SS 2012

Excercise Sheet 6, Lecture Theoretical Quantum and Atom Optics

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To be returned on 21/05/2012, in the tutorials

Exercise 12. Rabi oscillations I

Consider the expressions for the time-evolution of the original amplitudes (for a two-level system interacting with a classical light field) as given in the lecture:

$$b_1(t) = e^{-i(\omega_1-\triangle/2)t}\left[\cos(\Omega_g t/2) - i\frac{\triangle}{\Omega_g}\sin(\Omega_g t/2)\right]$$

$$b_2(t) = e^{-i(\omega_2+\triangle/2)t}\left[i\frac{\Omega}{\Omega_g}\sin(\Omega_g t/2)\right]$$

(a) Explicitely calculate $|b_1(t)|^2$ and $|b_2(t)|^2$ for a non zero detuning $\triangle$.

(b) Both $|b_1(t)|^2$ and $|b_2(t)|^2$ perform a harmonic oscillation. Explicitely determine the frequency and the amplitude of this oscillation.

2 Points

Exercise 13. Rabi oscillations II

For the rotated (with respect to the b_i) coefficients c_i, the following system of ordinary differential equations (ODE's) holds:

$$\dot{c}_1(t) = ic_2(t)\frac{\Omega^*}{2}e^{i\triangle t}$$

$$\dot{c}_2(t) = ic_1(t)\frac{\Omega}{2}e^{-i\triangle t}$$

(a) Show that the following is a solution of the ODE system:

$$c_1(t) = \left(a_1 e^{i\Omega_g t/2} + a_2 e^{-i\Omega_g t/2}\right)e^{i\triangle t/2}$$

$$c_2(t) = \left(a_3 e^{i\Omega_g t/2} + a_4 e^{-i\Omega_g t/2}\right)e^{-i\triangle t/2},$$

if suitable relations between the constants a_i are fulfilled.

(b) Determine how the constants a_i depend on the initial conditions $c_1(0)$ and $c_2(0)$.

(c) Show that for $c_1(0) = 1$, $c_2(0) = 0$ the solution reduces to the one given in the lecture:

$$c_1(t) = e^{i\triangle t/2}\left[\cos(\Omega_g t/2) - i\frac{\triangle}{\Omega_g}\sin(\Omega_g t/2)\right]$$

$$c_2(t) = e^{-i\triangle t/2}\left[i\frac{\Omega}{\Omega_g}\sin(\Omega_g t/2)\right]$$

4 Points

Exercise 14. Avoided crossing

Consider a two-level system governed by the Hamiltonian

$$\hat{H} = -\frac{\hbar}{2}\begin{pmatrix} 0 & \Omega^* \\ \Omega & 2\Delta \end{pmatrix}, \qquad \Delta \in \mathbb{R}, \Omega \in \mathbb{C}.$$

(a) Calculate the eigenvalues $E_\pm(\Omega, \Delta)$. It is useful to introduce $\Omega_g = \sqrt{\Delta^2 + |\Omega|^2}$.

(b) Sketch $E_\pm$ as a function of Δ for $\Omega = 0$ and $\Omega \neq 0$, respectively.

(c) Calculate the corresponding normalized eigenvectors $\psi_\pm(\Omega, \Delta)$.

(d) Simplify the results for $E_\pm$ and $\psi_\pm$ in the parameter regimes $|\Delta| \to 0$ and $|\Delta| \gg |\Omega|$ by carefully performing the appropriate limits.

4 Points