Anwesenheitsübung on 08.01.2018:

4.A Scattering length of the spherical potential well

During the lecture, we have seen a conceptually very important case for scattering problems, the spherical potential well. Using the general concept of differential continuity, we have found a solution for the radial Schrödinger equation, which was then used to find an expression for the scattering length. In this problem, we'll consider a somewhat simplified system using the 1D Schrödinger equation in the case of $l = 0$ to find an expression for the scattering length.

$$\chi_0'' + [k^2 - U(r)]\chi_0 = 0,$$

a. What do the general solutions of this equation look like?
Consider the difference between $r<r_0$ and $r>r_0$.

b. Using the logarithmic differential at the boundary condition χ_0'/χ_0 continuous at $r=r_0$, find an expression for the scattering phase shift η_0

c. From the lecture, you already know the expression for the scattering length
$$a = r_0\left(1 - \frac{\tan \kappa_0 r_0}{\kappa_0 r_0}\right)$$
At $\kappa_0 r_0 = (n + 1/2)\pi$ this expression diverges.
Show that this condition also means that a new **bound** state appears in the potential well. As in the lecture you should use in the above 1D Schrödinger equation, the replacement $k^2 \to -\kappa^2$.
What do the general solutions look like?
Again use the differential continuity concept to find an expression for the allowed states. (the general solution can be found graphically.)
Consider what it mathematically means 'to have a bound state appear' (what does it mean for κ?)

Hausübung:

4.1 Halo states (2p)

For extremely weakly bound states ($\kappa \to 0$), the expression 4.A.c can be further simplified.

a. What form does the scatterling length take in this case? 1p

b. Find an expression for the binding energy of these Halo states. 1p

4.2 Scattering theory (3p)

As a reminder, read in a Quantum mechanics book for your own choice (e.g. W. Nolting QM II; F. Schwabl, QM I, etc.) the introduction to scattering theory and answer the following questions:

a. Which problems can be described with the partial wave method?
What phenomena can't be described with the used approximations? 1p

b. Which consideration brings about the scattering phase shifts η_l? 1p

c. What is the advantage of using the partial wave expansion for scattering at low energies? 1p

4.3 Spherical potential well (5p)

Analogous to the in the lecture described spherical potential well, consider the spherical potential barrier given by $U(r) = V_0$ for $r < r_0$ and $U(r)=0$ for $r > r_0$. Consider the case Fall $E<V_0$. Use the wave number notation with $k^2 = 2mE/\hbar$ and $k_0^2 = 2m(E - V_0)/\hbar = -p^2 < 0$.

The dimensionless Schrödinger equation is the following:

$$\chi_0'' + q^2\chi_0 = 0$$

with

$$q^2 = \begin{cases} k_0^2 = -p^2, & r < r_0 \\ k^2 \quad\quad\quad , & r \geq r_0 \end{cases}$$

a. Make a sketch and mark k^2 and p^2. 0.5p

b. What are the solutions of the Schrödinger equation?
As always consider both the regions $r < r_0$ and $r > r_0$. 1.5p

c. Find an expression for the scattering length while using the differential continuity condition.
Use exercise 4.A.b as a blueprint. 2p

d. Make a sketch of the scattering length. Make sure that your solution is consistent by comparing with the hard sphere! Compare for small pr_0 i.e. $\kappa_0 r_0$ the potential barrier with the potential well. (→ Lecture). What interesting feature does the curve show? What can you learn from this about the sign of the scattering length? 1p

Abgabe am 15.01.2018