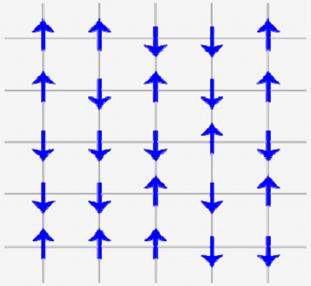
Unconventional dynamics in ultracold gases

Corinna Kollath

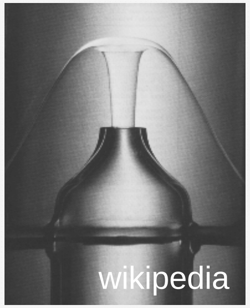
Quantum many body systems

many interesting quantum phases exist:

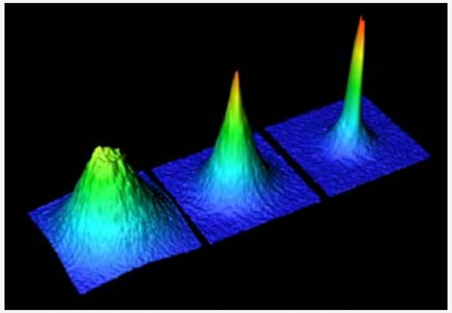


magnets

superconductor



superfluid He



Bose-Einstein condensation

theoretically very difficult due to
large number of degrees of freedom

-> use of model systems

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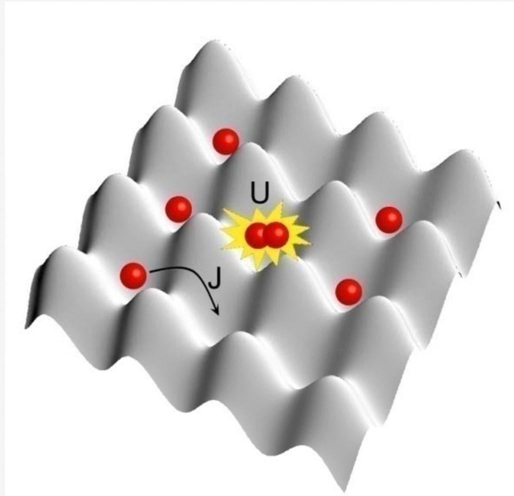
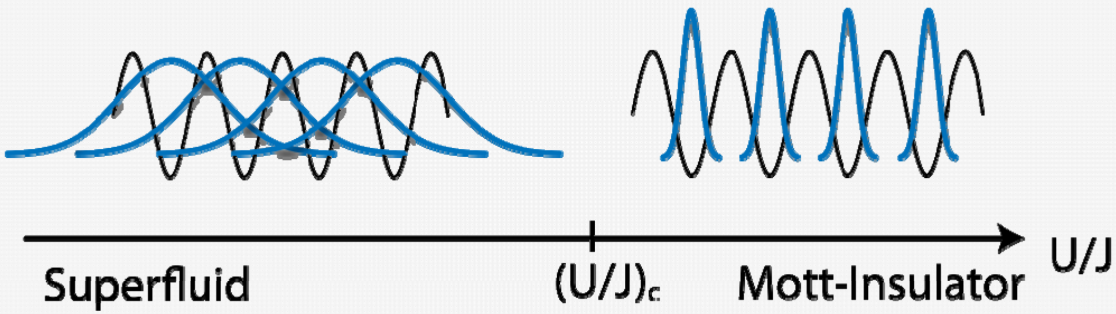
Bosonic atoms in optical lattices

kinetic energy | interaction energy

$$H = -J\sum_{<ij>}(b_i^+ b_j + h.c.) + U/2\sum_j n_j(n_j - 1)$$





dynamics very complicated

in experiments coupling to environment

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Lattice models with dissipative coupling

described by Markovian master equation:

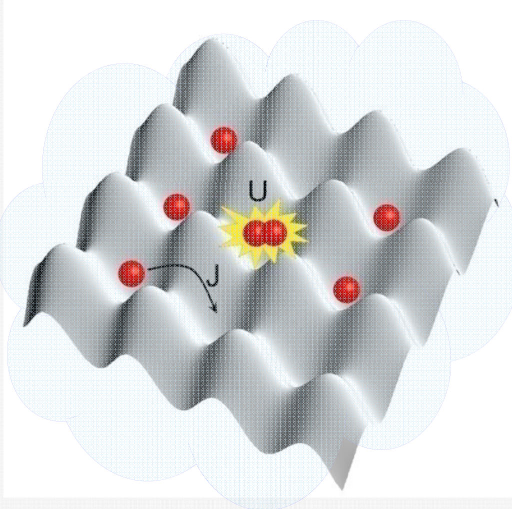
closed system dynamics

$$i\hbar\partial_t\hat{\rho} = [\hat{H},\hat{\rho}] + i\mathcal{D}(\hat{\rho})$$

dissipative dynamics

$$\mathcal{D}(\rho) = \frac{\gamma}{2}\sum_i 2K_i\rho K_i^\dagger - K_i^\dagger K_i\rho - \rho K_i^\dagger K_i$$

atom detection   no-detection event



ρ density matrix
γ dissipative coupling
H system Hamiltonian

- obtained from Hamiltonian dynamics for both the system and environment
- need short correlation times in environment
- here also used for strong dissipation

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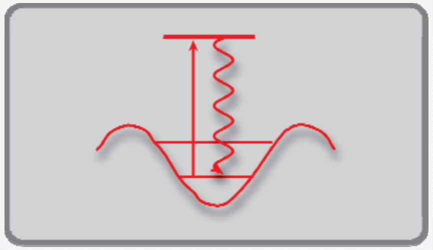
Outline

Methods:

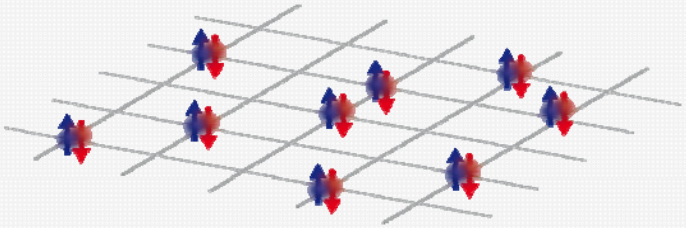
- Adiabatic elimination method for many body systems (well known in quantum optics)
- time-dependet DMRG (MPS) method (see talk of S. Manmana for DMRG)



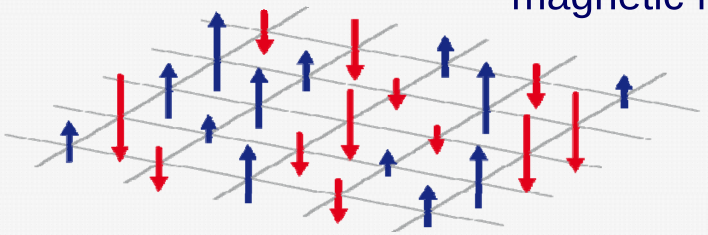
Applications:

- Unconventional dynamics of a Bose-gas subjected to light scattering
  two site model: adiabatic elimination + continuum mapping
  extended model: adiabatic elimination + mean-field coupling

- Formation of correlations by local dissipation in a fermionic model
  adiabatic elimination + mean-field decoupling of equations for observables
  DMRG approach combined with stochastic wave function method



‘magnetic field fluctuations’



Conditions for adiabatic elimination

split Lindbladian as

Syassen et al., Science (2008)
Garcia-Ripoll et al., New J. Phys. (2009)

$$\frac{\partial \rho}{\partial t} = (\nu + L_0)\rho$$

$\nu = -i\,[H_\nu, \cdot]$ 'perturbation' (but does not really need to be very small

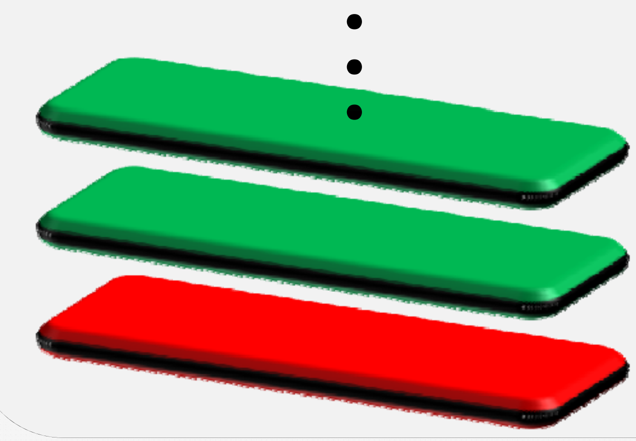
$L_0 = -i\,[H_D, \cdot] + D$ where we diagonalize the superoperator: $L_0 \rho_\alpha = (-\lambda_\alpha^{\mathrm{Re}} + i\,\lambda_\alpha^{\mathrm{Im}})\rho_\alpha$

for $\nu = 0$
linear solution of the equation of motion

$$\rho(t) = \sum_\alpha c_\alpha\, e^{(-\lambda_\alpha^{\mathrm{Re}} + i\,\lambda_\alpha^{\mathrm{Im}})t} \rho_\alpha$$

- typical exponential decay with decay rate $\lambda_\alpha^{\mathrm{Re}}$
- oscillations with $\lambda_\alpha^{\mathrm{Im}}$
- decoherence free subspace $\lambda_0^{\mathrm{Re}} = 0, \rho^F$

classify subspaces by their decay rate



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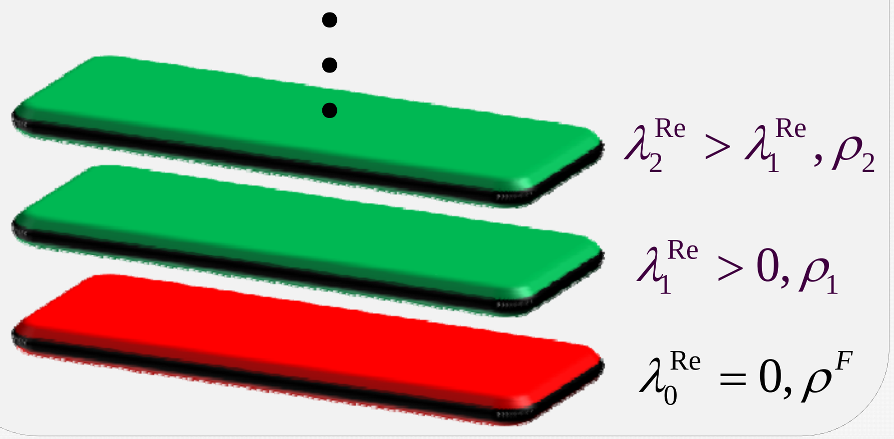
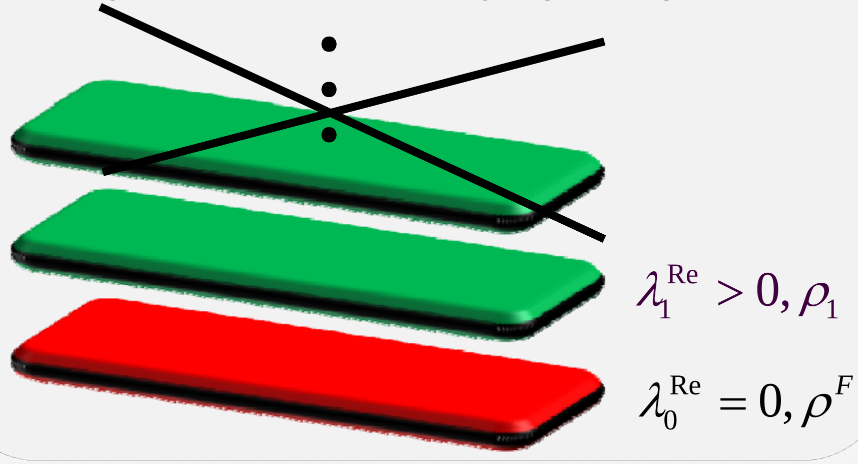
Idea of adiabatic elimination

