$

Selforganization of a BEC at T ~ 0



$$H = -\Delta_C\, a^\dagger a + \int_0^L \Psi^\dagger(x)\left[-\frac{\hbar}{2\,m}\frac{d^2}{dx^2} + U_0\, a^\dagger a \cos^2(kx) + i\eta_t \cos kx(a^\dagger - a)\right]\Psi(x)dx,$$



K Baumann *et al. Nature* 464, 1301 (2010)
+ many more papers since

Two-mode approximation
=> Tavis-Cummings model

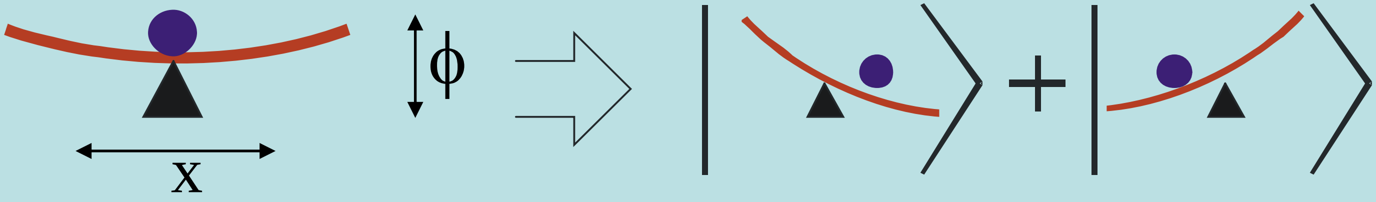
$$H = -\delta_C\, a^\dagger a + \omega_R \hat{S}_z + iy(a^\dagger - a)\hat{S}_x/\sqrt{N} + u a^\dagger a\left(\tfrac{1}{2} + \hat{S}_z/N\right)$$

Nagy, Domokos, PRL (2010), NJP 2011
Fernandez-Vidal, Morigi PRA (2010)

- „Dicke Superradiant Phase“ transition
- Formation of a supersolid ?

More refined theoretical models and numerical studies (tight binding):
Keldysh theory: Goldbart, Piazza, Strack, Zwerger, Diehl …
numerical studies: Vukics, Hofstetter, Bakhtiari, Thorwart,
Fermionic selfordering: Piazza, Keeling, ….

Very simple toy model for dynamics: “decay of a quantum seesaw “



classical

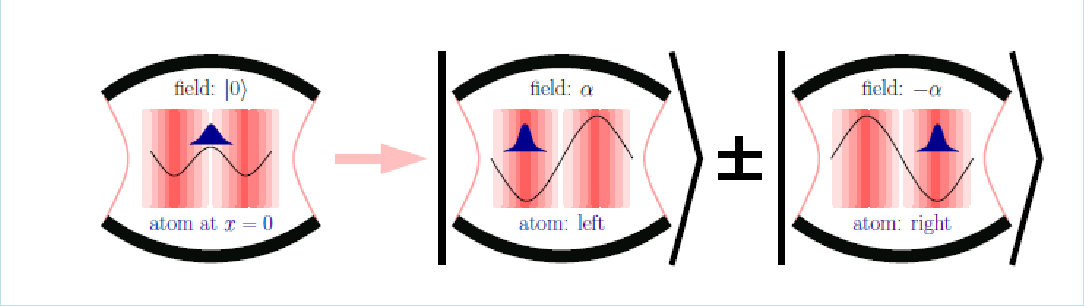
quantum

Two degrees of freedom: tilt angle ϕ and particle position x

Note: classical equilibrium point at x=ϕ=0

but

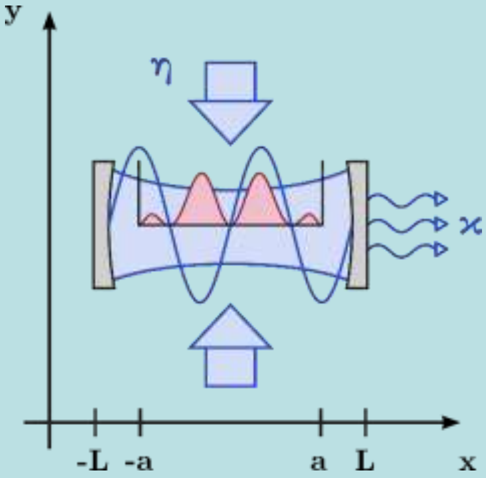
product state of oscillator ground states is not stationary



field phase replaces tilt angle <> occupation difference replaces position

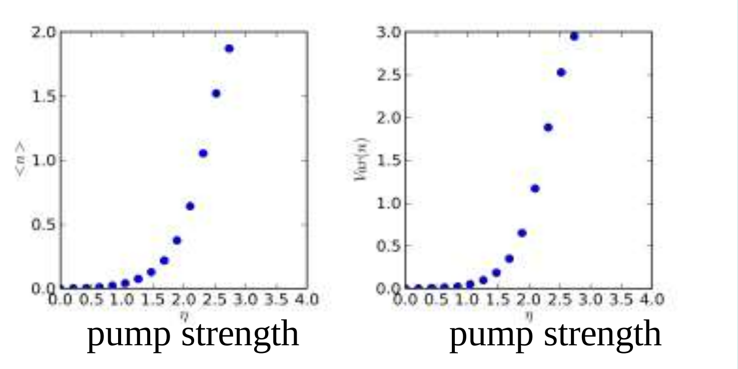
Selfordering of trapped particles within a cavity: a numerical study

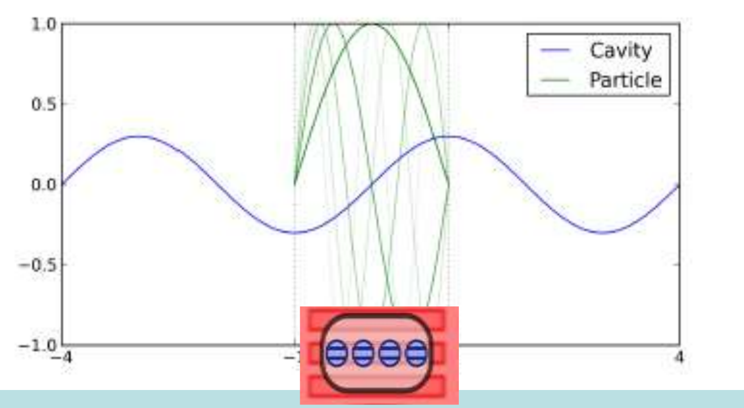
Flat box trap without prescribed lattice



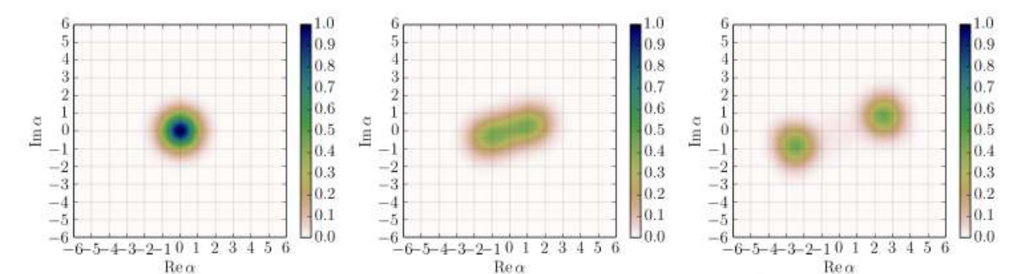
photons

photon variance





cavity field Q-function

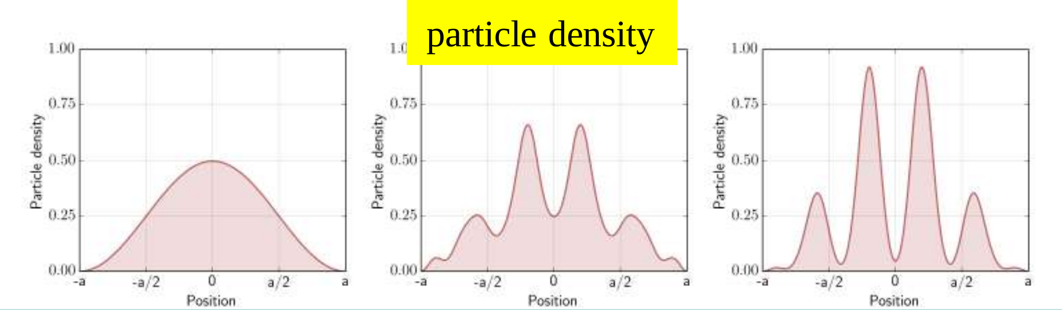


no pump

weak pump

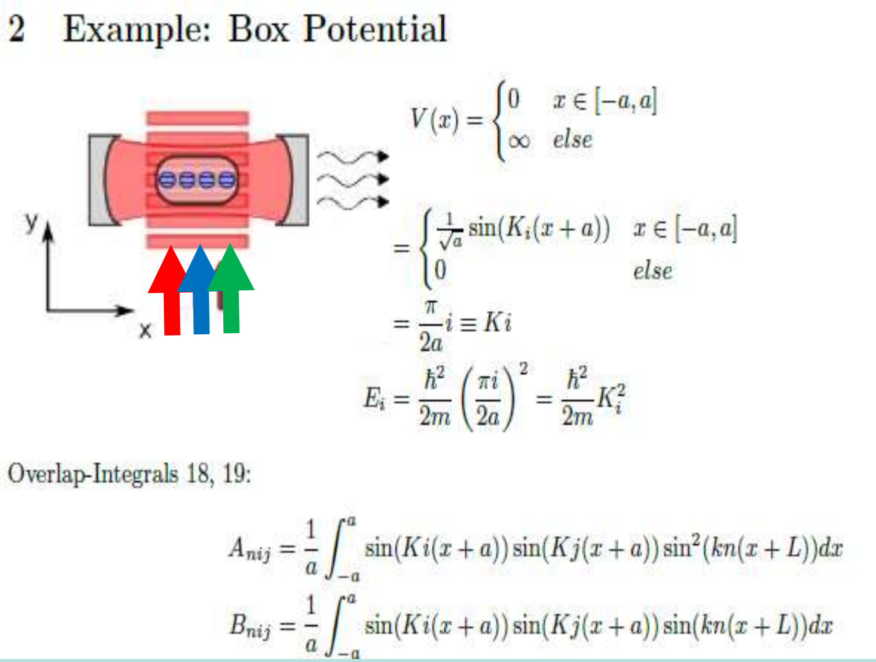
strong pump

particle density



several particle modes excited bi-modal Q-function of field

Selfordering with multicolor pump field using several cavity modes: => competitive phase transitions



2 Example: Box Potential

$$V(x) = \begin{cases} 0 & x \in [-a,a] \\ \infty & else \end{cases}$$

$$= \begin{cases} \frac{1}{\sqrt{a}} \sin(K_i(x+a)) & x \in [-a,a] \\ 0 & else \end{cases}$$

$$= \frac{\pi}{2a} i \equiv K i$$

$$E_i = \frac{\hbar^2}{2m} \left(\frac{\pi i}{2a} \right)^2 = \frac{\hbar^2}{2m} K_i^2$$

Overlap-Integrals 18, 19:

$$A_{nij} = \frac{1}{a} \int_{-a}^{a} \sin(Ki(x+a)) \sin(Kj(x+a)) \sin^2(kn(x+L)) dx$$

$$B_{nij} = \frac{1}{a} \int_{-a}^{a} \sin(Ki(x+a)) \sin(Kj(x+a)) \sin(kn(x+L)) dx$$

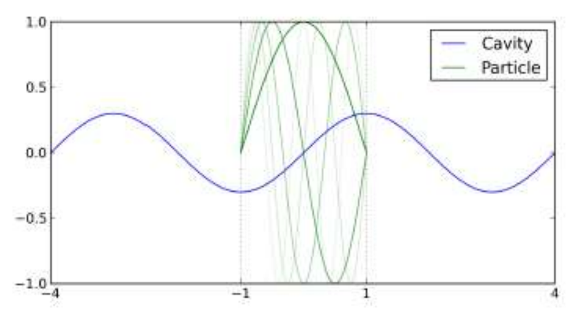
multimode Tavis Cummings model

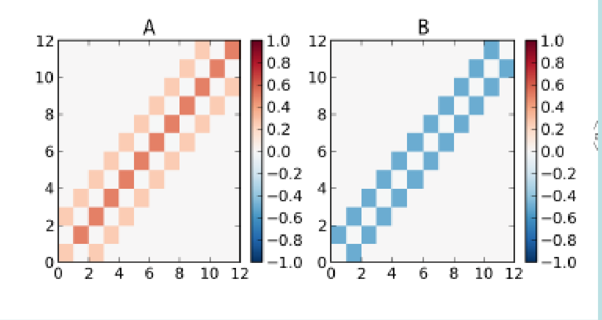
$$H = -\sum_n \Delta_p^n \hat{a}_n^\dagger \hat{a}_n + \int dx \hat{\Psi}^\dagger(x) \left(\frac{-\Delta}{2m} + V(x) \right) \hat{\Psi}(x)$$

$$+ \int dx \hat{\Psi}^\dagger(x) \sum_n T_0 \omega_n \sin^2(k_n(x+L)) \hat{a}_n^\dagger \hat{a}_n \hat{\Psi}(x)$$

$$+ \int dx \hat{\Psi}^\dagger(x) \sum_n \eta_n \sin(k_n(x+L)) (\hat{a}_n^\dagger + \hat{a}_n) \hat{\Psi}(x)$$

(„Hopfield model – associative memory“)





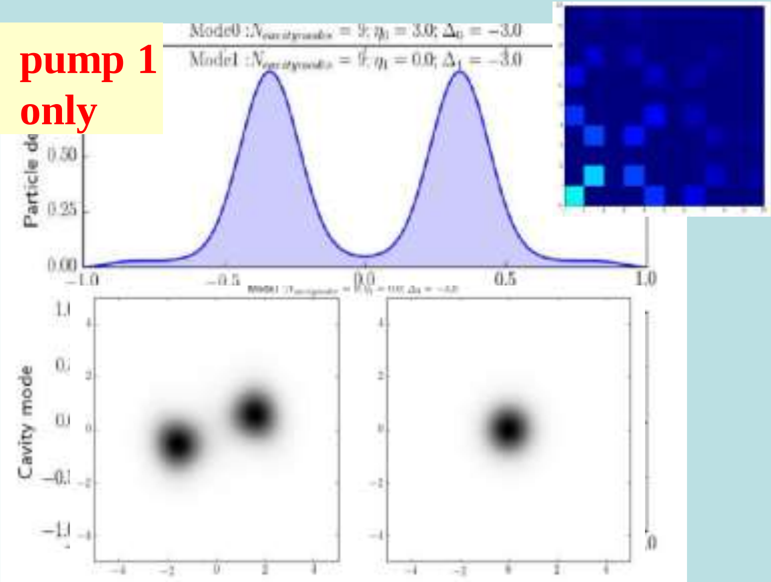
Expand particle operators in trap eigenmodes

