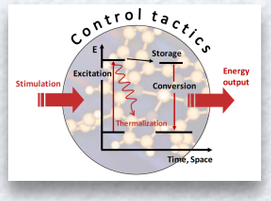
Nonequilibrium Dynamics with the Time-Dependent DMRG

Salvatore R. Manmana

Institute for Theoretical Physics
University of Göttingen



- A.C. Tiegel, S.R. Manmana, T. Pruschke, and A. Honecker, arXiv:1312.6044: *frequency-space dynamics at finite temperatures*
- F.H.L. Essler, S. Kehrein, S.R. Manmana, and N.J. Robinson, arXiv:1311.4557 (to appear in PRB): *controlled integrability breaking*
- J. Eisert, M. van den Worm, S.R. Manmana, and M. Kastner, PRL **111**, 260401 (2013): *Lieb-Robinson-Bound & long-range interactions*

Quantum Many-Body Systems in Nature and in the Lab

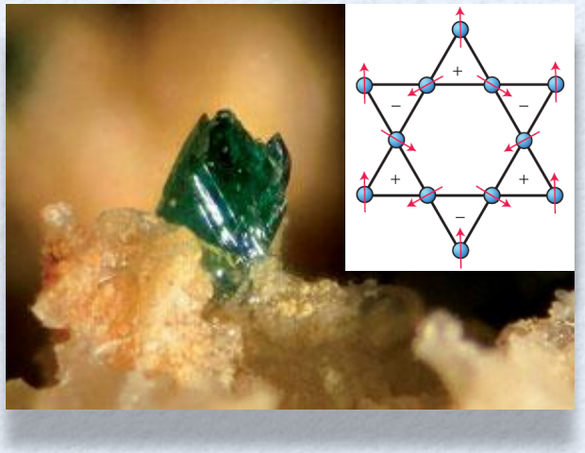
Schrödinger equation:

time dependent

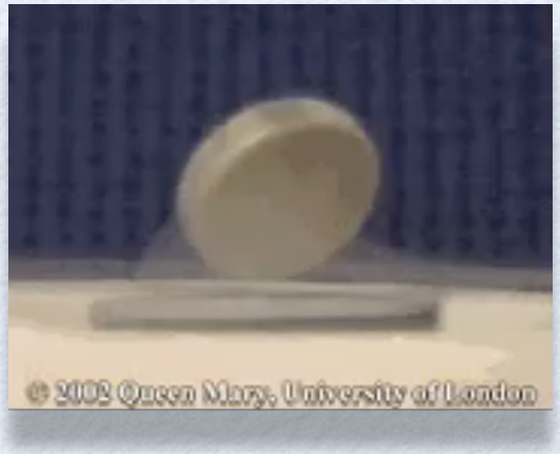
$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle$$

time independent

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

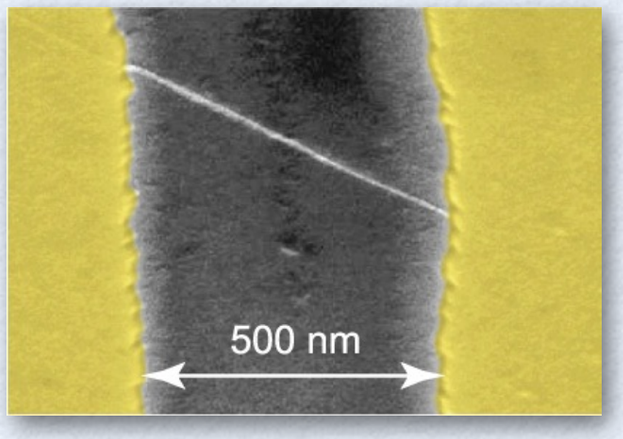


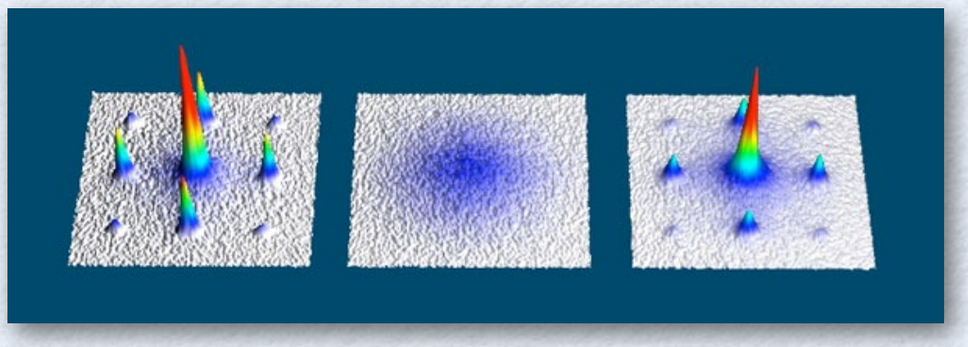
$$\hat{H} = -\sum_i \frac{\hbar^2}{2m_i}\vec{\nabla}_i^2 + \sum_{\langle i,j\rangle}\hat{V}(\vec{x}_i,\vec{x}_j)$$



Quantum Magnetism of natural Minerals
(Herbertsmithite, Azurite,...): “Spin-liquids”?

Synthetisized Materials:
High-Temperature Superconductors



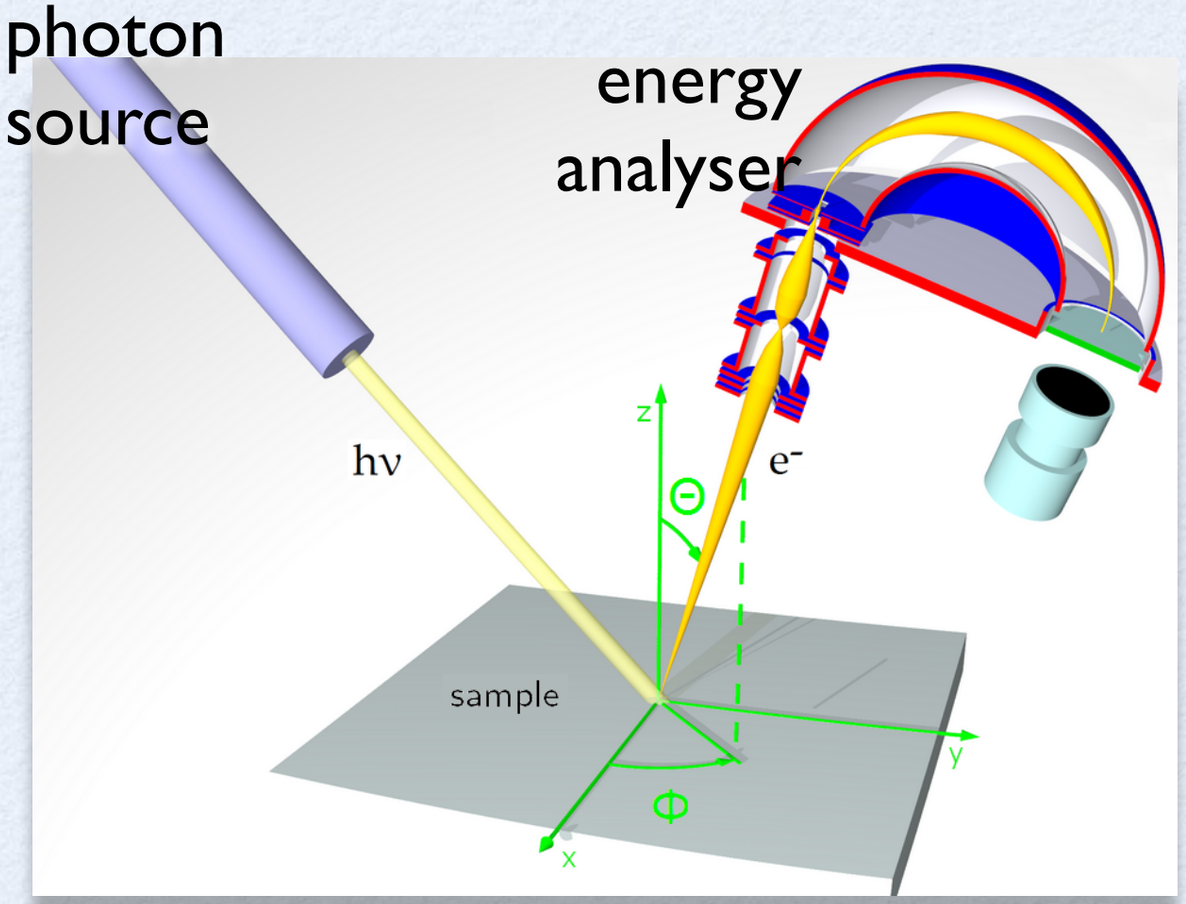


Low-dimensional systems
(e.g. TTF-TCNQ & further charge-transfer salts)

Ultracold Gases (Optical Lattices):
Bose-Einstein condensates & Mott-Insulators

Many-Body Systems Out-Of-Equilibrium: 1) Linear Response

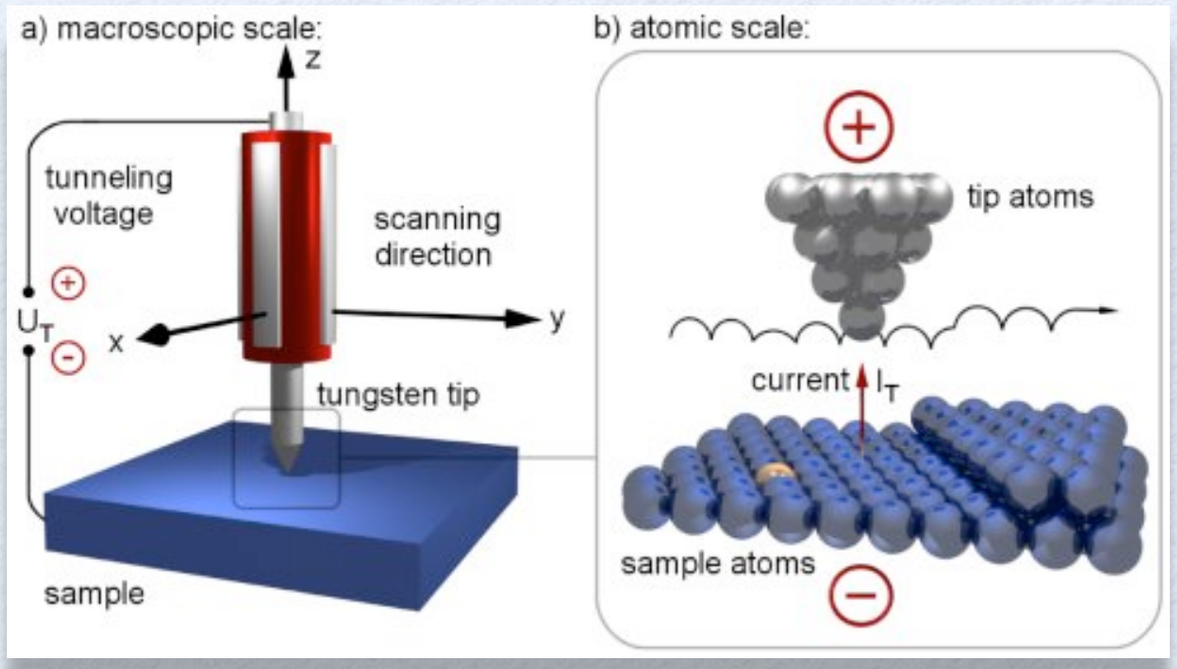
angle-resolved photoemission (ARPES)



(from Wikipedia)

☞ electronic density of states $A(k,\omega)$

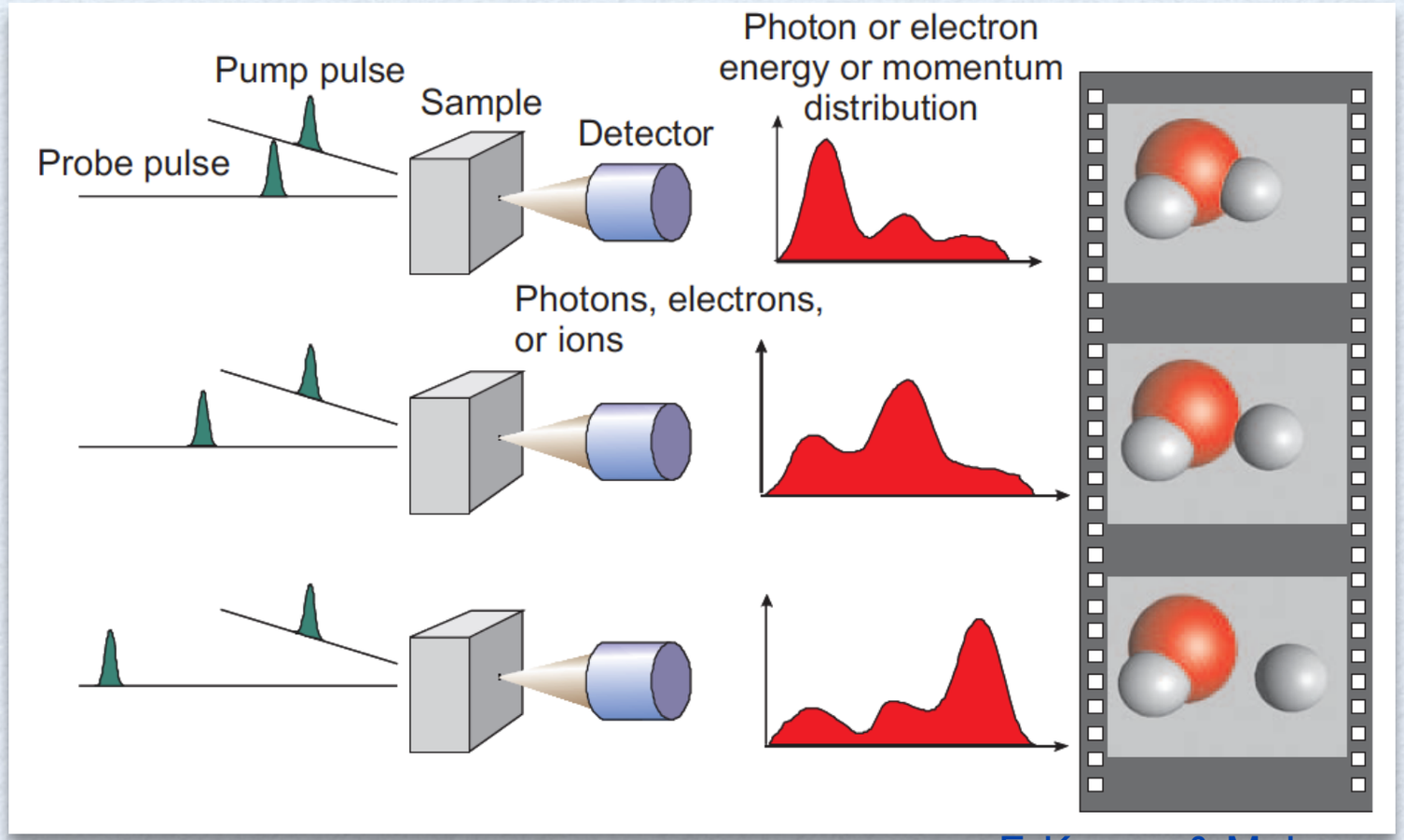
scanning-tunneling spectroscopy



(from www.physics.rutgers.edu/bartgroup/)

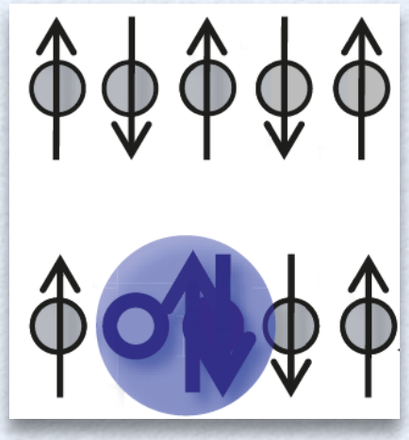
☞ **local** density of states $A(\omega)$

Many-Body Systems Out-Of-Equilibrium: 2) Highly Excited Materials



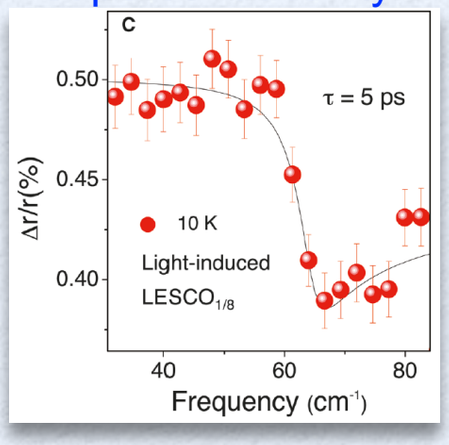
F. Krausz & M. Ivanov, RMP (2009)

