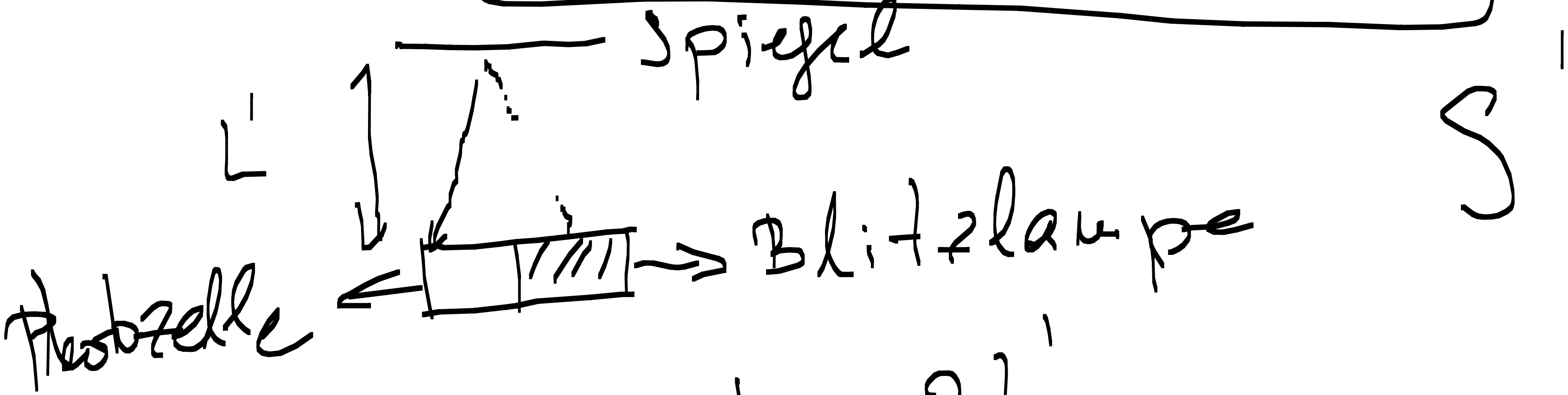
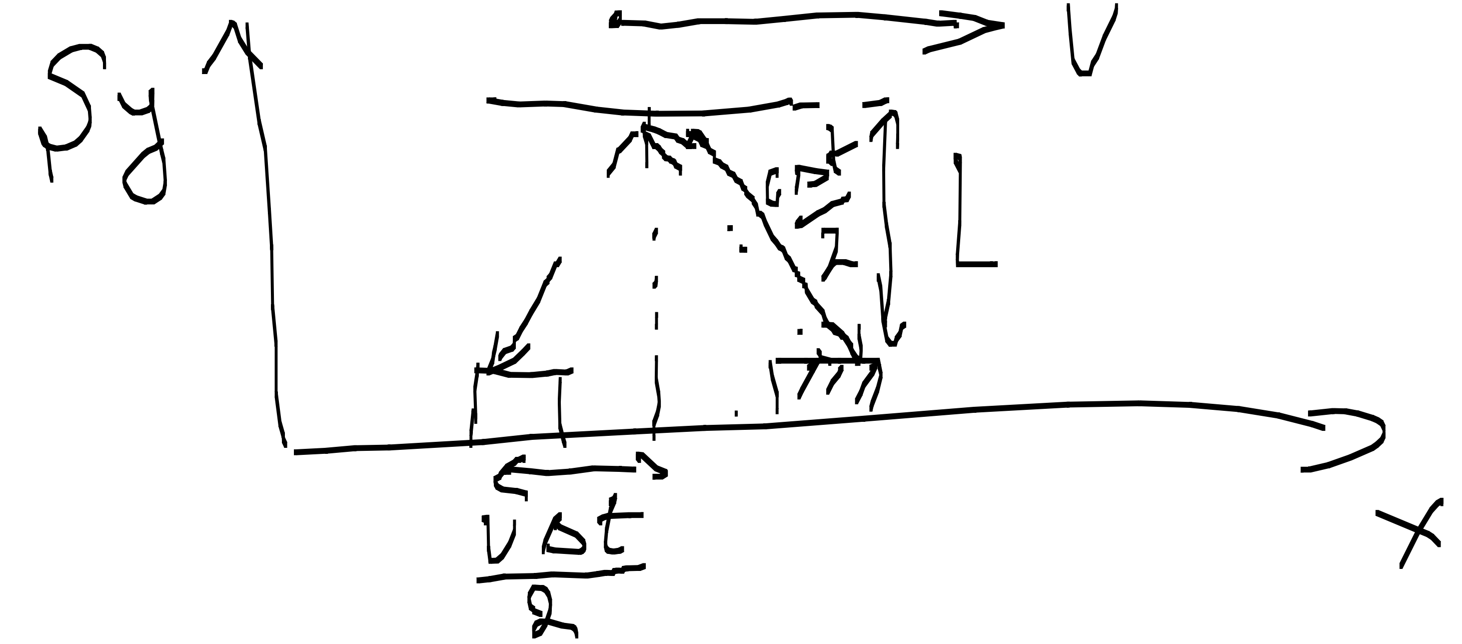
Bewegte Uhren



