SS 2012

**7. Exercise Sheet, Lecture Theoretical Quantum and Atom Optics**

University of Hamburg, Prof. P. Schmelcher

To be returned on 04/06/2012, in the tutorials

**Exercise 15.** Inversion

Starting from the probability amplitudes for a two-level system (levels 1 and 2) interacting with a single-mode light field (photon number $n$),

$$c_{2,n}(t) = c_n(0)\left[\cos\left(\frac{\Omega_n t}{2}\right) + \frac{i\Delta}{\Omega_n}\sin\left(\frac{\Omega_n t}{2}\right)\right] e^{-i\Delta\frac{t}{2}}$$

$$c_{1,n+1}(t) = -c_n(0)\left(\frac{2ig\sqrt{n+1}}{\Omega_n}\right)\sin\left(\frac{\Omega_n t}{2}\right) e^{+i\Delta\frac{t}{2}},$$

where $\Omega_n^2 = \Delta^2 + 4g^2(n+1)$ and $c_n(0) \equiv c_{2,n}(t=0)$, show that the *inversion*,

$$W(t) = \sum_{n=0}^{\infty}\left[|c_{2,n}(t)|^2 - |c_{1,n}(t)|^2\right],$$

for this system is given by

$$W(t) = \sum_{0}^{\infty}\rho_{nn}(0)\left[\frac{\Delta^2}{\Omega_n^2} + \frac{4g^2(n+1)}{\Omega_n^2}\cos(\Omega_n t)\right],$$

where $\rho_{nn}(0) = |c_n(0)|^2$ is the probability of having $n$ photons in the field at time $t=0$.

**3 Points**

**Exercise 16.** Mollow triplet

Consider the manifold $\mathcal{M}(n) = \{|1, n+1\rangle, |2, n\rangle\}$ consisting of the uncoupled basis states of a two-level atom (levels 1 and 2) in a single-mode light field (photon number $n$), and the full Hamiltonian describing the system:

$$H = H_A + H_F + H_{AF} = \frac{\hbar\omega}{2}\sigma_z + \hbar\omega_L a^\dagger a + \hbar g(\sigma_+ a + a^\dagger\sigma_-),$$

written in terms of the Pauli operators $\sigma_z = |2\rangle\langle 2| - |1\rangle\langle 1|$, $\sigma_+ = |2\rangle\langle 1|$, $\sigma_- = |1\rangle\langle 2|$ and field ladder operators $a, a^\dagger$, where $\hbar\omega = E_2 - E_1$ is the atomic level spacing, $\omega_L$ the driving field frequency, and $g$ the atom-field coupling strength.

(a) Find the eigenvalues $E_\pm$ of $H$, and show that the corresponding *dressed state* eigenvectors are written, in the representation $\mathcal{M}(n)$, as

$$|+, n\rangle = \sin\theta_n|1, n+1\rangle + \cos\theta_n|2, n\rangle$$
$$|-, n\rangle = \cos\theta_n|1, n+1\rangle - \sin\theta_n|2, n\rangle,$$

where the mixing angle $\theta_n$ is defined by $\tan(2\theta_n) = -\Omega_R/\Delta$, where $\Omega_R = 2g\sqrt{n+1}$ is the Rabi frequency and $\Delta = \omega_L - \omega$ the detuning.
*Hint*: Verify that the dressed state levels $E_\pm$ lie symmetrically above and below the unperturbed states $|1, n+1\rangle$ and $|2, n\rangle$, respectively, and are separated by an interval $\Omega_d = \sqrt{\Delta^2 + \Omega_R^2}$.

(b) Determine the transition matrix elements

$$\langle i, m|\sigma_+|j, n\rangle, \quad i, j \in \{+, -\}$$

of the spontaneous transitions between the two-level manifolds $\mathcal{M}(n) = \{|1, n+1\rangle, |2, n\rangle\}$ with $n = m, m\pm 1, m\pm 2, \ldots$ .
*Hint*: Verify that only transitions between adjacent manifolds are allowed.

(c) Show, using a sketch of the energy levels, that the central frequencies of these allowed spontaneous transitions form a fluorescence (Mollow) triplet structure around the field frequency $\omega_L$.

**7 Points**

# Edwin Thompson Jaynes

Edwin Thompson Jaynes (Waterloo, Iowa, July 5, 1922 – St. Louis, Missouri, April 30, 1998) was Wayman Crow Distinguished Professor of Physics at Washington University in St. Louis. He wrote extensively on statistical mechanics and on foundations of probability and statistical inference, initiating in 1957 the MaxEnt interpretation of thermodynamics, as being a particular application of more general Bayesian/information theory techniques (although he argued this was already implicit in the works of Gibbs). Jaynes strongly promoted the interpretation of probability theory as an extension of logic.

In 1963, together with Fred Cummings, he modeled the evolution of a two-level atom in an electromagnetic field, in a fully quantized way. This model is known as the Jaynes–Cummings model.

(text: `http://en.wikipedia.org/wiki/Edwin_Jaynes`)

[...] In the years immediately preceding his departure from Stanford (1960) he was becoming increasingly dissatisfied with the publish or perish mentality plaguing Stanford, a condition he talked about in "Backward Look to the Future". So in 1960 he packed his belongings, sold his house, and moved to St. Louis, Missouri where he joined the physics faculty of Washington University.

Upon arriving in St. Louis, Jaynes set out on his remaining research interest, reformulating quantum electrodynamics to avoid quantization of the electromagnetic field. Jaynes published his first paper on this subject in 1963 with Fred Cummings. It was titled "Comparison of Quantum and Semiclassical Radiation Theory with Application to the Beam Maser". Jaynes continued to publish articles on both semiclassical and neoclassical radiation theory more or less continuously until he retired.

[...] Jaynes retired in 1992 after a long and productive career. Jaynes' contributions to science were of the highest caliber. His work in reformulating statistical mechanics has illuminated the foundations of that theory and enabled extensions to non-equilibrium systems. His dedication to rooting out contradictions in quantum mechanics is legendary. He must have single-handedly sparked more debate in quantum mechanics than any other person in the last 50 years. The verdict on his neoclassical radiation theory is still not in, and may not be for many more years. It may yet prove to be a better description of nature than quantum electrodynamics. He also helped take an interpretation of probability theory from being virtually unknown to a healthy research area that is being applied daily in economics, biology, physics, nuclear magnetic resonance and many other disciplines. His writing helped to clarify the foundations of probability theory in a way never achieved before. He wrote profusely, in a warm and friendly way that enabled one to see complex points as if they were intuitively obvious. He spoke as he wrote. When he criticized someone's work, he always stuck to the facts; he never reverted to name calling. His friendship was hard to earn, and hard to keep, for he had little tolerance for incompetence. He would undoubtedly be uncomfortable with all of the attention being lavished on him now that he is dead.

(text: G. Larry Bretthorst, `http://bayes.wustl.edu/etj/etj.html`)