Photo-excitation of Mott insulators



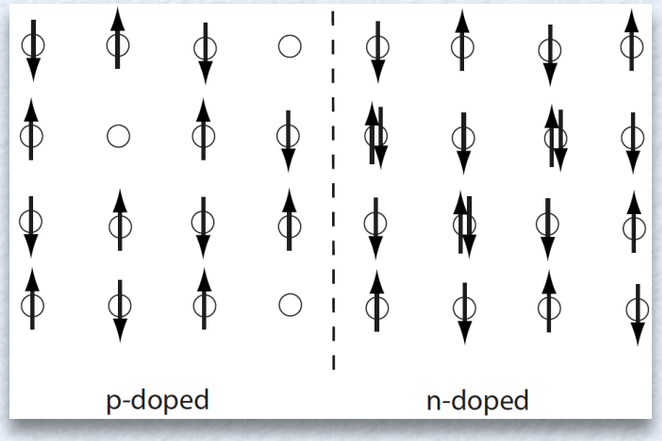
S. Wall et al., Nature Physics (2010)

"Light-induced superconductivity"



D. Fausti et al., Science (2011)

Photovoltaic effects

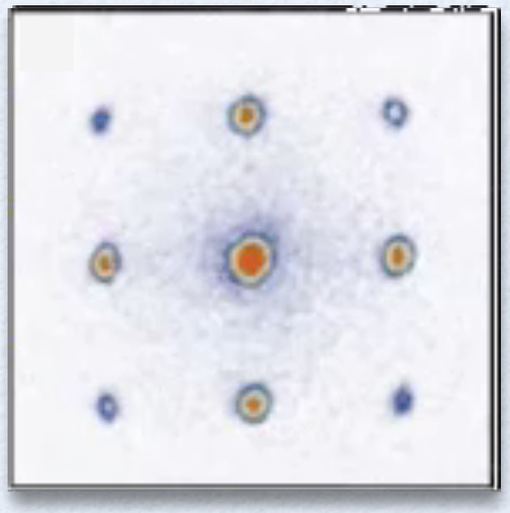


E. Manousakis PRB (2010)

Many-Body Systems Out-Of-Equilibrium: 3) Ultracold Gases & Optical Lattices

Out-of-Equilibrium

"Quantum Quenches"
➠ Sudden change of parameters
U_0 ➠ U



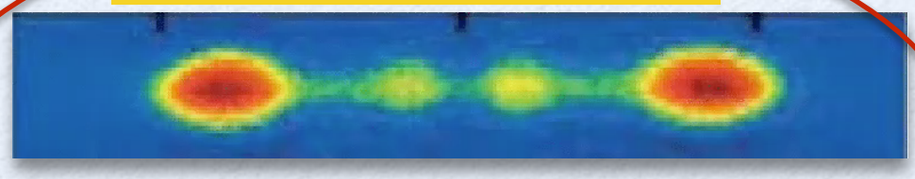
Collapse and Revival of a Bose-Einstein-Condensate

M. Greiner et al., Nature (2002)

Prepared states, Expansions

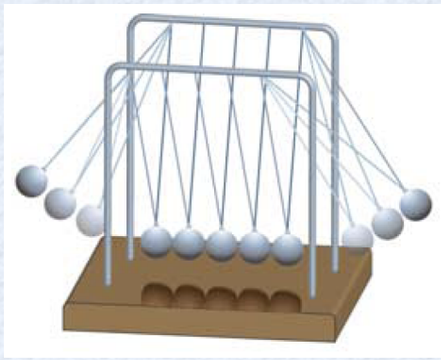
➠ "Release" atoms, remove a trapping potential

thermal state in 3D, not in 1D



'Quantum Newton Cradle'

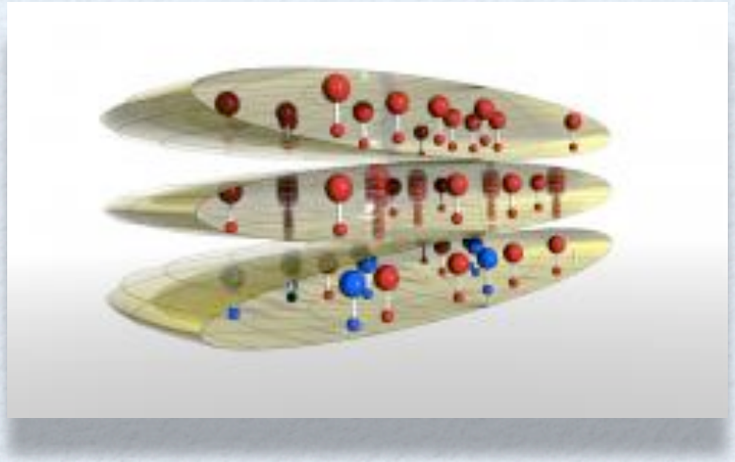
T. Kinoshita et al., Nature (2006)



➠ Relaxation behavior
➠ Time scales
➠ Novel (metastable) states?

Example Quantum Simulators: Polar Molecules

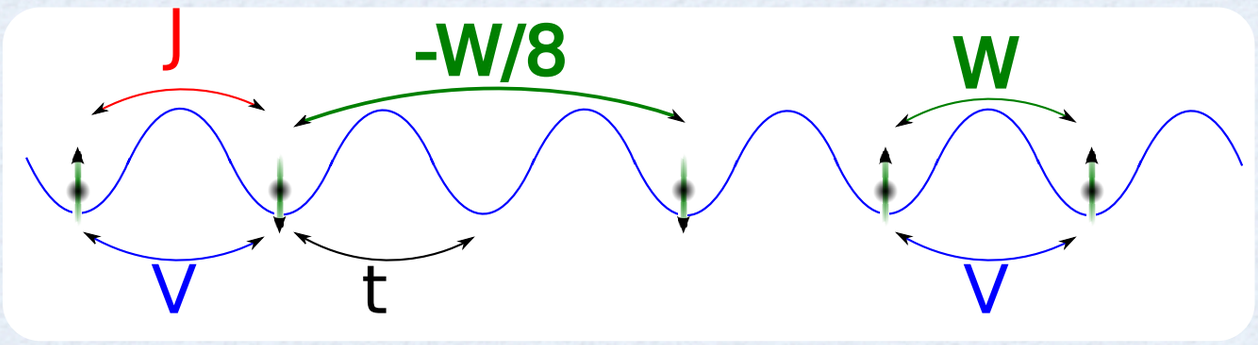
[A.V. Gorshkov, S.R. Manmana et al., PRL & PRA (2011)]



polar Molecules (e.g. KRb) in optical lattices:
2 Rotational states ⇔ two Spinstates

Effective Model:

$$\mathcal{H} = -t \sum_{j,\sigma} \left[c^{\dagger}_{j,\sigma} c_{j+1,\sigma} + h.c. \right] + \sum_{i,j} \frac{1}{|i-j|^3} \left[\frac{J_\perp}{2} \left(S^+_i S^-_j + S^-_i S^+_j \right) + J_z S^z_i S^z_j + V n_i n_j + W \left(n_i S^z_j + S^z_i n_j \right) \right]$$



t: nearest-neighbor hopping
V: Coulomb-repulsion (long-range)
W: density-spin-interaction (long-ranged)
J: Heisenberg coupling (anisotropic, long-ranged)

➥ generalized t-J model with dipolar long-range interactions

Example Optical Lattices: Polar Molecules

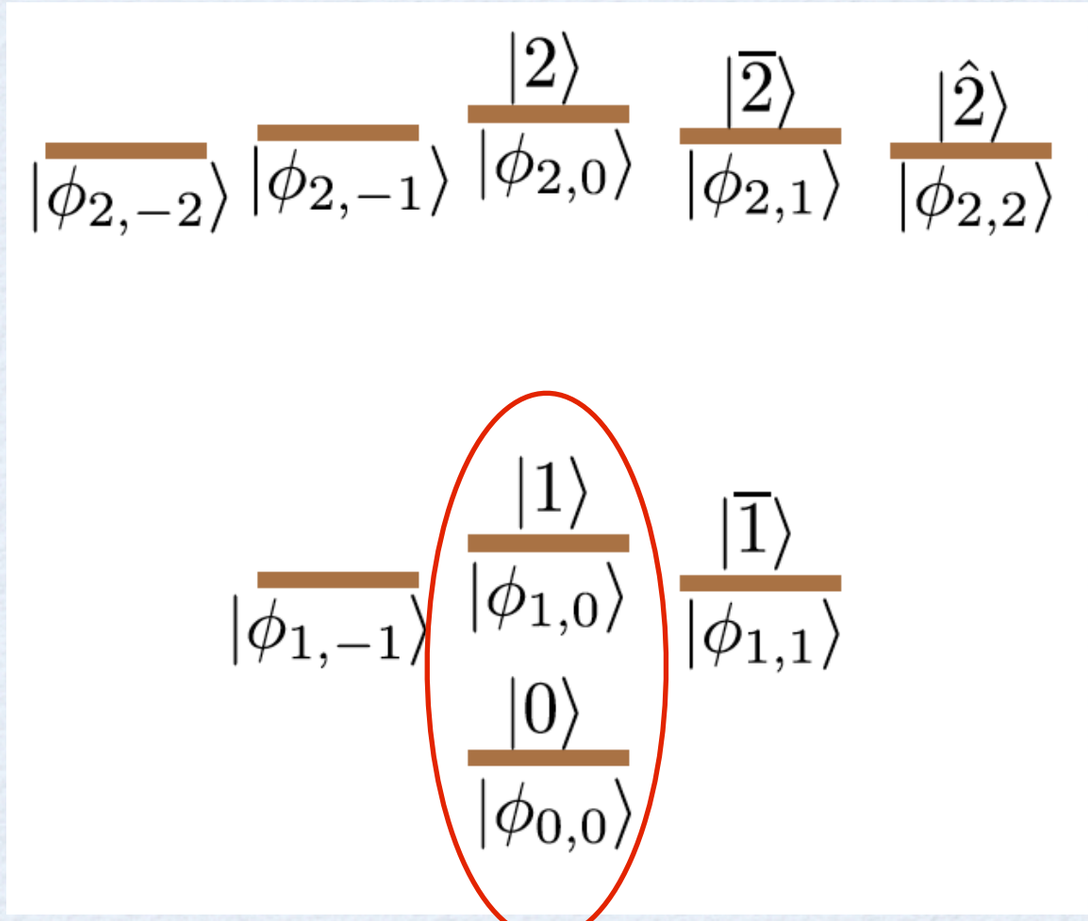
[A.V. Gorshkov, S.R. Manmana et al., PRL & PRA (2011)]

2 basic observations:

- polar molecules are rigid rotors, e.g., in electric field: $H_0 = B\mathbf{N}^2 - d_0\vec{E}$

- dipolar, long-ranged interactions: $H_{\rm dd} = \frac{1}{2}\sum_{i\neq j}|\mathbf{R}_i - \mathbf{R}_j|^{-3}[d_0^{(i)}d_0^{(j)} + \frac{1}{2}(d_+^{(i)}d_-^{(j)} + d_-^{(i)}d_+^{(j)})]$

level scheme for a rigid rotor in a field:

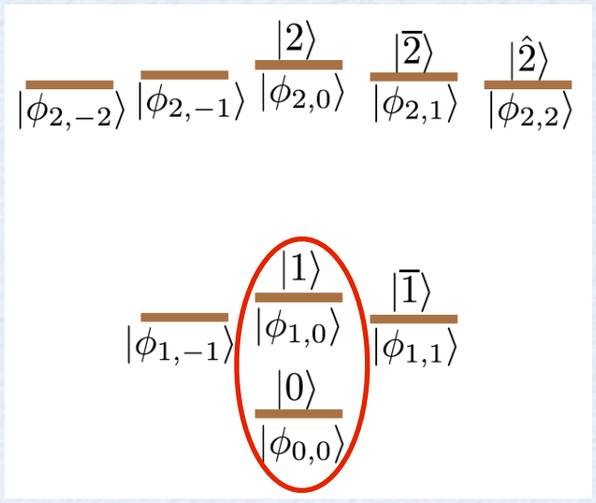


Idea: project dipolar operator onto two states ➠ effective S=1/2 system

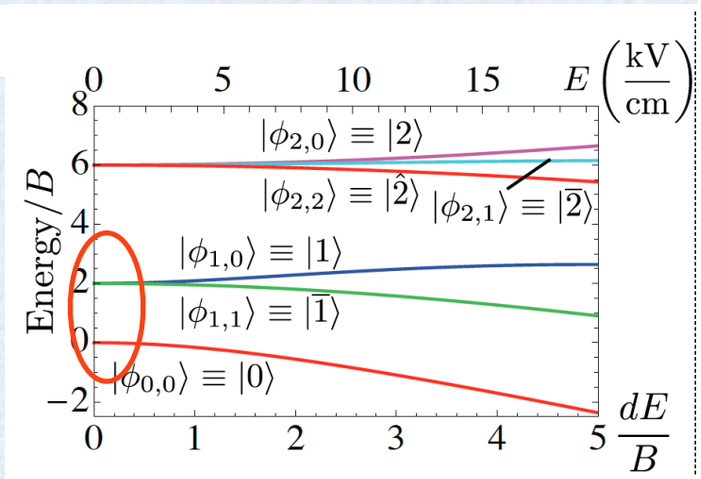
Polar molecules on optical lattices: effective models

[A.V. Gorshkov, S.R. Manmana et al., PRL & PRA (2011)]

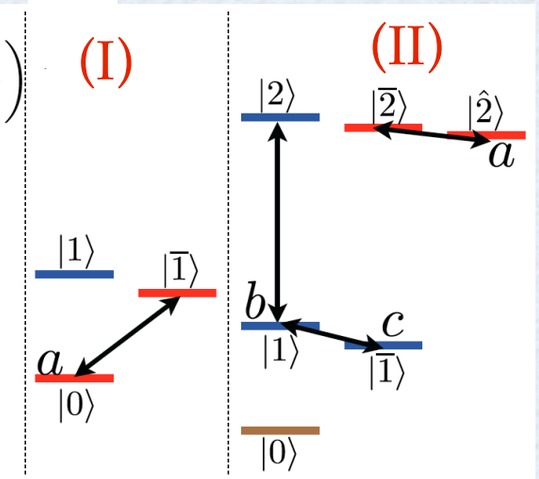
level scheme for a rigid rotor in a field:



energies in electric field:



microwaves ➠ dressed states:



More general: project dipolar operator onto two *dressed states* ➠ tunable parameters

useful choice of coefficients: (depend on details of dressed states $\{|m_0\rangle, |m_1\rangle\}$)

J_z	$=$	$[\langle m_0\|d\|m_0\rangle - \langle m_1\|d\|m_1\rangle]^2$	"Ising"
$J_\perp$	$=$	$2\langle m_1\|d\|m_0\rangle^2$	"spin flip"
V	$=$	$\frac{1}{4}[\langle m_0\|d\|m_0\rangle + \langle m_1\|d\|m_1\rangle]^2$	"density interaction"
W	$=$	$\frac{1}{2}[\langle m_0\|d\|m_0\rangle^2 - \langle m_1\|d\|m_1\rangle^2]$	"anisotropic interaction"

(I): Simplest case, leads to $J_z = V = W = 0$ ➠ This talk

(II): Arbitrary ratio between all coefficients ➠ Future research

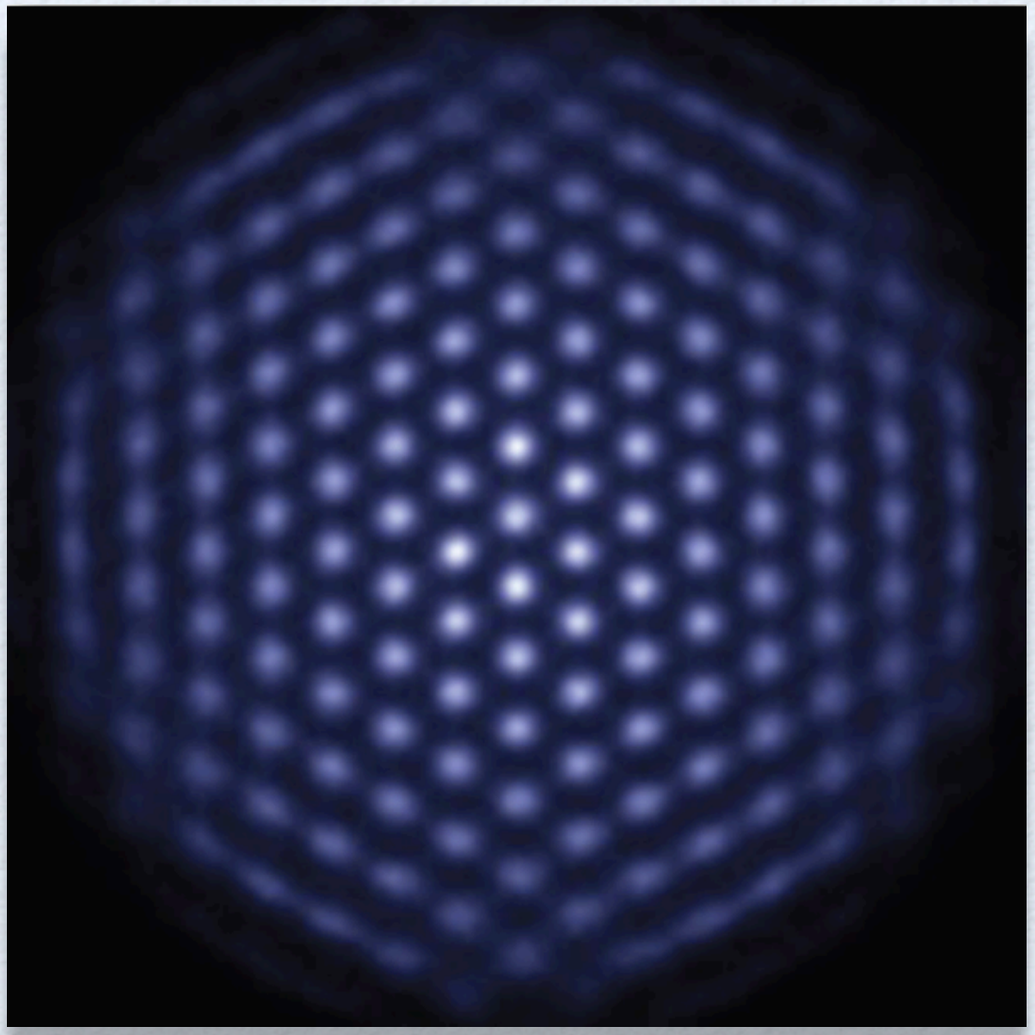
(III): Beyond S=1/2, spatial anisotropies,topological order:

S.R.M. et al., PRB (rapid comm.) **87,** 081106(R) (2013); A.V. Gorshkov, K. Hazzard & A.M. Rey, arXiv:1301.5636 (2013)

Example Ultracold Gases: Ions in a Trap

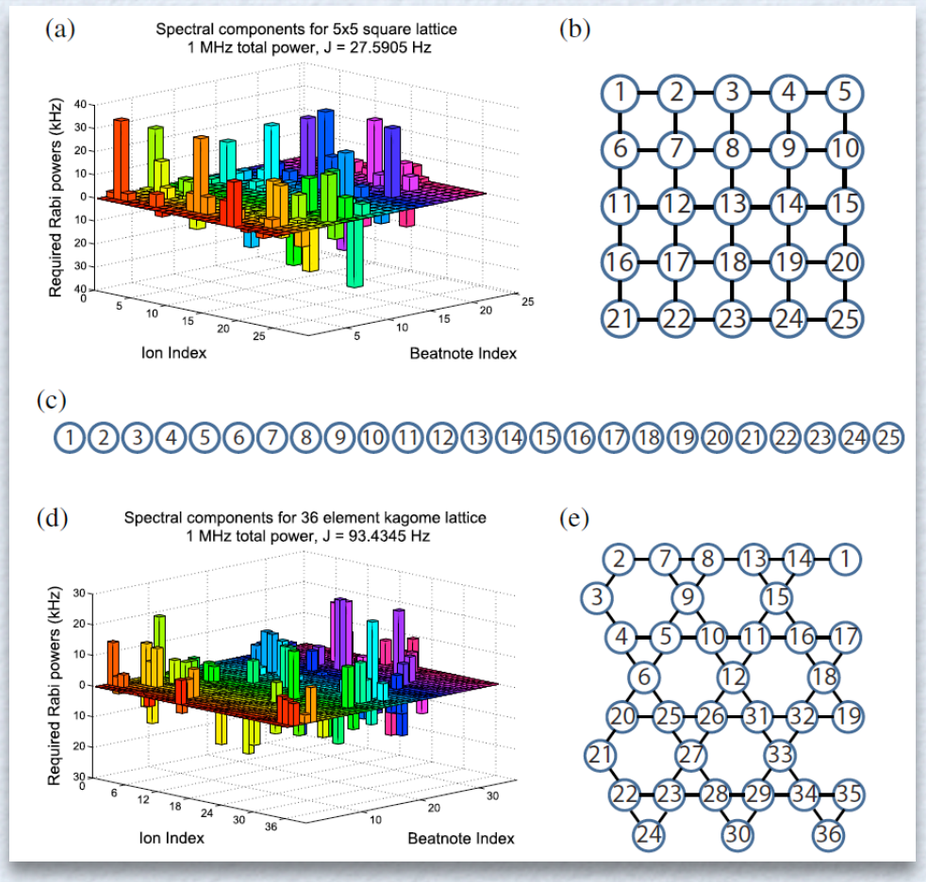
$^{9}Be^{+}$ ions in a Penning trap (NIST Boulder)

[J.W. Britton et al., Nature **484**, 489 (2012)]



$^{171}Yb^{+}$ ions (JQI/NIST Maryland)

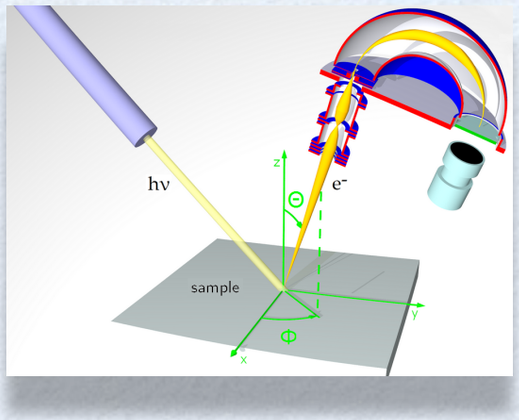
[K. Kim et al., Nature **465**, 590 (2010);
R. Islam et al., Nature Comm. **2**,377 (2011);
NJP and more...]



Realization of Ising models with transverse field on variety of lattices:

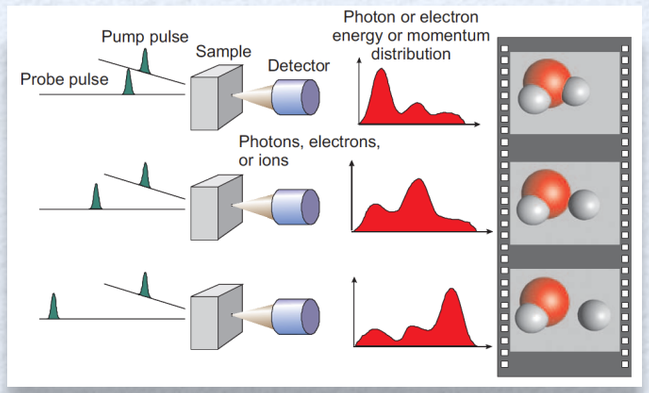
Interactions ~ $1/r^{\alpha}$

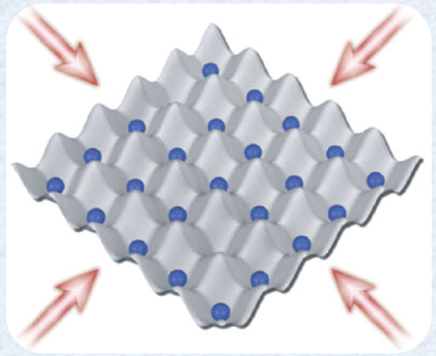
Numerical Methods for Many-Body Systems: Challenges



I) Dynamical spectral functions:

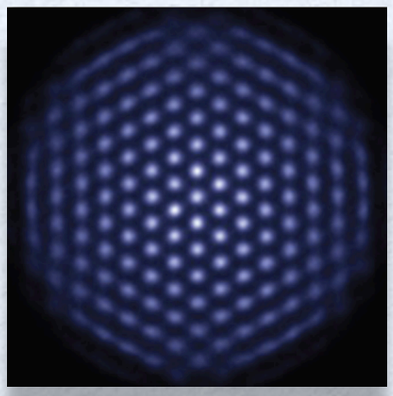
resolution, finite temperatures

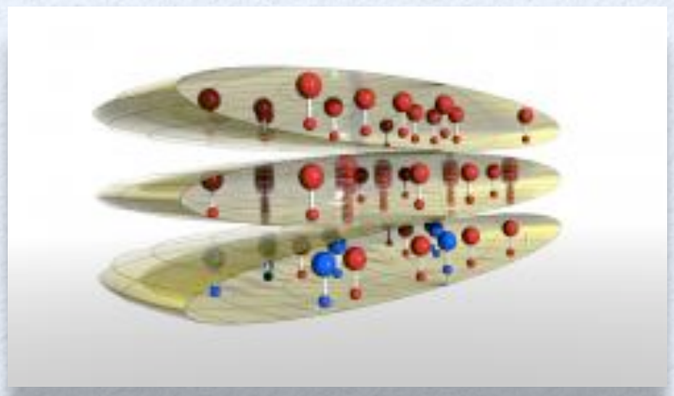




II) 'Highly excited systems':

long times, time evolution at finite temperatures





III) Recent development quantum simulators:

long-range interactions

Further important challenges: D>1, dissipation, infinite system size, ...

"Numerically Exact Dynamics": Exact Diagonalization

Direct approach:

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \Rightarrow |\psi(t)\rangle = \sum_n \langle n|\psi_0\rangle e^{-iE_n t/\hbar}|n\rangle$$

No limitations:

- arbitrary long times
- accuracy (machine precision)
- arbitrary geometry
- independent on details of system or initial state

Bad:

➠ Need the full spectrum...difficult ☹

"Numerically Exact Dynamics": Iterative Diagonalization

Lanczos procedure:
(Krylov space method)
K. Lánczos (1950)

$$|v_{n+1}\rangle = \mathcal{H}\,|v_n\rangle - a_n\,|v_n\rangle - b_n^2\,|v_{n-1}\rangle$$

$$a_n = \frac{\langle v_n|\,\mathcal{H}\,|v_n\rangle}{\langle v_n|v_n\rangle}\,, \qquad b_{n+1}^2 = \frac{\langle v_{n+1}|v_{n+1}\rangle}{\langle v_n|v_n\rangle}\,, \qquad b_0 = 0$$

Tridiagonalization of Hamiltonian matrix:

$$\mathbf{T}_n = \begin{pmatrix} a_0 & b_1 & & \\ b_1 & a_1 & b_2 & \mathbf{0} \\ & b_2 & a_2 & \ddots \\ & \mathbf{0} & \ddots & \ddots & b_n \\ & & & b_n & a_n \end{pmatrix}$$

Projection of time evolution operator:
T.J. Park and J.C. Light, J. Chem. Phys (1986)

$$e^{-i\Delta\tau/\hbar\,\hat{H}}\,|\psi(\tau)\rangle \approx \mathbf{V}_n(\tau)\,e^{-i\Delta\tau/\hbar\,\mathbf{T}_n(\tau)}\,\mathbf{V}_n^{+}(\tau)\,|\psi(\tau)\rangle$$

Error estimate:
M. Hochbruck and C. Lubich, SIAM (1997)

$$\begin{aligned}\varepsilon_n &:= ||\hat{U}|\psi\rangle - \hat{U}_{\text{approx}}|\psi\rangle|| \\ &\leq 12\,\exp\left\{-\frac{(w\,\Delta\tau)^2}{n}\right\}\left(\frac{e\,w\,\Delta\tau}{n}\right)^n,\ n \geq 2\,w\,\Delta\tau\end{aligned}$$

Usually n<20 is sufficient

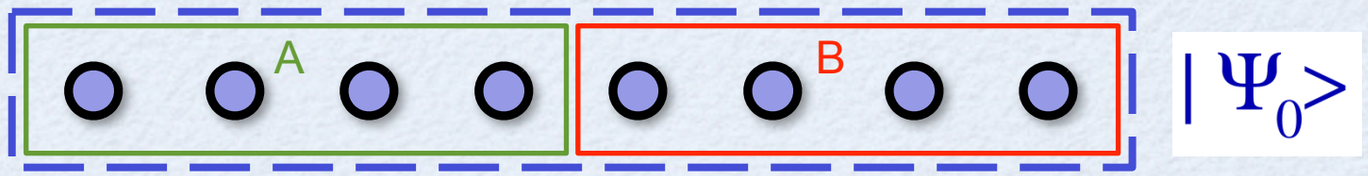
Larger systems possible

Pro's/Con's similar to 'full diagonalization'

➠ Need to store n vectors with dimension of H ☹

"Numerically Exact Dynamics": The DMRG

S.R. White, PRL (1992); U. Schollwöck, RMP (2005)/Ann. Phys. (2011); R.M. Noack & S.R. Manmana, AIP (2005)



- Obtain ground state of finite, small lattice (e.g., using Lanczos)
- Reduced density matrix of subsystem ("system block")

➠ *Schmidt decomposition* (1907)

$$|\psi\rangle = \sum_{j=1}^{\dim\mathcal{H}} w_j \, |\alpha\rangle_j \, |\beta\rangle_j \approx \sum_{j=1}^{m} w_j \, |\alpha\rangle_j \, |\beta\rangle_j$$

Approximation: $m \ll \dim\mathcal{H}$

$|\alpha\rangle_j$, $|\beta\rangle_j$: Eigenstates of reduced density matrices of A or B
typically (1D) m ~ 1000, error (discarded weight) ~ 10^{-9}

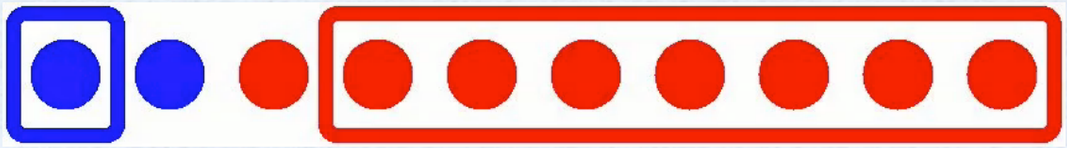
➥ central quantity: entanglement entropy $S = -\sum_j w_j^2 \log w_j^2$

The larger the entanglement, the larger m for a desired accuracy.

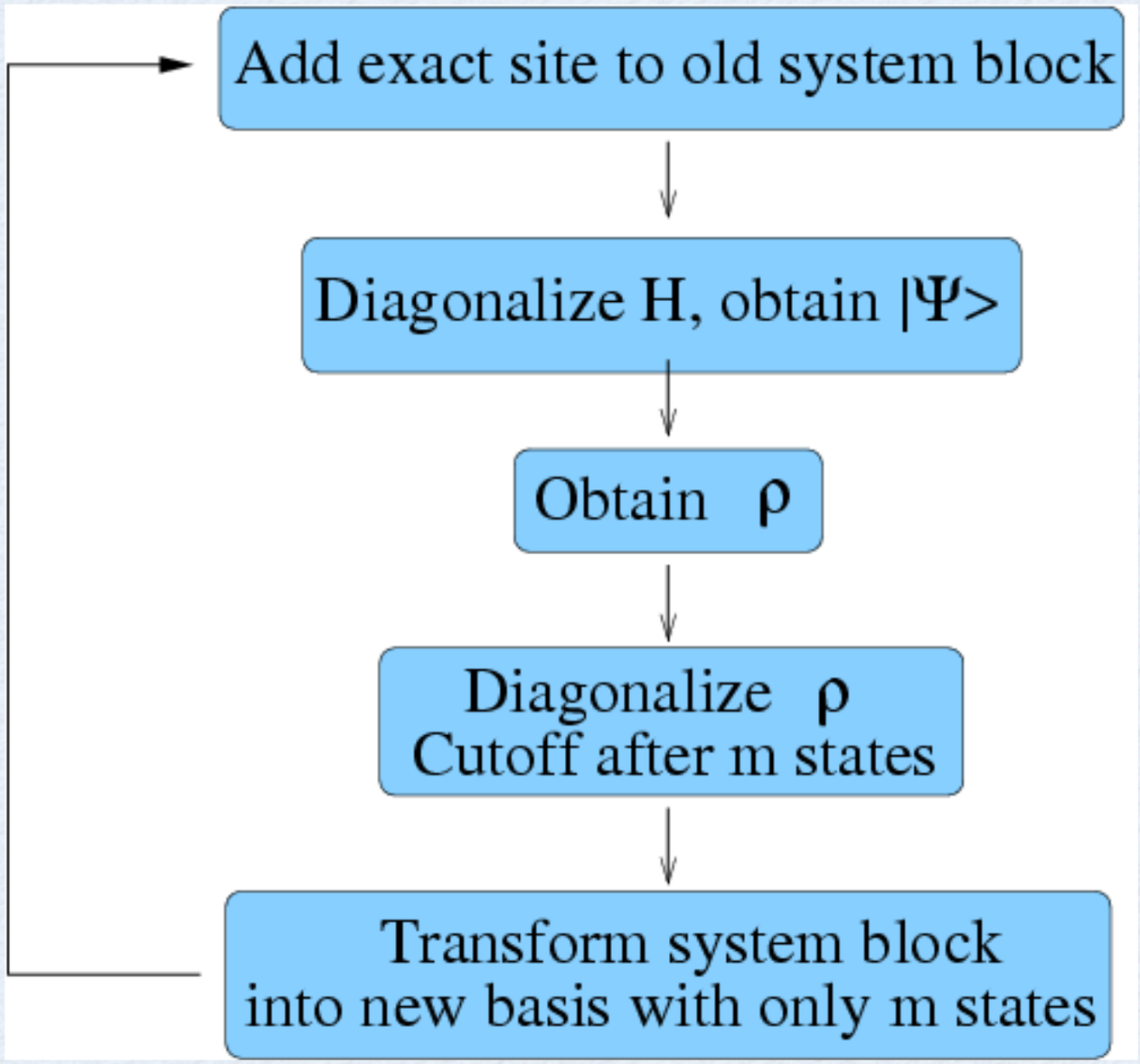
- Problematic for D > 1 ('area law of entanglement')
- Entanglement grows with time - inhibits (very) long times