Taktperiode $\Delta t' = \frac{2L'}{c}$



$$L^2 + \left(\frac{v\,\Delta t}{2}\right)^2 = \left(\frac{c\,\Delta t}{2}\right)^2 \rightarrow \boxed{c = \text{Const}}$$

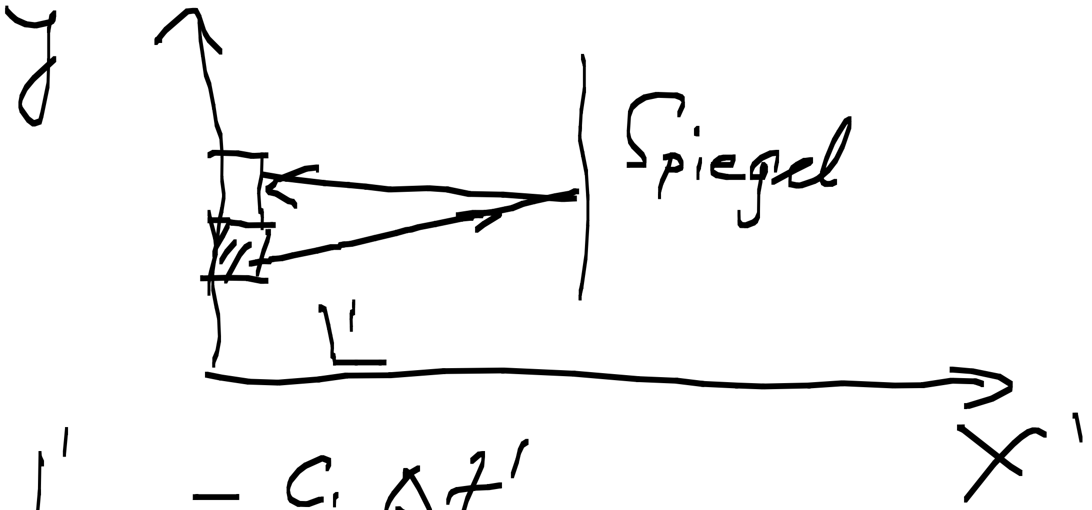
$$L^2 = \frac{\Delta t^2}{4}\left(c^2 - v^2\right)$$

$$\Delta t = \frac{2L/c}{\sqrt{1 - v^2/c^2}}$$

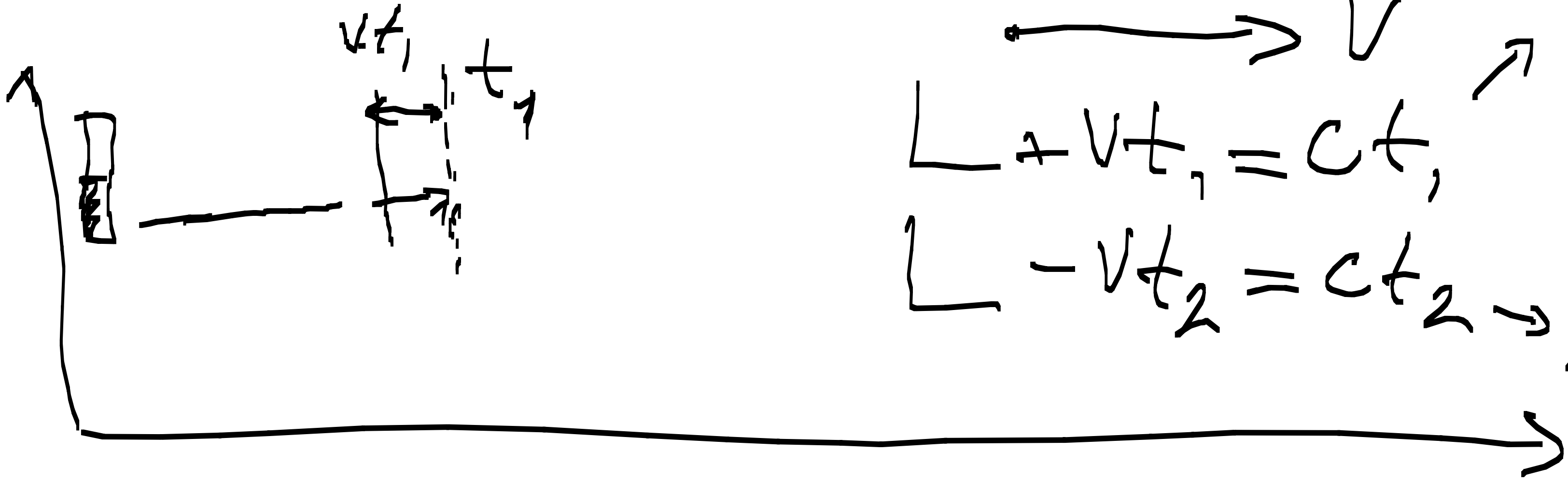
$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} = \gamma\,\Delta t' > \Delta t'$$