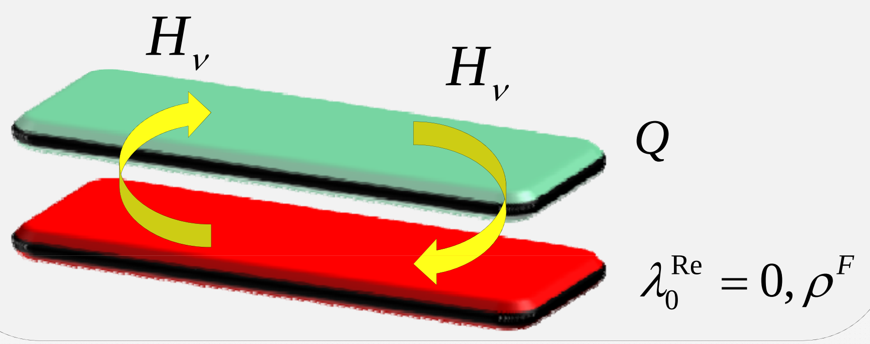
classify subspaces by their decay rate



only take slowest decaying subspaces



only virtual occupation
of decaying subspace allowed



effective dynamics
within decoherence free subspace

$$\frac{\partial \rho^F}{\partial t} = \nu_{FF}\rho^F + \frac{1}{\lambda_1}\sum_1 \nu_{F1}\nu_{1F}\rho_F$$

this equation is often still very complicated!

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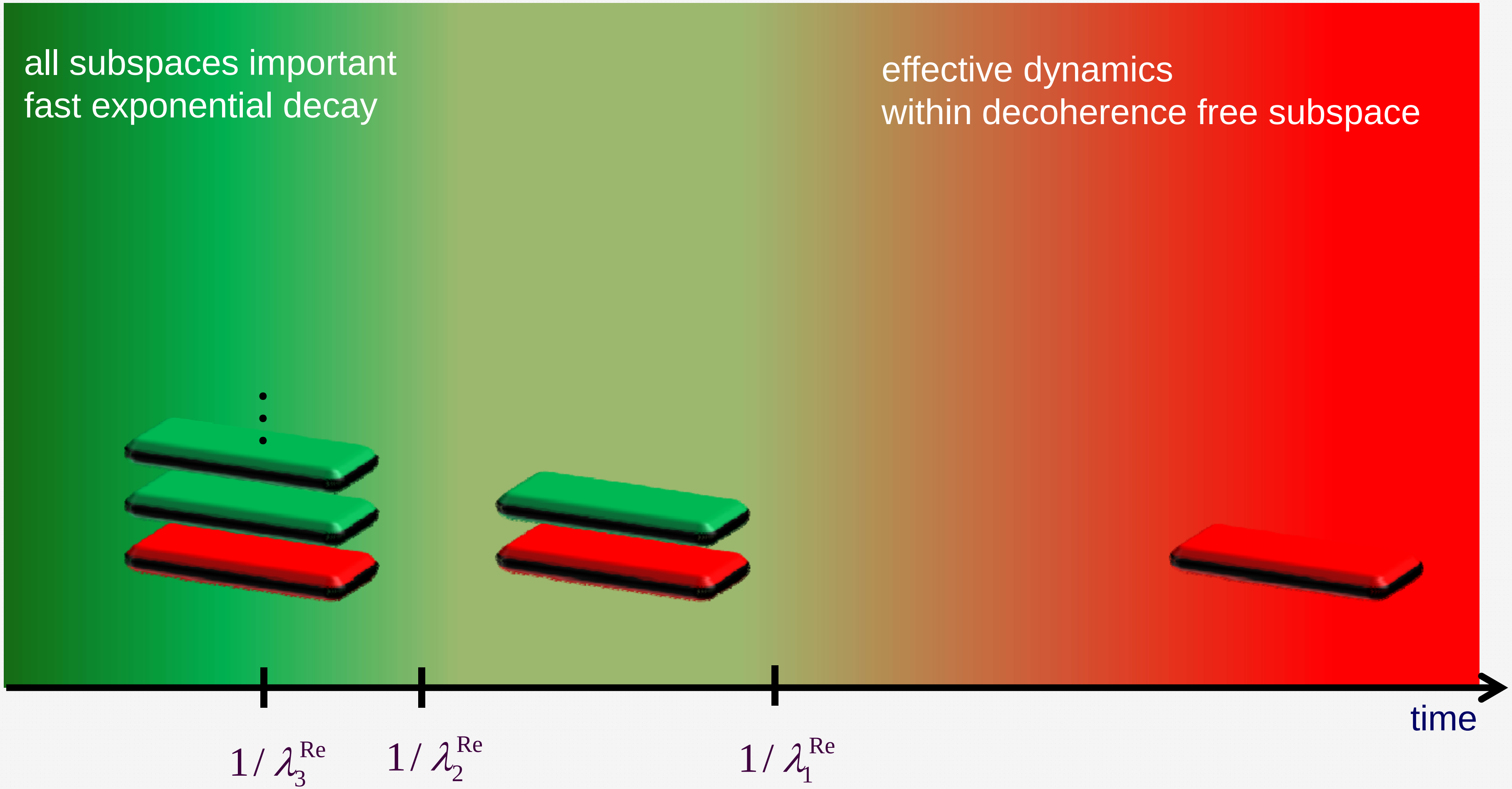
Validity of adiabatic elimination



- very flexible
- any dimension
- complements well short time methods

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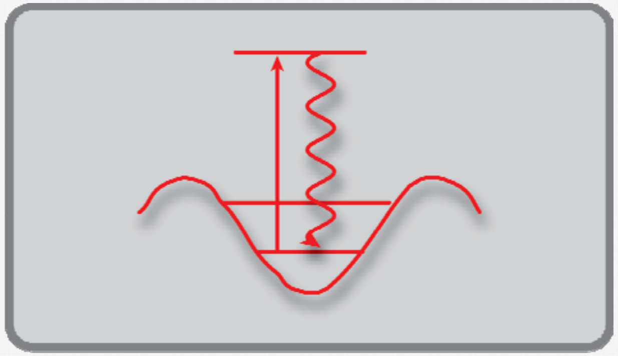
Application: Bosons with dissipative coupling

dissipative effects:

scattering with thermal atoms

flourescence scattering with light fields

fluctuating noise field



described by Markovian master equation:

closed system dynamics

$$i\hbar\partial_t\hat{\rho} = [\hat{H},\hat{\rho}] + i\mathcal{D}(\hat{\rho})$$

dissipative dynamics

$$\mathcal{D}(\hat{\rho}) = \hbar\gamma\sum_j\left(\hat{n}_j\hat{\rho}\hat{n}_j - \frac{1}{2}\hat{n}_j^2\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{n}_j^2\right)$$

ρ density matrix
γ dissipative coupling
H Bose-Hubbard model (any D)

F. Gerbier and Y. Castin (2010)
S. Pichler et al (2010)

switch on dissipation at time t=0

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Supression of decoherence by interaction

large interaction

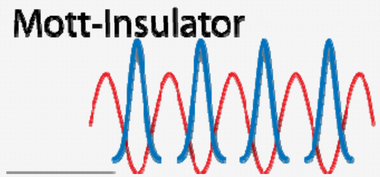
&

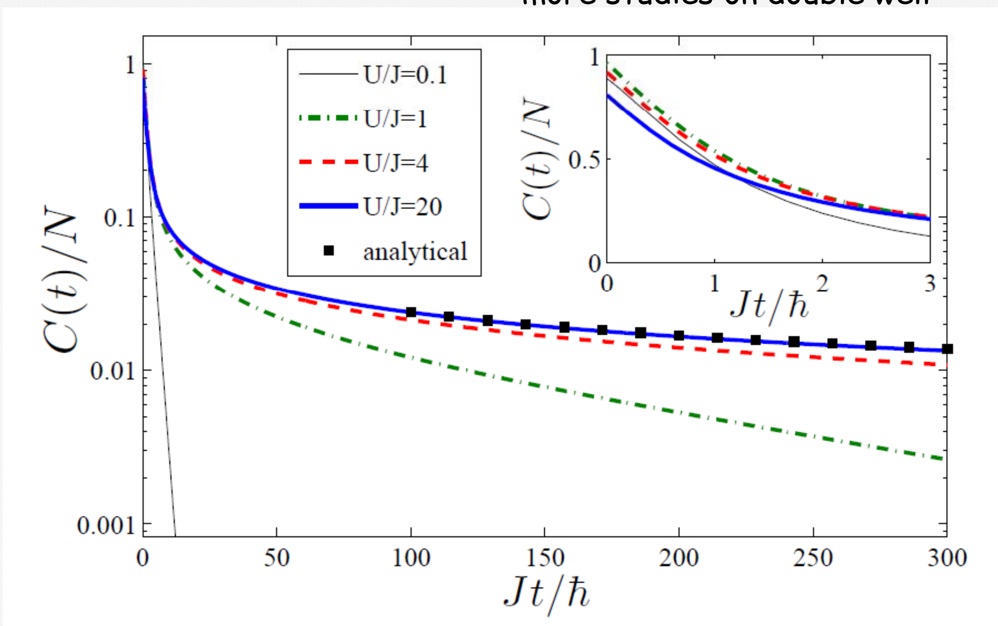
dissipation:
local measurement
of particle number

interaction impedes decoherence

first slowing down seen in
S. Pichler et al (2010)
more studies on double well

localization of atoms
destruction of coherence





exponentially fast
destruction of coherence
with rate γ

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Adiabatic elimination for bosonic atoms

