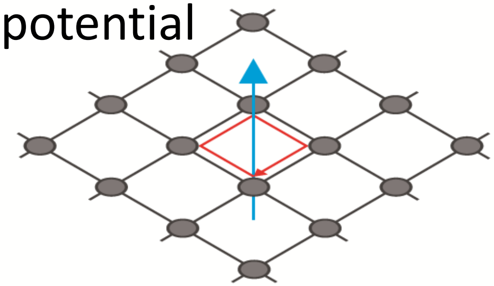
Ultracold Quantum Gases

Part 3: Artificial gauge potentials

Ultracold Quantum Gases

3.4 Artificial magnetic field on a lattice

- Magnetic phenomena in the presence of a spatially periodic potential



  » Competition between two length scales

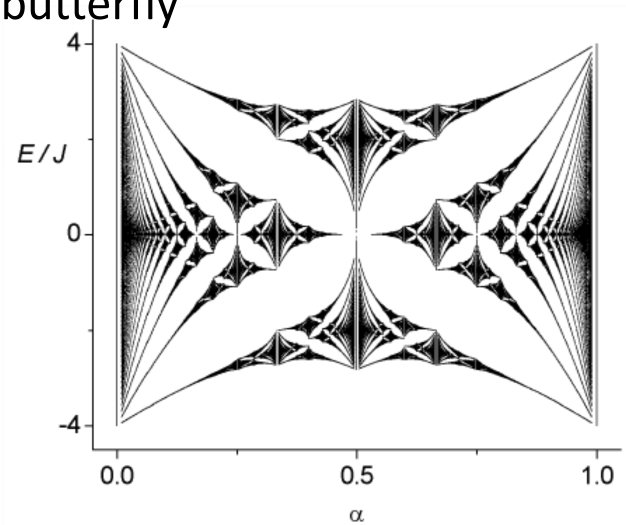
  Lattice spacing: *a*

  Magnetic length: $l_{\mathrm{mag}} = \sqrt{\frac{\hbar}{eB}}$

  » New phenomena when $l_{\mathrm{mag}} \approx a$

  Fractal structure for the energy spectrum “Hofstadter butterfly”

$$\frac{a^2}{l_{\mathrm{mag}}^2} = \frac{e\,Ba^2}{\hbar} = 2\pi\frac{\Phi}{\Phi_0} \qquad \Phi_0 = h/e$$



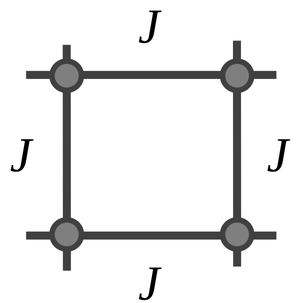
- Experimental realization with quantum gases

  allows reaching strong fields in optical lattices

$$l_{\mathrm{mag}} \approx a \Leftrightarrow \Phi \approx \Phi_0$$

Hubbard Model

- Hubbard model
  - » 2D square lattice
  - » Single-band
  - » Nearest-neighbor hopping J



$$\hat{H}_{\text{Hubbard}} = -J \sum_{j,l} (|j+1,l\rangle\langle j,l| + |j,l+1\rangle\langle j,l|) + hc$$

- Eigenstates and eigenenergies
  - » Bloch states

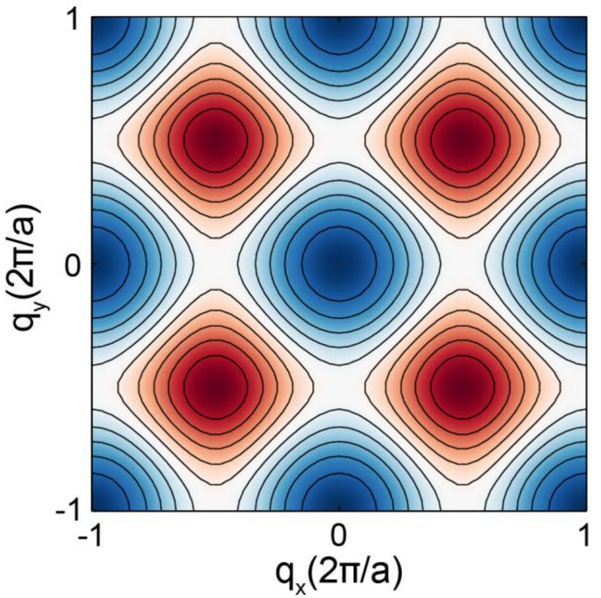
$$|\psi(\boldsymbol{q})\rangle = \sum_{j,l} e^{ia(jqx+lqy)} |j,l\rangle$$

  - » Eigenenergies

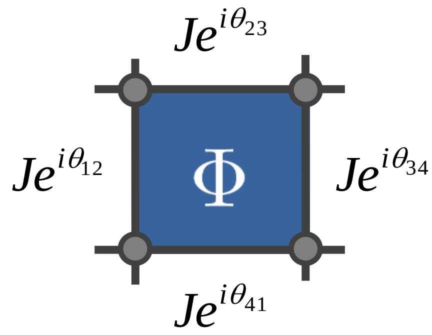
$$E(q) = -2J\left[\cos(aq_x) + \cos(aq_y)\right]$$

Reduction to the 1st Brillouin zone

Band centered around *E=0* with a full width of *8J*



Gauge potentials on a lattice - Peierls phase

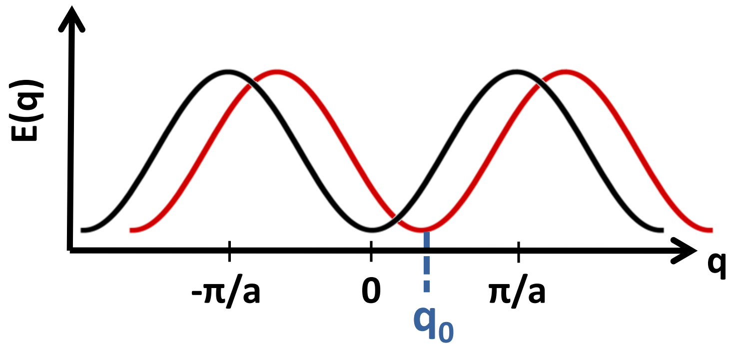


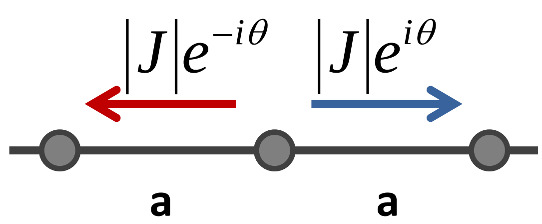
- Peierls substitution
  - » Presence of a gauge potential

    $\leftrightarrow$ Complex tunneling matrix element
  - » Peierls phase $\theta_{i,j} = \frac{e}{\hbar} \int_{R_i}^{R_j} \boldsymbol{A}(\boldsymbol{r}) \cdot d\boldsymbol{r}$
  - » Magnetic flux through a plaquette - Aharonov-Bohm phase

$$\sum \theta_{ij} = \frac{e}{\hbar} \oint \boldsymbol{A}(\boldsymbol{r}) \cdot d\boldsymbol{r} = \frac{e}{\hbar} \iint \boldsymbol{B}(\boldsymbol{r}) \cdot d\boldsymbol{S} = 2\pi \frac{\Phi}{\Phi_0}$$