"Numerically Exact Dynamics": The DMRG

Iterative Procedure:



[Webpage E. Jeckelmann]



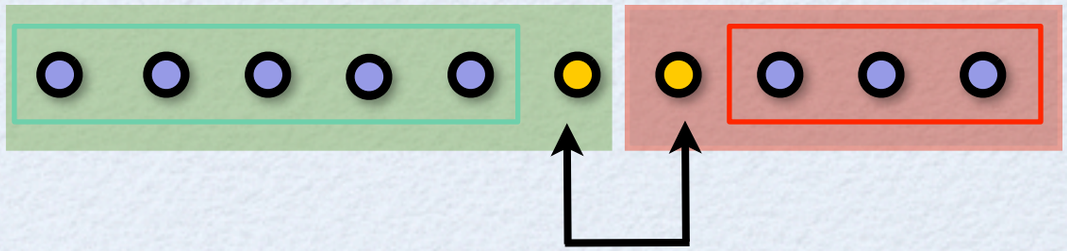
"Numerically Exact Dynamics": The adaptive t-DMRG

Basic idea:

– Approximate time evolution operator

• Suzuki-Trotter decomposition [Vidal (2003/2004); S.R. White & A. Feiguin (2004); A. Daley *et al.* (2004)]

$$e^{-i\,dt\,\hat{H}/\hbar} = \prod_{i\,\mathrm{odd}} e^{-i\,dt\,\hat{H}_i/\hbar} \prod_{i\,\mathrm{even}} e^{-i\,dt\,\hat{H}_i/\hbar} + \mathcal{O}(dt^2)$$

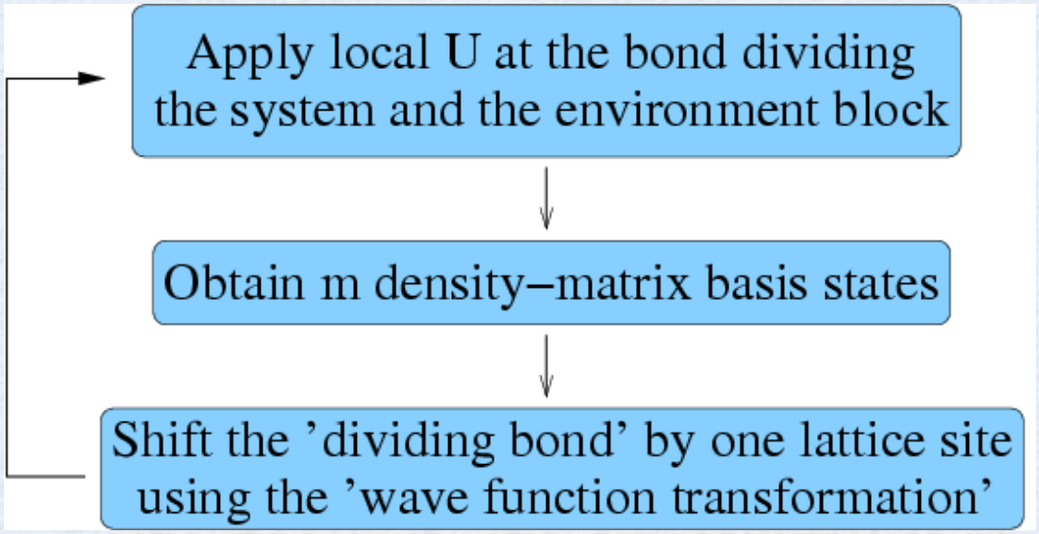


• Lanczos projection [P. Schmitteckert (2004); S.R. Manmana *et al.* (2005)]

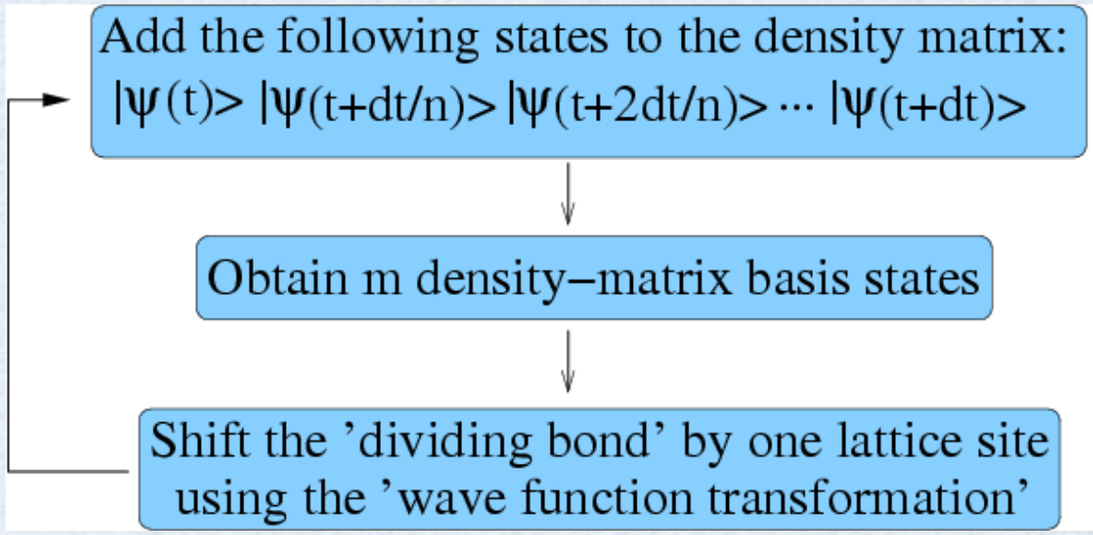
$$e^{-i\Delta\tau/\hbar\,\hat{H}}\,|\psi(\tau)\rangle \approx \mathbf{V}_n(\tau)\; e^{-i\Delta\tau/\hbar\,\mathbf{T}_n(\tau)}\; \mathbf{V}_n^{+}(\tau)\;|\psi(\tau)\rangle$$

– Adapt basis of density-matrix eigenvectors at each time step

Trotter approach (n.n. interactions):



Lanczos approach (arbitrary geometry)



"Numerically Exact Dynamics": Matrix Product States

U. Schollwöck, Ann. Phys. (2011)

Matrix product state (MPS) representation of wave functions:

$$|\psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L} c_{\sigma_1\ldots\sigma_L} |\sigma_1,\ldots,\sigma_L\rangle$$

$$|\psi\rangle = \sum_{\sigma_1,\ldots,\sigma_L} \sum_{a_0,\ldots,a_L} M^{[1]\sigma_1}_{a_0,a_1} \cdots M^{[L]\sigma_L}_{a_{L-1},a_L} |\sigma_1,\ldots,\sigma_L\rangle$$

local complex-valued matrix

➠ underlying structure of the wave function in the DMRG

Convergence: optimize M-matrices via variational principle

Matrix product operator (MPO) representation of operators:

$$\hat{O} = \sum_{\substack{\sigma_1,\ldots,\sigma_L \\ \sigma'_1,\ldots,\sigma'_L}} W^{\sigma_1\sigma'_1} \ldots W^{\sigma_L\sigma'_L} |\sigma_1,\ldots,\sigma_L\rangle\langle\sigma'_1,\ldots,\sigma'_L|$$

Linear Response Dynamics at $T>0$

Linear Response: Dynamical correlation functions

- time-dependent perturbation

$$\mathcal{H}(t) = \mathcal{H}_0 - h_A \, \mathrm{e}^{i\,\omega\,t}\, A$$

- linear response:

$$\frac{\mathrm{d}}{\mathrm{d}h_A} \int\limits_{-\infty}^{\infty} \mathrm{d}t \, \langle B(t) \rangle \Big|_{h_A=0} = \int\limits_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{i\,\omega\,t} \, \langle \mathcal{T}\, B(t)\, A \rangle_0 = \int\limits_{-\infty}^{\infty} \mathrm{d}t \sum_n \langle \Psi_0 | B | n \rangle \, \langle n | A | \Psi_0 \rangle \, \mathrm{e}^{i\,t\,(\omega - (E_n - E_0))}$$

$$= 2\pi \sum_n \langle \Psi_0 | B | n \rangle \, \langle n | A | \Psi_0 \rangle \, \delta(\omega - (E_n - E_0))$$

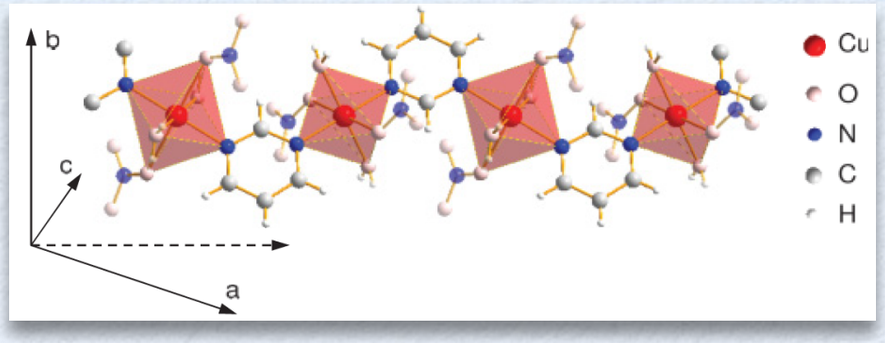
with $\mathcal{H}_0 \, |n\rangle = E_n \, |n\rangle$

- express via Green's functions

$$C_{A^\dagger,A}(\omega) = \mathrm{Im} G_A(\omega + i\,\eta + E_0)\,, \quad G_A(z) = \langle \Psi_0 |\, A^\dagger \, (z - \mathcal{H})^{-1} \, A \, |\Psi_0\rangle$$

Dynamical properties of quantum magnets: ESR on Cu-PM in magnetic fields

Copper pyrimidine dinatrate:



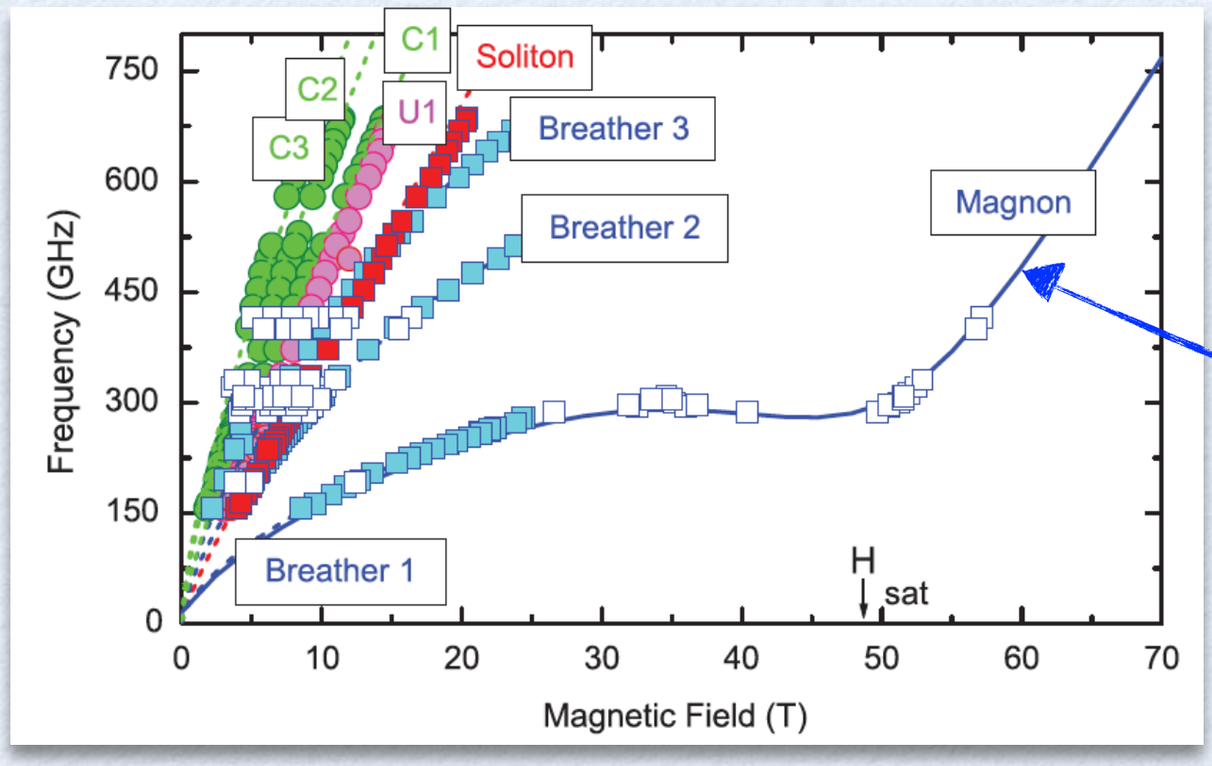
[S. Zvyagin et al., PRB(R) (2011)]

(Quasi-)1D Heisenberg AFM, described by

$$\mathcal{H} = \sum_j \left[J\mathbf{S}_j \cdot \mathbf{S}_{j+1} - HS_j^z - h\,(-1)^j\, S_j^x \right]$$

effect of staggered g-tensor + DM interaction

ESR spectrum in magnetic field:

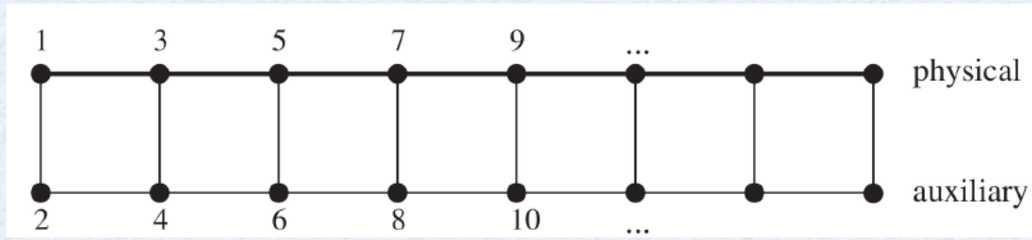


DMRG results

Finite temperature methods: purification with matrix product states

☞ Compute thermal density matrix via a pure state in an extended system:

[U. Schollwöck, Annals of Physics (2011)]



formal replacement
$H \to H \otimes \mathbb{1}_Q$

$$|\Psi_T\rangle = e^{-(H_P \otimes I_Q)/(2T)} \left[\otimes_{j=1}^{L} |\mathrm{rung-singlet}\rangle_j \right]$$

$$\Rightarrow \varrho_T = e^{-H/T} = \mathrm{Tr_Q} |\Psi_T\rangle \langle \Psi_T|$$

☞ Real time evolution at finite temperature:

$$|\Psi_T\rangle (t) = e^{-i(H_P \otimes U_Q)t} |\Psi_T\rangle \Rightarrow G_A(T,t) \overset{\text{Fourier}}{\Rightarrow} G_A(T,\omega)$$

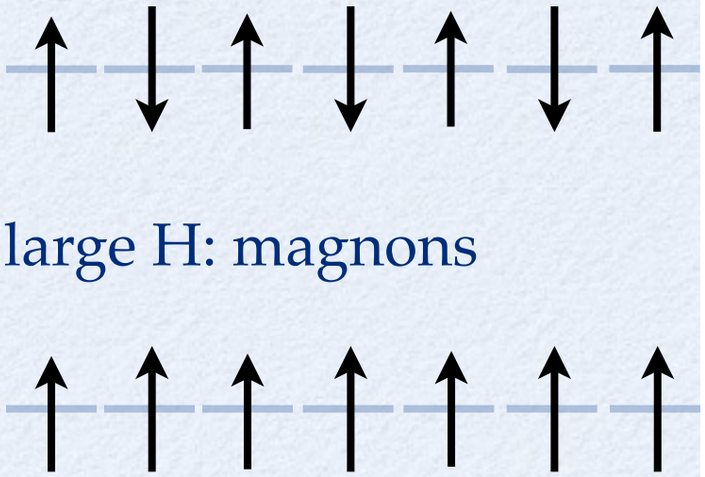
- Problem: reach long times for large systems
- Ways out: linear prediction, backward time evolution in Q

[T. Barthel, U. Schollwöck & S.R. White, PRB (2009); C. Karrasch, J.H. Bardarson & J.E. Moore, PRL (2012)]