Poletti, Bernier, Georges, Kollath, PRL (2012)
Poletti, Barmettler, Georges, Kollath, PRL (2013)

split Lindbladian as

$$\frac{\partial\rho}{\partial t} = (\nu + L_0)\rho$$

$\nu = -i\,[H_K,\cdot]$ kinetic term

$L_0 = -i\,[H_I,\cdot] + D$ interaction +dissipation
diagonalized by Fock states

$$\mathcal{D}(\hat{\rho}) = \hbar\gamma \sum_j \left(\hat{n}_j\hat{\rho}\hat{n}_j - \tfrac{1}{2}\hat{n}_j^2\hat{\rho} - \tfrac{1}{2}\hat{\rho}\hat{n}_j^2\right)$$

decoherence free subspace: diagonal matrices $\rho^F = \left|\{n_j\}\right\rangle\left\langle\{n_j\}\right|$

infinite time = infinite temperature state $\rho^F(t=\infty) = 1$

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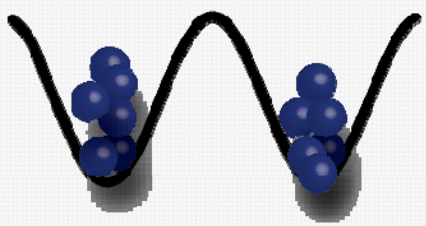
Effective equation for two sites



n $\qquad$ $N-n$

decoherence free subspace:

$$\rho^F = \sum_n \rho_{n,n} |n\rangle\langle n|$$

effective (classical) Master equation for diagonal matrix elements:

two sites two particles

$$\frac{d}{dt}(\rho_{0,0}+\rho_{1,1}+\rho_{2,2}) = 0$$
$$\frac{d}{dt}(\rho_{0,0}-\rho_{2,2}) \approx \lambda_O(\rho_{0,0}-\rho_{2,2})$$
$$\frac{d}{dt}(\rho_{0,0}-2\rho_{1,1}+\rho_{2,2}) \approx \lambda_E(\rho_{0,0}-2\rho_{1,1}+\rho_{2,2})$$

$$\lambda_E = -12\frac{\gamma J^2}{U^2+\gamma^2}$$

large γ: $\quad \lambda_E \propto \frac{J^2}{\gamma}$ $\quad$ ‘normal’ Zeno effect

large U: $\quad \lambda_E \propto \frac{\gamma J^2}{U^2}$ $\quad$ ‘interaction’ Zeno effect
->impeding of decoherence

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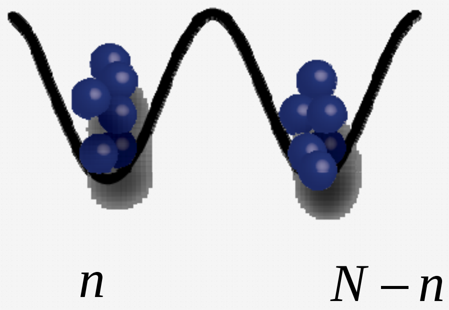
Effective equation for two sites, N atoms



decoherence free subspace:

$$\rho^F = \sum_n \rho_{n,n} |n\rangle\langle n|$$

effective (classical) Master equation for diagonal matrix elements (here 'interaction Zeno')

$$\partial_\tau \rho_{n,n} = 2\left(W_{n+1}\Delta\rho_n - W_n \Delta\rho_{n-1}\right)$$

$$\Delta\rho_n = N^2(\rho_{n+1,n+1} - \rho_{n,n})$$

occupation dependent rates $W_{n+1} = \frac{(n+1)(N-n)}{(n-N/2+1/2)^2}$

parameters only enter in rescaled time: $\tau = t\frac{2N^2U^2}{\gamma J^2}$

-> even without solution 'interaction' Zeno effect

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Mapping to classical diffusion equation (large U)

continuum mapping (large N limit x=n/N)

$$N\rho_{n,n} = \phi(x)$$

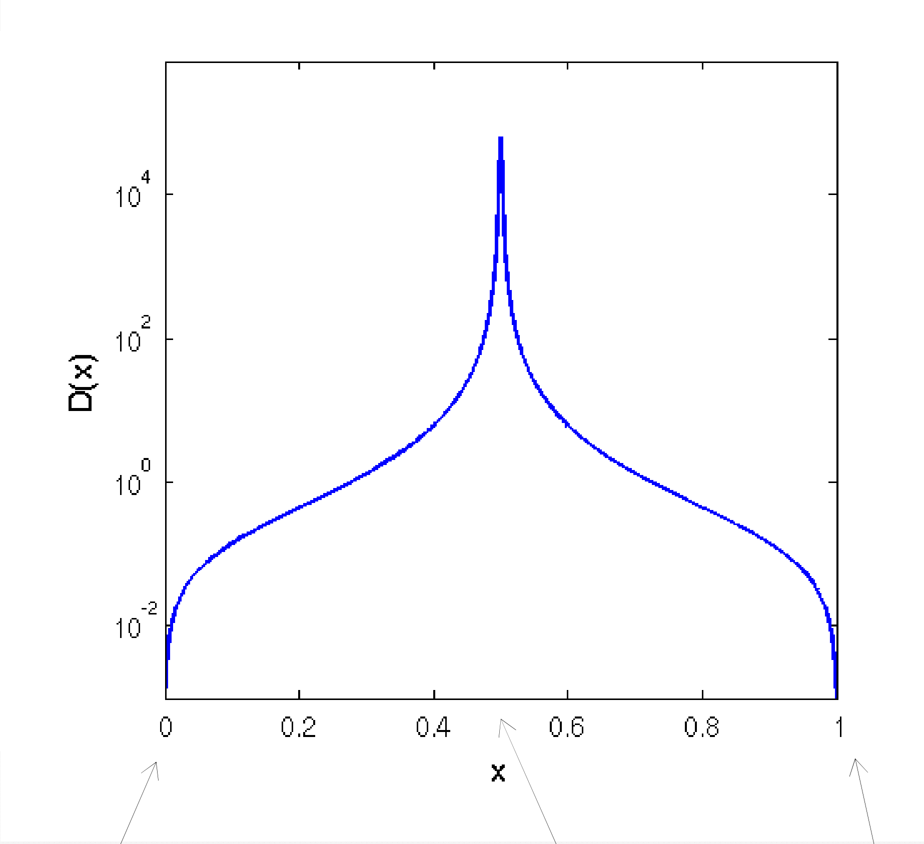
$$\frac{d}{dx}\left(D(x)\frac{d}{dx}\phi(x,\tilde{t})\right) = \frac{\hbar U^2}{2J^2\gamma}\frac{\partial}{\partial \tilde{t}}\phi(x,\tilde{t})$$

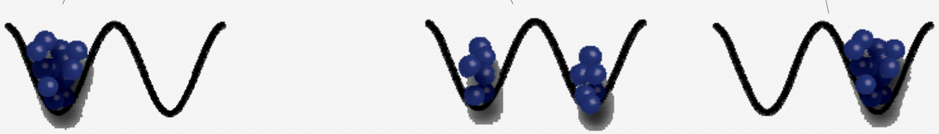
$$D(x) = \frac{x(1-x)}{(2x-1)^2}, \qquad \tilde{t} = t/N^2$$

slowly diffusing states at the boundary correspond to large imbalance

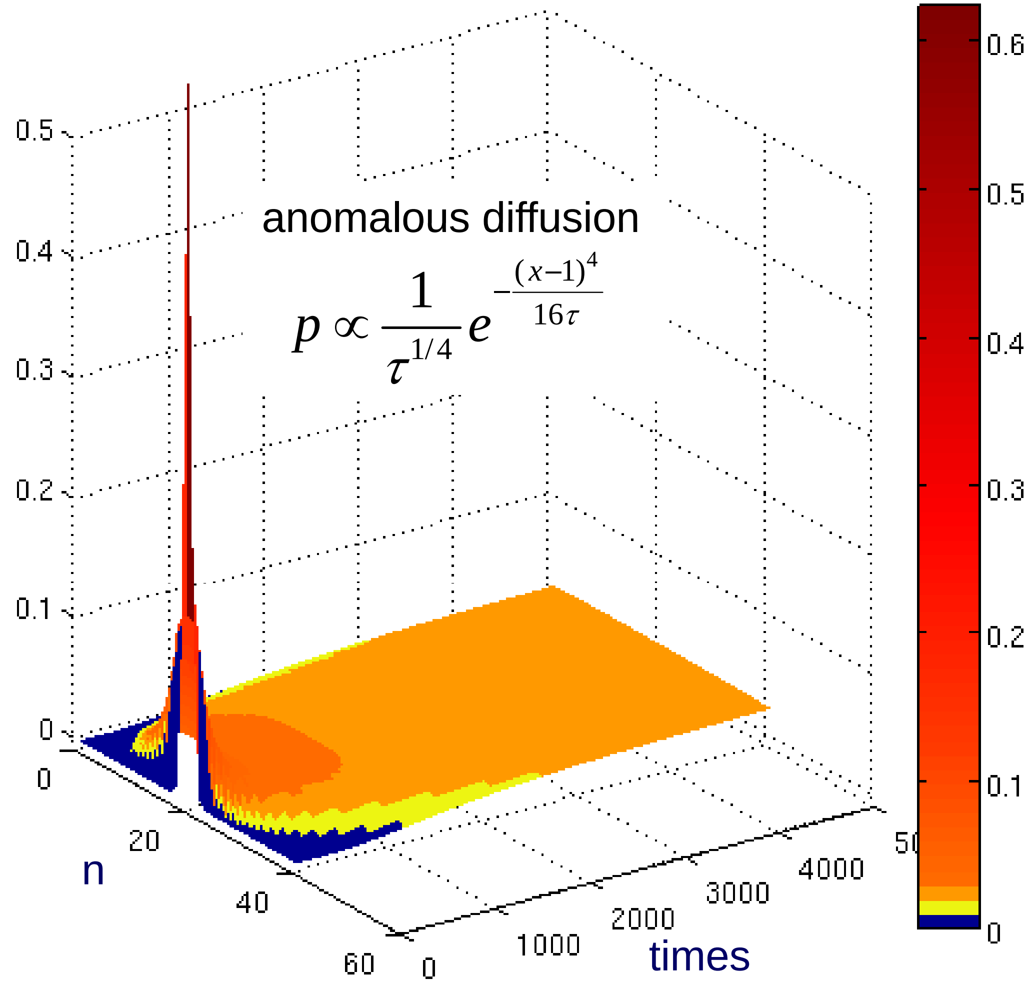
rescaled time-scale:

$$\tau = t\frac{2N^2U^2}{\gamma J^2}$$





Diffusion of initial balanced state

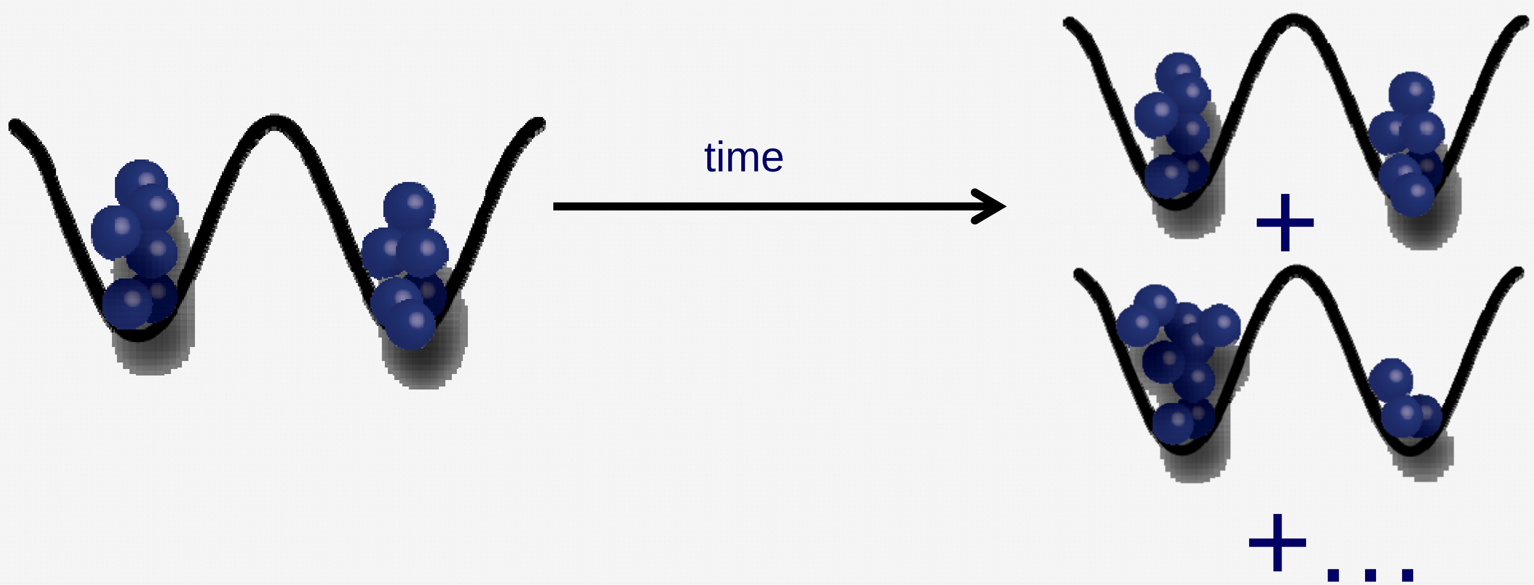


$$p \propto \frac{1}{\tau^{1/4}} e^{-\frac{(x-1)^4}{16\tau}}$$

time-scale

$$\tau = t \frac{2N^2U^2}{\gamma J^2}$$

Slow evolution towards totally mixed state



ground state close to balanced

build up of strongly imbalanced states
energetically very costly

-> impeding of decoherence due to interaction
(not due to Mott-state)

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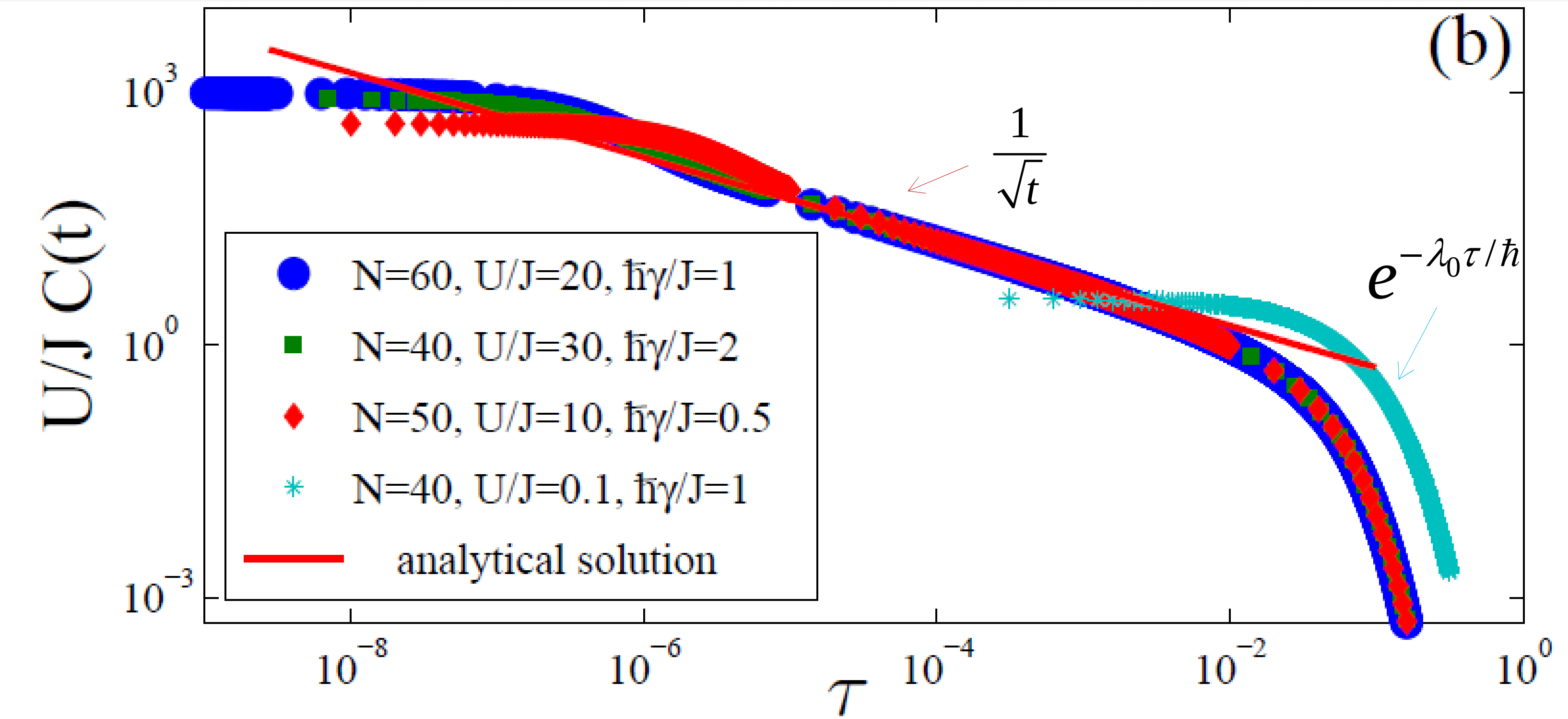
Scaling behaviour in coherence

$$C = \langle \hat{b}_1^\dagger \hat{b}_2 + \hat{b}_2^\dagger \hat{b}_1 \rangle$$

symbols: exact diagonalization

line: adiabatic elimination + continuum approximation $C(t)/N \sim \frac{1}{\sqrt{t}}$



$$\tau = t\frac{2N^2U^2}{\gamma J^2}$$

adiabatic elimination works well at large times

not too small U and γ

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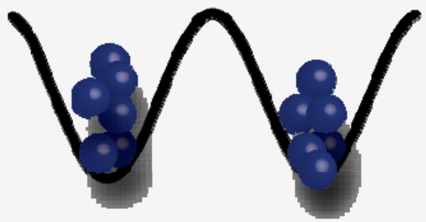
Interplay of interaction and dissipation



n $\qquad N-n$

decoherence free subspace:

$$\rho^F = \sum_n \rho_{n,n} |n\rangle\langle n|$$

impeding of decoherence by

‘normal’ Zeno effect
strong coupling to the environment $\lambda_E \propto \frac{J^2}{\gamma}$ ‘measurement’ freezes state

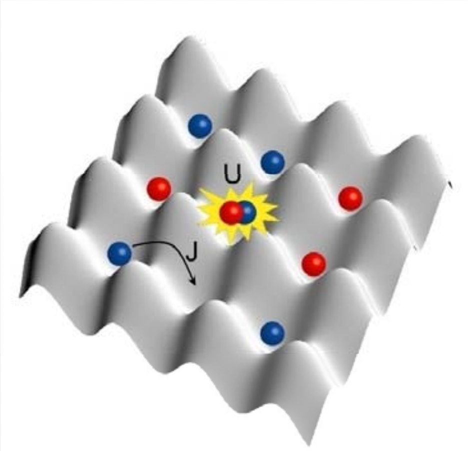
‘interaction’ Zeno effect:
strong interaction $\lambda_E \propto \frac{\gamma J^2}{U^2}$ build up of strongly imbalanced states energetically very costly

effective dynamics ~ anomalous diffusion in occupation space
algebraic decay

Effective dynamics in extended lattice

effective (classical) Master equation very complicated

(classical Monte-Carlo, see Gabriel Kocher diploma thesis)



mean-field decoupling for classical density

continuum limit

-> diffusion like equation for single site density distribution p

$$\partial_\tau p(x,\tau_f) = \partial_x \left[D(x,\tau_f)\partial_x p(x,\tau_f) - F(x,\tau_f)p(x,\tau_f) \right]$$

$$\tau_f = \tau/f^2,\ D(x,\tau_f) = \int_0^\infty \frac{xy \quad p(y,\tau_f)}{(x-y)^2+\epsilon^2} dy,\ F(x,\tau_f) = \int_0^\infty \frac{xy \quad \partial_y p(y,\tau_f)}{(x-y)^2+\epsilon^2} dy \text{ and } \epsilon = \hbar\gamma/fU.$$