Bewegte Uhren gehen langsamer!

Längekontraktion



$$L' = \frac{c\,\Delta t'}{2}$$



$$L + vt_1 = ct_1 \rightarrow \text{Hinflug}$$

$$L - vt_2 = ct_2 \rightarrow \text{Rückflug}$$

S

$$\Delta t = t_1 + t_2$$

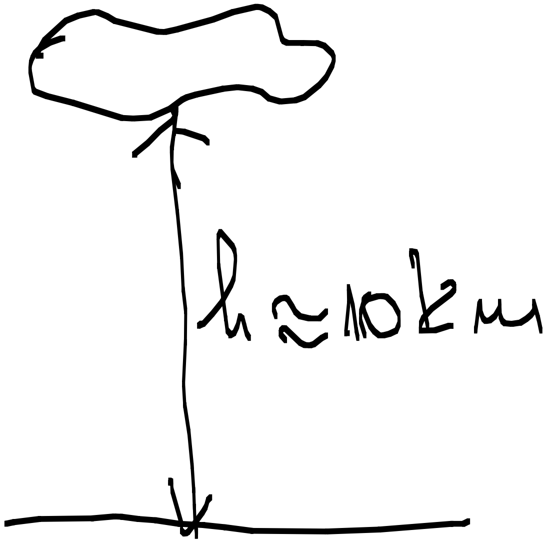
$$\Delta t = \frac{L}{c-v} + \frac{L}{c+v} = L\,\frac{c-v+c+v}{c^2-v^2} =$$

$$= \frac{2Lc}{c^2-v^2} \Rightarrow L = \Delta t\,\frac{c^2-v^2}{2c} = \frac{c\Delta t}{2\gamma^2}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} \Rightarrow \Delta t = \gamma\,\Delta t'$$