- Gauge potential in momentum space

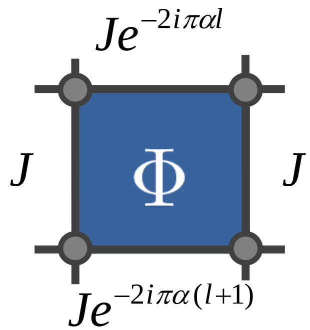




$$|\psi(q_0)\rangle = \sum_j e^{iajq_0}|j\rangle \Rightarrow q_0 a = \theta$$

Harper Hamiltonian - Hofstadter butterfly

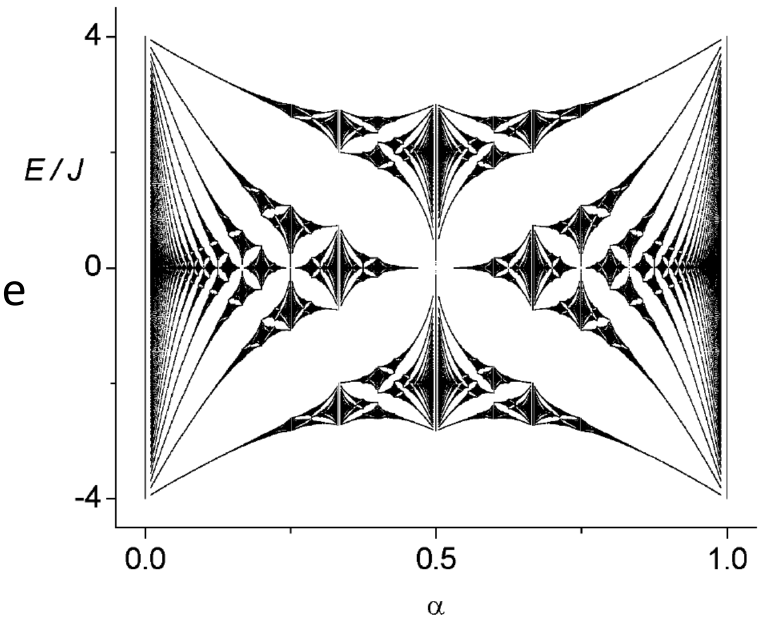
- Particle moving on a square lattice in presence of a magnetic field
  - » Same flux through each plaquette $\Phi = \alpha\Phi_0$
  - » Landau gauge $\boldsymbol{A} = -By\,\boldsymbol{e}_x$
  - » Peierls phase $\theta(|j,l\rangle \rightarrow |j,l+1\rangle) = 0$
    $\theta(|j,l\rangle \rightarrow |j+1,l\rangle) = -2\pi\alpha l$



- Harper Hamiltonian

$$\hat{H}_{\text{Harper}} = -J\sum_{j,l}\left(e^{-i2\pi\alpha l}|j+1,l\rangle\langle j,l| + |j,l+1\rangle\langle j,l|\right) + hc$$

- Energy spectrum: Hofstadter butterfly
  - » Invariant under $\alpha \rightarrow \alpha + 1$
    → study of the spectrum for $0 \leq \alpha < 1$
  - » Magnetic field breaks the translational invariance along *y*
  - » Fractal structure

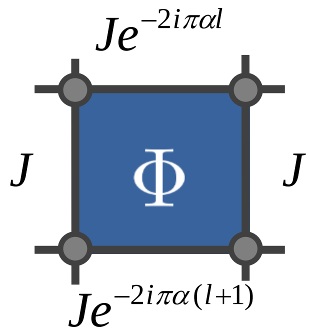


Harper Hamiltonian - Hofstadter butterfly

- Rational values of the flux $\alpha = p'/p$
  - » Translational symmetry restored along y

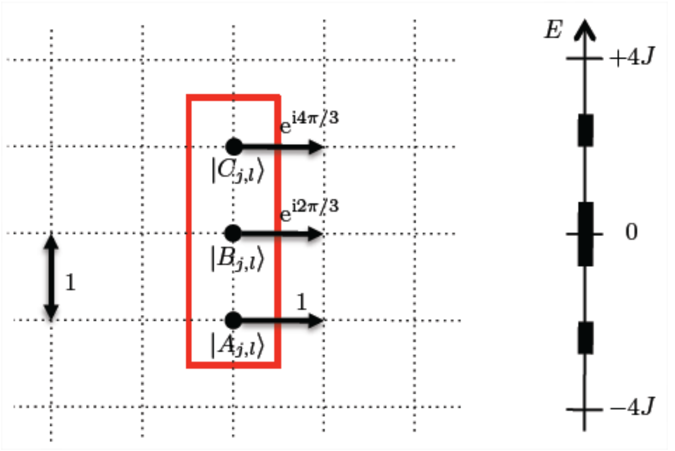
$$\theta(|j, l+p\rangle \rightarrow |j+1, l+p\rangle) = -2\pi\alpha(l+p)$$
$$= \theta(|j, l\rangle \rightarrow |j+1, l\rangle) \quad \text{modulo } 2\pi$$

  - » Increased spatial period *pa*: magnetic unit cell



- Case α=1/3
  - » Magnetic unit cell: length of *a* along *x* and *3a* along *y*
  - » Each unit cell contains 3 sites

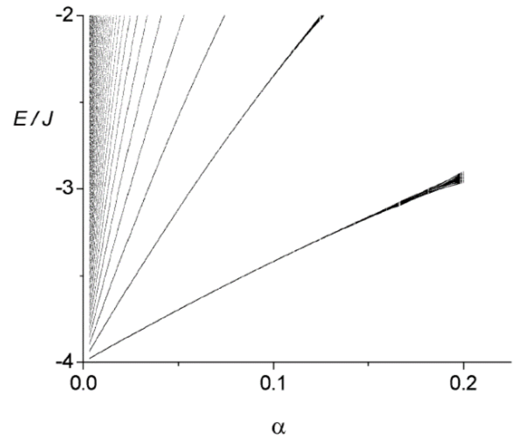
    → Splitting of the energy spectrum in 3 sub-bands