- $H_{particles} \Psi_k(x) = E_k \Psi_k(x)$
- Field operators: $\hat{\Psi}(x) = \sum_k \Psi_k(x) \hat{c}_k$

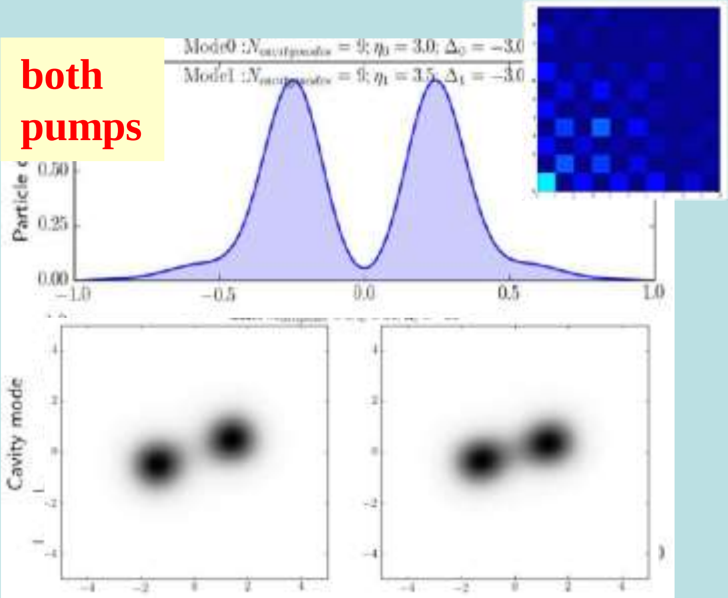
Nonlinear coupled oscillator model
with tailorable coupling:
pump amplitudes + detunings as control

„BEC“ - in a box with two pump frequencies

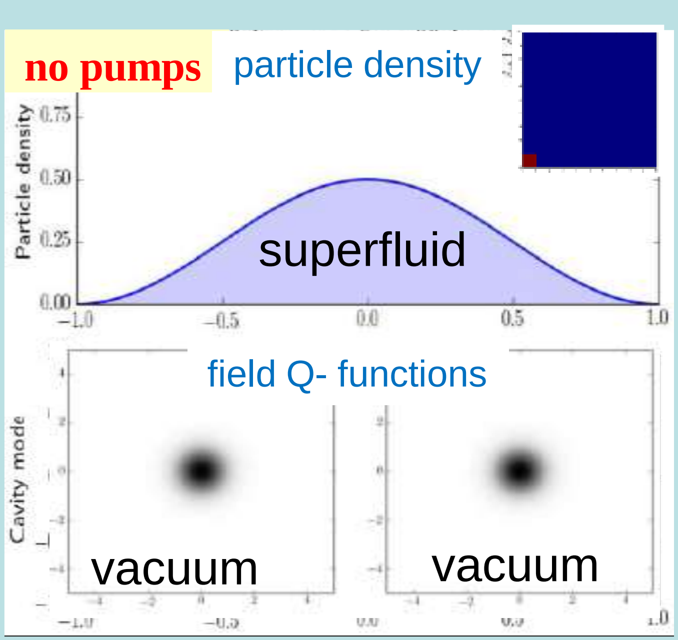
pump 1 only



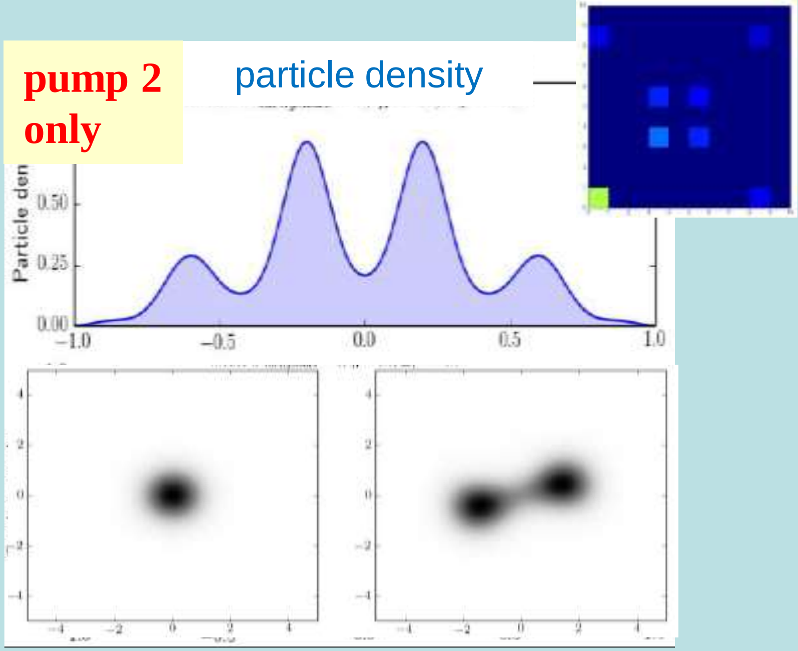
both pumps



no pumps

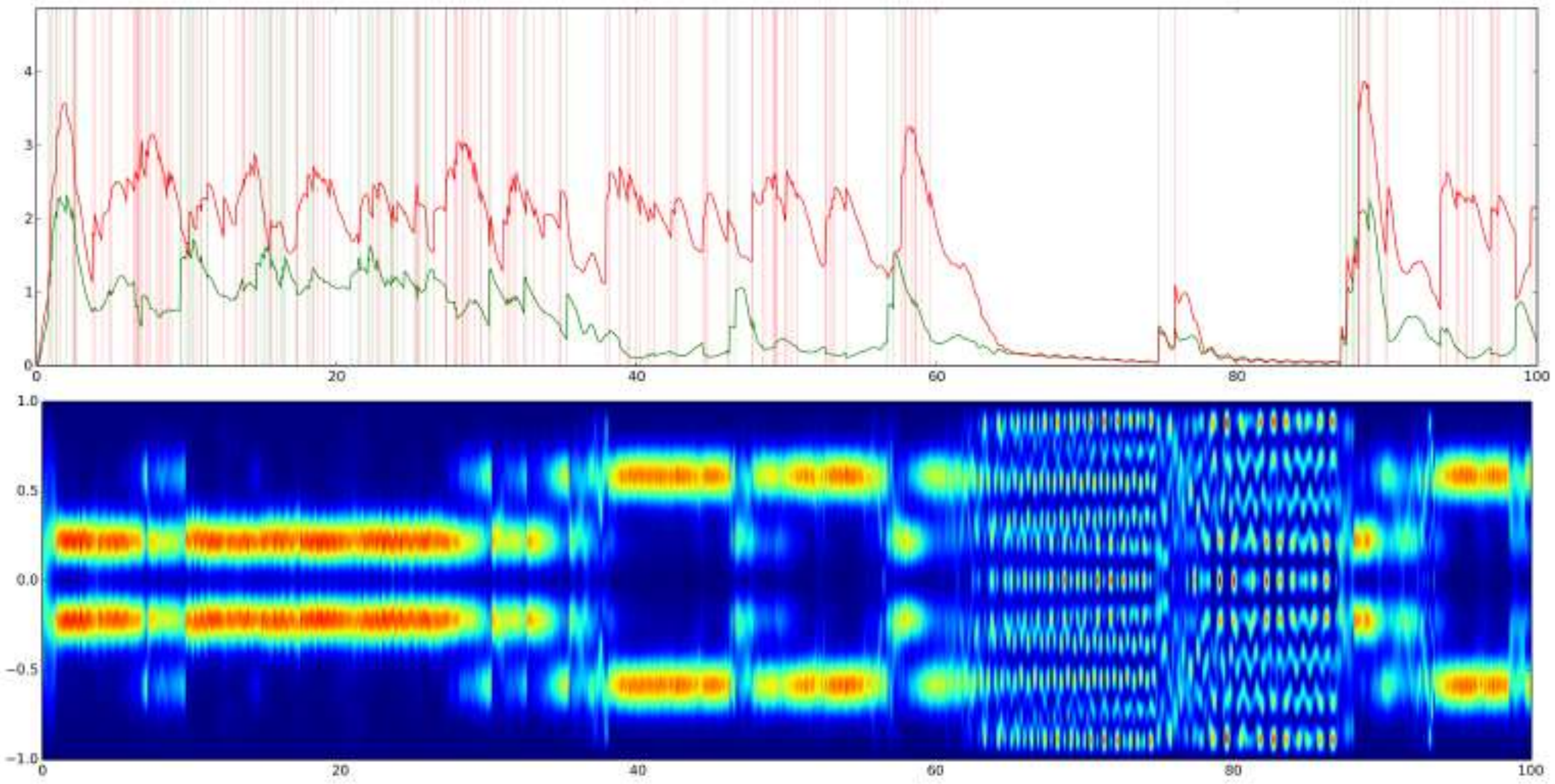


pump 2 only



many possible quasi-steady states !

Single trajectory dynamics:
particels and field jump between different configurations



formation and melting of order in time ….

realistic scenarios far beyond our computing power …

Summary and Outlook

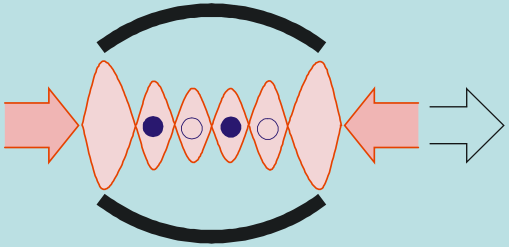
- Cavities can be used to confine, cool and control particle motion
- Replace evaporation to reach degeneracy and CW atom lasing
- implement tailorable long range interactions
- dynamic model of crystallisation of quantum systems
- study associative memory and neural network models even in the quantum regime

Thanks for your attention !

Innsbruck University – visitors welcome !

Selfordering beyond mean field

multiparticle quantum description of selforganization in a lattice



- pump creates optical lattice with atoms in lowest band
- cavity field from scattered lattice light

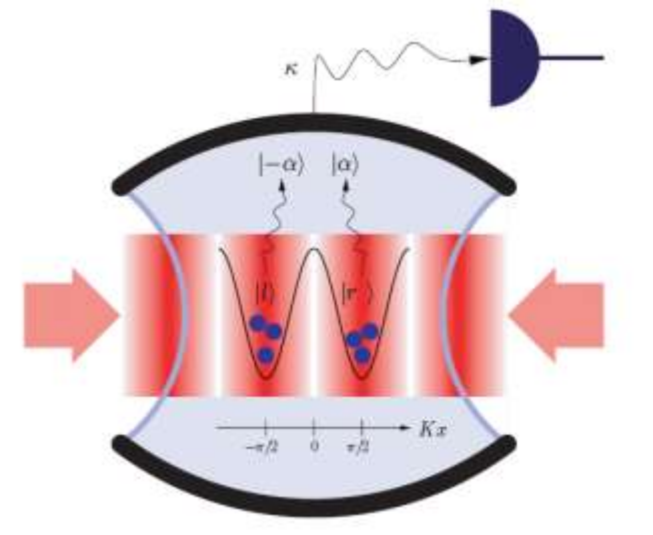
Effective Hubbard type Hamiltonian:

$$H=\sum_{k,l} E_{k,l} b_k^\dagger b_l + \hbar U_0 \eta' g \sum_{k,l} J_{k,l} b_k^\dagger b_l + \hbar\eta' \left(a+a^\dagger\right) \sum_{k,l} \tilde{J}_{k,l} b_k^\dagger b_l - \hbar\left(\Delta_c - U_0\right) a^\dagger a$$

pump amplitude determined by atomic distribution operator

How and when will selforganization happen here ?

microscopic dynamics beyond mean field at simple example: two-effective lattice sites (1,2)



Lowest energy states for atoms at two sites ...

$$a = -i\frac{\eta'}{\kappa - i(\Delta_c - U_0)}\tilde{J}_0\left(b_1^\dagger b_1 - b_2^\dagger b_2\right)$$

$$a^\dagger a \sim \left(b_1^\dagger b_1 - b_2^\dagger b_2\right)^2$$

$$\frac{1}{\sqrt{2}}\left(|\mathrm{left}\rangle\,|\alpha\rangle \pm |\mathrm{right}\rangle\,|-\alpha\rangle\right)$$

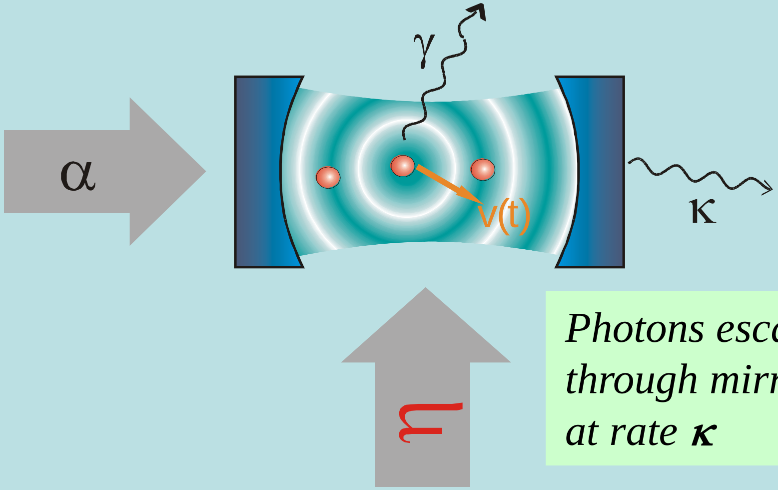
... show atom field entanglement

- Note: strongly entangled state
- Symmetry leads to zero field but nonzero intensity (photons)
- How does entanglement and intensity grow ?

real Cavity QED = open system

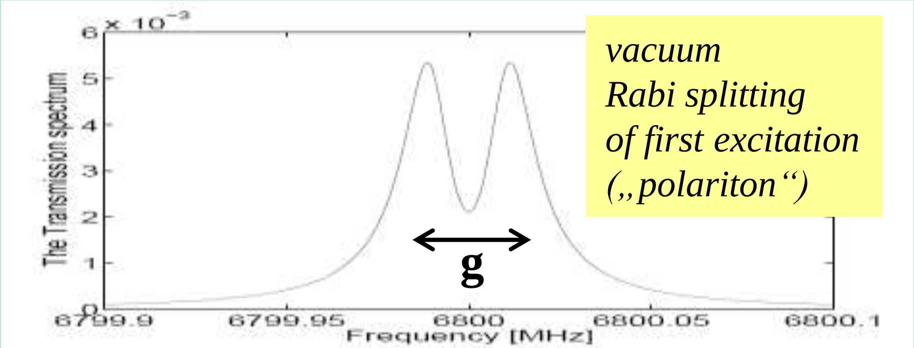
input and output channels => damping + fluctuations + decoherence
but
allow measurement + (feedback) control

particles scatter spontaneously at rate γ



Photons escape through mirror at rate κ

Cavity QED (=strong coupling) limit, if
$\omega_f, \omega_a \gg g \gg (\kappa, \gamma)$



vacuum
Rabi splitting
of first excitation
(„polariton“)

Consequences :

- Nonlinear atomic response at less than a single photon : n_0
- Single atom shifts cavity by more than a linewidth **N_0**

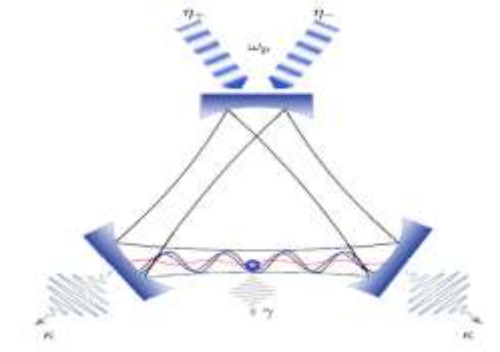
Experiments in wide range:

Microwave cavity to Optical cavity

ω ~ 10^9 Hz to ω~ 10^14 Hz

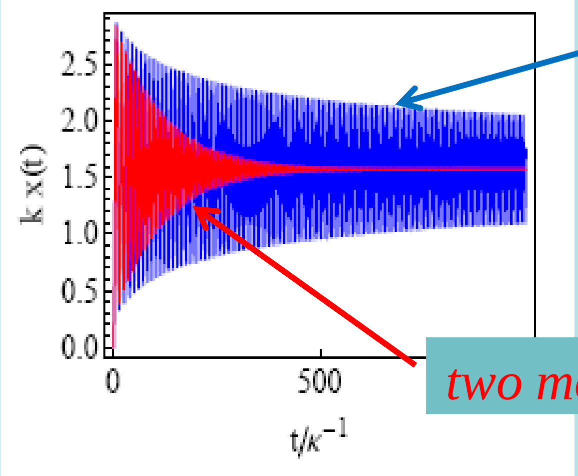
Gedankenexperiments of Quantum Mechanics realized :
e.g. Haroche, Walther, Kimble, Rempe,… + many more recently (circuit QED)

Trapped particle in a ring cavity with symmetric pump



$E(x,t) \sim a_c \cos(kx)+a_s \sin kx$

point particle motion :



single mode

two modes = ring cavity

Efficient trapping and cooling towards very low velocities !