$$L = \frac{L'}{\gamma}$$

Muonenzerfall



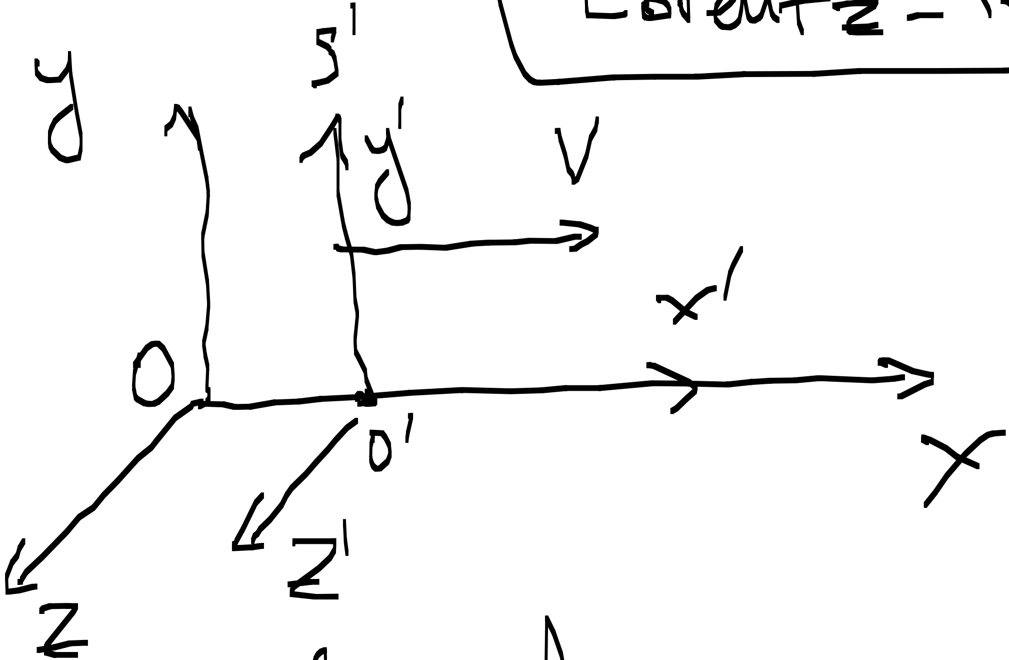
Lebensdauer $\tau \simeq 2{,}2\,\mu s$

$v \simeq 0{,}9996\,c$

$s = v \cdot \tau \simeq 660\,m$

$\tau_{Erde} = \gamma \cdot \tau$

Lorentz-Transformationen



$S: x, y, z, t$

$S': x', y', z', t'$

$O \equiv O' \quad t = t' = 0$

Klassisch:

$$x = x' + vt'$$
$$y = y'$$
$$z = z'$$
$$t = t'$$

Relativistisch:

Ansatz:

$$x = \gamma (x' + vt')$$
$$\gamma = \gamma(v, c)$$
$$x' = \gamma (x - vt)$$

$x = \gamma(x' + vt')$ Lichtblitz bei $t = 0$

$x' = \gamma(x - vt)$ $c = c' = \text{const}$

$x = ct = \gamma(ct' + vt')$ 1

$x' = ct' = \gamma(ct - vt)$ 2

$$\Rightarrow \frac{t'}{t} = \frac{ct - vt}{ct' + vt'} = \frac{t}{t'} \cdot \frac{c - v}{c + v}$$

$$\Rightarrow t'^2 = t^2 \frac{c - v}{c + v}$$

aus 1 $c^2 \cancel{t^2} = \gamma^2 (c+v)^2 t'^2 =$

$$= \gamma^2 (c+v)^2 \cancel{t^2} \frac{c - v}{c + v}$$

$$\Rightarrow c^2 = \gamma^2 (c^2 - v^2)$$

$$\Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

Zeittransformation

$$x' = \gamma(x - vt) = \gamma(\gamma(x' + vt') - vt)$$

$$x = \gamma(x' + vt')$$

$$x' = \gamma^2(x' + vt') - \gamma vt$$

$$t = \frac{\gamma^2(x' + vt') - x'}{\gamma v} = \frac{\gamma(x' + vt')}{v} - \frac{x'}{\gamma v}$$

$$t = \gamma \left[\frac{x' + vt'}{v} - \frac{x'}{\gamma^2 v} \right] =$$

$$= \gamma \frac{x'(\gamma^2 - 1) + \gamma^2 v t'}{\gamma^2 v} = \gamma \left(x' \frac{\gamma^2 - 1}{\gamma^2 v} - t' \right)$$

$$\boxed{t = \gamma \left(x' \frac{\gamma^2 - 1}{\gamma^2 v} - t' \right)}$$

2 Ereignisse zu Zeitpunkten t_1', t_2'

<u>am selben Ort</u> x_0'

$$t_1 = \gamma \left(x_0' \frac{\gamma^2 - 1}{\gamma^2 v} + t_1' \right)$$

$$t_2 = \gamma \left(x_0' \frac{\gamma^2 - 1}{\gamma^2 v} + t_2' \right)$$

$$\Delta t = t_2 - t_1 = \gamma \left(t_2' - t_1' \right) = \gamma \Delta t'$$

$$\underbrace{\text{Längekontraktion}}$$

$$x' = \gamma (x - vt)$$

$$L = x_2 - x_1$$

$$x_2' = \gamma (x_2 - v t_2) \qquad t_1 = t_2$$

$$x_1' = \gamma (x_1 - v t_1)$$

$$L' = x_2' - x_1' = \gamma L \Rightarrow L = \frac{L'}{\gamma}$$

Addition von Geschwindigkeiten

$$v_x' = \frac{dx'}{dt'} \; ; \qquad v_x = \frac{dx}{dt}$$

$$\boxed{dx = \gamma (dx' + v\, dt')}$$

$$t = \gamma \left(t' + \frac{v x'}{c^2} \right)$$

$$dt = \gamma\left(dt' + \frac{v\,dx'}{c^2}\right)$$

$$v_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v\,dx'}{c^2}} =$$

$$= \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \boxed{\frac{v_x' + v}{1 + \frac{v v_x'}{c^2}} = v_x}$$

Beispiel: $v_x' = c$

$$V_x = \frac{c + V}{1 + \frac{V \cdot c}{c^2}} = \frac{c + V}{1 + \frac{V}{c}} = c$$