note: direct use of Gutzwiller decoupling leads to incorrect steady states

freezing different steady states can occur

-> Gutzwiller only good for short time

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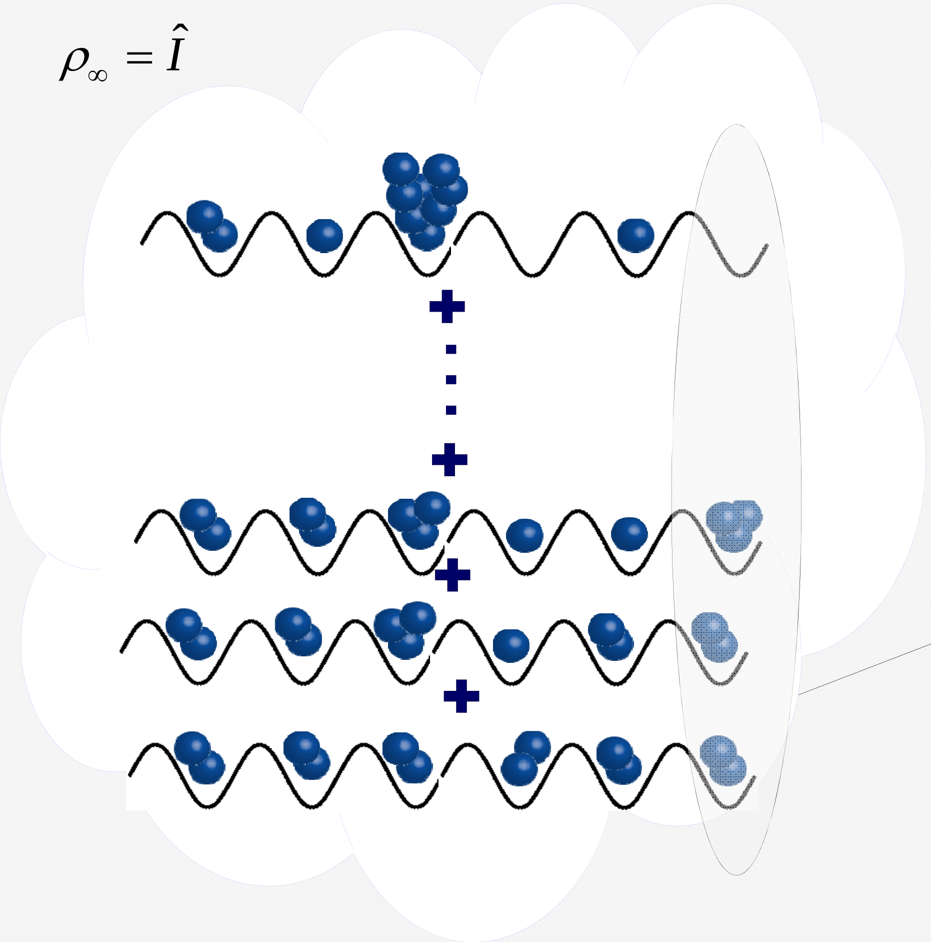
Extended optical lattice: infinite time limit

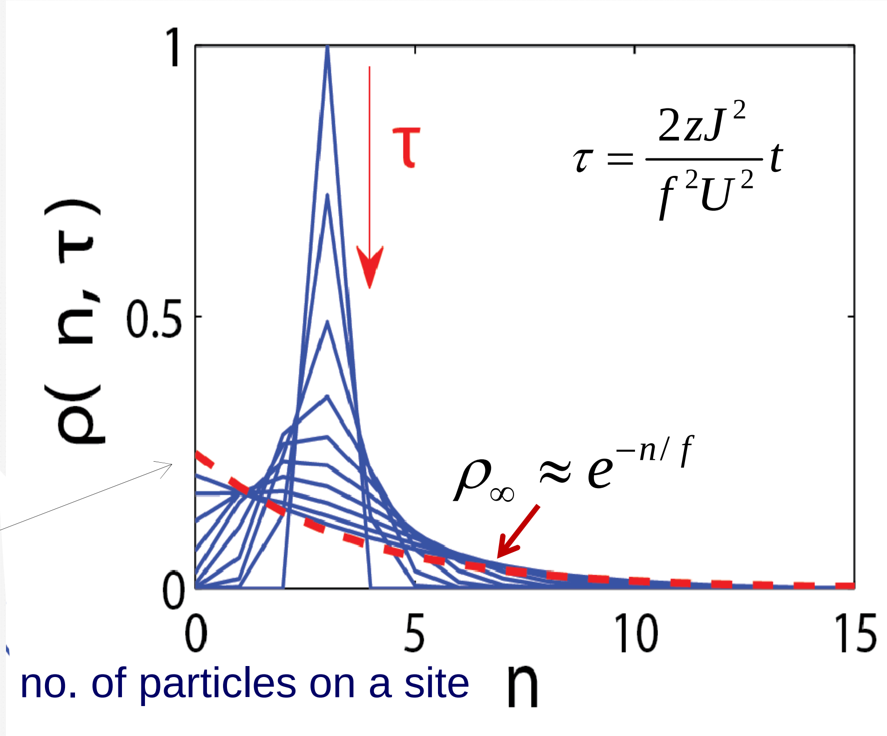
large time -> infinite temperature state

heating probes ***'all'*** configurations

reduced single site density matrix:

$\rho_\infty = \hat{I}$





no. of particles on a site n

adiabatic elimination gives correct infinite time limit

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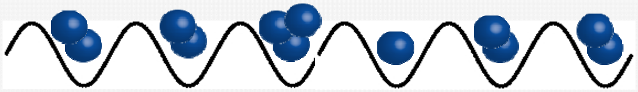
two unconventional regimes

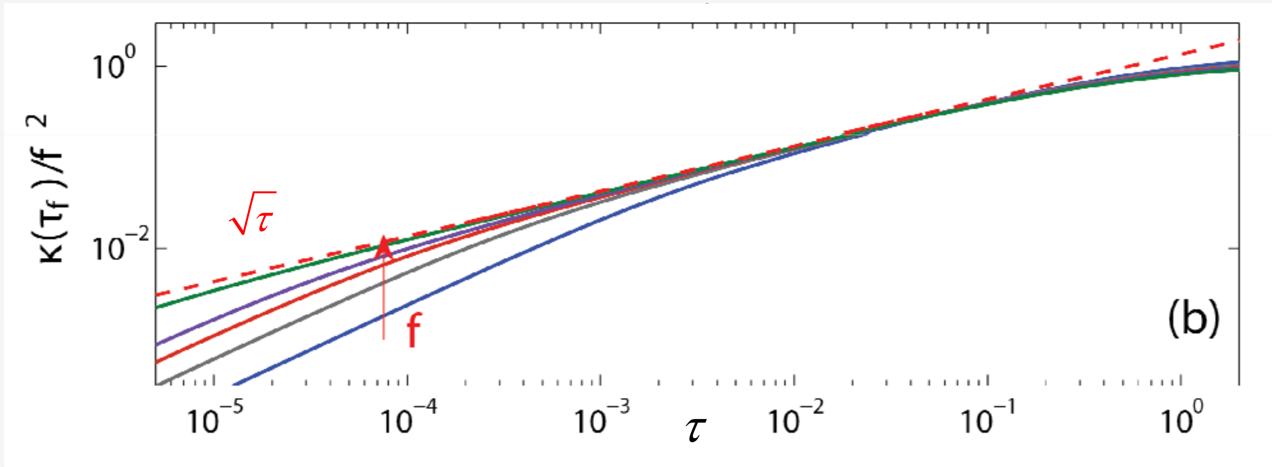
method: single site mean-field decoupling after adiabatic elimination

short time: power law regime

many almost equally costly processes

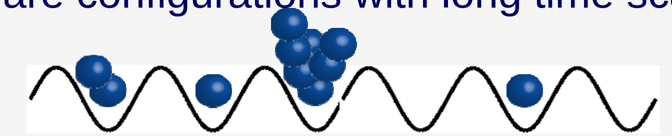
(resembles two site dynamics)

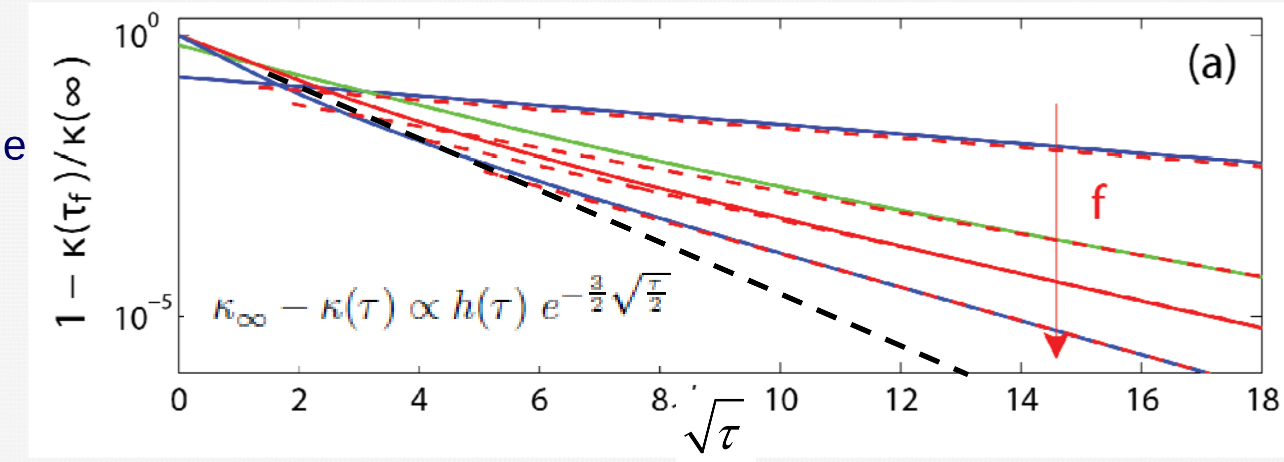




long times: stretched exponential

rare configurations with long time scale





typical for disorder and frustration

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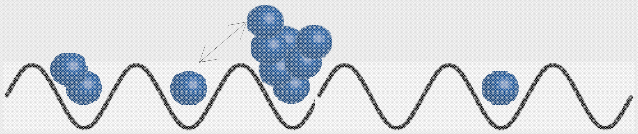
Idea of origin of stretched exponential

physical interpretation:

exponentially rare states ~exp(-x)

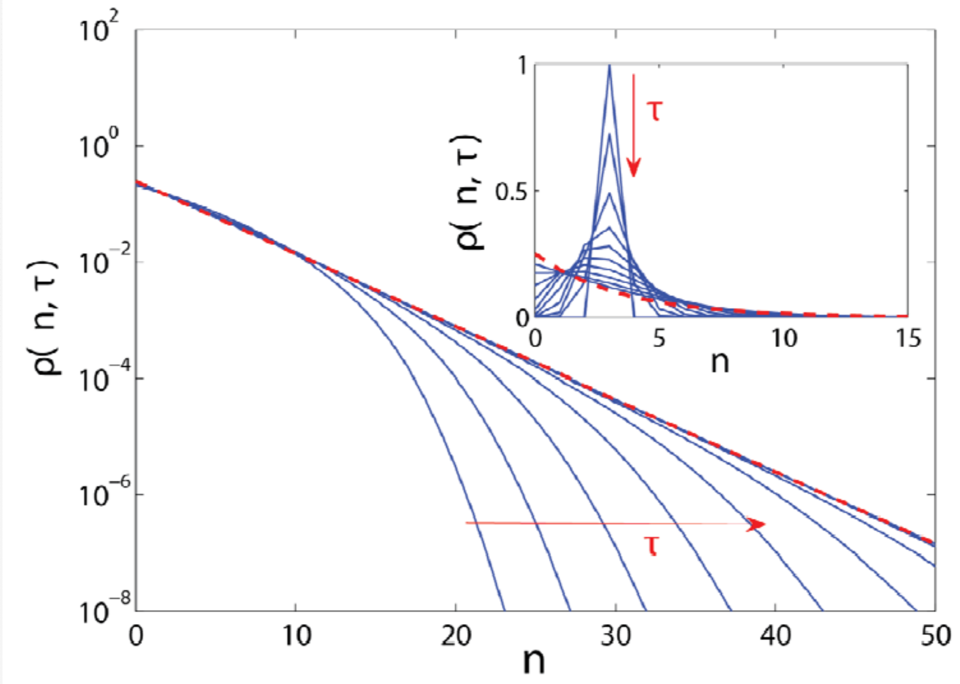
dominate evolution by low diffusion rates γ~ 1/x