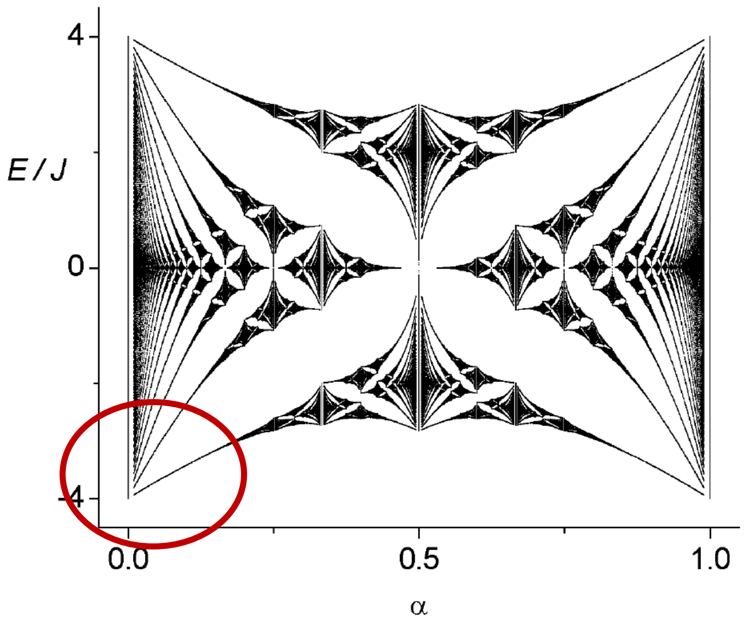


- Origin of the fractal structure
  - » α=1/3 and α=10/31: very close values of α
  - » But very different results as 3 or 31 sub-bands!

Harper Hamiltonian - Hofstadter butterfly

- Recovering the Landau levels
  - » For low magnetic fluxes: $l_{\text{mag}} \gg a$
  - » Analog to a free particle in a static magnetic field
  - » Landau levels?



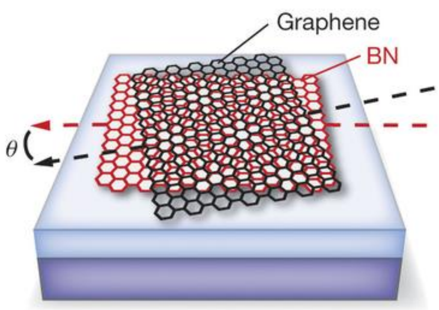


Übungsblatt 9

- Measurement of the Hofstadter butterfly

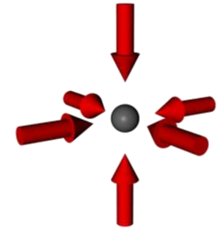
$$\Phi = \Phi_0 \Leftrightarrow B \approx \Phi_0/a^2$$

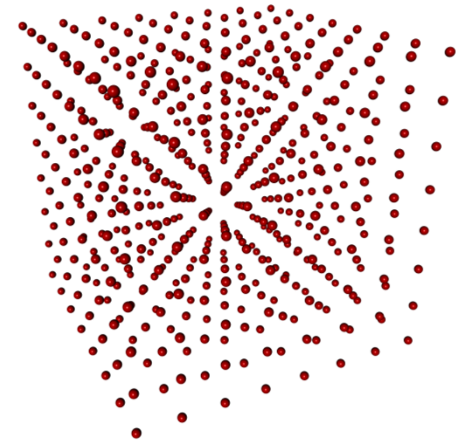
  - » Solid state systems $a \approx 1\,A \Rightarrow B \approx 4\,10^5 T$
  - » Realized using the Moiré pattern in monolayer graphene
  - » Quantum gases?



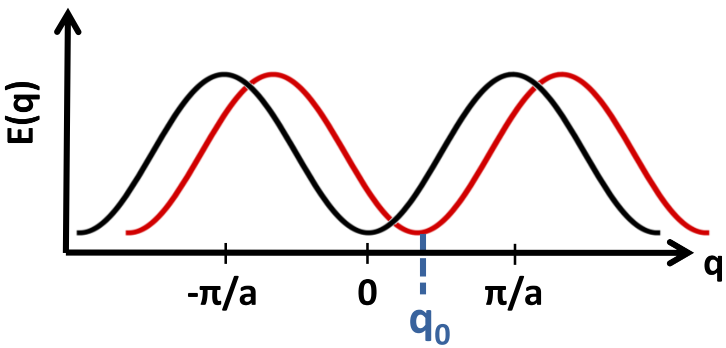
Generating *artificial* gauge potentials on a lattice

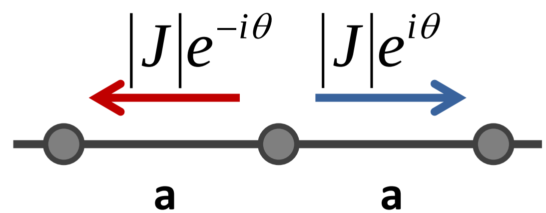
- Natural tunneling in an optical lattice
  - » Well controlled with the lattice depth
  - » Tunneling = hopping probability $J \geq 0$





- Getting complex tunneling
  - » Shift the dispersion relation





$$|\psi(q_0)\rangle = \sum_j e^{iajq_0}|j\rangle \Rightarrow q_0 a = \theta$$

  - » Strong field regime reachable as one simulates directly the Peierls phase

$$0 \leq \theta < 2\pi$$

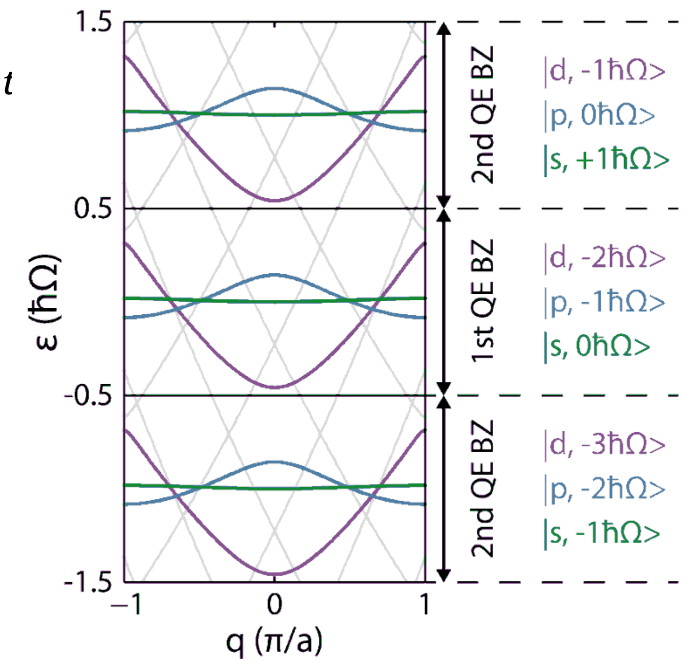
Generating *artificial* gauge potentials on a lattice

- Band engineering via periodic driving: “Floquet engineering”
  - » Periodic driving of the quantum system $\hat{H}(t+T) = \hat{H}(t)$
  - » Analog to the Bloch theorem in time