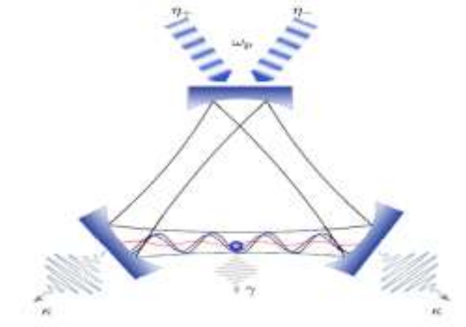
Gangl, PRA 99, Exp. by Hemmerich (Hamburg) and Zimmermann (Tübingen)

trapping and cooling to the quantum limit in a ring cavity

ultracold + localized particles =>
atom-field Hamiltonian for quantized motion:

$$H = \frac{\hat{p}^2}{2m} - \hbar\Delta\left(a_c^\dagger a_c + a_s^\dagger a_s\right) - \hbar U(\hat{x}) + i\hbar\left(\eta a_c^\dagger - \eta^* a_c\right)$$

$$U(\hat{x}) = a_c^\dagger a_c \underbrace{U_c(\hat{x})}_{\text{cosine\^{}2}} + a_s^\dagger a_s \underbrace{U_s(\hat{x})}_{\text{sine\^{}2}} + \left(a_c^\dagger a_s + a_c a_s^\dagger\right) \underbrace{U_{cs}(\hat{x})}_{\text{cosine*sine}}$$



E(x,t) ~ a_c cos(kx)+a_s sinkx

generic setup:
strong pump of cosine mode:
=> deep trap for particle
=> mode in coherent state α

trap frequency of particle proportional cos-field amplitude

$$\omega_m^2 = 4\omega_R U_0 \alpha^2$$

$$H = \left[\frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_m^2 \hat{x}^2\right] - \hbar\Delta a^\dagger a - \hbar U_0'(a_s + a_s^\dagger)\hat{x}$$

linear coupling '$(a_s^+ + a_s)$ x'
=> "optomechanical cooling"

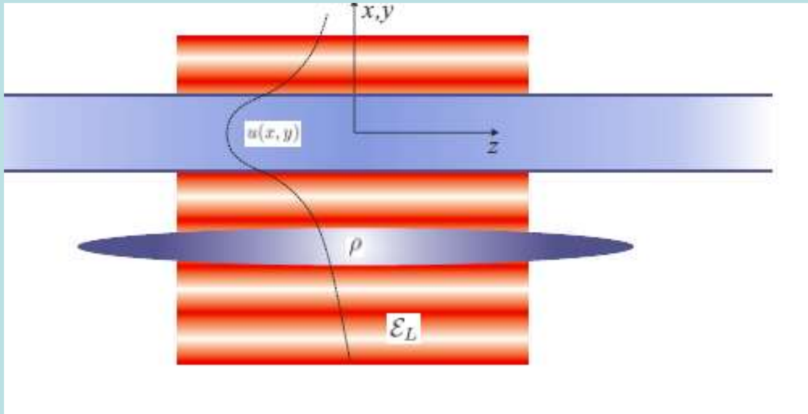
quadratic coupling '$a_c^+ a_c$ x^2'
=> trap + x^2 nonlinearity

Selfordering via a continuum of light modes

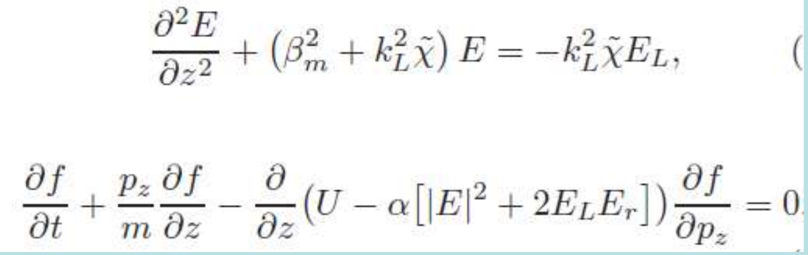
trapped particles interact by collective scattering and dipole-dipole exchange

Example: dipole-dipole interaction
enhanced by optical fiber or microstructure

A. Rauschenbeutel

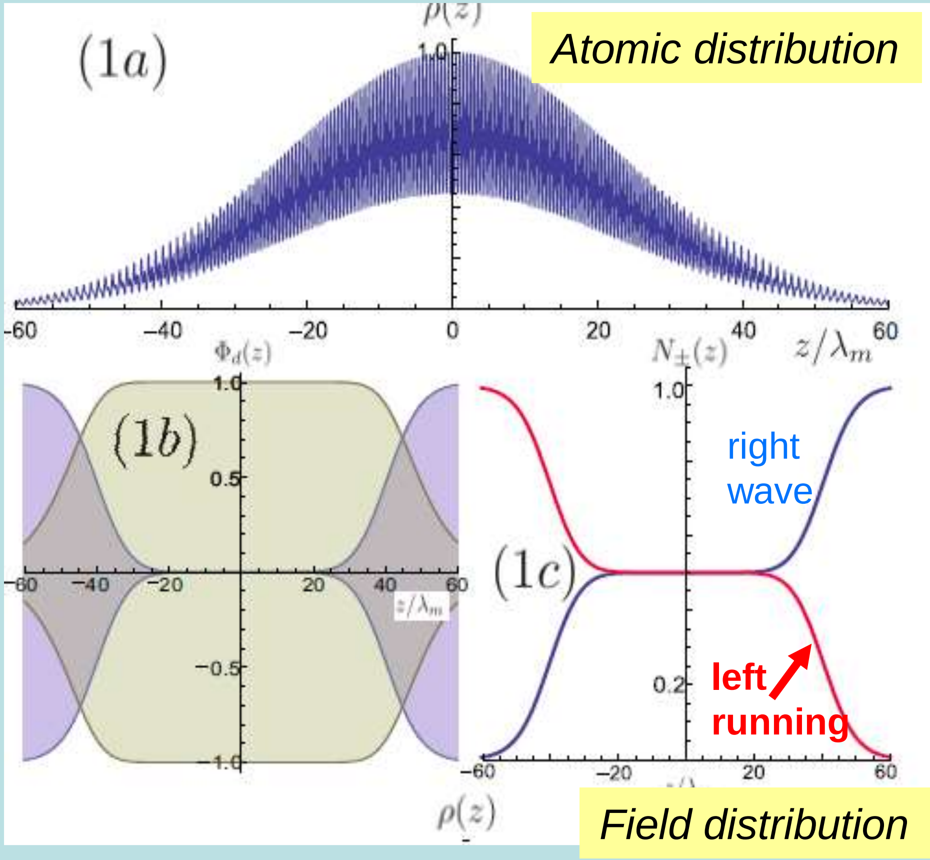


: Cigar-shaped atomic gas alongside optical nanofiber.



$$\frac{\partial^2 E}{\partial z^2} + \left(\beta_m^2 + k_L^2\tilde{\chi}\right) E = -k_L^2\tilde{\chi}E_L, \qquad ($$

$$\frac{\partial f}{\partial t} + \frac{p_z}{m}\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\left(U - \alpha\left[|E|^2 + 2E_L E_r\right]\right)\frac{\partial f}{\partial p_z} = 0$$



Atomic distribution

right wave

left running

Field distribution

- *particles order to confine light and themselves*
- *long range phononlike excitations*
- *self optimized light harvesting structure*

T. Griesser, PRL 2013

(see also: Chang et.al , PRL 2013)

Analytic model in the linear regime:

Dynamic equations:

$$\begin{aligned}
\frac{d}{dt}\langle a^\dagger a\rangle &= -2\kappa\langle a^\dagger a\rangle - i\bar{U}_0(\langle aQ\rangle - \langle a^\dagger Q\rangle) \\
\frac{d}{dt}\langle Q^2\rangle &= \omega_m\langle \mathcal{A}\rangle \\
\frac{d}{dt}\langle \mathcal{A}\rangle &= 2\omega_m(\langle P^2\rangle - \langle Q^2\rangle) + \\
&\quad 2\bar{U}_0(\langle aQ\rangle + \langle a^\dagger Q\rangle) \\
\frac{d}{dt}\langle P^2\rangle &= -\omega_m\langle \mathcal{A}\rangle + 2\bar{U}_0(\langle aP\rangle + \langle a^\dagger P\rangle) \\
\frac{d}{dt}\langle aQ\rangle &= \omega_m\langle aP\rangle - (\kappa - i\Delta)\langle aQ\rangle + i\bar{U}_0\langle Q^2\rangle \\
\frac{d}{dt}\langle aP\rangle &= (-\kappa + i\Delta)\langle aP\rangle - \omega_m\langle aQ\rangle + \\
&\quad \bar{U}_0\left(\langle a^\dagger a\rangle + 1/2 + \langle a^2\rangle + \frac{i}{2}\langle \mathcal{A}\rangle\right) \\
\frac{d}{dt}\langle a^2\rangle &= -2(\kappa - i\Delta)\langle a^2\rangle + 2i\bar{U}_0\langle aQ\rangle,
\end{aligned}$$

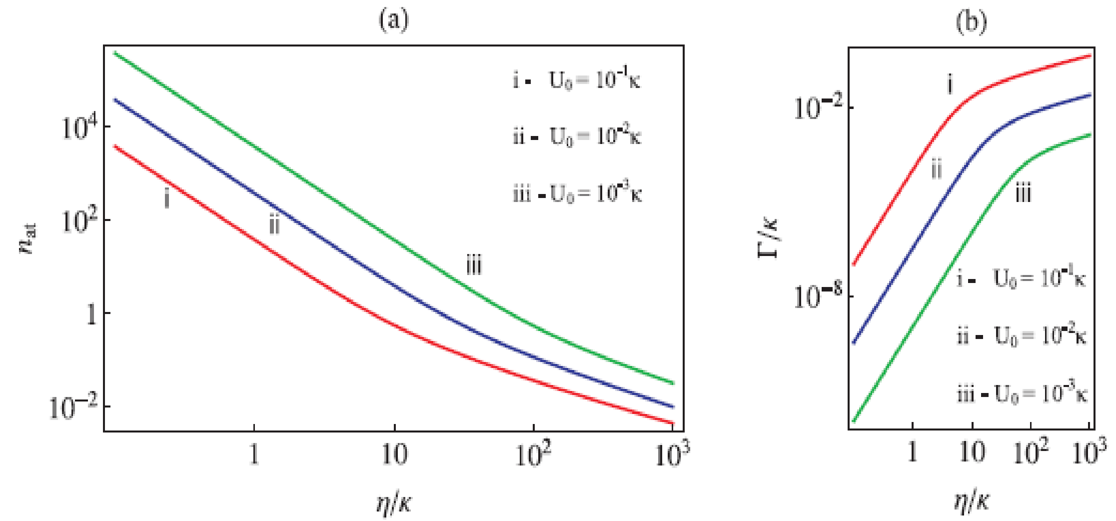
Steady states:

$$\langle a^\dagger a\rangle = -\frac{U_0^2(\Delta^2+\kappa^2)}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2},$$

$$\langle Q^2\rangle = -\frac{(\kappa^2+\omega_m^2+\Delta^2)(\kappa^2+\Delta^2)+2\bar{U}_0^2\omega_m\Delta}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2} \qquad (10)$$

$$\langle P^2\rangle = -\frac{(\kappa^2+\omega_m^2+\Delta^2+2\bar{U}_0^2\Delta/\omega_m)(\kappa^2+\Delta^2)+2\bar{U}_0^2\omega_m\Delta}{4\omega_m\Delta(\kappa^2+\Delta^2)+8\bar{U}_0^2\Delta^2}$$

Examples:

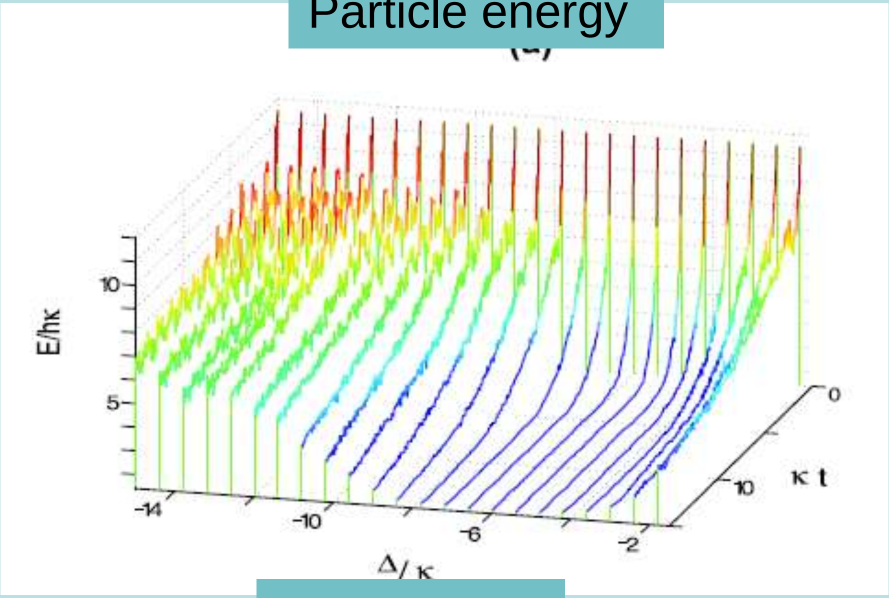


* Ground-state cooling !
* acceleration by strong power !