Spectral functions at finite field

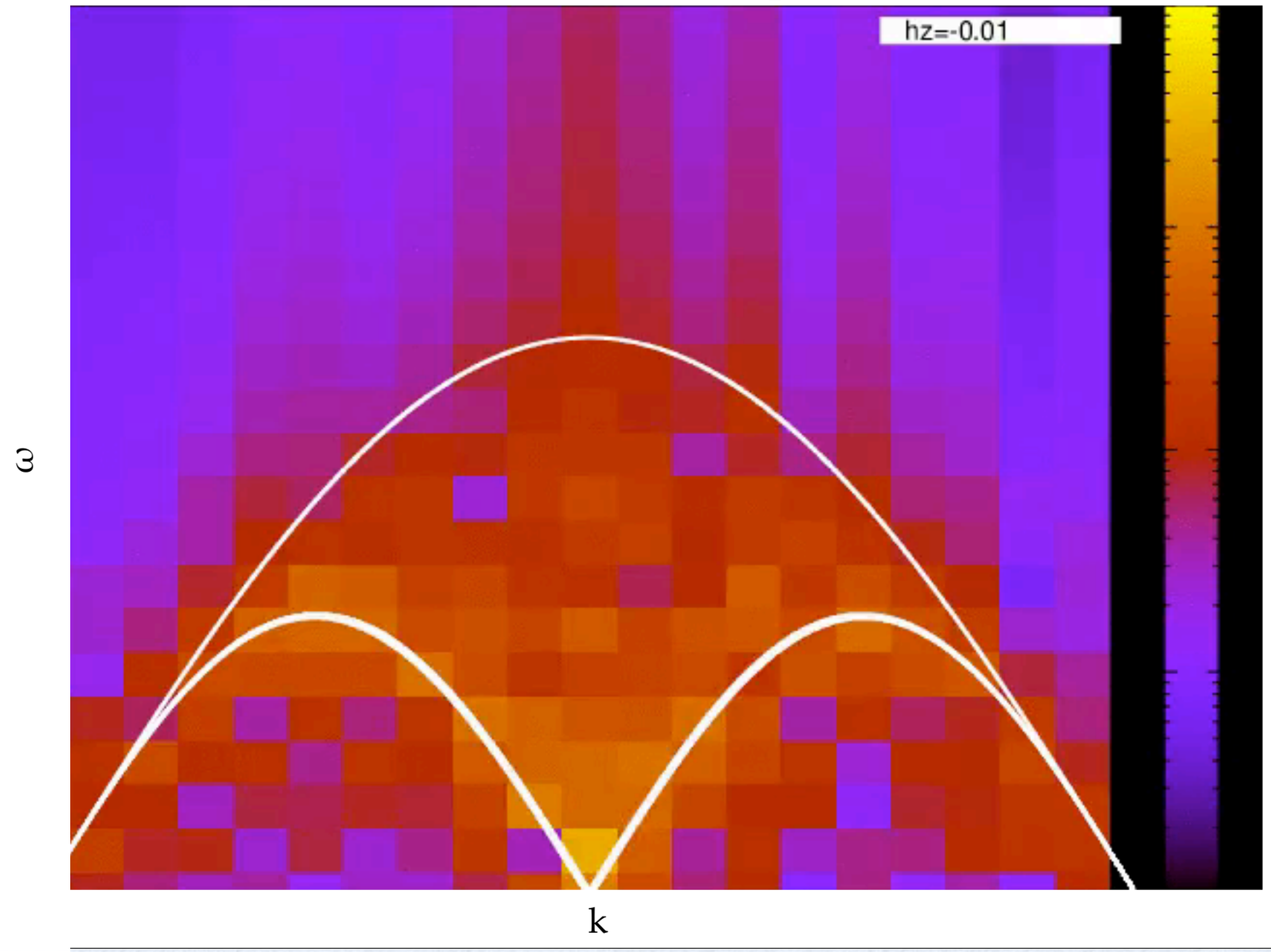
Finite-T dynamics
in strong magnetic fields:

small H: spinons



large H: magnons

Time evolution at finite T + Fourier transform
(non-optimized code, no linear prediction)



[T. Köhler, Master thesis, Univ. Göttingen 2013]

Dynamical correlation functions

Dynamical correlation functions at T = 0:

$$G_A(\omega) = -\tfrac{1}{\pi}\mathrm{Im}\left\langle \psi_0 \left| A^\dagger \tfrac{1}{\omega+E_0+i\varepsilon-H} A \right| \psi_0 \right\rangle = \sum_n \left|\langle n|A|\psi_0\rangle\right|^2 \delta\left(\omega-(E_n-E_0)\right)$$

$$\mathcal{H}_0 |n\rangle = E_n |n\rangle$$

Dynamical correlation functions at T > 0:

$$G_A(\omega,T) = \frac{1}{Z}\sum_{n,m} e^{-\beta E_m}\langle m|A|n\rangle\langle n|A|m\rangle\delta(\omega-(E_n-E_m))$$

➠ Need the full spectrum...difficult ☹

Way out: continued fraction expansion

Dynamical correlation functions: Lanczos recursion

[E. Dagotto, RMP (1994)]

☞ use continued fraction expansion (CFE)

$$G_A(z) = -\frac{1}{\pi}\mathrm{Im}\left\langle \psi_0 \left| A^\dagger \frac{1}{z-H} A \right| \psi_0 \right\rangle = -\frac{1}{\pi}\mathrm{Im}\,\frac{\langle \Psi_0 |\, A^\dagger\, A\, | \Psi_0 \rangle}{z-a_0-\dfrac{b_1^2}{z-a_1-\dfrac{b_2^2}{z-..}}}$$

via Lanczos recursion

Note: $|\Psi_T\rangle$ is a vector in the Liouville space spanned by $\mathcal{H}_P \otimes \mathcal{H}_Q$

➠ Dynamics is actually governed by Liouville equation

$$\frac{\partial}{\partial t}|\Psi_T\rangle = -i\mathcal{L}|\Psi_T\rangle, \qquad \mathcal{L} = \mathcal{H}_P \otimes I_Q - I_P \otimes H_Q$$

(backward evolution in Q by Karrasch et al.)

☞ At finite temperatures:

$$G_A(k,\omega) = -\frac{1}{\pi}\mathrm{Im}\left\langle \Psi_T \left| A^\dagger \frac{1}{z-\mathcal{L}} A \right| \Psi_T \right\rangle$$

+ evaluation via CFE, correction vector, etc...

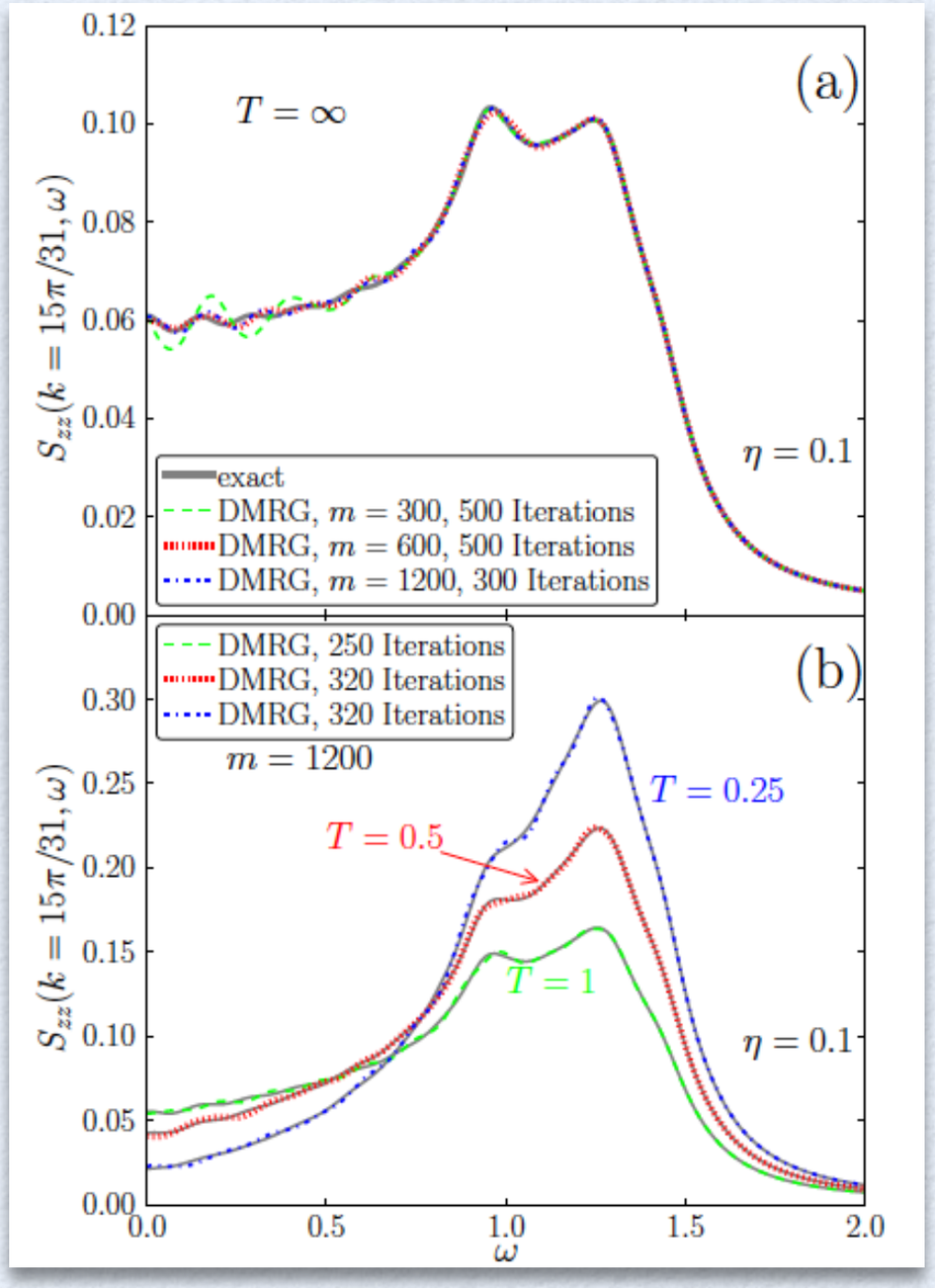
[A.C. Tiegel et al., arXiv:1312.6044]

Liouvillian finite-T approach: comparison to exact results

$$H_{XX} = J \sum_{i}^{L-1} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y \right)$$

$$S_k^\alpha = \sqrt{\frac{2}{L+1}} \sum_{i=1}^{L} \sin(ki)\, S_i^\alpha$$

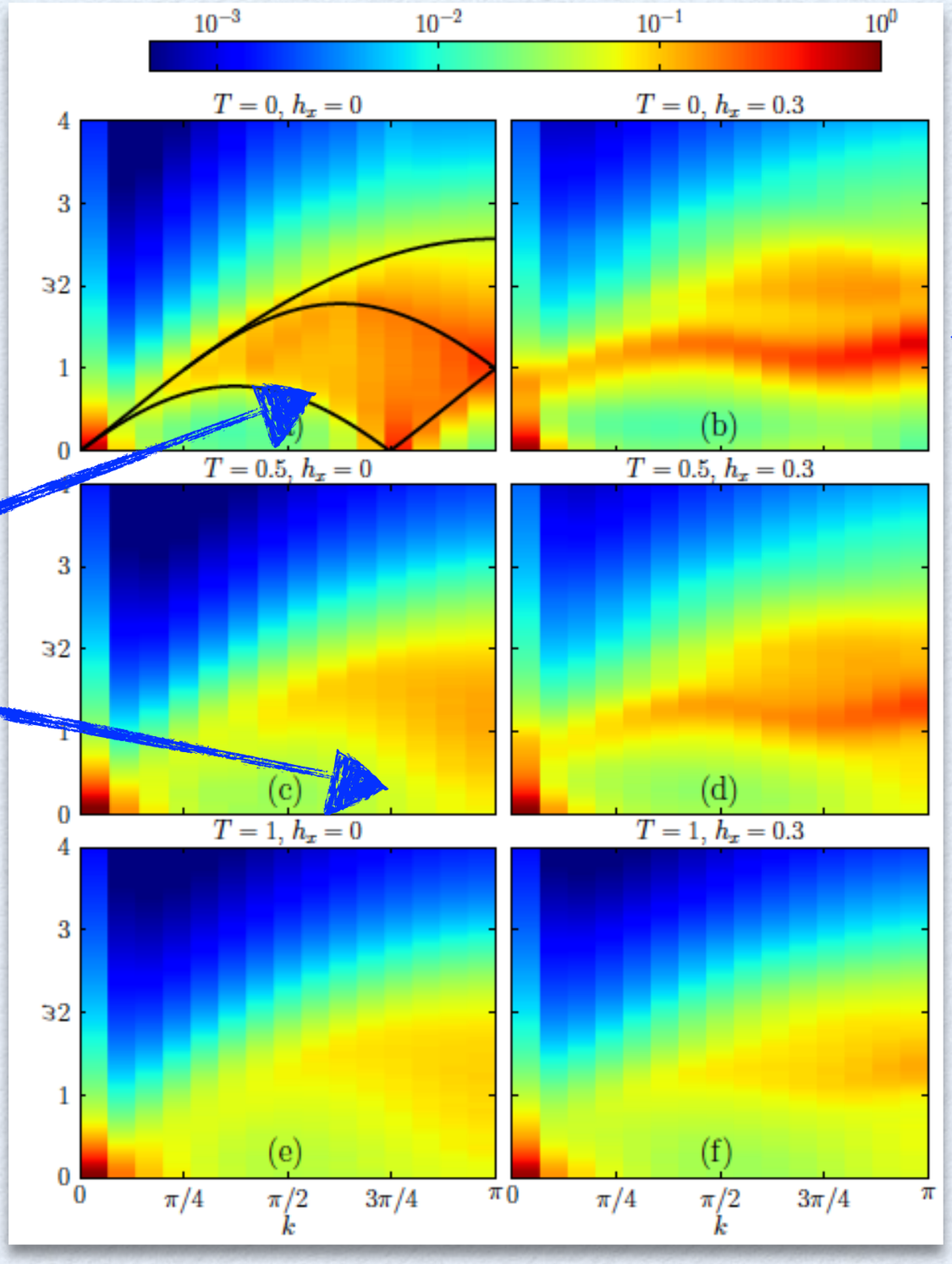
Excellent agreement with exact results!