    Eigenstate: Floquet states $|\psi_n(t)\rangle = |u_n(t)\rangle e^{-i\epsilon_n t}$

    $$|u_n(t+T)\rangle = |u_n(t)\rangle$$
  - » Floquet theorem

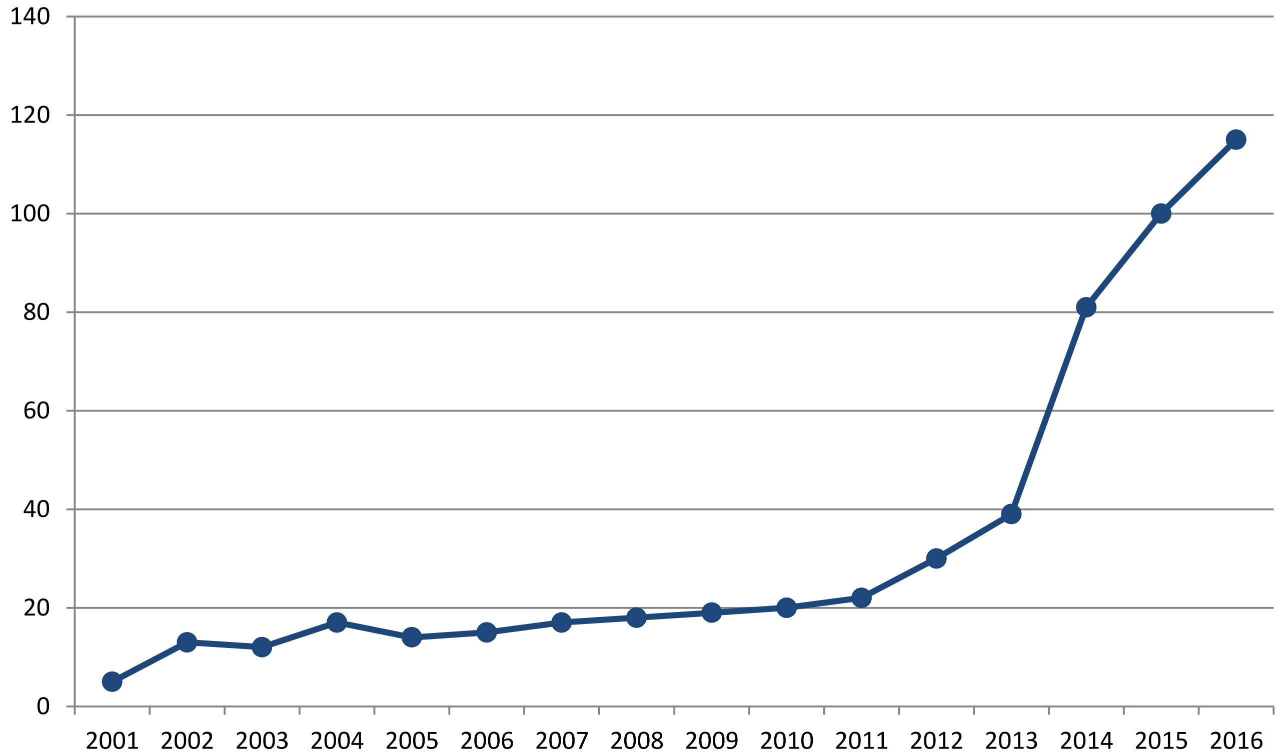
    $$|U(t_1,t_2)\rangle = P(t_2) e^{iH_{\text{eff}}(t_1-t_2)} P^{+}(-t_1)$$



- High frequency limit
  - » Faster than all other timescales in the system
  - » Effective Hamiltonian is time independent

    $$\hat{H}_{\text{eff}} = \langle \hat{H}(t) \rangle_T$$
  - » New properties can emerge in the effective Hamiltonian, especially gauge fields

Band engineering via periodic driving



Publications with the word „Floquet" in the abstract:
(Source: arXiv:condensed matter section)

Realization of artificial magnetic fields on a lattice

» Periodic acceleration of the optical lattice

» Periodic modulation of the lattice depth

Band engineering via lattice shaking

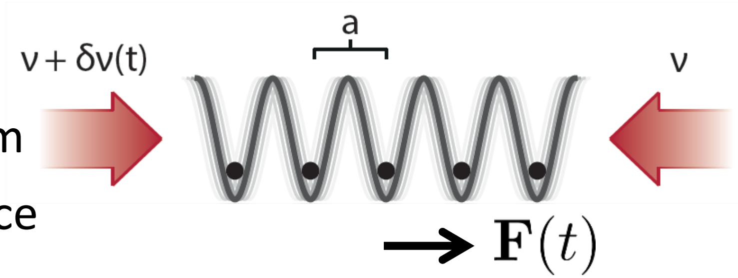
- Lattice shaking
  - » Modification of the frequency of one lattice beam
  - » Acceleration of the lattice in space → inertial force

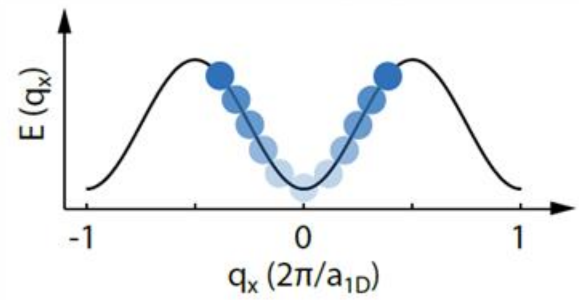
$$\boldsymbol{F}(t) = -m\ddot{\boldsymbol{r}}(t)$$

  - » Semi-classical equation for the quasi-momentum

$$\hbar\dot{\boldsymbol{q}}_k(t) = \boldsymbol{F}(t)$$

  - » Time-periodic force with zero mean value $\langle \boldsymbol{F}(t)\rangle_T = 0$



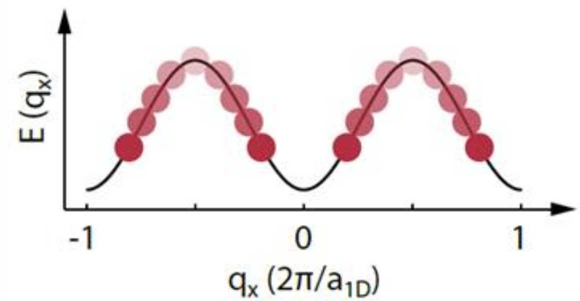


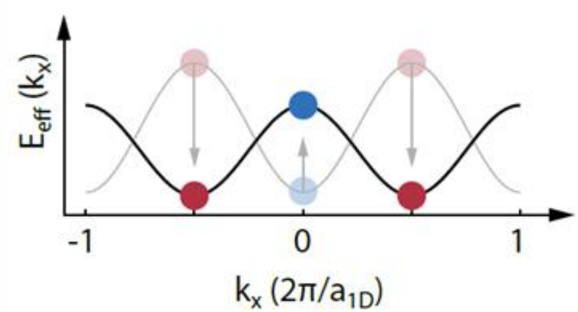
- Renormalization of the band structure in 1D
  - » Sinusoidal shaking $F(t) = F_0 \sin(\omega t)$