Numerical Monte Carlo wave function simulations
beyond linearized potential (C++QED- package)

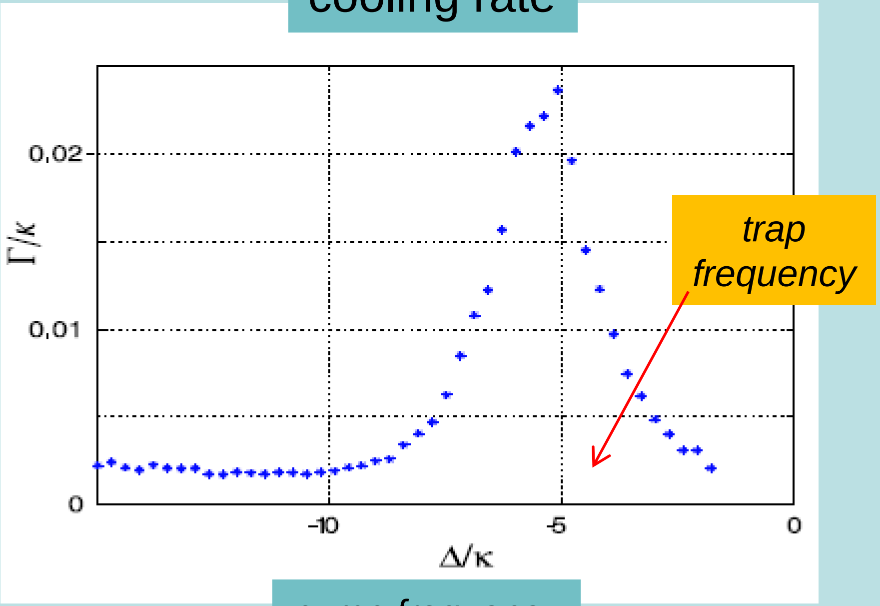
Single quantum particle in a ring cavity with dispersive interaction:
frequency dependence of cooling

Particle energy



pump frequency

cooling rate



trap frequency

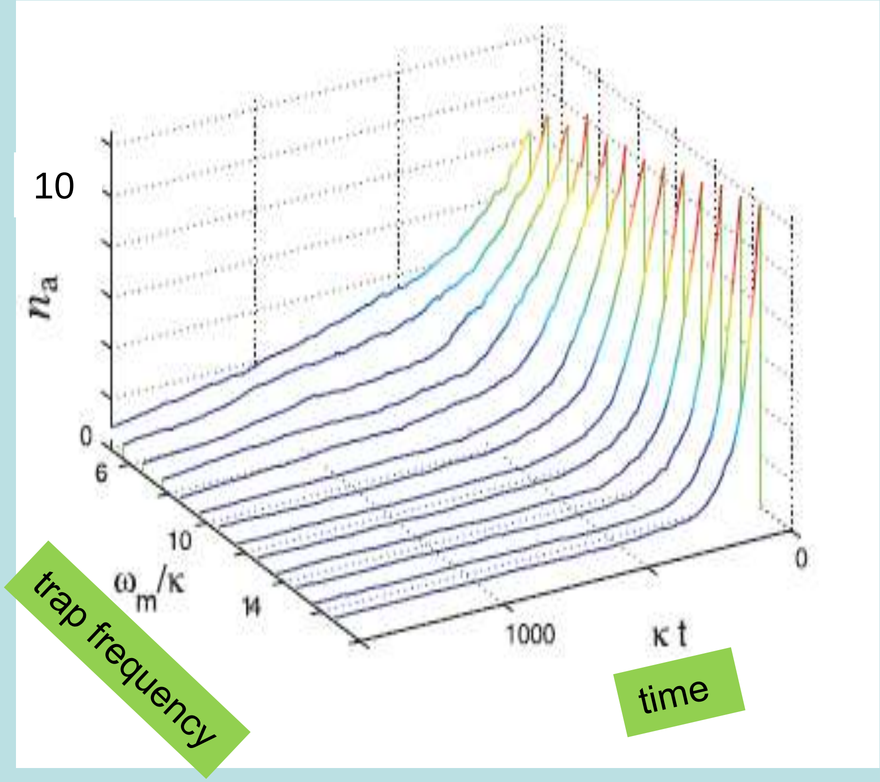
pump frequency

ground state cooling when mode is tuned the antistokes line : $\Delta \sim \omega >> \kappa$

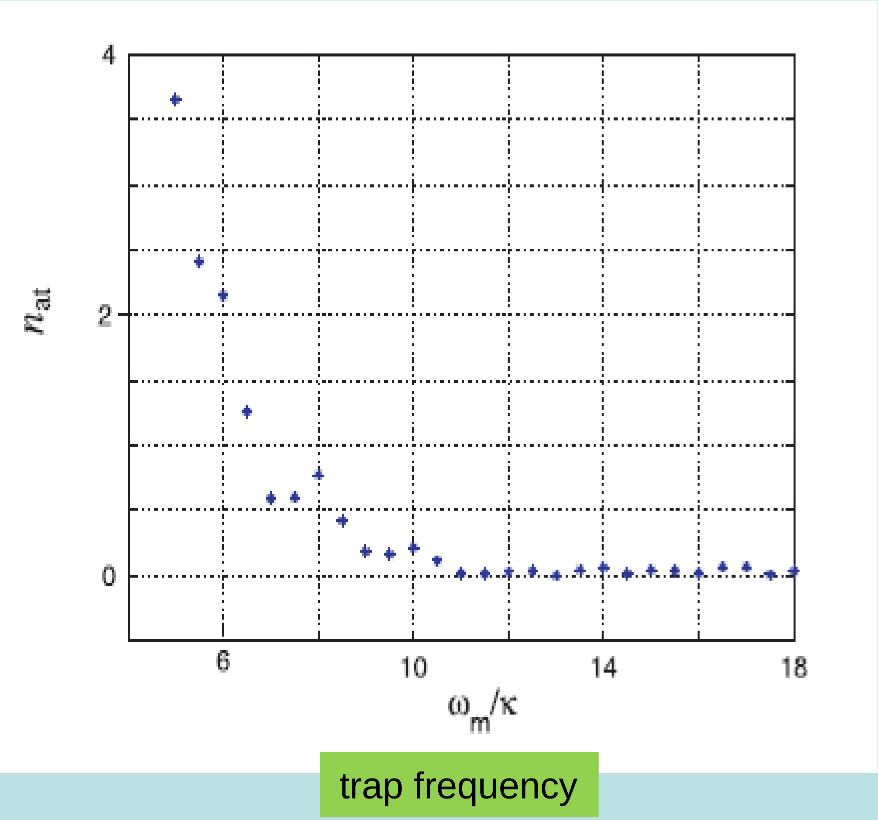
Optomechanics: all you need is power !

$$\omega_m^2 = 4\omega_R U_0 \alpha^2$$

Time evolution of quantum number



final quantum number



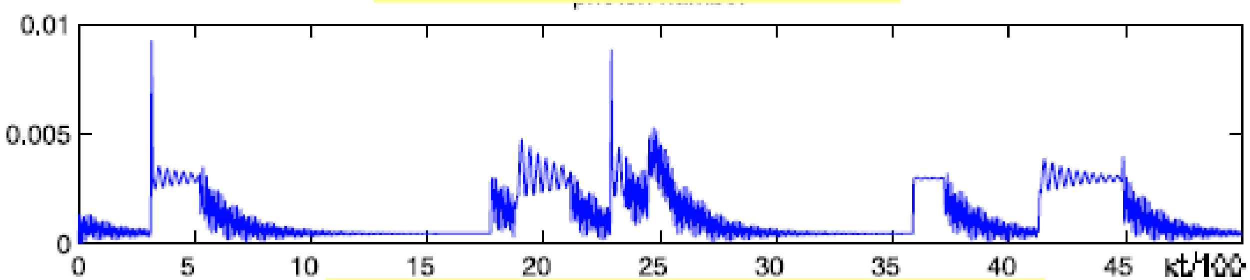
any polarizable, nonabsorptive particle can be cooled with sufficient power !

(-> Romero-Isart)

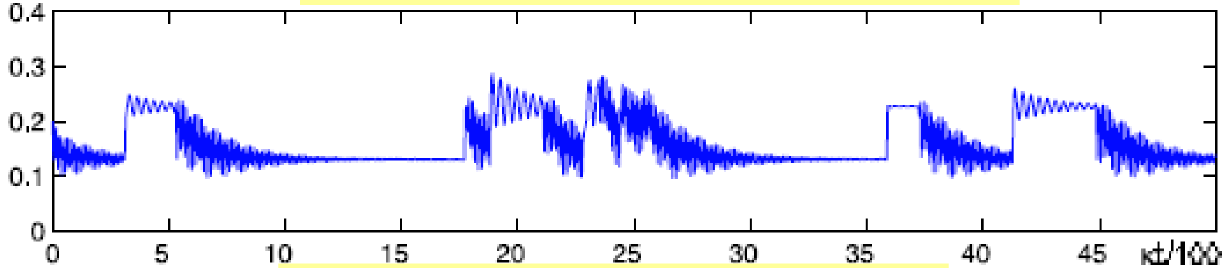
(new proposed experiments with beads : P. Barker, M. Raizen, M. Aspelmeyer, J. Kimble)

Single trajectory analysis of groundstate cooling :
Quantum jumps of particle and field near ground state !

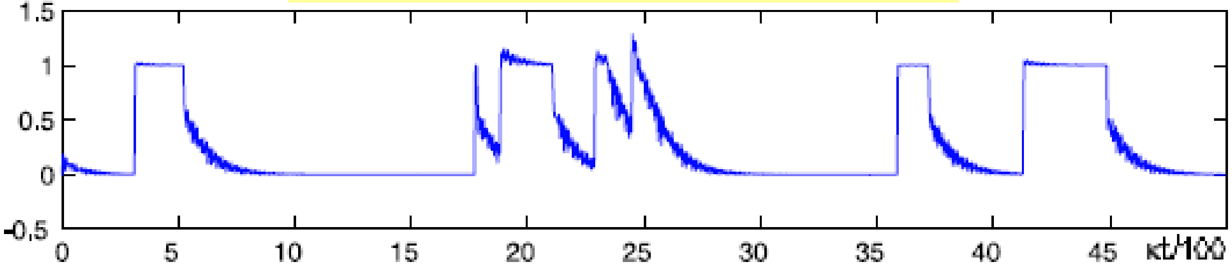
sin-mode photon number



particle state position uncerntainty



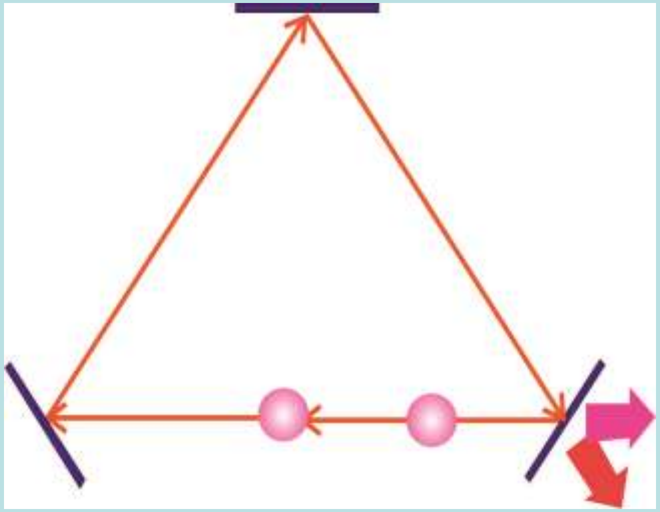
particle state occupation number

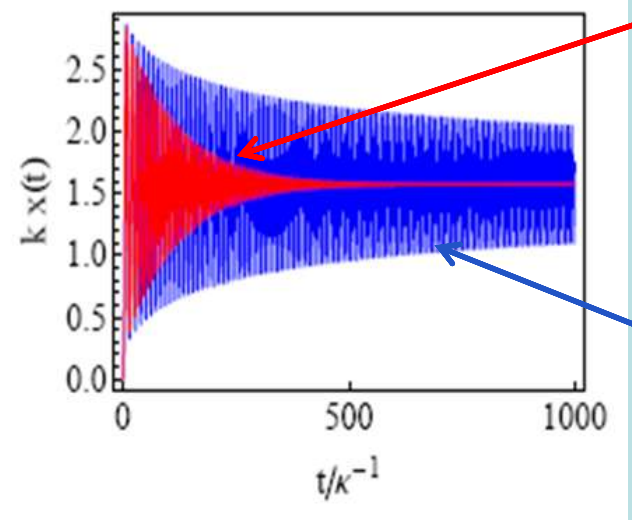


Particle jumps between two lowest states (parity change !)