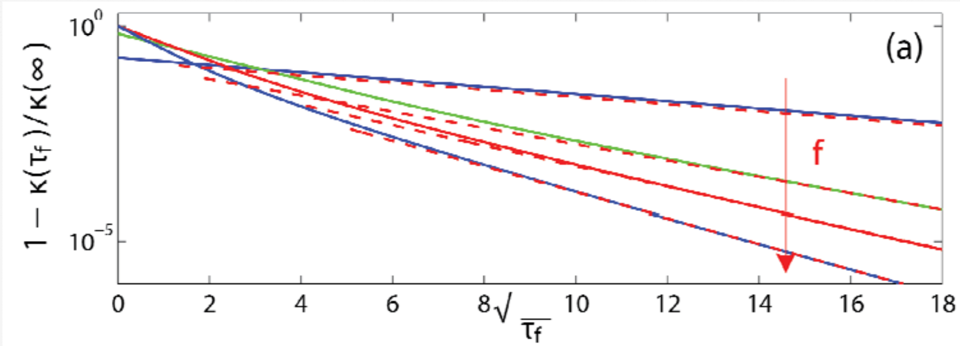


example: local particle fluctuations (scaled)

$$\kappa \approx \int_0^\infty x^2 e^{-x} e^{-\frac{At}{x}} \approx e^{-\sqrt{At}}$$

stretched exponential behaviour!





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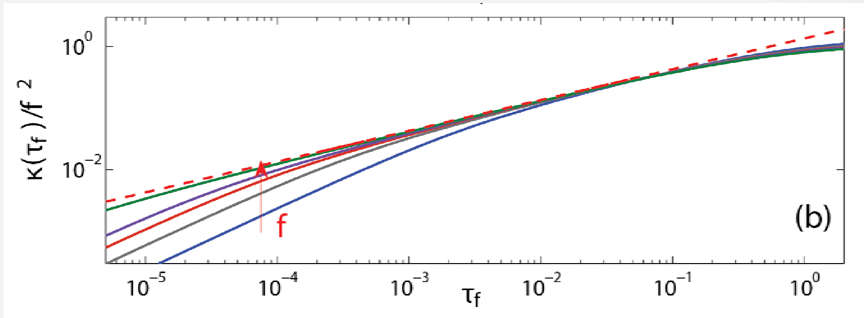
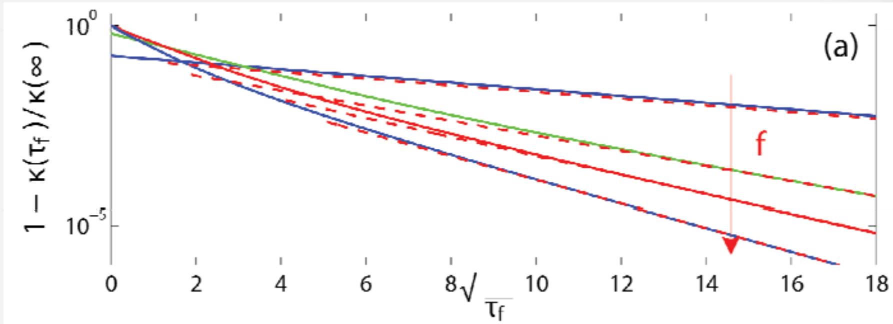
Dissipation in interacting Bose-gases

interplay: dissipation and interaction

- impeding of decoherence due to interaction blocking
- glass like dynamics due to rare states with long time scales
- dissipation probe of entire energy structure





open questions:

- can interaction be used to prevent dissipation?
- implications for heating of complex states in cold gases?

all this by adiabatic elimination + X

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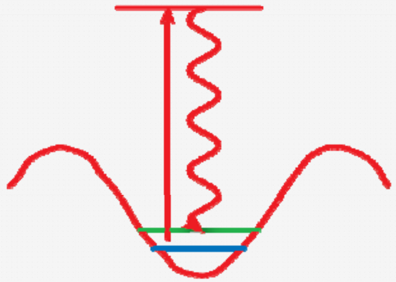
Fermions in a noisy, incoherent 'magnetic field'

dissipative effects:
polarization measurements, eg. by light scattering
or in solids local magnetic field fluctuations



described by Markovian master equation:

closed system dynamics

$$i\hbar \partial_t \hat{\rho} = [\hat{H}, \hat{\rho}] + i\hbar\, \mathcal{D}(\hat{\rho})$$

dissipative dynamics

$$\mathcal{D}(\hat{\rho}) = \gamma \sum_{l=1}^{L} \left(\hat{n}_{s,l} \hat{\rho} \hat{n}_{s,l} - \frac{1}{2} \hat{n}_{s,l}^2 \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{n}_{s,l}^2 \right)$$

ρ density matrix
γ dissipative coupling
n_s spin density
H system Hamiltonian:
Fermi-Hubbard model

procedure: prepare system in ground state
switch on dissipation

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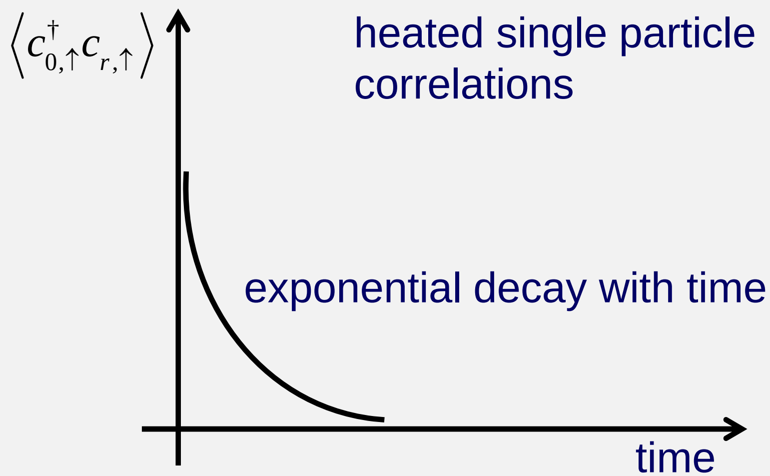
Only dissipation (H = 0)

single particle correlations:

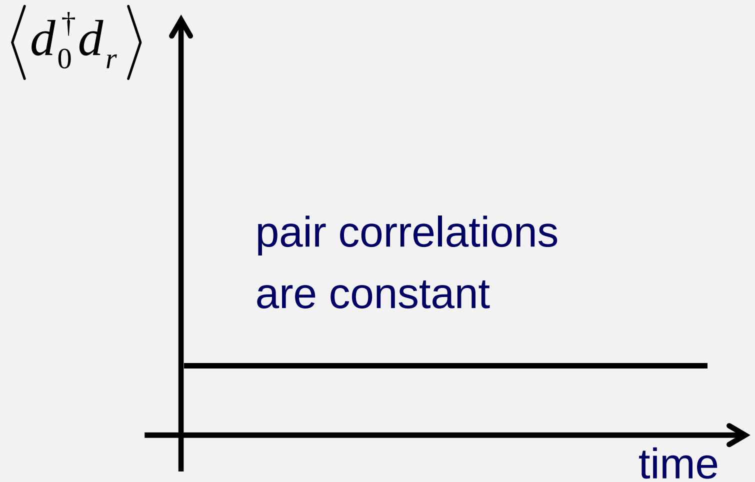
$$\frac{\partial}{\partial t}\left\langle c_{0,\uparrow}^{\dagger} c_{r,\uparrow} \right\rangle = -\Gamma \left\langle c_{0,\uparrow}^{\dagger} c_{r,\uparrow} \right\rangle$$



pair correlations are ***immune***! $n_s \left|\uparrow\downarrow\right\rangle = 0$

$$\frac{\partial}{\partial t} f_r = 0 \qquad f_r = \left\langle d_0^{\dagger} d_r \right\rangle$$

$$d_j = c_{j,\uparrow} c_{j,\downarrow}$$



decoherence free subspace:

all combinations of doubly occupied sites

+ diagonal states

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Effective equations

complicated effective dynamics

go to equation of motion for special operators $d_j = c_{j,\uparrow} c_{j,\downarrow}$

$$\frac{d}{dt}\hat{d}_{\mathbf{r}} = \sum_{\substack{\mathbf{r}',\,|\mathbf{r}-\mathbf{r}'|=1 \\ \alpha,\alpha'=1,\ldots,4}} \frac{J^2}{\hbar^2} \frac{\mathcal{P}^0_{\mathbf{r}} \mathcal{P}^0_{\mathbf{r}'} \left[\hat{K}_{\mathbf{r},\mathbf{r}'} , \mathcal{P}^{\alpha}_{\mathbf{r}} \mathcal{P}^{\alpha'}_{\mathbf{r}'} \left[\hat{K}_{\mathbf{r},\mathbf{r}'} , \hat{d}_{\mathbf{r}} \right] \right]}{\lambda_\alpha + \lambda_{\alpha'} + i\frac{U}{\hbar}}$$

$$= \sum_{\mathbf{r}',\,|\mathbf{r}-\mathbf{r}'|=1} \hat{A}_{\mathbf{r}'} \hat{d}_{\mathbf{r}} + \hat{A}_{\mathbf{r}} \hat{d}_{\mathbf{r}'} \,,$$

$$\text{with } \hat{A}_{\mathbf{r}} = -2\frac{J^2}{\hbar^2} \left\{ \frac{(\hat{p}_{\mathbf{r},\uparrow} + \hat{p}_{\mathbf{r},\downarrow})}{\Gamma} + \frac{\hat{p}_{\mathbf{r},\downarrow\uparrow}}{\Gamma + i\frac{U}{\hbar}} + \frac{\hat{p}_{\mathbf{r},0}}{\Gamma - i\frac{U}{\hbar}} \right\}$$

mean field decoupling for alternating pairing correlation $\tilde{f}_r = (-1)^{r+1} f_r \quad with \; f_r = \langle d_0^\dagger d_r \rangle$

$$\frac{\partial}{\partial t} \tilde{f} = \frac{d}{dr}\left(D(f_0) \frac{d}{dr} \tilde{f} \right), \quad D(f_0) = -8\left(\frac{J^2}{\Gamma}(1 - 2f_0) + 2\frac{J^2 \Gamma}{\Gamma^2 + U^2} f_0 \right)$$