$$\Rightarrow q_k(t) = k + \frac{F_0}{\hbar\omega}\cos(\omega t)$$

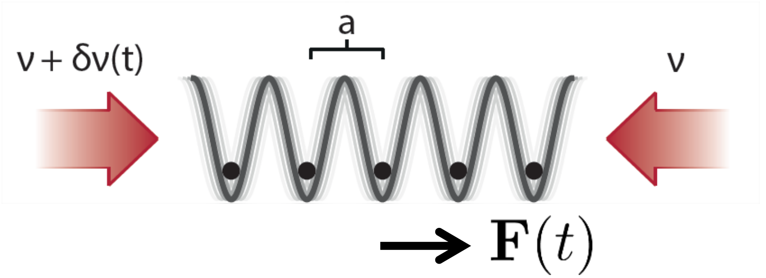
  - » Effective band-structure

$$\mathrm{E_{eff}}(k) = \frac{1}{T}\int_0^T E\big(q_k(\tau)\big)d\tau$$





Band engineering via lattice shaking



- Effective tunneling
  - » Band structure and tunneling

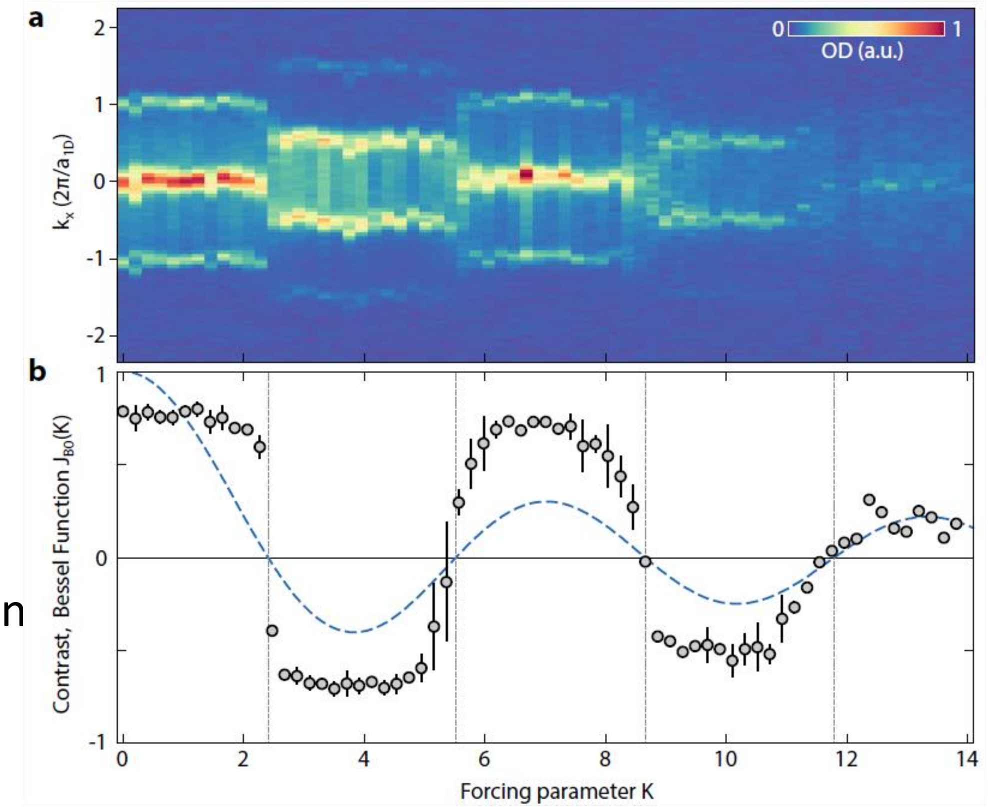
    $$E(q) = -2J_{\text{bare}}\cos(qa)$$

  - » Effective tunneling

    $$\mathrm{E}_{\text{eff}}(k) = \frac{1}{T}\int_0^T E\big(q_k(\tau)\big)d\tau = -2J_{\text{eff}}\cos(ka)$$

    $$J_{\text{eff}} = J_{\text{bare}}J_0(K)$$

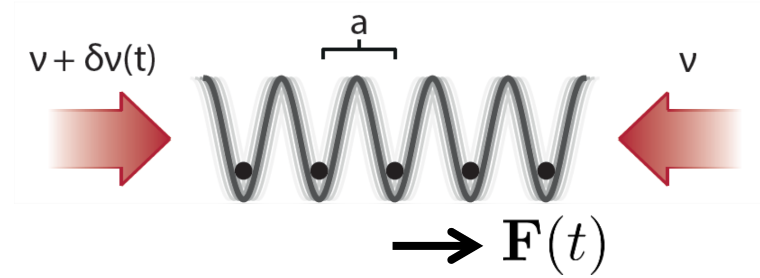
    $$K = \frac{F_0 a}{\hbar\omega}$$



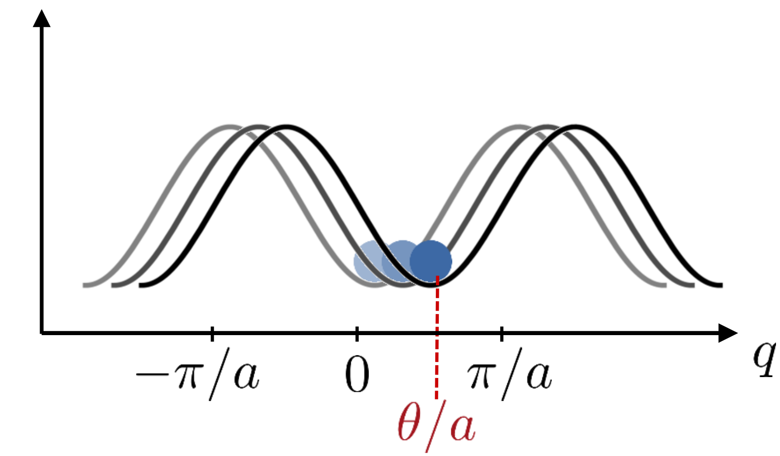
- Measurement with a condensate
  - » BEC: occupies the minimal energy quasi-momentum *k*
  - » Quasi-momentum retrieved after time-of-flight expansion for different forcing amplitude K

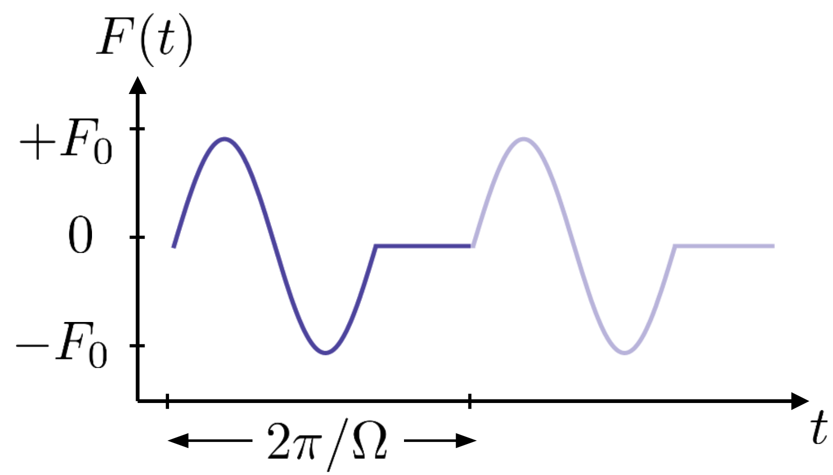
Band engineering via lattice shaking

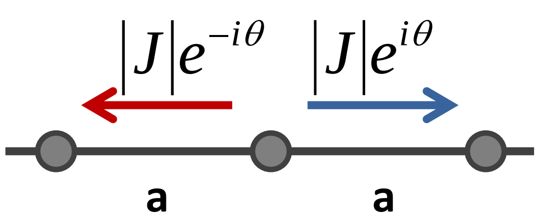
- Realization of complex tunneling
  - » Inertial force asymmetric around q=0