Two particles in a ring cavity with dispersive interaction:
near their ground state: T ~ 0

classical point particles





Center of mass damps fast

relative motion:
pairing
+
momentum
anti-correlation

- * Strongly pumped Cos(kx) mode => mean field α
- * Scattered photons in Sin(kx) mode => quantum operator a
- * two particles : classical description of motion

Dynamical effects of a quantum potential:

bad cavity limit :
effective Hamiltonian with eliminated field

$$a_0^\dagger a_0 = \frac{|\eta_0|^2}{(\Delta_p - \delta_0 \hat{D}_{00})^2 + \kappa^2}$$

$$\begin{aligned} H &= \left[E + J\left(V_{cl} - \hbar U_0 \eta^2 \frac{\kappa^2 - {\Delta_c'}^2}{\left(\kappa^2 + {\Delta_c'}^2\right)^2}\right)\right]\hat{B} \\ &+ 3\hbar U_0^2 \eta^2 \Delta_c' \frac{3\kappa^2 - {\Delta_c'}^2}{\left(\kappa^2 + {\Delta_c'}^2\right)^4} J^2 \hat{B}^2 + \frac{U}{2}\sum_k \hat{n}_k\left(\hat{n}_k - 1\right) \end{aligned} \qquad (13)$$

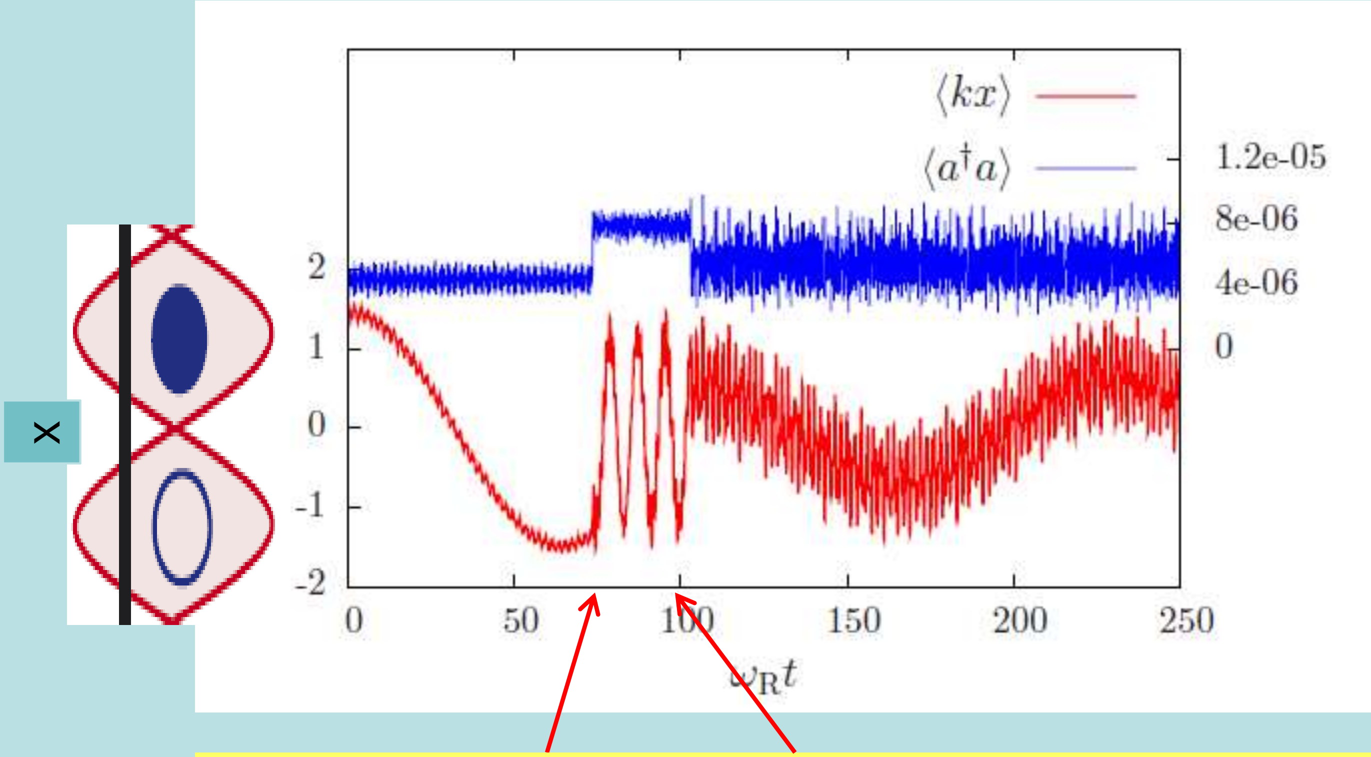
rescaled hopping terms

Nonlocal atom-atom interaction
via nonlocal correlated hopping

Cavity parameters can be used to effectively tune size and type of interactions !

Simulated single atom dynamics for two wells : photon-assisted or photon blocked tunneling

at t=0 atom prepared at right well:

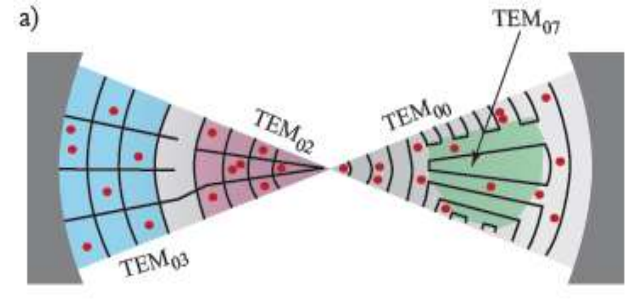


- jumps in photon number + atomic state
- effective model contains weighted average of tunnel amplitudes

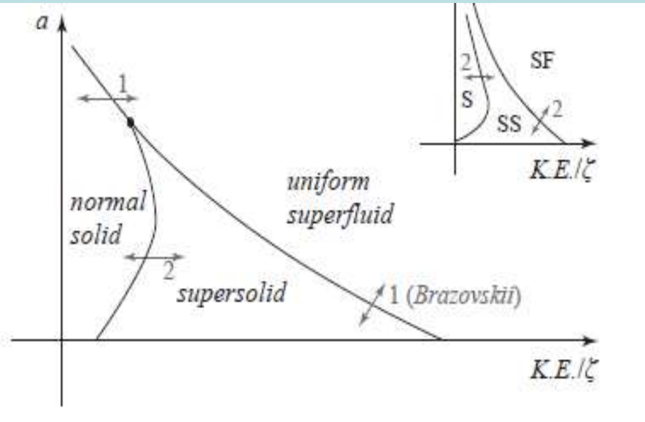
Generalization to multimode confocal cavity :

S.Gopalakrishnan, B. L.Lev, P. M.Goldbart
Nat.Phys. 5, 845 (2009).

„Quantum Brazovskii transition“



$$\frac{\Omega_{\mathrm{th}} - \Omega_{\mathrm{th}}^{\mathrm{mf}}}{\Omega_{\mathrm{th}}^{\mathrm{mf}}} \simeq 2.5 \left[\frac{\alpha U \sqrt{\hbar^2 K_0^2 / 2M}}{(\hbar \zeta N \chi)^{3/2}} \right]$$

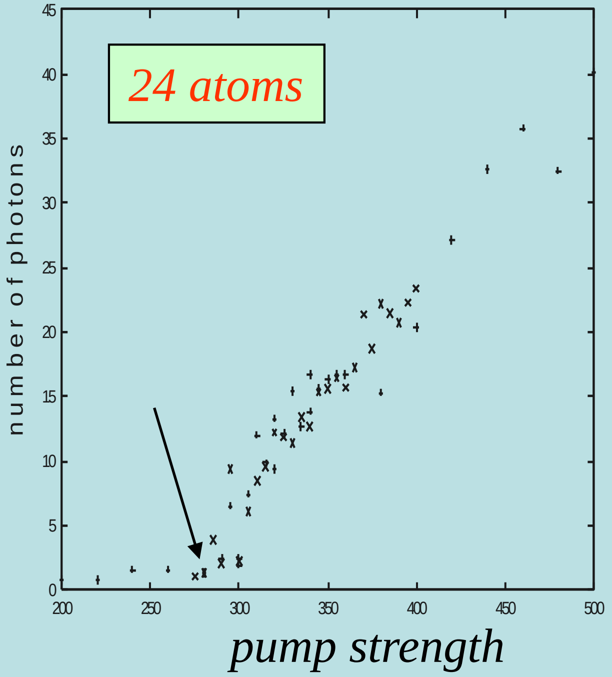


P. **Strack** and S. **Sachdev**

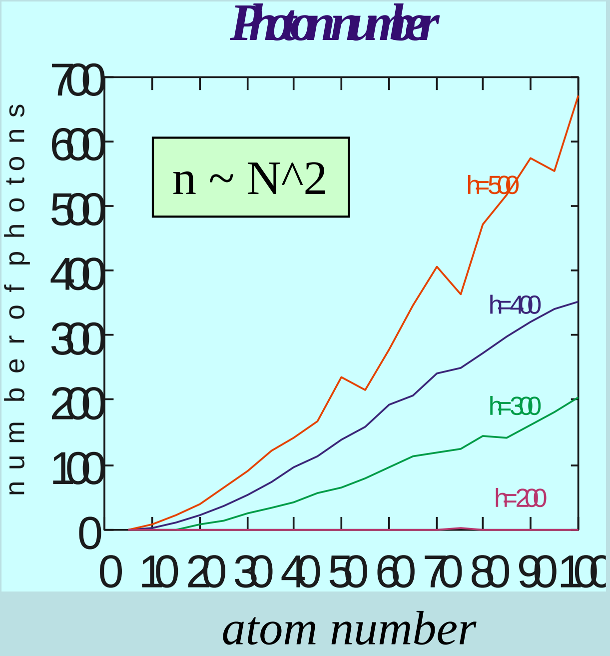
- Dicke quantum spin glass of atoms and photons
- Exploring models of associative memory via cavity quantum electrodynamics

Numerical simulations of coupled dynamics including atomic motion (start with random distribution at Doppler temperature)

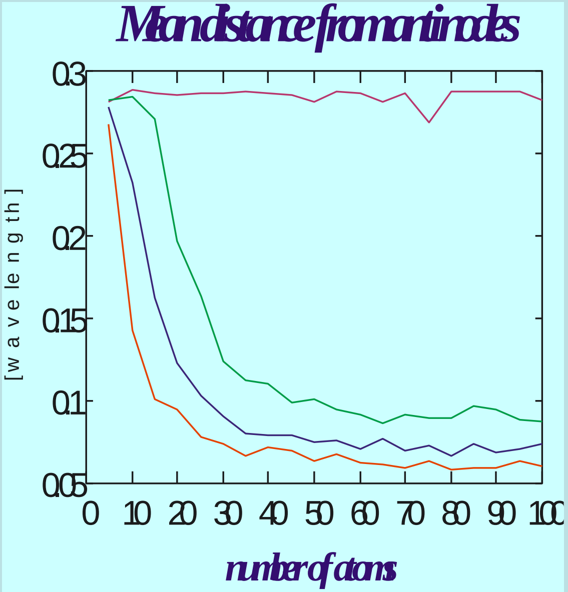
clear threshold !



Superradiance !



Selforganization!



quadratic dependence of cavity photons on atom number
=> stimulated emission dominates over spontaneous emission for large N
=> internal state unchanged (no repumper required)

diffusion and temperature : kinetic equation for fluctuations

$$\frac{\partial \langle f_l \rangle}{\partial t} + v \frac{\partial \langle f_l \rangle}{\partial x} - \frac{1}{m_l} \frac{\partial \langle \Phi_l \rangle}{\partial x} \frac{\partial \langle f_l \rangle}{\partial v} = \frac{1}{m_l} \left\langle \frac{\partial \delta \Phi_l}{\partial x} \frac{\partial \delta f_l}{\partial v} \right\rangle$$

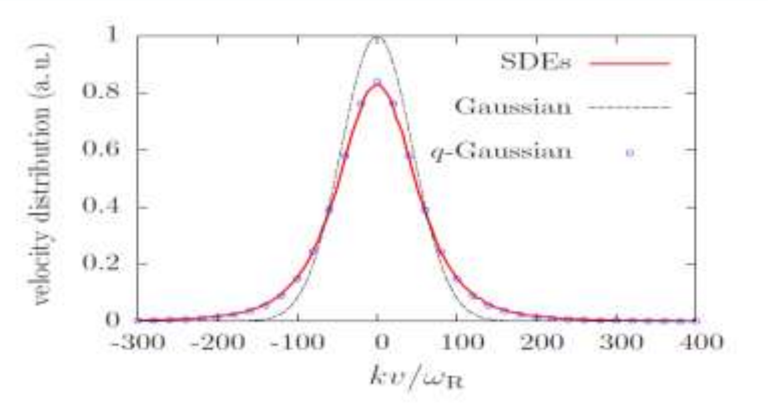
Below self consistent threshold:

$$\langle F(v) \rangle \propto \left(1 - (1-q)\frac{mv^2}{2k_\mathrm{B}T}\right)^{\frac{1}{1-q}}$$

$$q_s = 1 + \frac{\omega_{R,s}}{|\delta|}$$

$$k_\mathrm{B}T = \hbar \frac{\kappa^2 + \delta^2}{4|\delta|} = \frac{\hbar\kappa}{2}$$

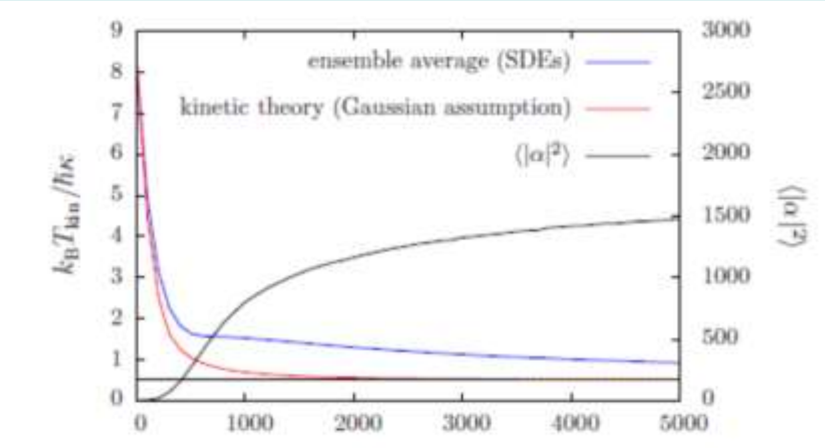
stationary velocity distribution



Above self consistent threshold:

$$N|U_0|V_\mathrm{opt} \overset{!}{<} \left(\frac{\kappa^2 + \delta^2}{2|\delta|}\right)^2 \frac{2}{3-q} \overset{\delta=-\kappa}{=} \frac{2}{3-q}\kappa^2$$

time evolution of ‚hot‘ ensemble



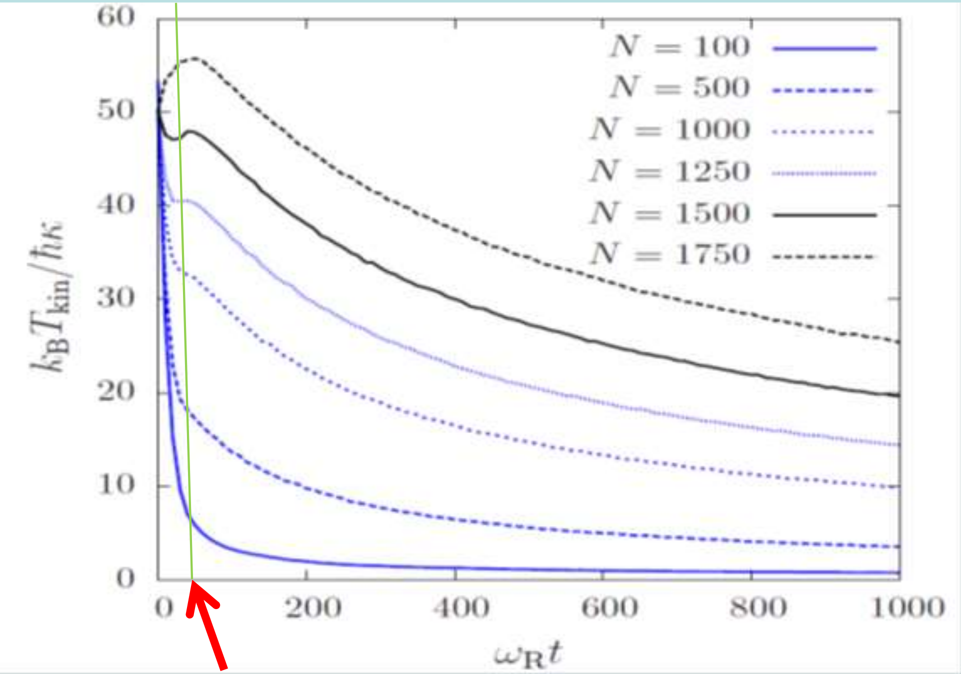
spatial average: kinetic equation for velocity distribution

$$\frac{\partial}{\partial t}\langle F\rangle + \frac{\partial}{\partial v}\Big(A[\langle F\rangle]\,\langle F\rangle\Big) = \frac{\partial}{\partial v}\left(B[\langle F\rangle]\frac{\partial}{\partial v}\langle F\rangle\right)$$

$$A[\langle F\rangle] := \frac{2\hbar k\delta\kappa\eta^2}{m}\frac{kv}{|D(ikv)|^2}$$

$$B[\langle F\rangle] := \frac{\hbar^2k^2\eta^2\kappa}{2m^2}\frac{\kappa^2+\delta^2+k^2v^2}{|D(ikv)|^2}$$