Liouvillian finite-T approach: Heisenberg antiferromagnet in magnetic field

no DM

with DM



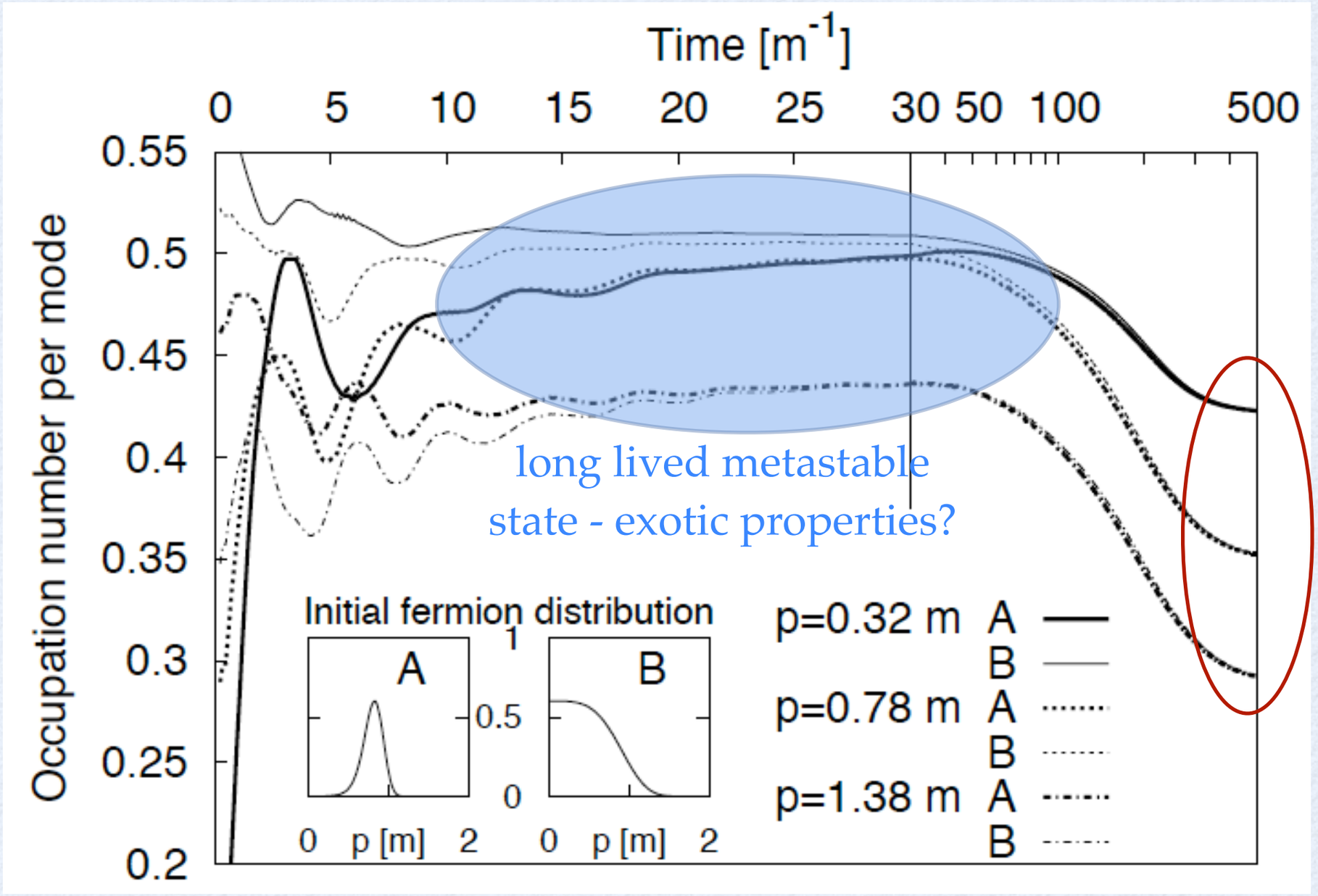
Formation of bands

Melting of a Luttinger liquid

Relaxation Behavior of Isolated Systems

Unconventional states: Out-of-Equilibrium Dynamics

High-energy physics: "Prethermalisation" in heavy ion collisions

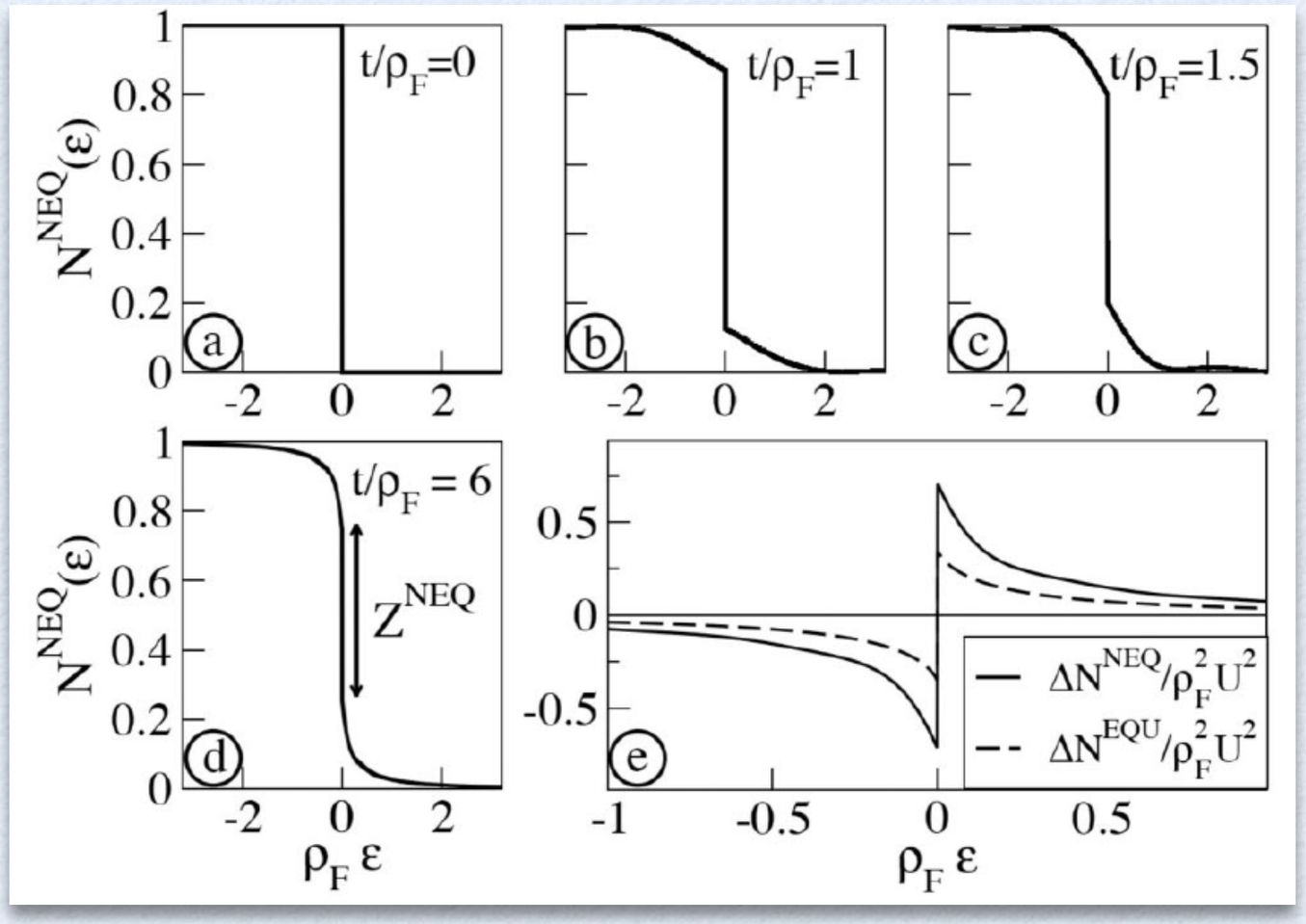


thermal final state

[Berges *et al.*, PRL 2004]

Out-of-Equilibrium Dynamics: prethermalization in Hubbardmodel

M. Moeckel & S. Kehrein, PRL (2008):



At weak interactions (U << 1, not 1D):

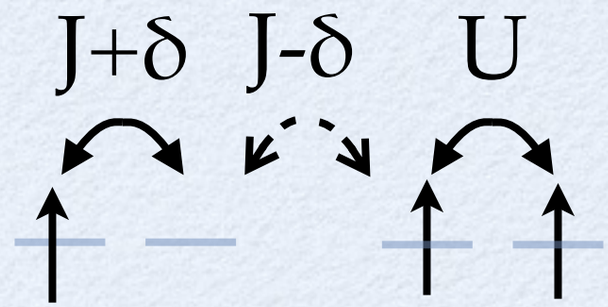
- Time scale $\rho^{-1}\, U^{-2} \lesssim t \lesssim \rho^{-3}\, U^{-4}$: metastable prethermalized state with "wrong" quasiparticle residue
  ➥ agrees with scenario from particle physics
- Larger times: "wrong" quasiparticle residue allows for scattering:
  Boltzmann dynamics & thermalization

Controlled breaking of integrability: Dynamics of a 1D dimerised state

Convenient model:

"Spinless fermions with dimerisation" on a chain (1D):

$$H(\delta, U) = -J \sum_{l=1}^{L} \left[1 + (-1)^l \delta\right] \left(c_l^\dagger c_{l+1} + h.c.\right) + U \sum_{l=1}^{L} n_l n_{l+1}$$

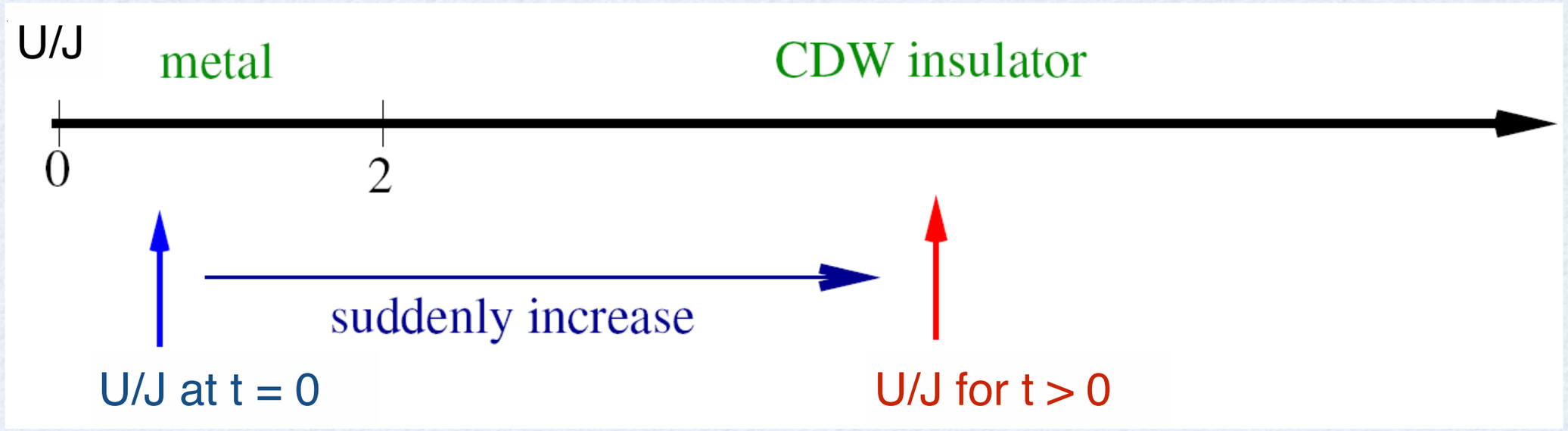


- U = 0: free fermions, exactly solvable and integrable — should not thermalise
- δ = 0: exactly solvable and integrable via Bethe ansatz — should not thermalise
- U,δ ≠ 0: no exact solution, non-integrable — should thermalise

⇒ Control integrability breaking and look for qualitative differences at long times

Out-of-Equilibrium Dynamics: Relaxation

δ=0: integrable using Bethe Ansatz, phase diagram:



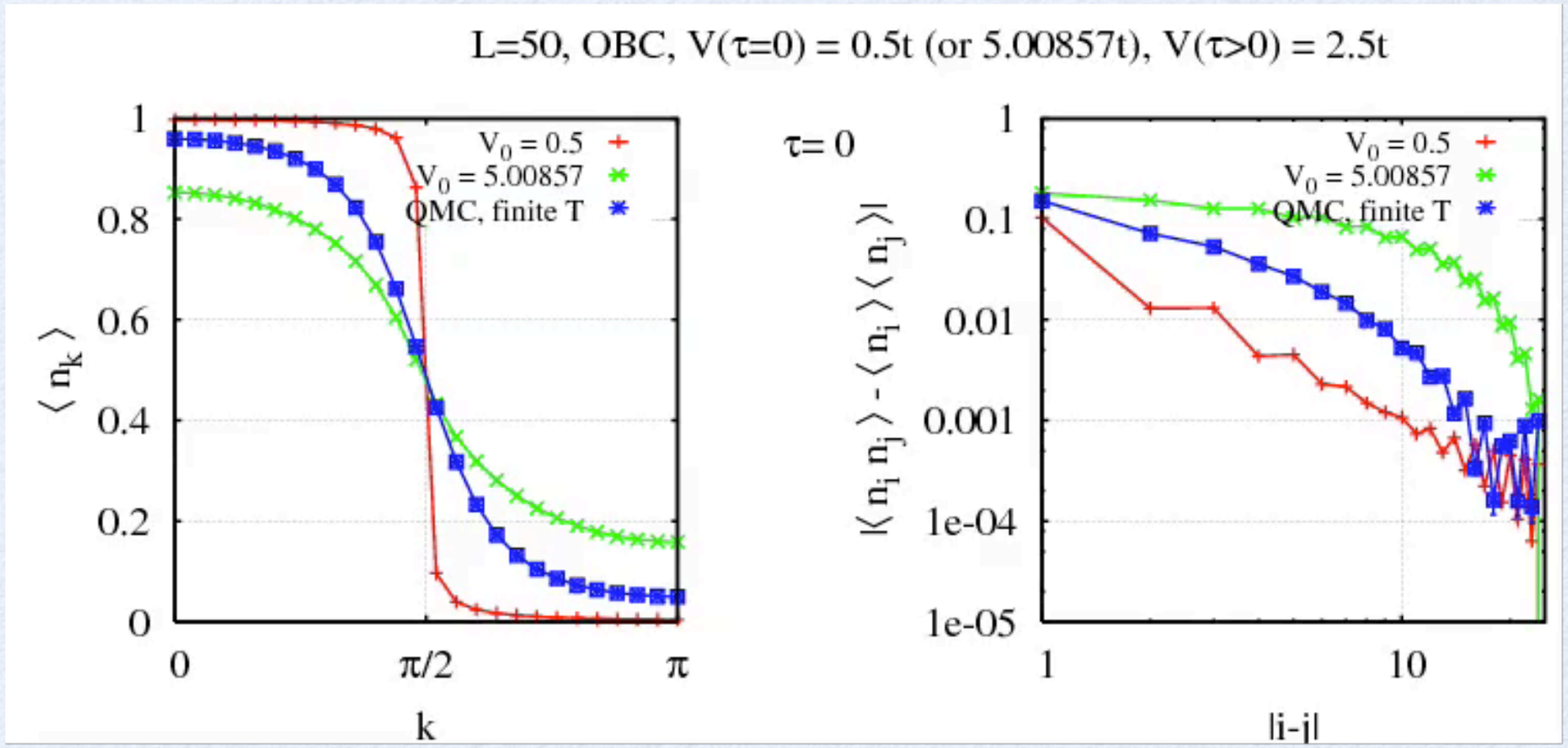
Rich behaviour:

- Relaxation behavior of the momentum distribution function: non-thermalization?
  [S.R. Manmana, S. Wessel, R.M. Noack, and A. Muramatsu, PRL **98**, 210405 (2007)]
- Density correlations: 'horizon-effect' vs. creation of domain walls
  [S.R. Manmana, S. Wessel, R.M. Noack, and A. Muramatsu, PRB **79**, 155104 (2009)]

Out-of-Equilibrium Dynamics: Relaxation

[S.R. Manmana et al., PRL (2007)]

Momentum distribution function and density correlations:



Integrable system

U=0.5 to 2.5

U=5.01 to U = 2.5

Blue line: finite-T QMC results

- Two completely different initial states ‘relax’ to a similar state
- Relaxation to a thermal state?
  - ➠ Controlled breaking of integrability in dimerized system

Quench in dimerised state

CUT: approximately conserved quasi-particles described by U-dependent "number operators" (U<<1):

$$\mathcal{Q}_\alpha(k) = a^\dagger_\alpha(k)a_\alpha(k) - U\sum_{q_j>0} N^\gamma_{\alpha\alpha}(\boldsymbol{q}|k,k,B=\infty) \times a^\dagger_{\gamma_1}(q_1)a_{\gamma_2}(q_2)a^\dagger_{\gamma_3}(q_3)a_{\gamma_4}(q_4).$$

$$[\mathcal{Q}_\alpha(k),\mathcal{Q}_\beta(p)] = \mathcal{O}(U^2)$$

$$[\mathcal{Q}_\alpha(k),H(\delta_f,U)] = \mathcal{O}(U)$$

➠ "deformed" generalized Gibbs ensemble

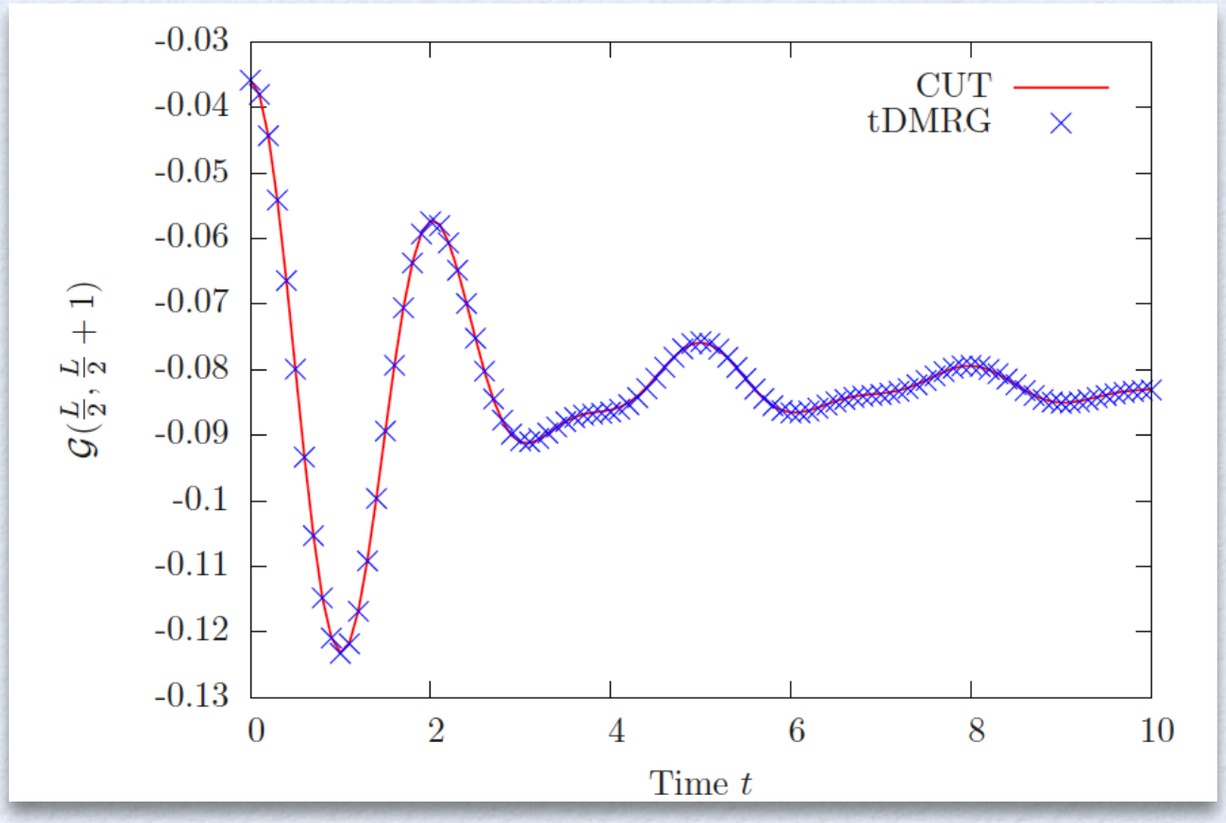
$$\varrho_{\mathrm{PT}} = \frac{1}{Z_{\mathrm{PT}}}\exp\left(\sum_{k,\alpha}\lambda^{(\alpha)}_k\mathcal{Q}_\alpha(k)\right)$$