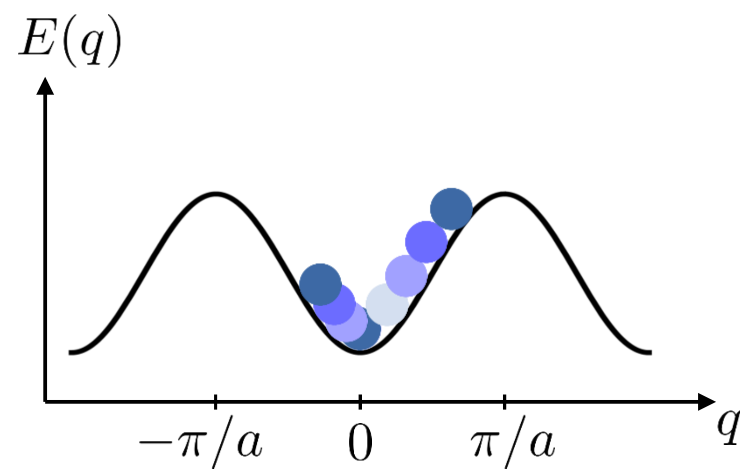


$$E_{\text{eff}}(q) = E(q + \theta/a)$$



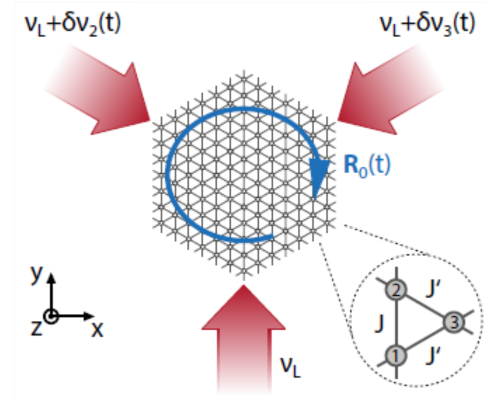




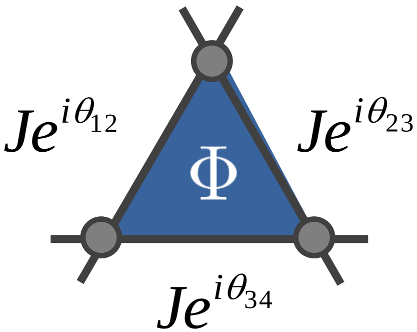


Sengstock, Simonet, Phys. Rev. Lett. **108**, 225304 (2012)

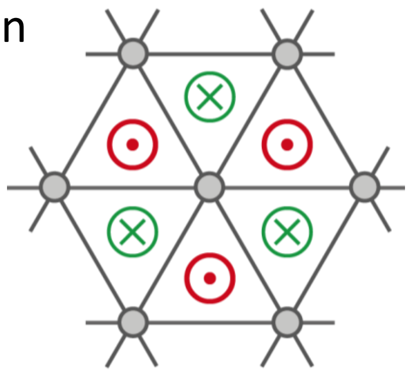
Band engineering via lattice shaking



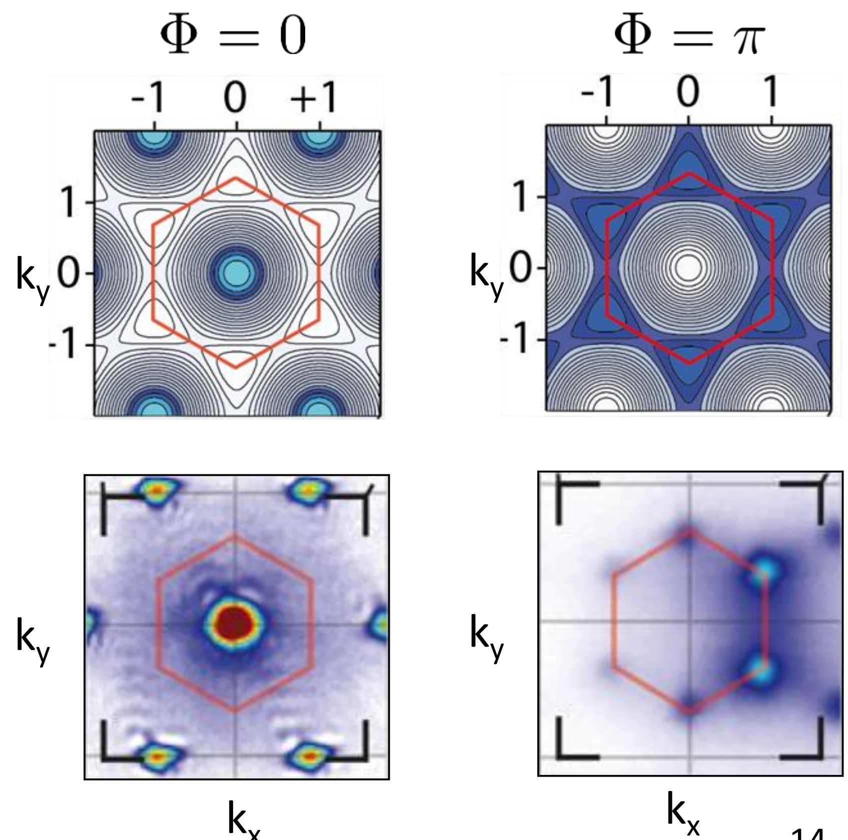
- Realization of artificial magnetic fluxes
  » Shaking of a triangular lattice → complex tunneling



