- vollständig elastischer Stoß

hier gilt 2. Beziehung Erhaltung

d. kinet. Energie:

$$\frac{1}{2} m_1 v_{1,A}^2 + \frac{1}{2} m_2 v_{2,A}^2 = \frac{1}{2} m_1 v_{1,E}^2 + \frac{1}{2} m_2 v_{2,E}^2$$

$\exists$ 2 Gleichungen für 2 Unbekannte

$\rightarrow$ vereinfacht über Relativgeschw.

$$\Rightarrow \quad m_2 \left(v_{2,E}^2 - v_{2,A}^2 \right) = m_1 \left(v_{1,A}^2 - v_{1,E}^2 \right)$$

$$m_2 \left(v_{2,E} - v_{2,A} \right) \left(v_{2,E} + v_{2,A} \right) = m_1 \left(v_{1,A} - v_{1,E} \right) \left(v_{1,A} + v_{1,E} \right) \quad (*)$$

wg. Impulserhaltung

$$m_1 v_{1,E} + m_2 v_{2,E} = m_1 v_{1,A} + m_2 v_{2,A}$$

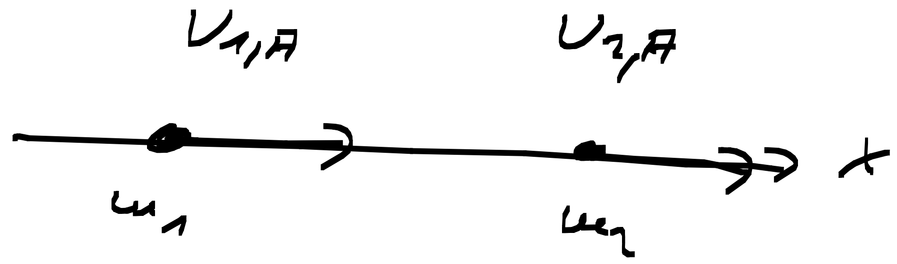
$$m_2 \left(v_{2,E} - v_{2,A} \right) = m_1 \left(v_{1,A} - v_{1,E} \right) \quad (**)$$

$$\Rightarrow \quad \frac{(*)}{(**)} \Rightarrow v_{2,E} + v_{2,A} = v_{1,A} + v_{1,E}$$

$$\Rightarrow \underbrace{v_{2,E} - v_{1,E}}_{\equiv\, v_{\text{relativ},E}} = - \left(v_{2,A} - v_{1,A} \right) \qquad = - v_{\text{rel},A}$$

$$v_2 - v_1 = v_2^{(1)} = \text{Geschw. des 2. Körpers relativ zum 1. Körper}$$

↳ Betrachte:



Sei $v_{2,A} > v_{1,A}$ $\Rightarrow$ kein Stoß

$\Rightarrow$ $v_{1,A} = v_{1,E}$

$v_{2,A} = v_{2,E}$

$\Rightarrow$ (**) : $m_2 \cdot 0 = m_1 \cdot 0 = 0$

$\Rightarrow$ $\dfrac{(*)}{(**)}$ nicht erfasst

↳ Spezialfall: 2. Teilchen in Ruhe

$$v_{2,A} \equiv 0$$

$$\Rightarrow \quad (1) \quad m_1 v_{1,A} = m_1 v_{1,E} + m_2 v_{2,E}$$

$$(2) \quad v_{2,E} - v_{1,E} = -(v_{2,A} - v_{1,A}) =$$

(s. oben)

$$= v_{1,A}$$

$$\Rightarrow \quad v_{2,E} = v_{1,E} + v_{1,A} \quad \text{in } (1)$$

$$\Rightarrow \quad m_1 v_{1,A} = m_1 v_{1,E} + m_2 (v_{1,E} + v_{1,A}) =$$

$$= v_{1,E} (m_1 + m_2) + m_2 v_{1,A}$$

$$\Rightarrow \quad v_{1,E} = \frac{m_1 v_{1,A} - m_2 v_{1,A}}{m_1 + m_2}$$

$$\Rightarrow \quad \boxed{v_{1,E} = v_{1,A} \cdot \frac{m_1 - m_2}{m_1 + m_2}}$$