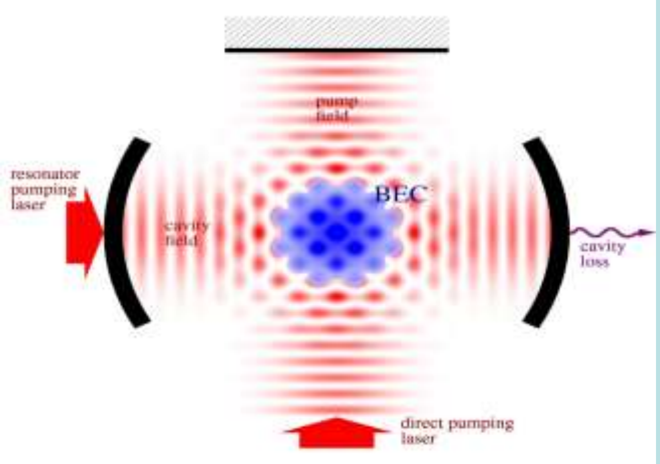
numerical solution:

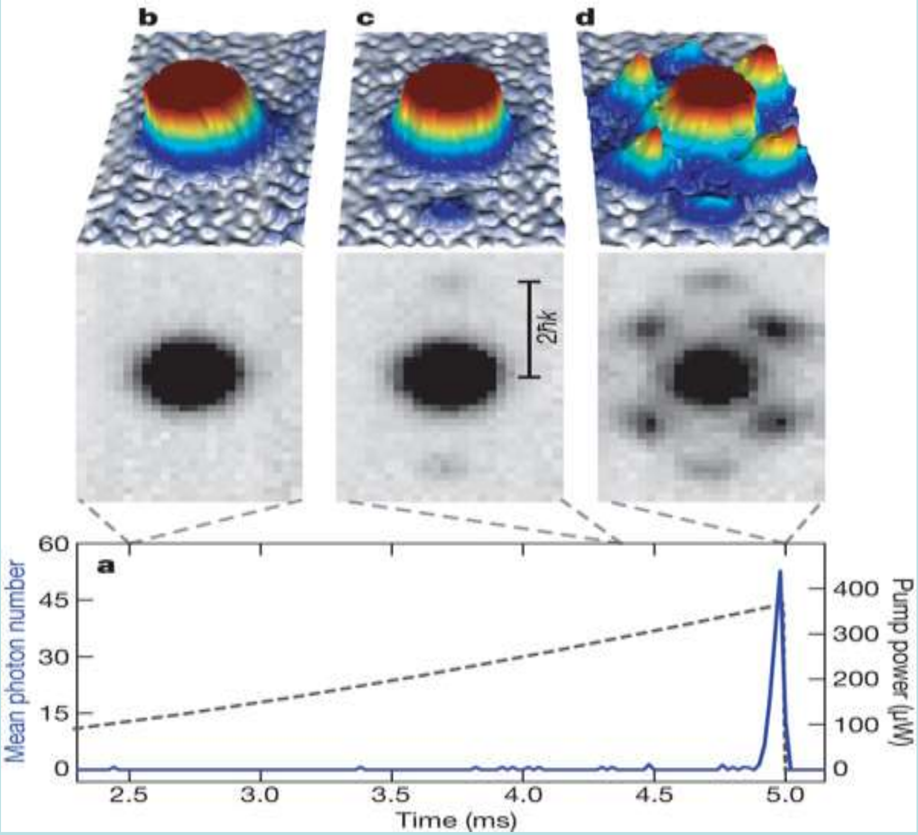


selforganization time

very weak particle number dependenceon selforganization time + cooling !

Eperiment ETH:
Observation of the phase transition to new phase
with coherence + ordering present (“supersolid phase”)



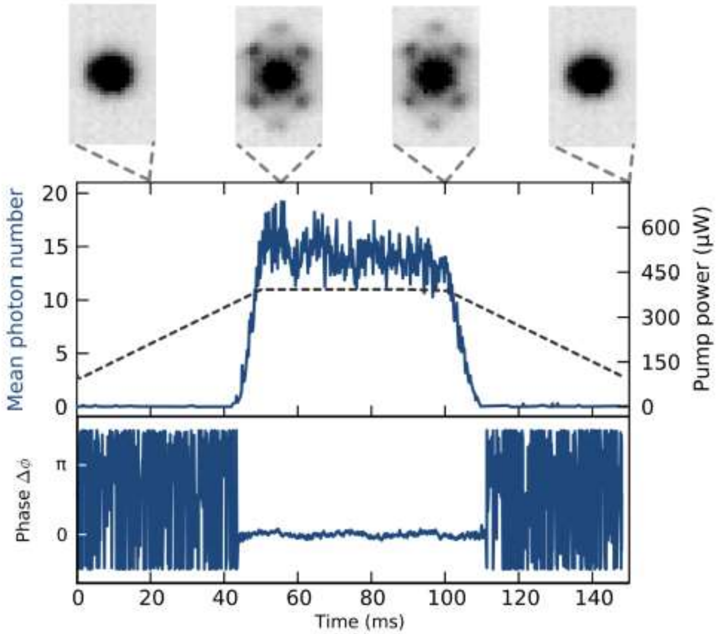


$$\Psi(x) = \frac{1}{\sqrt{L}} c_0 + \sqrt{\frac{2}{L}} c_1 \cos kx$$

Implementation of „Dicke Superradiant Phase“ transition

K Baumann *et al.* *Nature* 464, 1301-1306 (2010) doi:10.1038/nature09009

Measurement of phase diagram :



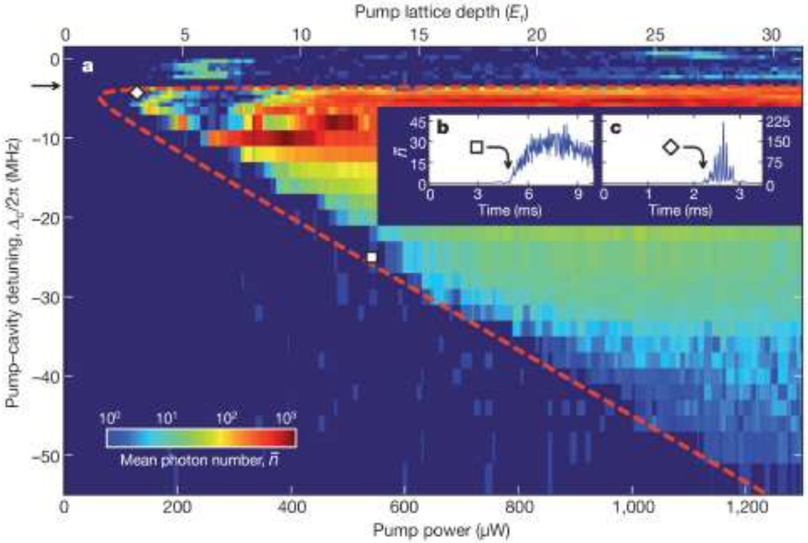


FIG. 40 *Phase diagram of the Dicke model*, from (Baumann *et al.*, 2010).

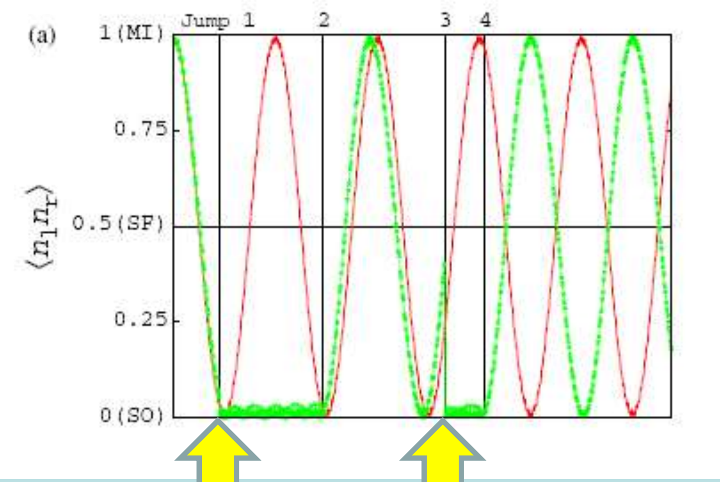
in ordered region:
coherence + ordering present: “supersolid phase”

Two atoms

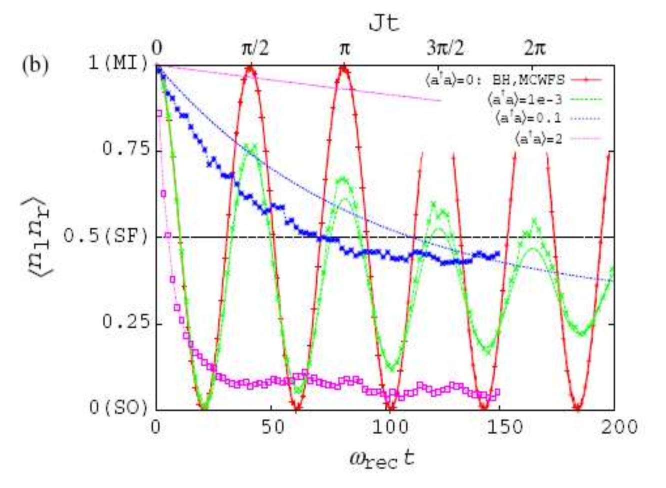
$\langle n_l n_r \rangle$ measures ordering:

- 1 „Mott“-insulator (1,1)
- 0 Ordered states {(2,0) +/-(0,2)}

Single trajectory



ensemble average



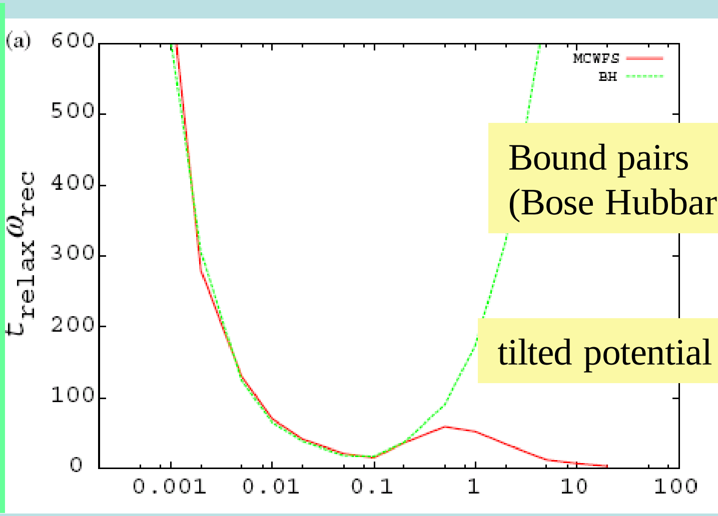
„spontaneous „ ordering
via photon scattering by sign change

$$|\Psi(t)\rangle = (|-,-2\alpha\rangle + |+,2\alpha\rangle)$$

$$|\Psi'(t)\rangle \propto a\,|\Psi(t)\rangle \propto |-,-2\alpha\rangle - |+,2\alpha\rangle$$

|-> =[(2,0)-(0,2)] state does not tunnel !

Time scale of ordering

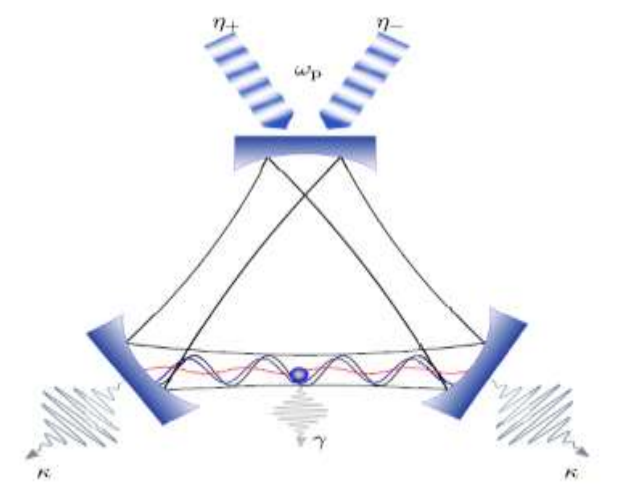


Bound pairs
(Bose Hubbard)

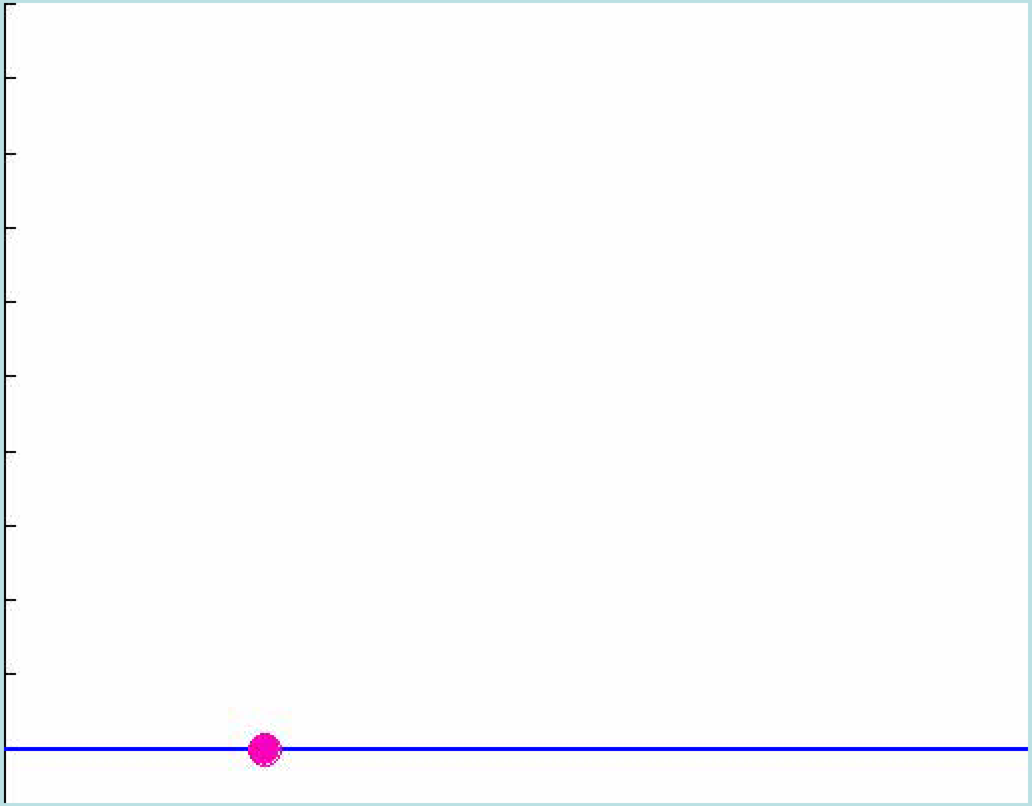
tilted potential

Part III) Quantum dynamics and controlled interactions of few partilces in multimode“ cavities