  » Alternating flux pattern

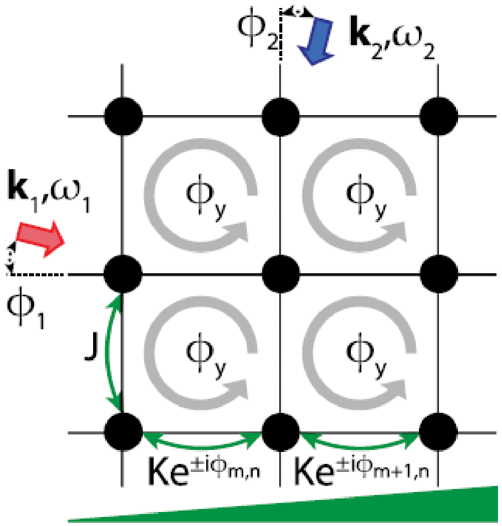


  » Modification of the band structure can be retrieved after time-of-flight

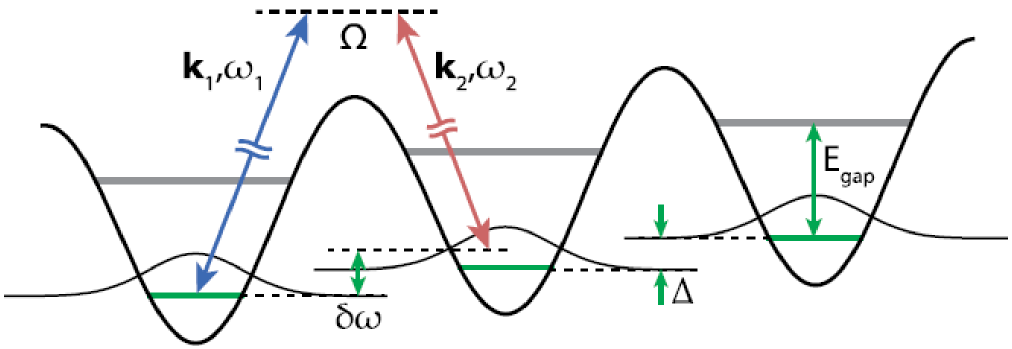


Sengstock, Simonet, Nature Physics **9**, 738 (2013)

Band engineering via amplitude modulation



- Tilted optical lattice
  - » Along *y*: standard lattice potential, tunneling J
  - » Along *x*: tilted potential using a magnetic field gradient
    → tunneling suppressed by the energy offset Δ
- Raman coupling
  - » Restore the tunneling along *x (photon-assisted tunneling)*



$$\omega_1 - \omega_2 = \Delta/\hbar$$

$$\delta\boldsymbol{k} = \boldsymbol{k}_1 - \boldsymbol{k}_2$$

$$\boldsymbol{R}_{m,n} = m\boldsymbol{d}_x + n\boldsymbol{d}_y$$

- » Realization of complex tunneling

$$K_{\text{pert}} = \frac{\Omega}{2}\int d^2\boldsymbol{r}\, w^*(\boldsymbol{r} - \boldsymbol{R}_{m,n})e^{-i\,\delta\boldsymbol{k}\cdot\boldsymbol{r}}w(\boldsymbol{r} - \boldsymbol{R}_{m,n} - a\boldsymbol{e}_x) = K\,e^{-i\,\delta\boldsymbol{k}\cdot\boldsymbol{R}_{m,n}}$$

Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

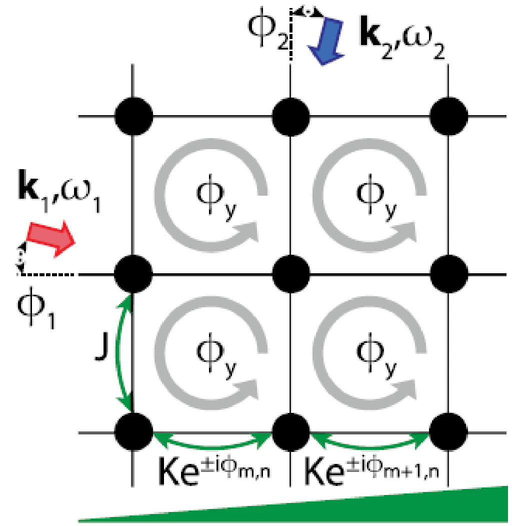
Band engineering via amplitude modulation

- Realization of the Harper Hamiltonian
  - » Accumulated phase around a closed path $\Phi_y = \delta k_y a$
  - » Raman beams propagating along x and y

$$\Phi_y = k_L a = \pi \Rightarrow \alpha = 1/2$$

  - » Tuning the flux: alignment of the Raman beams

$$\hat{H}_{\text{Harper}} = -J \sum_{j,l} \left(e^{-i2\pi\alpha l} |j+1,l\rangle\langle j,l| + |j,l+1\rangle\langle j,l| \right) + hc$$



- Amplitude modulation
  - » This scheme can be understood in the frame of Floquet theory
  - » Raman beams create a local optical potential

$$V_K(\boldsymbol{r}) = V_K^0 \cos^2\left(\frac{\delta\boldsymbol{k}\cdot\boldsymbol{r}}{2} + \frac{\omega t}{2}\right)$$

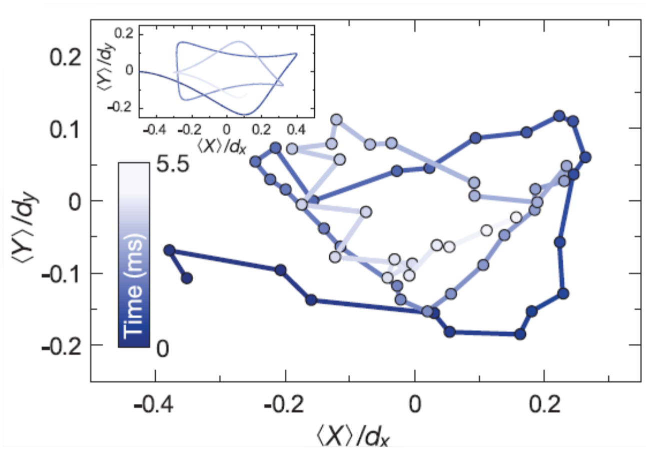
Induce a time-periodic on-site modulation of the lattice depth

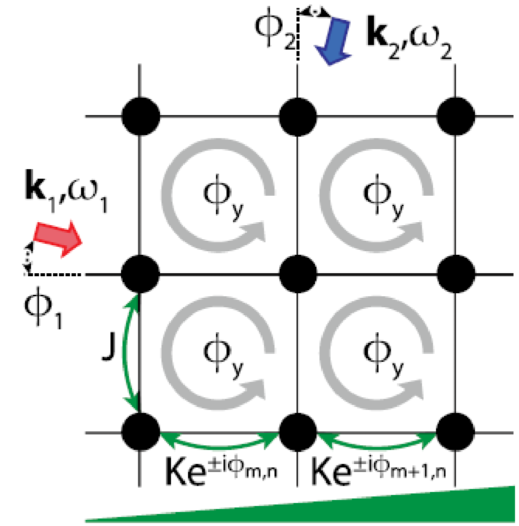
  - » Spatially dependent phases not necessary in the minimal implementation

Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

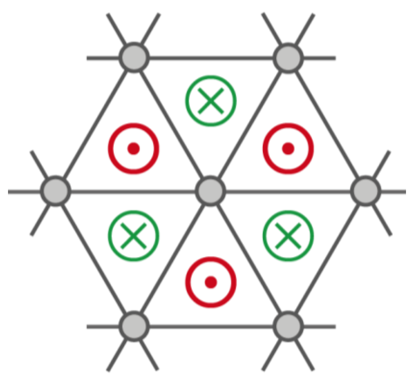
Band engineering via amplitude modulation