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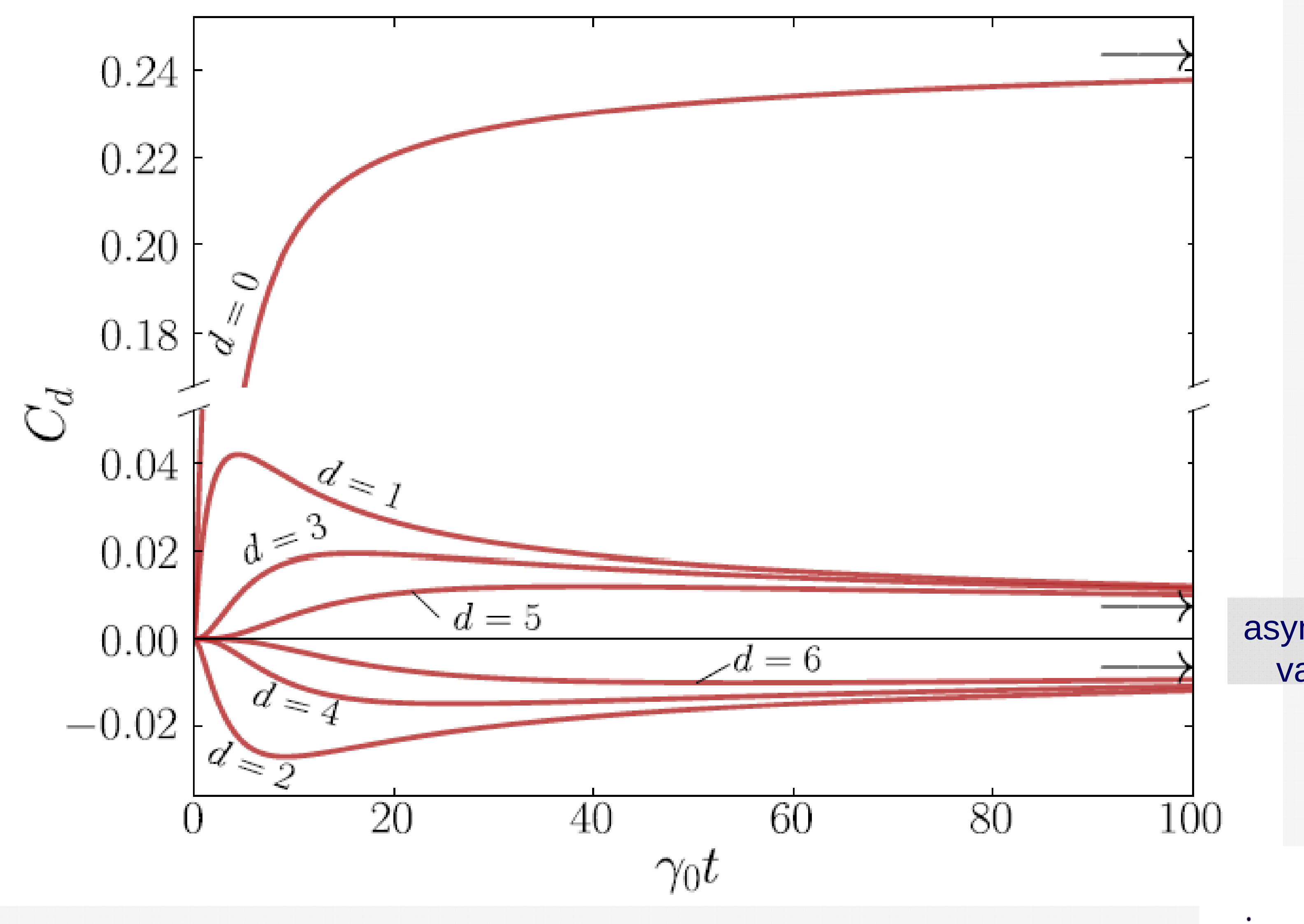
Formation of pair correlations

procedure: prepare system in ground state and switch on dissipation



asymptotic value ~1/L

Symmetries of the Hubbard model

U/J=12

γ/J=8, J=1

number of η-pairs is a conserved quantity

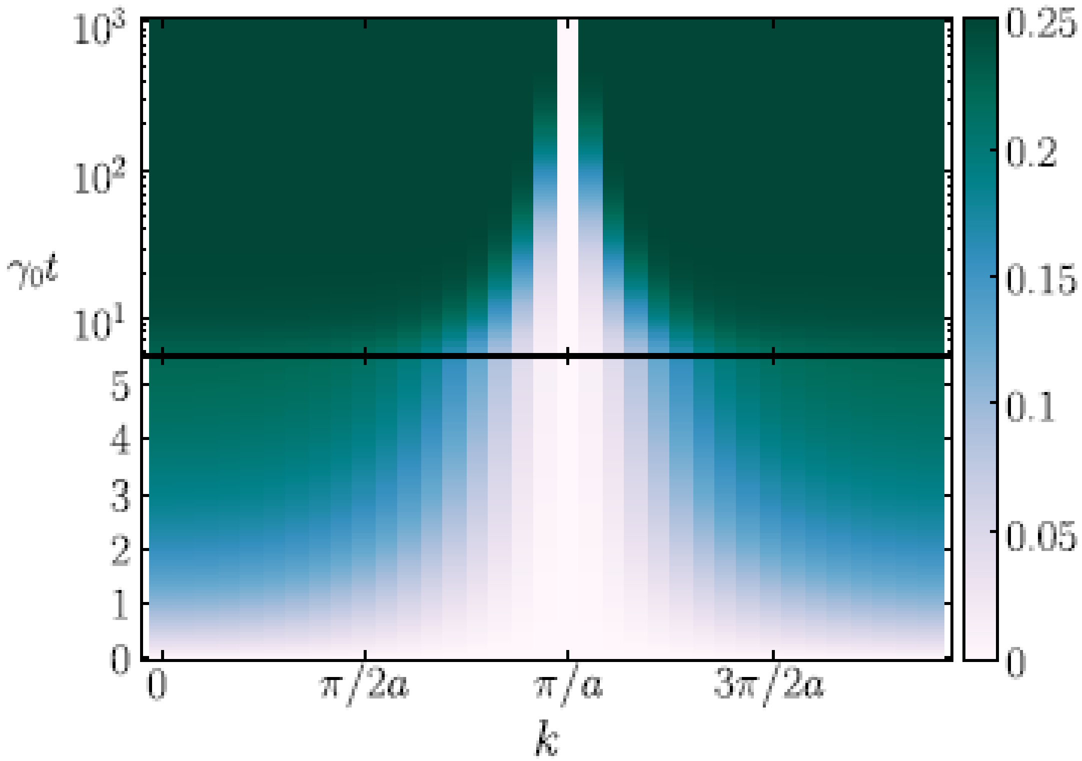
$$\langle \eta^\dagger \eta \rangle = f_{k=\pi/a} = const.$$

$$\eta^\dagger = \sum_r e^{i\pi r} c^\dagger_{r\uparrow} c^\dagger_{r\downarrow}$$

related to momentum distribution

$$f_k = \frac{1}{V} \sum_d e^{-ikd} f_d$$

pair momentum distribution



no. of local pairs
~ longer distance correlations

$$f_{\pi/a} = \frac{1}{V}\left(f_0 + \sum_{d>0} e^{-ikd} f_d\right)$$

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Diffusion of staggered correlations $\tilde{f}_r = (-1)^{r+1} f_r$

diffusion equation (U,Γ>>1): $\frac{\partial}{\partial t}\tilde{f} = \frac{d}{dr}\left(D(f_0)\frac{d}{dr}\tilde{f}\right), \quad D(f_0) = -8\left(\frac{J^2}{\Gamma}(1-2f_0) + 2\frac{J^2\Gamma}{\Gamma^2+U^2}f_0\right)$