Expectations:

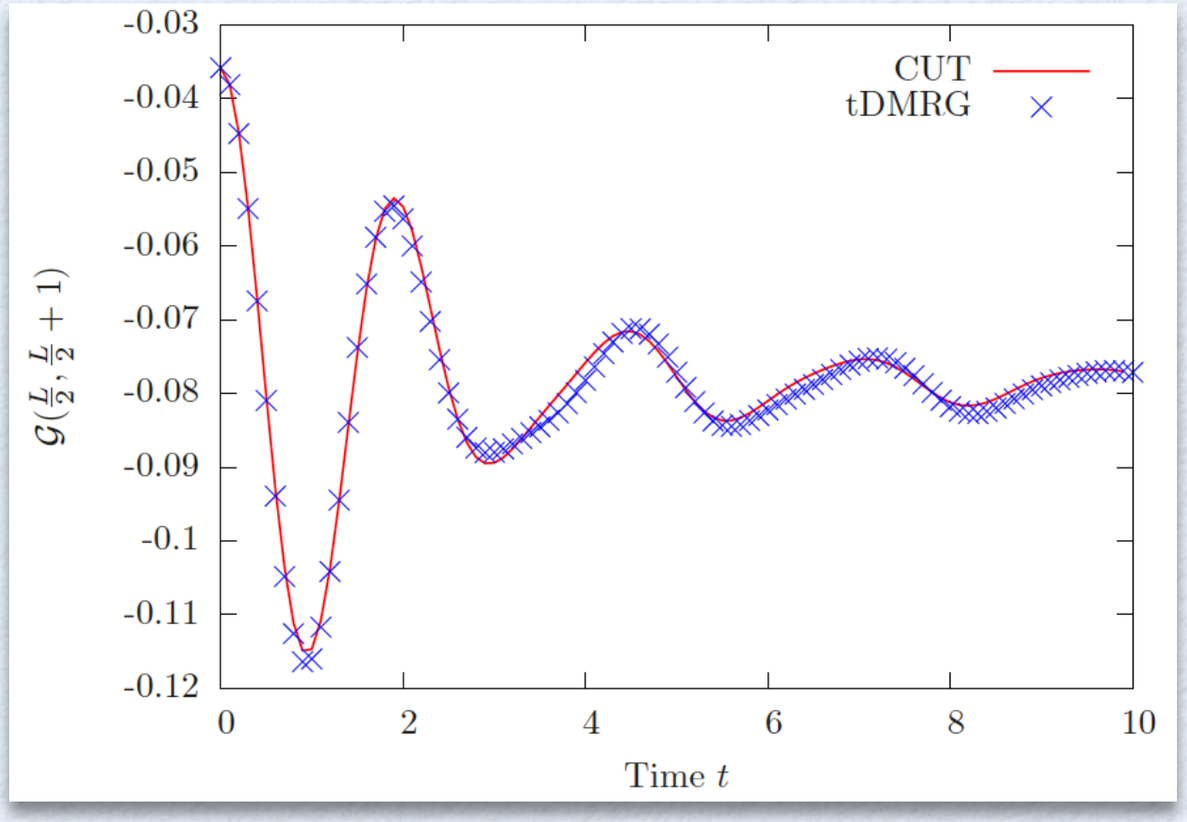
- Metastable state described by this ensemble during life time of quasi-particles
- Scattering between quasi-particles leads to thermalization, as in D>1 Hubbard model

Quench in dimerised state

$$\delta = 0.75 \to \delta = 0.5,$$
$$U = 0 \to U = 0.15$$



$$\delta = 0.75 \to \delta = 0.5,$$
$$U = 0 \to U = 0.5$$



Excellent agreement between tDMRG and CUT!

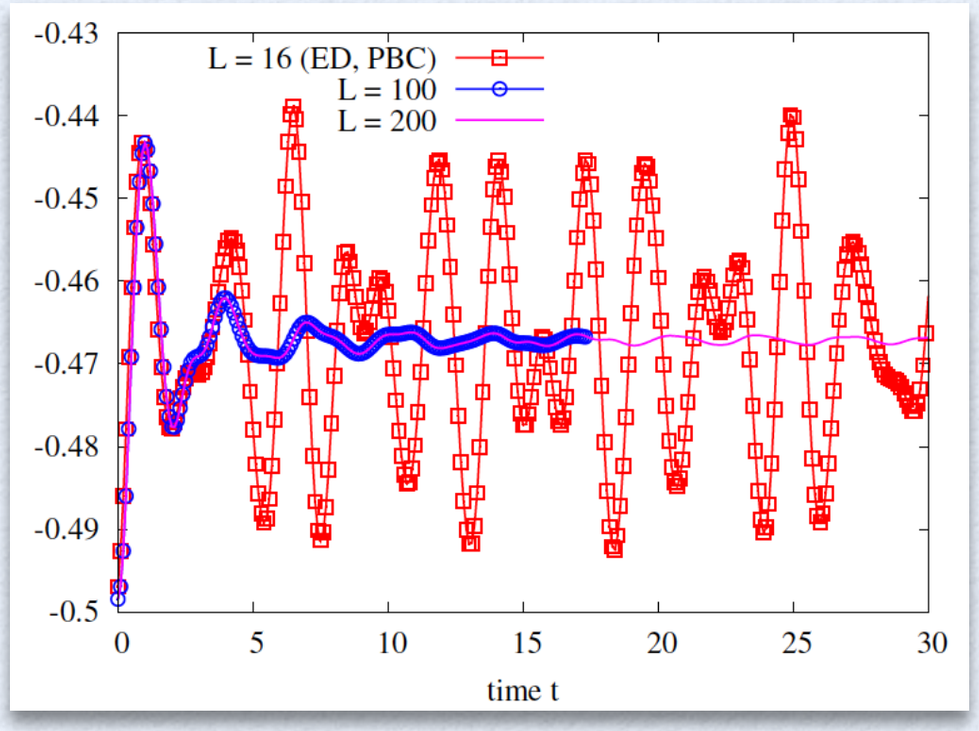
➠ Confirms formation of quasi-particles & prethermalization plateau

Do we see thermalisation for long enough times and large enough systems?

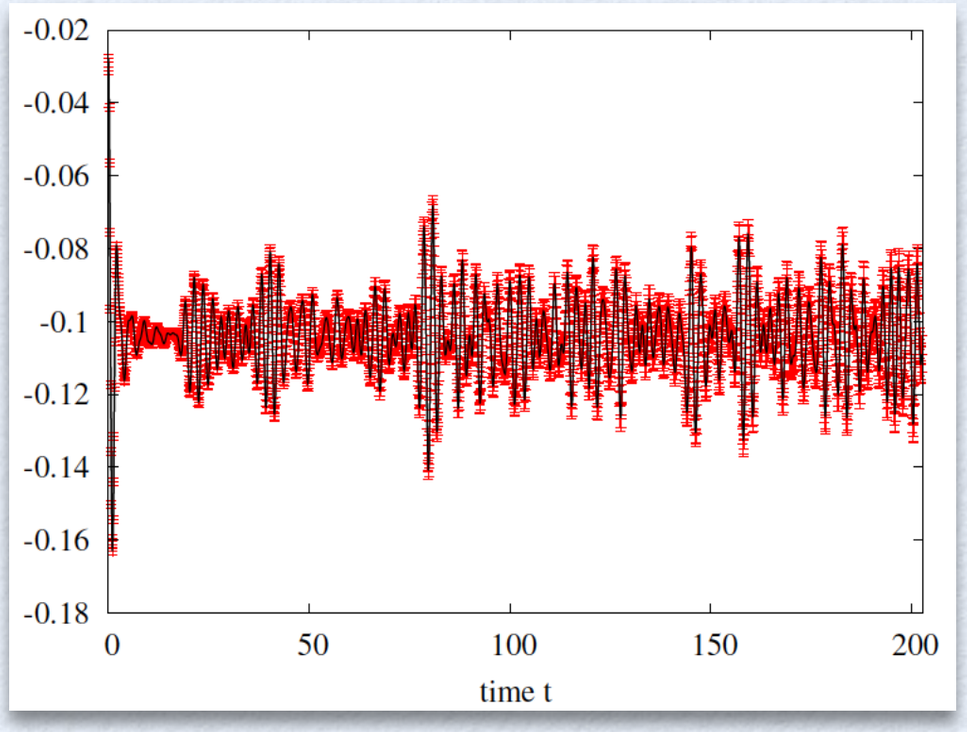
Quench in dimerised state

$$\delta = 0.8 \to \delta = 0.4,$$
$$U = 0 \to U = 0.4$$

Change system size:



long(er) times:

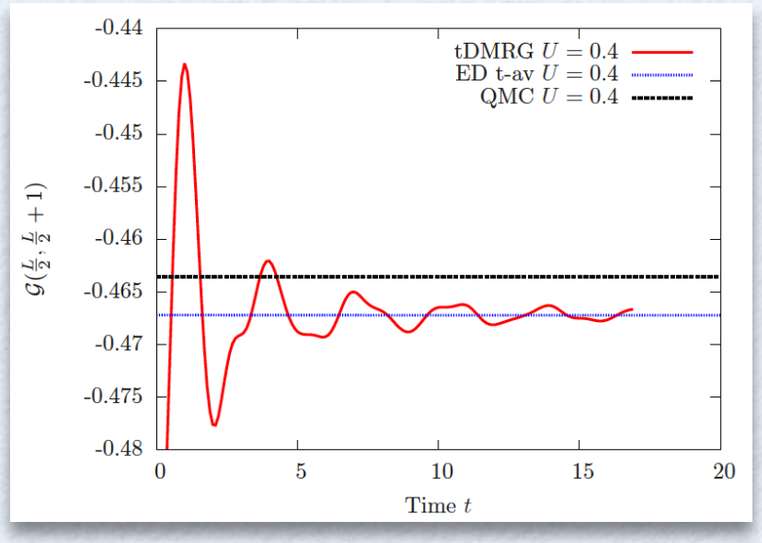


- Weak or no system size dependence
- Up to the times reached: dynamics stays on the prethermalization plateau (also for L=16, t=1000)
- Large thermalisation time scale due to weak breaking of integrability?

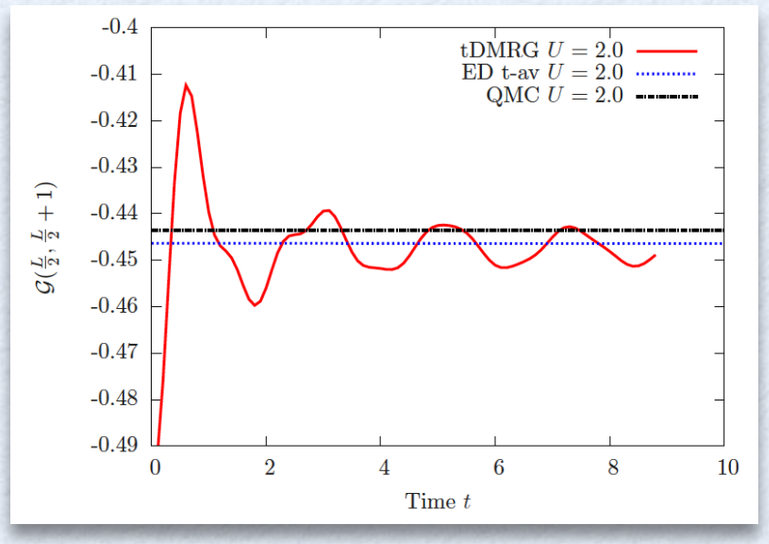
Quench in dimerised state

$$\delta = 0.8 \rightarrow \delta = 0.4$$

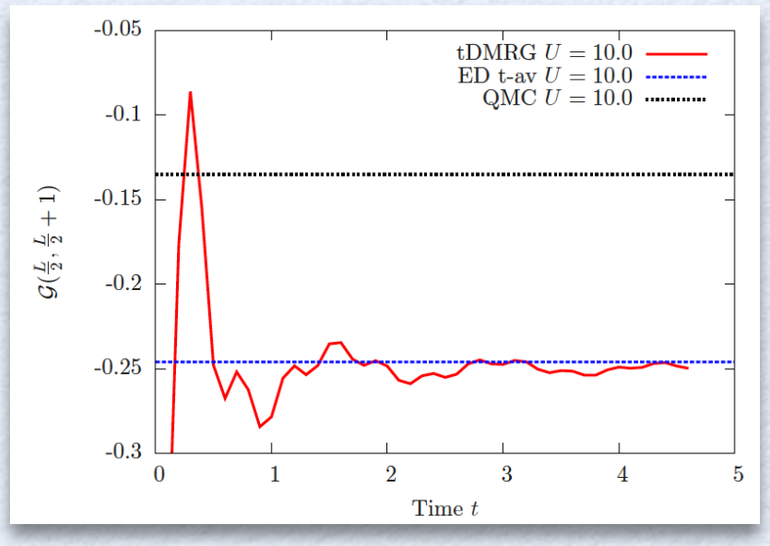
$U = 0 \rightarrow U = 0.4$



$U = 0 \rightarrow U = 2$



$U = 0 \rightarrow U = 10$



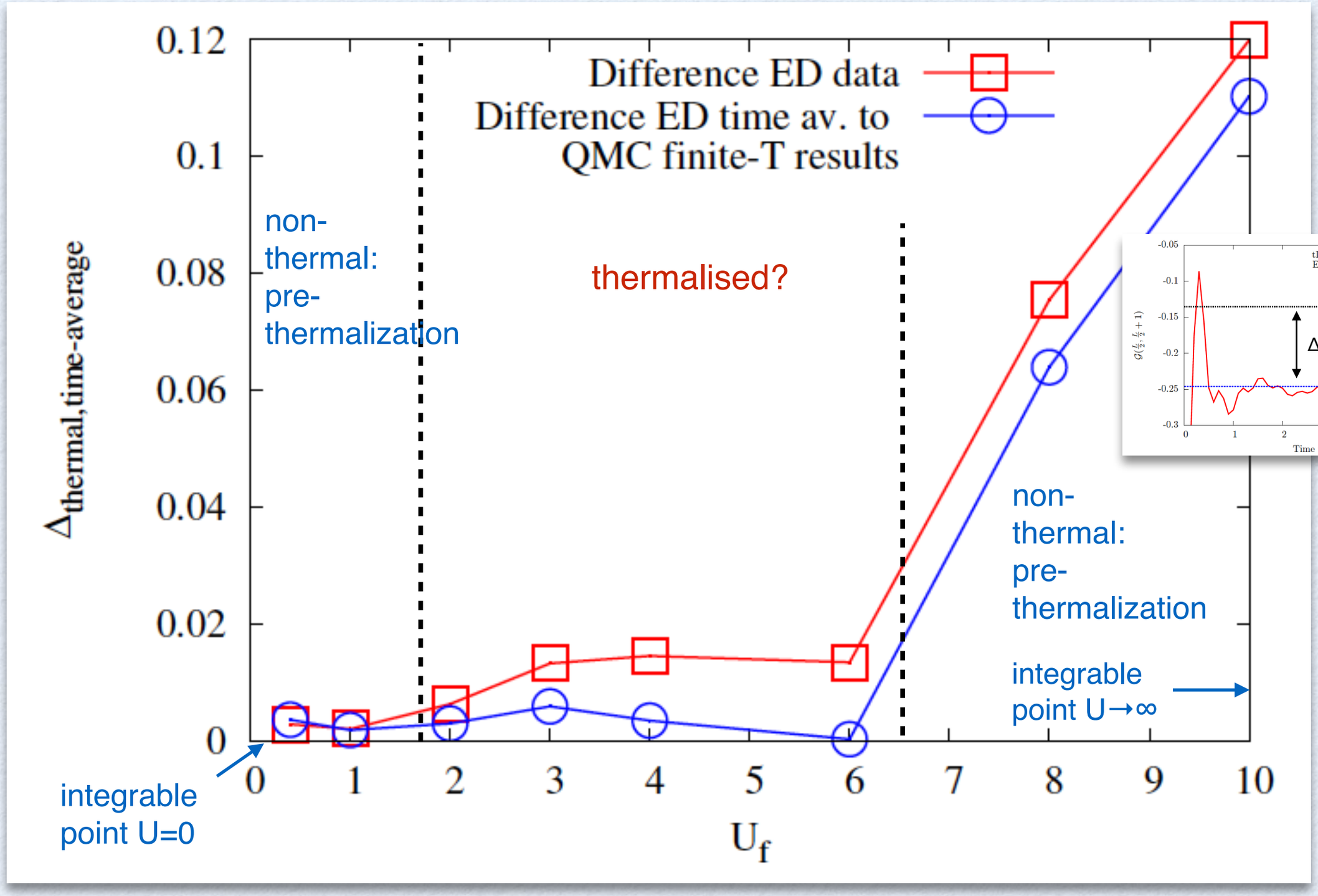
Strong interactions:

- Prethermalization plateau seems to appear
- Difference between time-averages and thermal expectation values
- Difference minimal for intermediate strength of U — due to finite size effects?