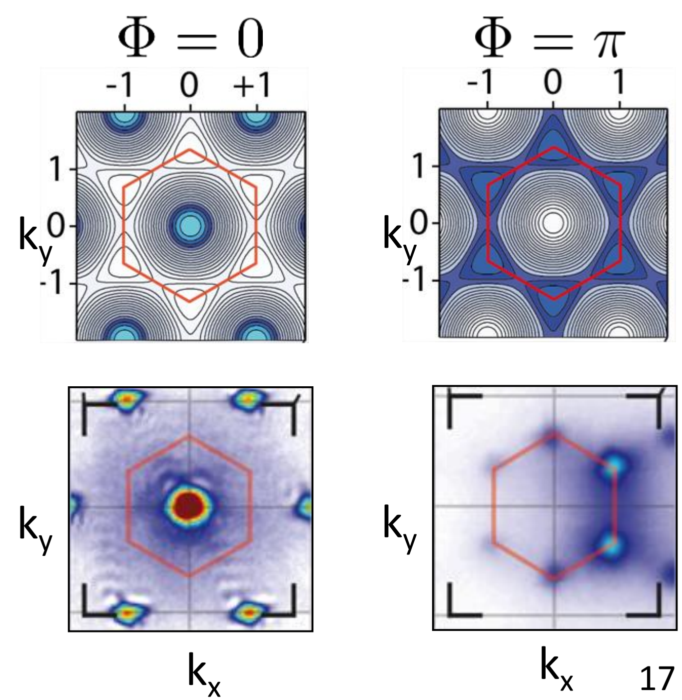
- Cyclotron motion of mass currents
  - » Cyclotron orbit around a square plaquette





  - » Finite mass current ↔ finite quasi-momentum





Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

Artificial magnetic fields on lattices - Status

- Harper Hamiltonian
  - » Realized

Selected for a Viewpoint in *Physics*

PRL **111,** 185302 (2013) PHYSICAL REVIEW LETTERS week ending 1 NOVEMBER 2013

Realizing the Harper Hamiltonian with Laser-Assisted Tunneling in Optical Lattices

Hirokazu Miyake, Georgios A. Siviloglou, Colin J. Kennedy, William Cody Burton, and Wolfgang Ketterle

  - » Realized with quantum gases

nature physics ARTICLES

PUBLISHED ONLINE: 10 AUGUST 2015 | DOI: 10.1038/NPHYS3421

Observation of Bose–Einstein condensation in a strong synthetic magnetic field

Colin J. Kennedy*, William Cody Burton, Woo Chang Chung and Wolfgang Ketterle

Fluxes fully tunable but heating as to be taken care of…

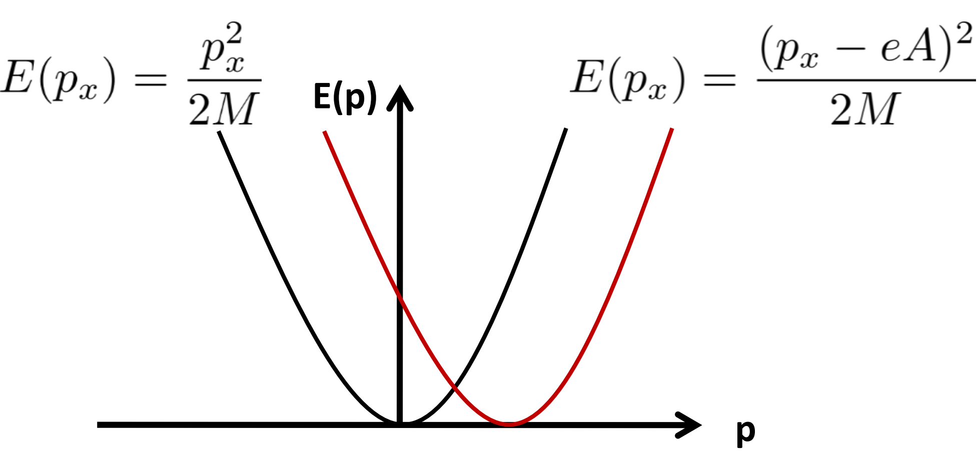
Bloch / Ketterle, Phys. Rev. Lett. **111** (2013)

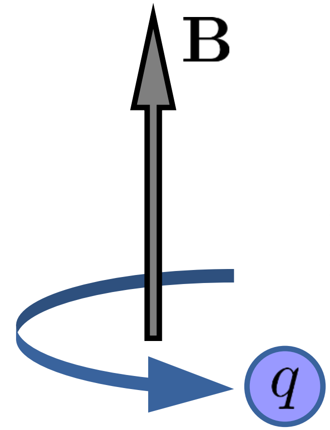
Generating *artificial* gauge potentials on a lattice

Momentum space

Real space

Free particle

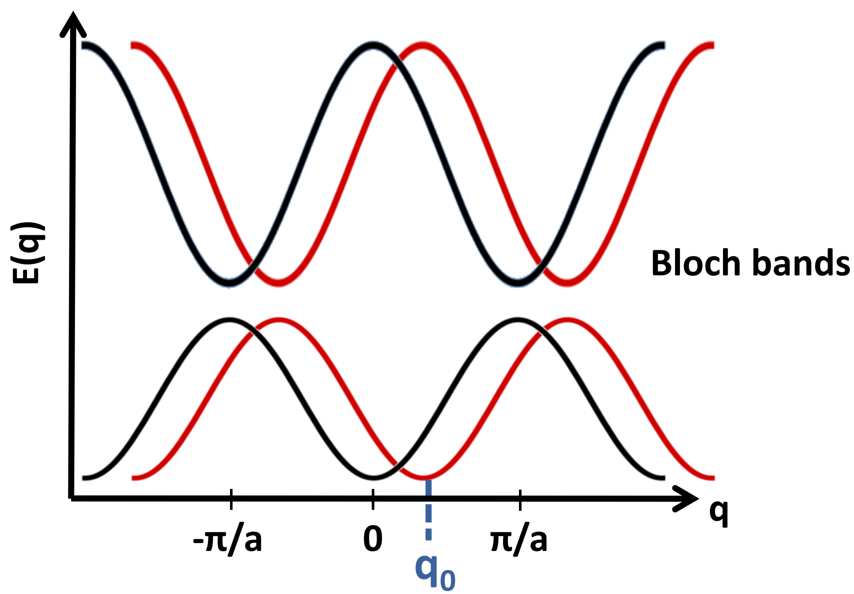


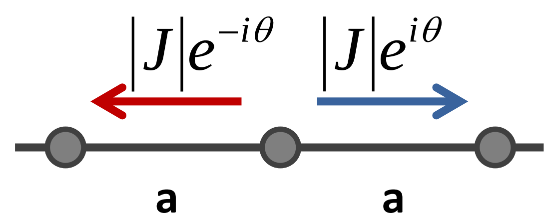


$$\vec{B}(\vec{r},t) = \vec{\nabla} \times \vec{A}(\vec{r},t)$$

$$\vec{E}(\vec{r},t) = -\frac{\partial \vec{A}(\vec{r},t)}{\partial t}$$

Particle on a lattice





$$q_0 a = \theta$$