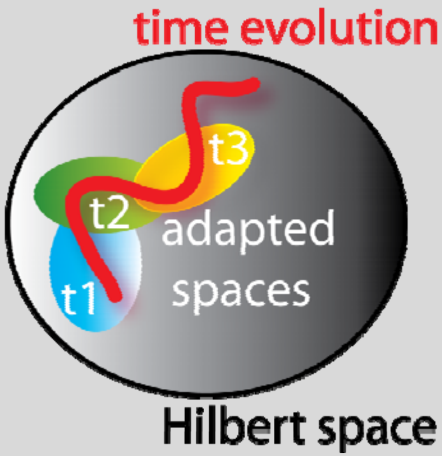
meta stable state with pair coherence

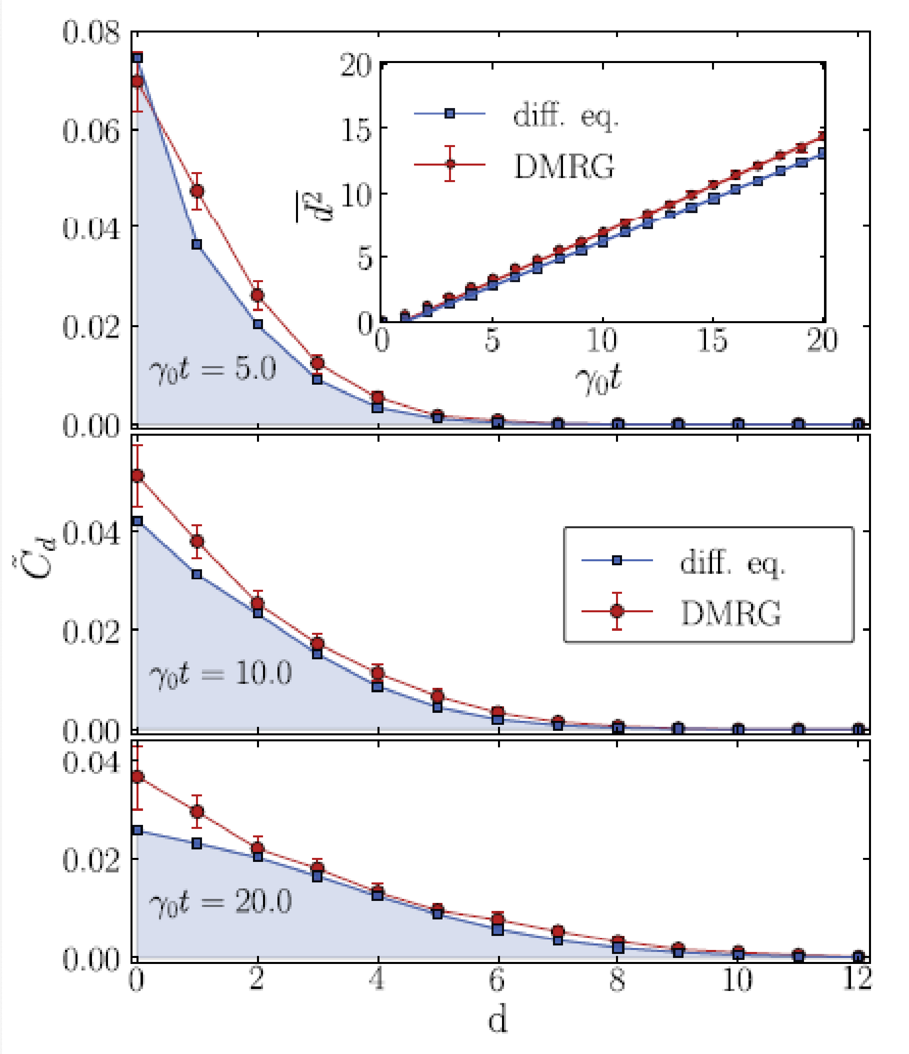
here 1D

comparison to numerical stochastic wave function with time-dependent DMRG

A. Daley, C.Kollath, U. Schollwöck, G. Vidal (2004)

S. White and A. Feiguin (2004)

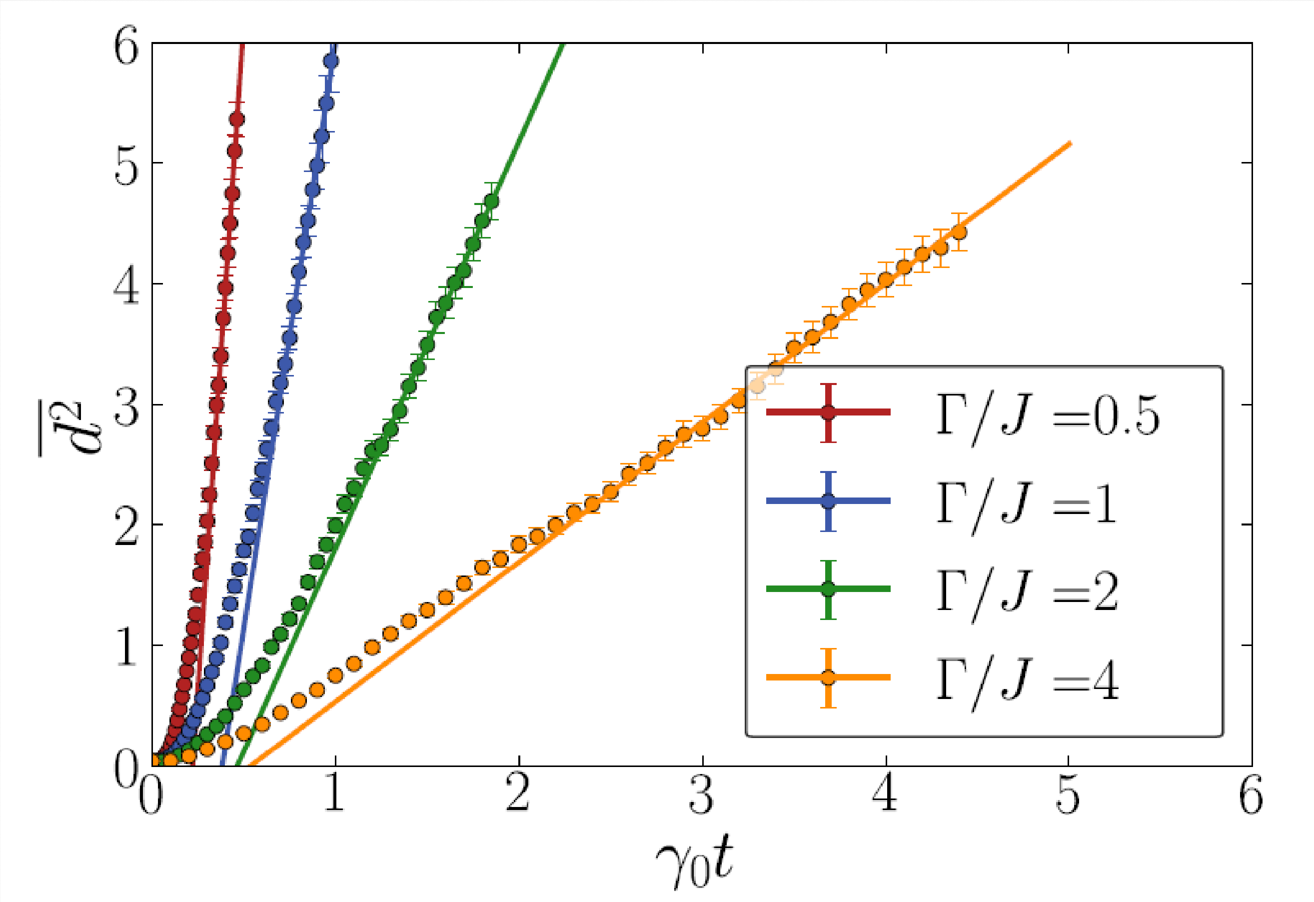




(see talk of S. Manmana for DMRG)

Variance of distribution of pair correlations

$$\left\langle d^2 \right\rangle = \frac{1}{N}\sum_r r^2 \tilde{f}_r \qquad \text{for diffusion} \qquad \left\langle d^2 \right\rangle = zD(t)t$$



- diffusion far beyond validity of adiabatic elimination approximation

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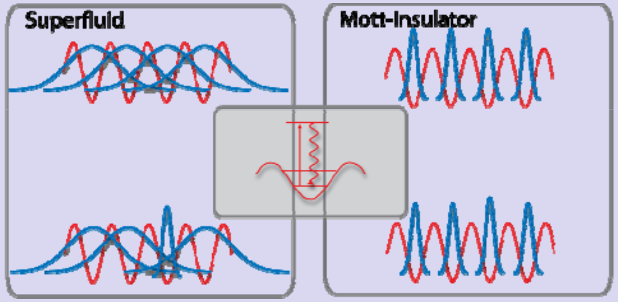
Dissipative dynamics in interacting systems

interplay: dissipation and interaction

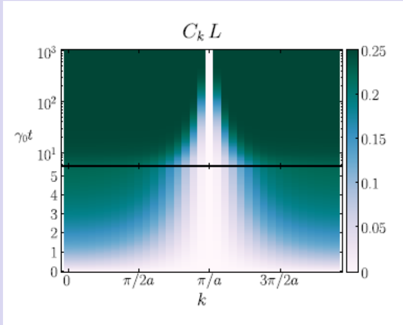


- impeding of decoherence due to interaction blocking
- glass like dynamics due to rare states with long time scales

open questions:

- can interaction be used to prevent dissipation?
- suppression of heating?

➢ ***create pair correlations by local dissipation***



- creation of metastable state with pair coherence
- diffusive dynamics
- long time pair coherence ~1/L in finite systems

adiabatic elimination is a powerfull method, (any D)

complement to short time methods as e.g. mean-field methods or time-dependent DMRG

Thanks to

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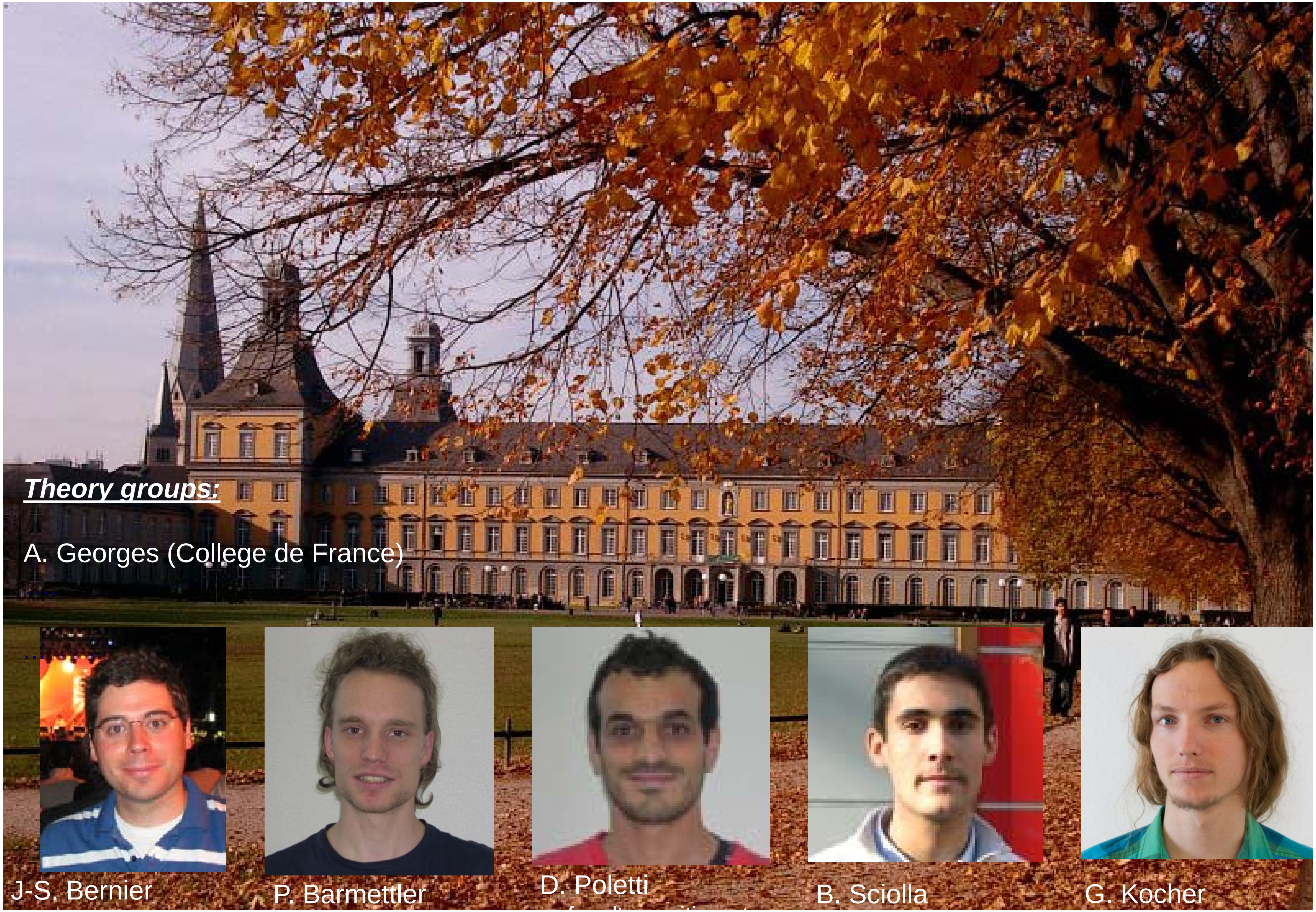
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