Symmetrically pumped ring cavity:



- **cosine-mode as trap**
- **sine–mode for cooling + control**



atom phase locks field modes and changes intensity distribution

=>

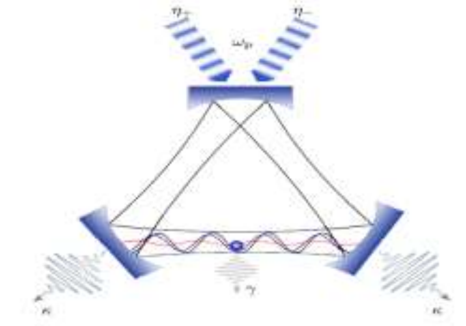
atom drags node along ist path to stay at intensity maximum = field minimum

Quantum model in a ring cavity

atom-field Hamiltonian for quantized motion:

$$H = \frac{\hat{p}^2}{2m} - \hbar\Delta\left(a_c^\dagger a_c + a_s^\dagger a_s\right) - \hbar U(\hat{x}) + i\hbar\left(\eta a_c^\dagger - \eta^* a_c\right)$$

$$U(\hat{x}) = \underbrace{a_c^\dagger a_c U_c(\hat{x})}_{\text{cosine^2}} + \underbrace{a_s^\dagger a_s U_s(\hat{x})}_{\text{sine^2}} + \left(a_c^\dagger a_s + a_c a_s^\dagger\right) \underbrace{U_{cs}(\hat{x})}_{\text{cosine*sine}}$$



E(x,t) ~ a_c cos(kx)+a_s sinkx

pumped cosine mode:
=> mode in coherent state α
⇒ deep harmonic trap for particle

$$\omega_m^2 = 4\omega_R U_0 \alpha^2$$

unpumped sine mode:
=> mode near vacuum state
⇒ linear coupling
⇒ cooling + measurement

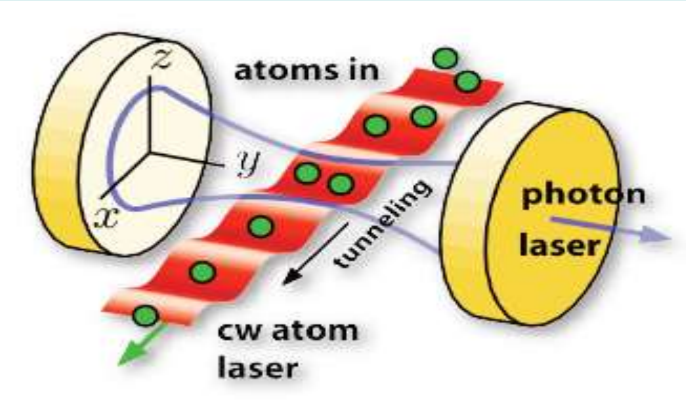
deep trap limit:

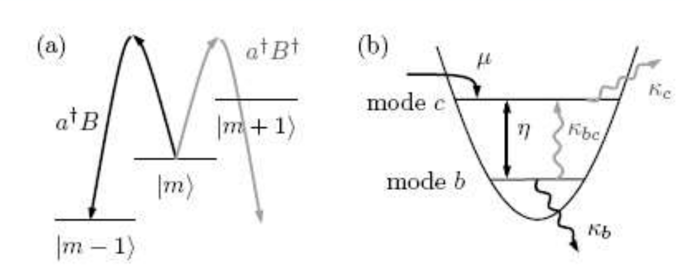
$$H = \left[\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2\right] - \hbar\Delta a_s^\dagger a_s - \hbar U_0'(a_s + a_s^\dagger)\hat{x}$$

quadratic coupling ' $a_c^+ a_c$ x^2'
=> trap + x^2 nonlinearity

linear coupling '$(a_s^+ + a_s)$ x'
=> "optomechanical cooling"

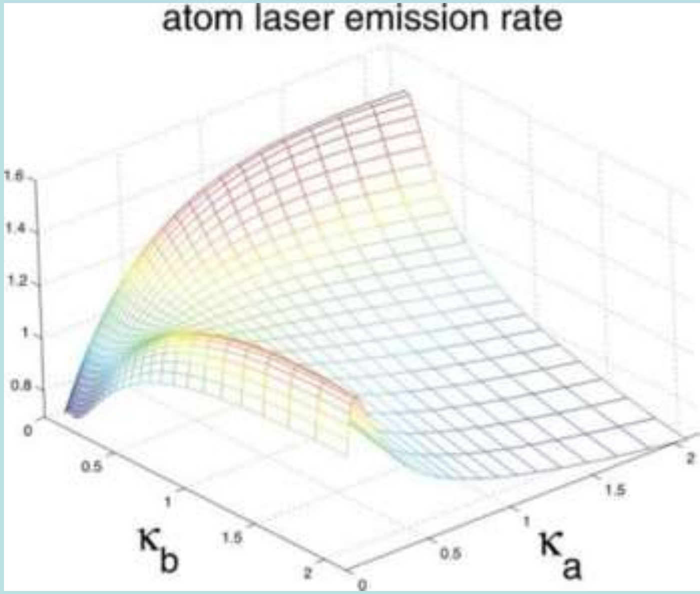
CW- operation of atom-photon pair laser: *=> add CW 'hot' atom source or incoherent pump*





$$\mathcal{H}_{\rm int} = \eta\left(a^\dagger b^\dagger c + a\,b\,c^\dagger\right)$$

parametric interaction

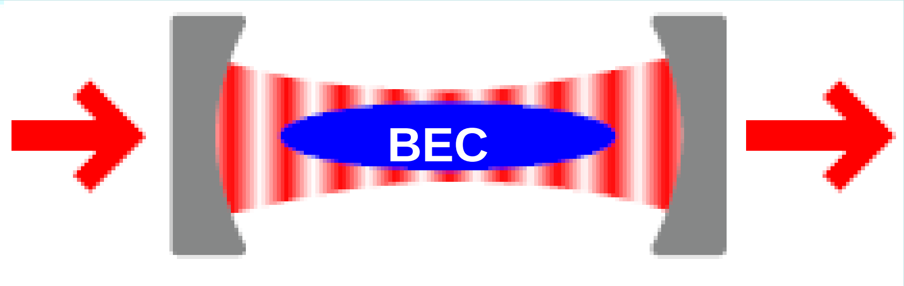

atom laser emission rate

Stimulated amplification of
light and atomic ground state population
via blue Raman sideband

T. Salzburger et. al. , PRL

*Cold gas in a **quantum** optical potential*
(mean field limit)

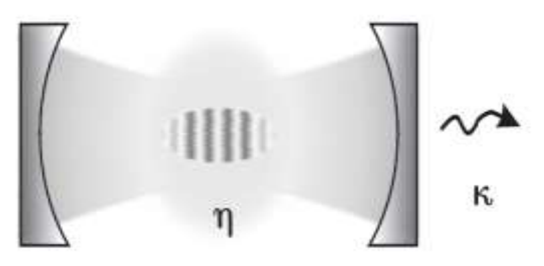
* *Cavity field generates **dynamical optical lattice with quantum properties***
* *Atoms act on the cavity field depending on their quantum state*



Correlated / entangled dynamics of
field amplitude and particle wave-function
Coupled Maxwell - Schrödinger equation

Cold dilute gas in a cavity generated optical potential at finite T (Vlasov mean field approach)

Continuous density approximation for cold cloud: dynamic refractive index



$$f(x,v,t) = \frac{m}{2N\pi\hbar}\int e^{-izmv/\hbar}\rho_{P,1}\left(x+\frac{z}{2},x-\frac{z}{2},t\right)dz$$

Kinetic limit-Vlasov equation

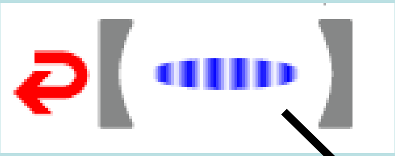
$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{U_0|\alpha|^2}{2}\sin(2kx)\Big(f(x,v+v_R) - f(x,v-v_R)\Big) = 0$$

field dynamics:

$$\dot{\alpha} = [-\kappa + i(\Delta_c - NU_0/2)]\alpha - i\frac{NU_0}{2}\alpha\int_{-\infty}^{\infty}dv\int_0^{\lambda} f(x,v,t)\cos(2kx)dx + \eta$$

nonlinear coupled dynamics

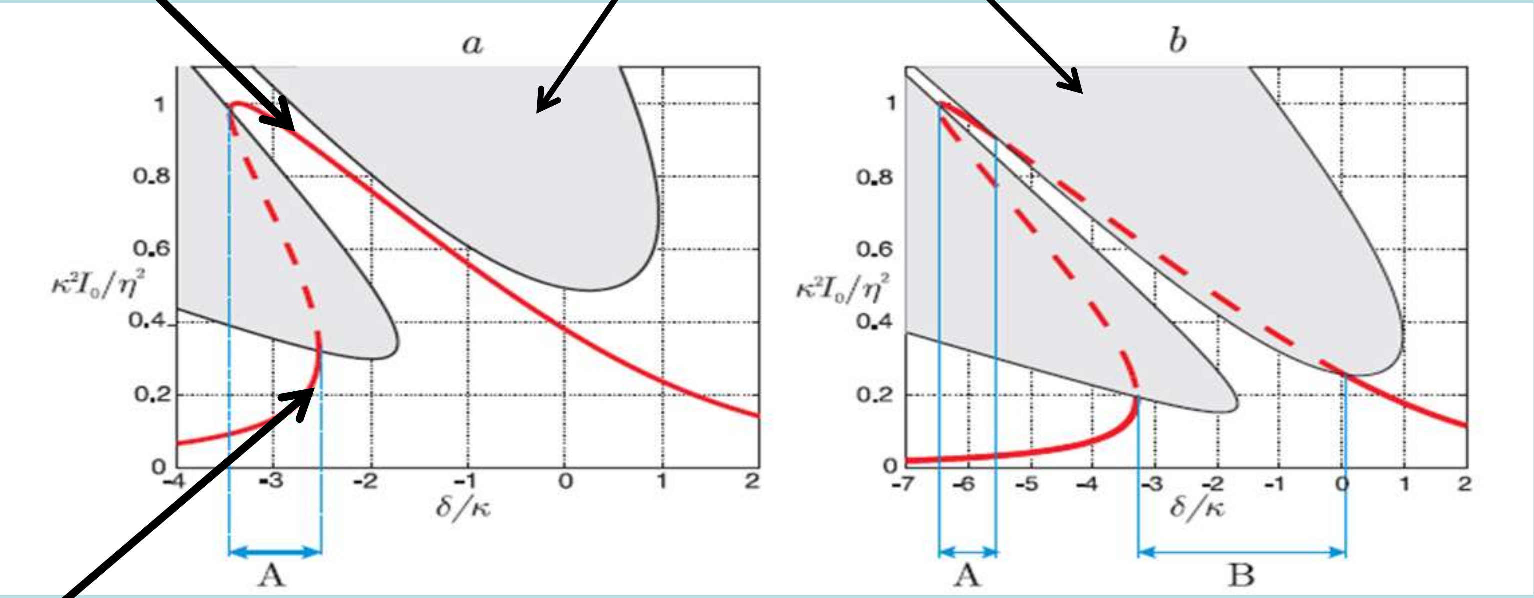
Stationary solution for strong field and many particles: bistability

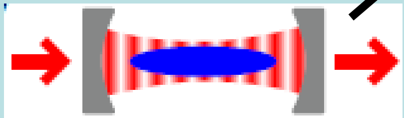


moderate pump

strong pump

instable regime





two stable solutions: nonlinear optics

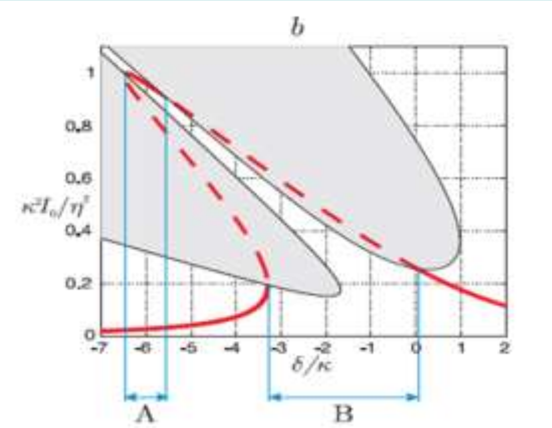
(a) weak field and homogeneous density (kT >> V_opt)

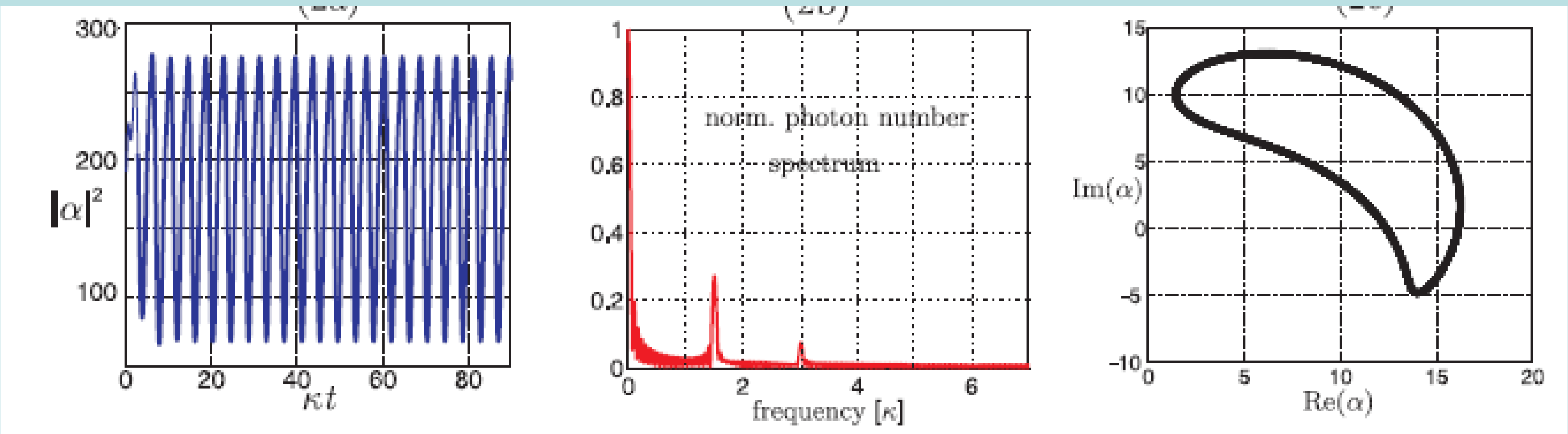
(b) strong field and modulated density (kT << V_opt)

no stable solution: limit cycle

unstable regime for strong pump:

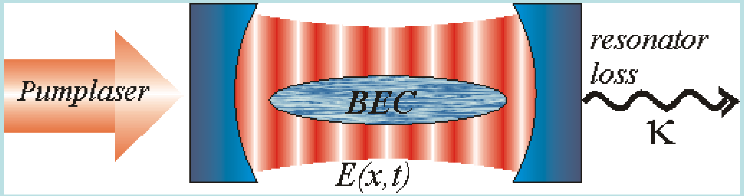
dynamic solution shows density waves with limit cycle behaviour





related experiments:
A. Hemmerich (transverse motion)
J. Eschner (thermal cloud)

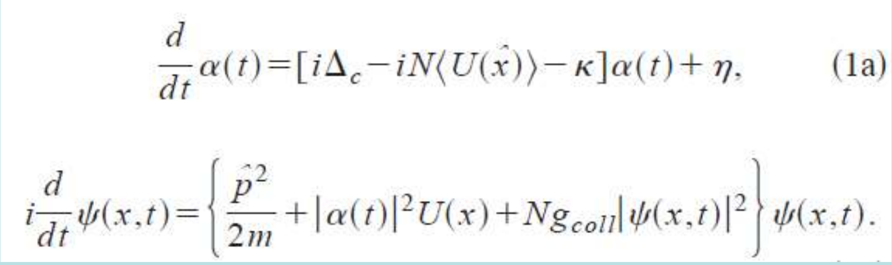
*Quantum **dynamics** of many particles and field near T~0*



BEC** in **optical lattice
with dynamic (quantum) properties

*Me**an field** description of many particles and field*

Gross-Pitajevski ⬄ Maxwell



$$\frac{d}{dt}\alpha(t)=[i\Delta_c-iN\langle U(\hat{x})\rangle-\kappa]\alpha(t)+\eta, \qquad (1a)$$

$$i\frac{d}{dt}\psi(x,t)=\left\{\frac{\hat{p}^2}{2m}+|\alpha(t)|^2U(x)+Ng_{coll}|\psi(x,t)|^2\right\}\psi(x,t).$$

coupled nonlinear and nonlocal equations with a wealth of dynamic effects

Refs: **Horak, Barnett, Hammerer, Zoller, Meystre, Liu, Bhattacherjee…**

Experiments: ***Esslinger, Reichel, Zimmermann, Hemmerich, Stamper-Kurn,Vuletic, Treutlein ...***

BEC in a cavity: quantum T=0 limit

two relevant momentum modes :
1. homogeneous state
2. diffracted wave: cos(2kx)

$$\psi(x,t) = c_0(t) + c_2(t)\sqrt{2}\,\cos(2kx)$$

two coupled oscillators

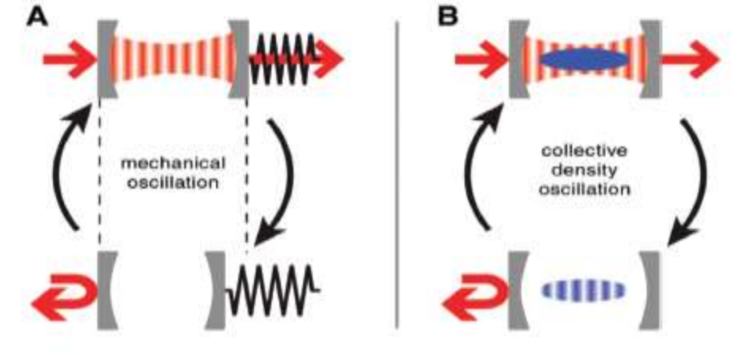
$$X = 2\sqrt{1/N}\,\mathrm{Re}(c_0{}^*c_2)$$

⇒ optomechanics – Hamiltonian at T=0

$$\ddot{X} + (4\omega_{\mathrm{rec}})^2 X = -\omega_{\mathrm{rec}} U_0 \sqrt{8N}\langle\hat{a}^\dagger\hat{a}\rangle$$

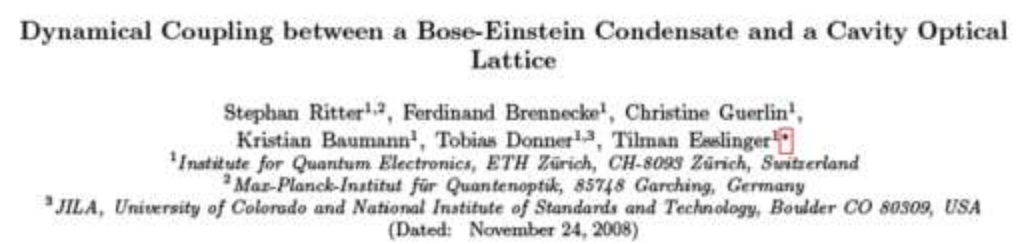
mirror-position ⬄ field-intensity

ideal „optomechanics“ test toolbox => next talk!



mirror position = amplitude of density wave

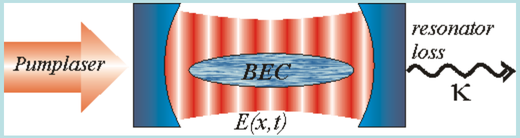
BEC parameter regime :
- start at ground state (T=0)
- strong single photon coupling
- atoms and light can be measured
- nonlinear regime: bistable response

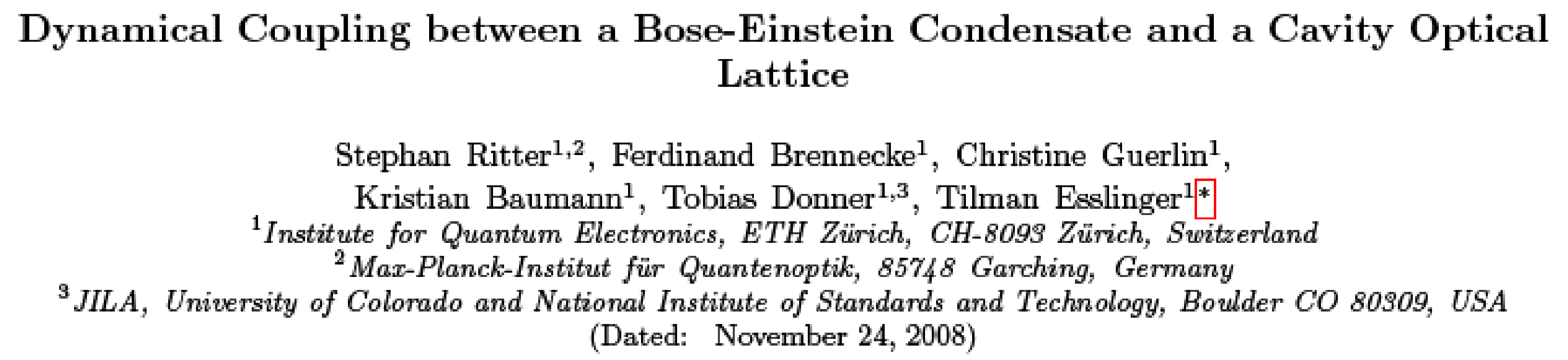


Dynamical Coupling between a Bose-Einstein Condensate and a Cavity Optical Lattice

Stephan Ritter[1,2], Ferdinand Brennecke[1], Christine Guerlin[1],
Kristian Baumann[1], Tobias Donner[1,3], Tilman Esslinger[1,*]
[1] *Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3] *JILA, University of Colorado and National Institute of Standards and Technology, Boulder CO 80309, USA*
(Dated: November 24, 2008)

Experiment ETH Zürich: BEC in high-Q cavity

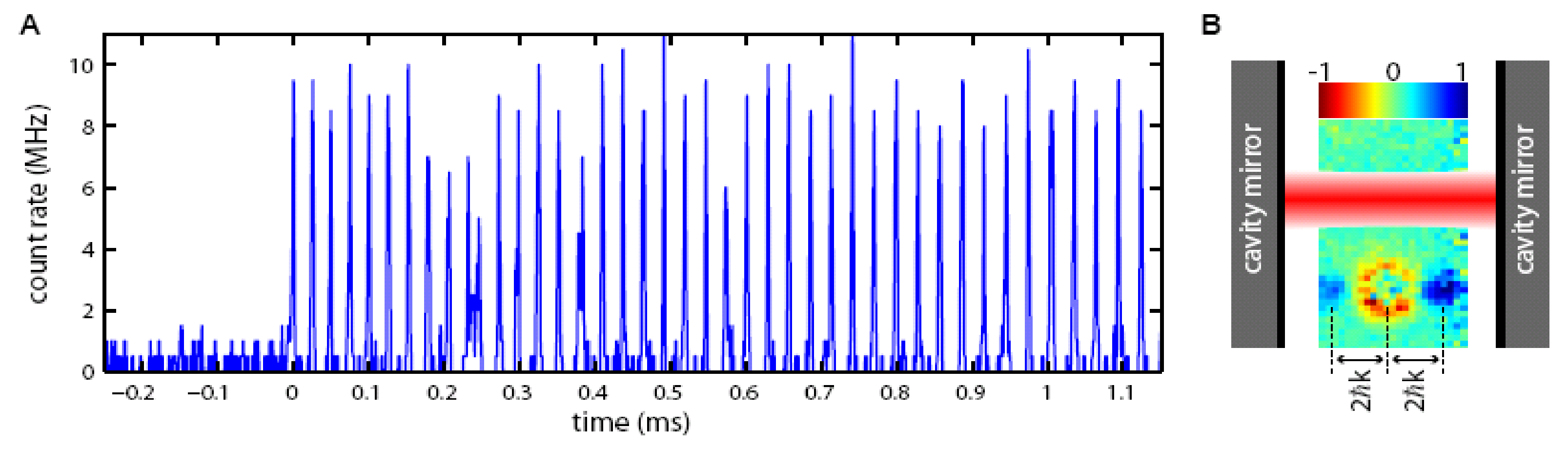




Dynamical Coupling between a Bose-Einstein Condensate and a Cavity Optical Lattice

Stephan Ritter[1,2], Ferdinand Brennecke[1], Christine Guerlin[1],
Kristian Baumann[1], Tobias Donner[1,3], Tilman Esslinger[1]*
[1] *Institute for Quantum Electronics, ETH Zürich, CH-8093 Zürich, Switzerland*
[2] *Max-Planck-Institut für Quantenoptik, 85748 Garching, Germany*
[3] *JILA, University of Colorado and National Institute of Standards and Technology, Boulder CO 80309, USA*

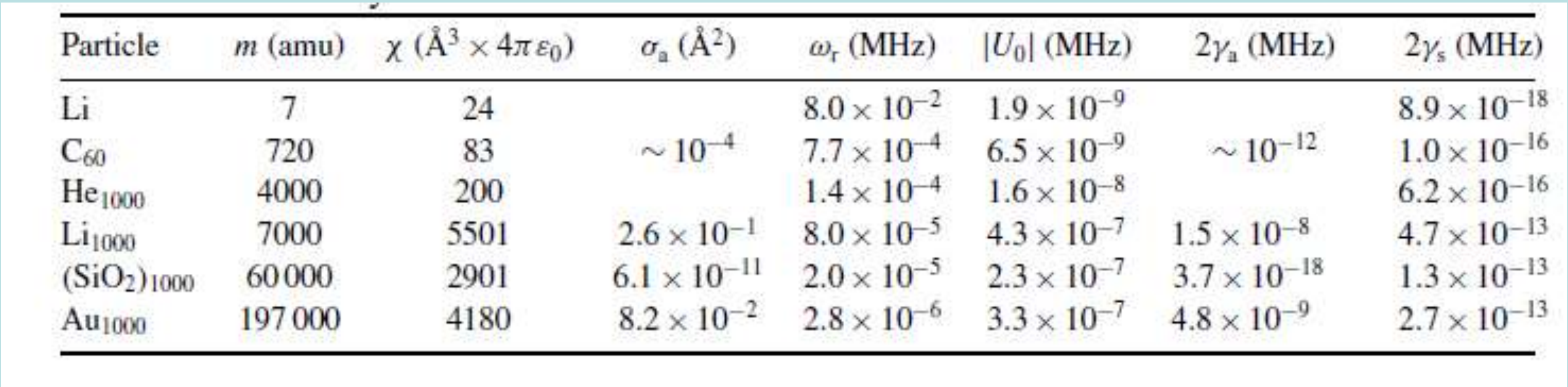
(Dated: November 24, 2008)



- * two mode BEC expansion works very well
- * study of zero T optomechanics in the instable regime
- * nonlinear oscillations + atom field entanglement

Cavity cooling of molecules or heavier particles

Examples: Cavity of V=1 mm^3 at 1.5 μ laser



Particle	m (amu)	χ ($\text{Å}^3 \times 4\pi\varepsilon_0$)	σ_a (Å^2)	ω_r (MHz)	$\lvert U_0\rvert$ (MHz)	$2\gamma_a$ (MHz)	$2\gamma_s$ (MHz)
Li	7	24	$\sim 10^{-4}$	8.0×10^{-2}	1.9×10^{-9}	$\sim 10^{-12}$	8.9×10^{-18}
C_{60}	720	83		7.7×10^{-4}	6.5×10^{-9}		1.0×10^{-16}
He_{1000}	4000	200		1.4×10^{-4}	1.6×10^{-8}		6.2×10^{-16}
Li_{1000}	7000	5501	2.6×10^{-1}	8.0×10^{-5}	4.3×10^{-7}	1.5×10^{-8}	4.7×10^{-13}
$(SiO_2)_{1000}$	60 000	2901	6.1×10^{-11}	2.0×10^{-5}	2.3×10^{-7}	3.7×10^{-18}	1.3×10^{-13}
Au_{1000}	197 000	4180	8.2×10^{-2}	2.8×10^{-6}	3.3×10^{-7}	4.8×10^{-9}	2.7×10^{-13}

- + *single setup for wide range of species*
- - *but need to prepare useful initial confinement in mode*
- - *Watts of pump power at finesse F > 10^4*

Note: speed up cooling for weak coupling by more power !

(S. Nimmrichter, NJP 12, 2010, Salzburger 2009, Deachapunya EPJD 08)