Possible scenarios:

- no prethermalization plateau, direct relaxation to thermal value for intermediate U
- relaxation from prethermalization plateau to thermal on longer time scales
- no thermalisation?

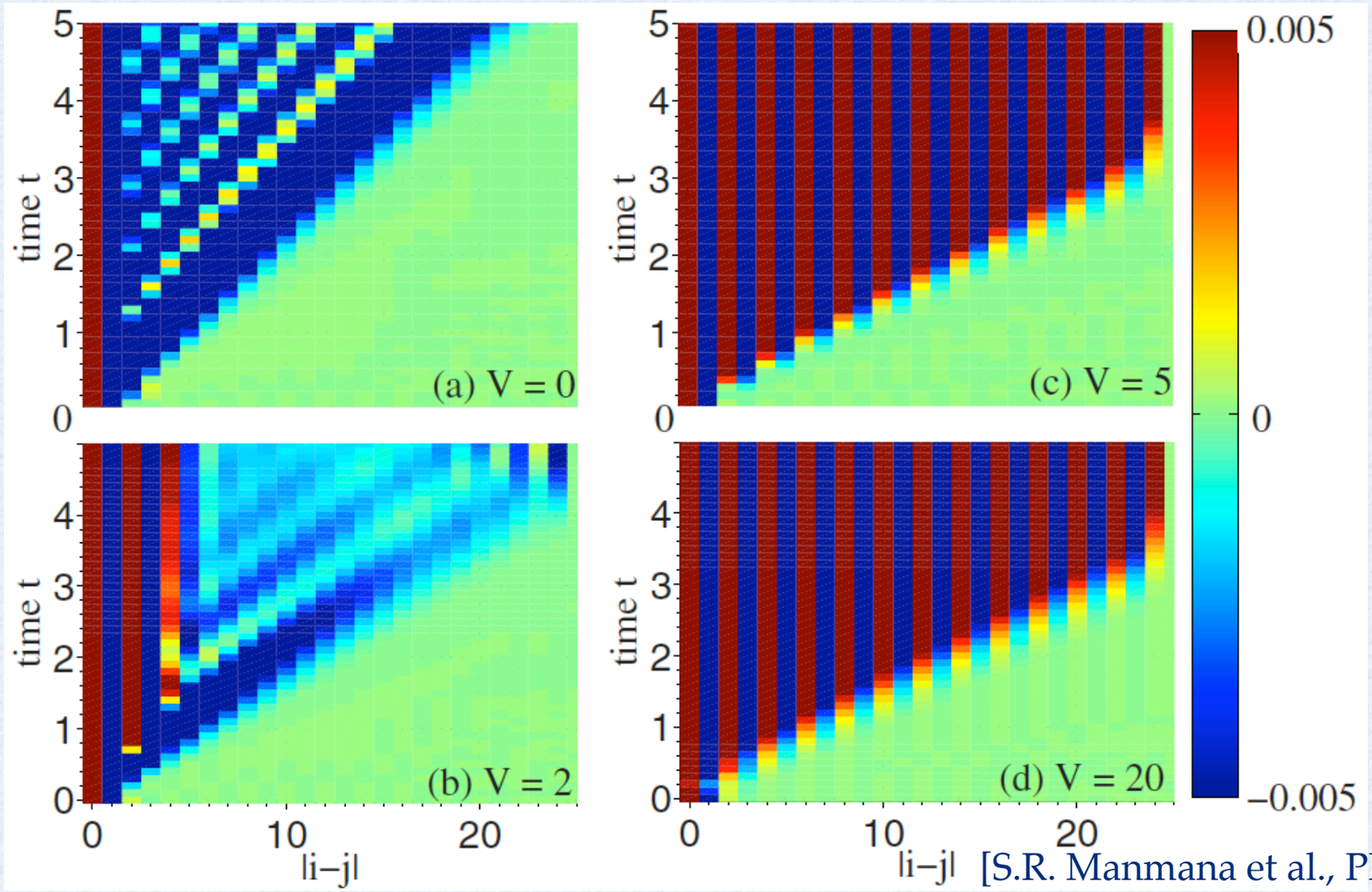
Quench in dimerised state



Systems with long-range interactions

Spread of Information: Lieb-Robinson-Bound

Correlation functions after a global quench:



[S.R. Manmana et al., PRB (2009)]

Lieb-Robinson-Bound for short-range interactions (lattice systems):

$$\|[O_A(t), O_B(0)]\|$$

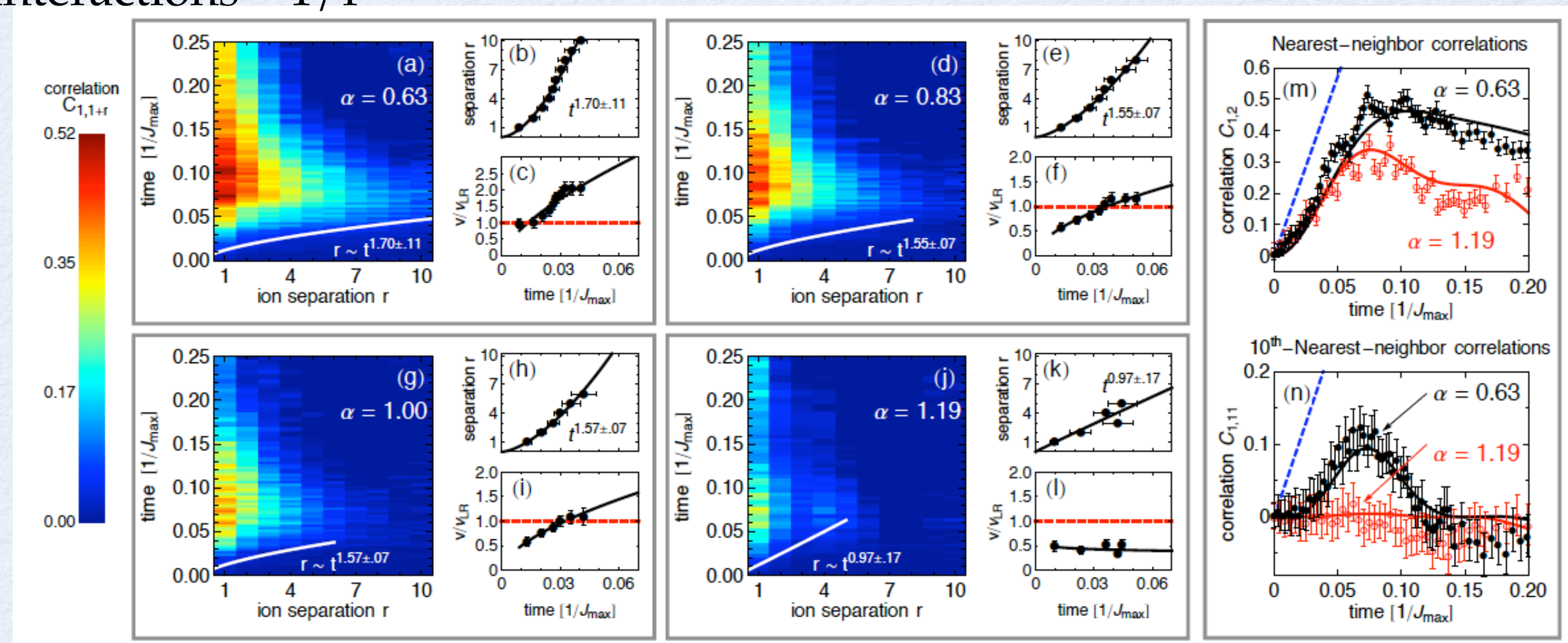
$$\leq C\|O_A\|\|O_B\| \min(|A|, |B|)\mathrm{e}^{[v|t|-d(A,B)]/\xi}$$

→ Linear spread of information

Spread of Information: Ion-Trap-Experiments

[P. Richerme et al., arXiv:1401.5088]

Interactions ~ $1/r^{\alpha}$



Not a linear 'bound of causality', but curved!

Proposed Lieb-Robinson-Bounds for algebraic long-range interactions:

$$\|[O_A(t), O_B(0)]\| \leq C\|O_A\|\|O_B\| \frac{\min(|A|,|B|)(e^{v|t|}-1)}{(\mathrm{d}(A,B)+1)^{\alpha}}$$

(Hastings & Koma 2006)

$$\frac{\|[A(t),B]\|}{2\,\|A\|\,\|B\|} \leq \left(c_1 \frac{e^{v_1 t}-1}{e^{\mu r}} + c_2 \frac{e^{v_2 t}-1}{[(1-\mu)r]^{\alpha}} \right)$$

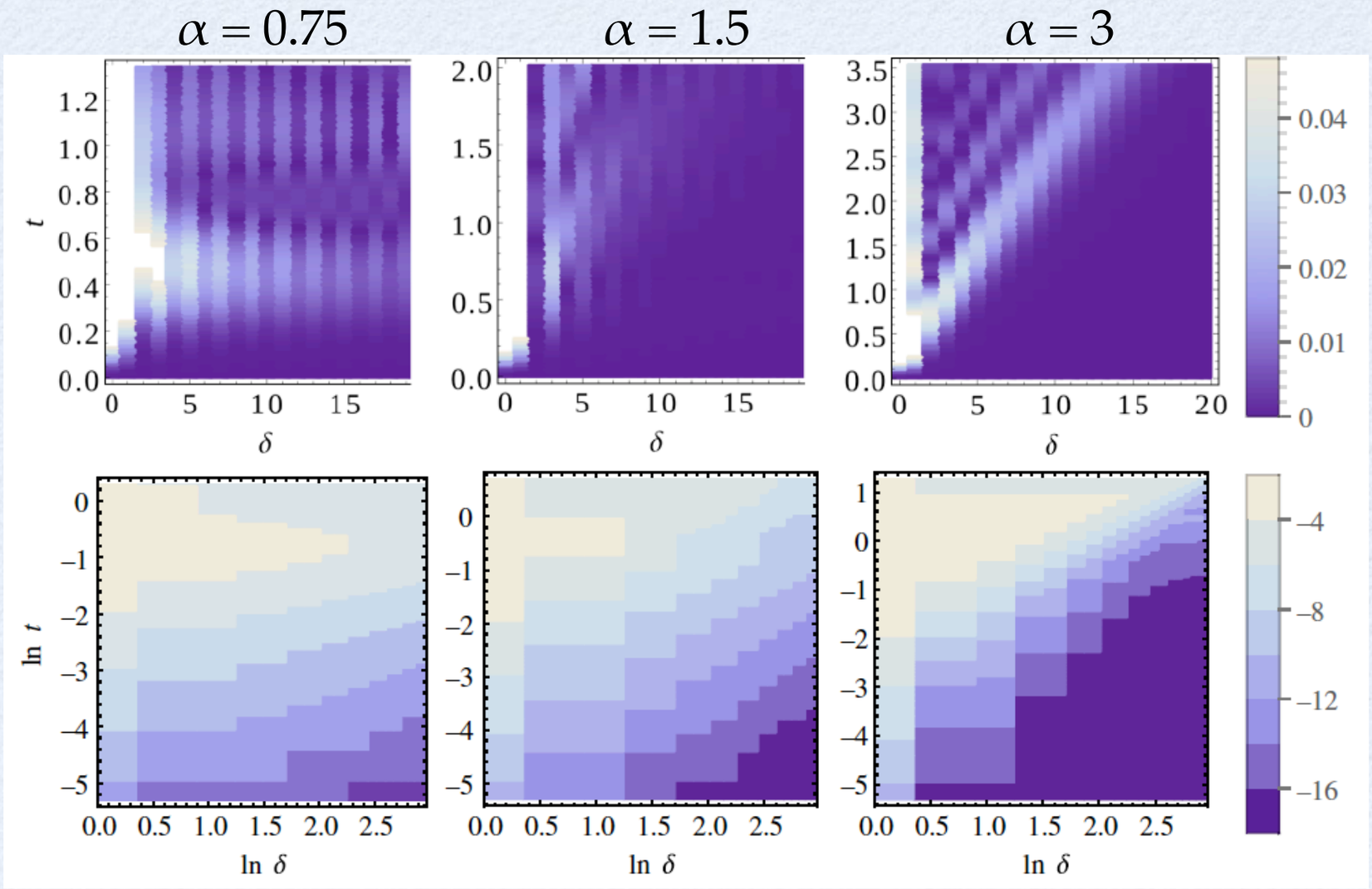
Z.-X. Gong et al., arXiv:1401.6174

Algebraically Decaying Interactions: Causal Horizon vs. Immediate Spread

When do these bounds apply?

[J. Eisert et al., PRL (2013)]

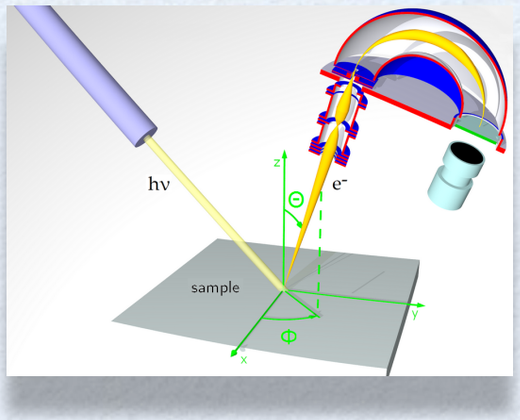
t-DMRG results for a 'XXZ' chain:



generic initial state: causal region appears for $\alpha > D$

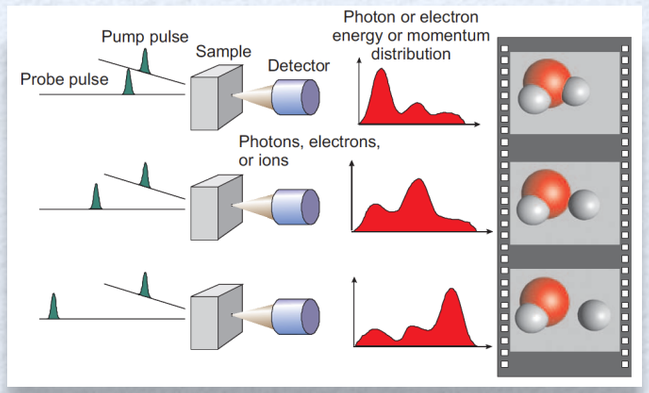
product initial state: causal region appears for $\alpha > D/2$

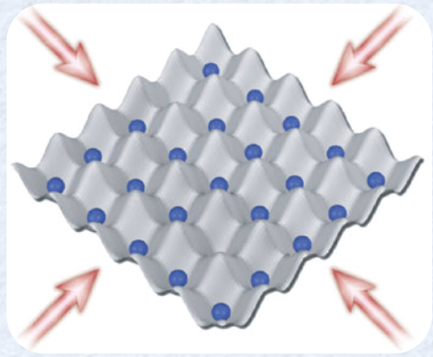
Conclusions and Outlook



I) Dynamical spectral functions at finite T:

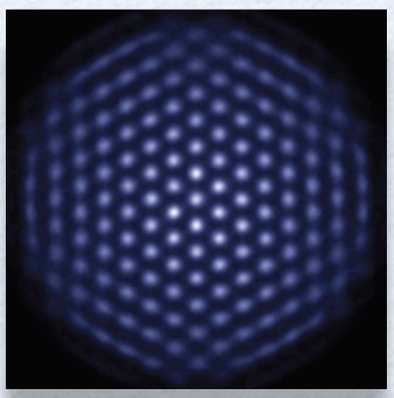
- multitude of interesting effects, compare to experiments
- Liouville-approach

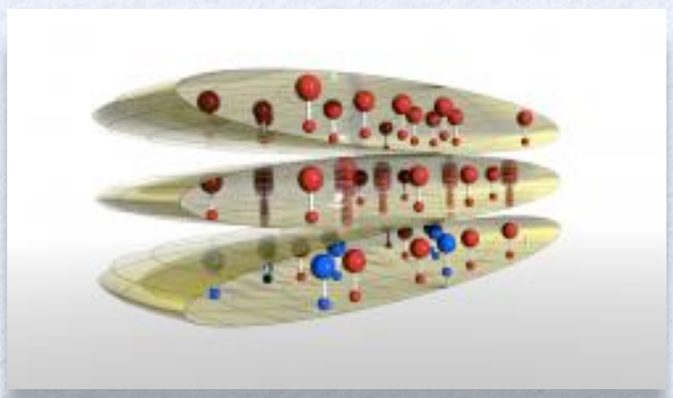




II) Quantum Quenches:

- Relaxation behavior, metastable states
- t-DMRG (Trotter or Krylov variant)





III) Long-range interactions:

- information spread (Lieb-Robinson)
- Krylov t-DMRG, MPOs

Thank you!

Salvatore R. Manmana