$$\Rightarrow \quad v_{2,E} = v_{1,E} + v_{1,A} =$$

$$= v_{1,A} \left(1 + \frac{m_1 - m_2}{m_1 + m_2} \right)$$

$$\Rightarrow \quad \boxed{v_{2,E} = v_{1,A} \cdot \frac{2 m_1}{m_1 + m_2}}$$

$$\rightarrow \quad m_1 = m_2 \quad \Rightarrow \quad v_{1,E} = 0$$

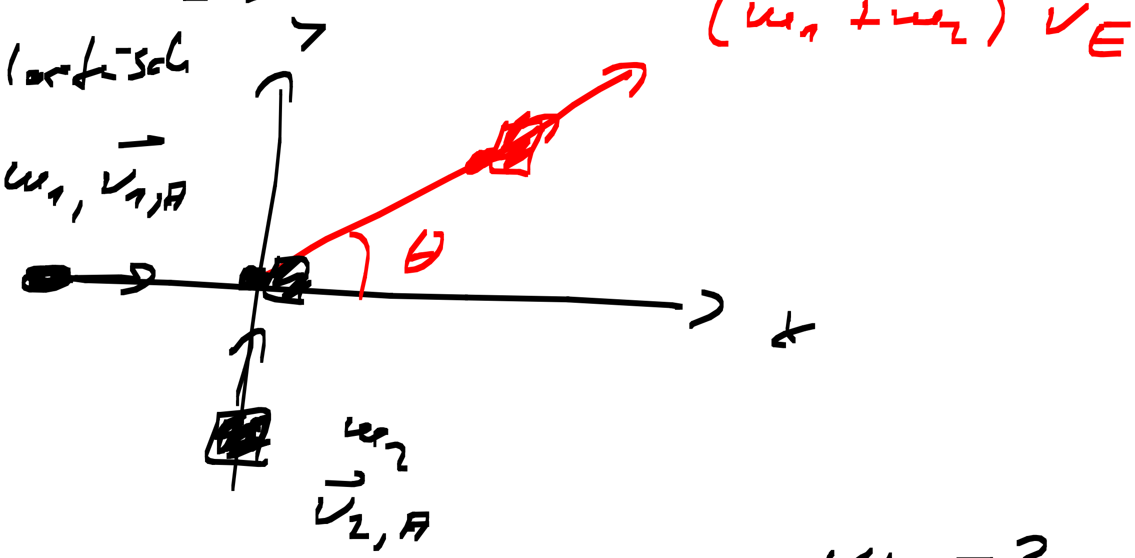
$$v_{2,E} = v_{1,A}$$

$$\rightarrow \quad m_2 = 2 m_1 \quad \Rightarrow \quad v_{1,E} = -\frac{1}{3} v_{1,A}$$

$$v_{2,E} = \frac{2}{3} v_{1,A}$$

→ Stöße in 2D

Bsp 1: inelastisch



→ ges: $\vec{v}_E = ?$ — $v_E = ?$, $\theta = ?$

→ komponentenweise:

(1) $m_1 v_1 + m_2 \cdot 0 = (m_1+m_2) v_E \cdot \cos\theta$

(2) $m_1 \cdot 0 + m_2 v_2 = (m_1+m_2) v_E \cdot \sin\theta$

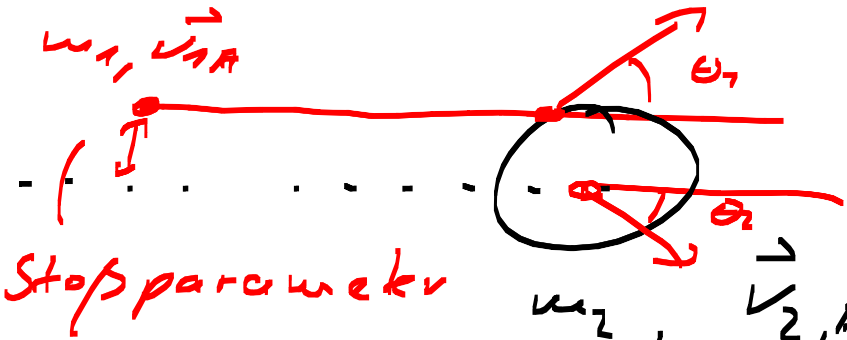
$\Rightarrow \frac{(2)}{(1)} \Rightarrow \frac{m_2 v_2}{m_1 v_1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \theta = \tan^{-1} \frac{m_2 v_2}{m_1 v_1}$

$\Rightarrow v_E$ aus (1) oder (2)

Bsp 2: elastischer Stoß

nicht-zentral, sonst 1D



$m_2, \vec{v}_{2,A} \equiv 0$

$\rightarrow \quad \vec{p} = m_1 \vec{v}_{1A} = m_1 \vec{v}_{1,E} + m_2 \vec{v}_{2,E}$

$\hookrightarrow \exists$ 4 Unbekannte: $v_{1,E}$, $v_{2,E}$, θ_1, θ_2 $= ?$

$\hookrightarrow \exists$ 3 Gleichungen:

2 aus Impulserhaltung (vektor!!!)

1 aus Energieerhaltung

$\Rightarrow$ fehlt 4. Bedingung

$\Rightarrow$ Spezialfall: $m_1 = m_2 = m$

$$\Rightarrow \cancel{m}\,\vec{v}_{1A} = \cancel{m}\,\vec{v}_{1,E} + \cancel{m}\,\vec{v}_{2,E}$$

aus E-Erhaltung:

$$\cancel{\tfrac{1}{2}m}\,v_{1A}^2 = \cancel{\tfrac{1}{2}m}\,v_{1,E}^2 + \cancel{\tfrac{1}{2}m}\,v_{2,E}^2$$

$$\Rightarrow v_{1,A}^2 = v_{1,E}^2 + v_{2,E}^2$$

$\Rightarrow$ Pythagoras

→ Schwerpunktsystem:

Teilchen mit Geschw. $\vec{v}$ hat im SP-System die Geschw.

$$\vec{v}^{(S)} = \vec{v} - \vec{v}_S$$

mit $\vec{v}_S = \dfrac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$

$$\vec{v}_1^{(S)} = \vec{v}_1 - \vec{v}_S$$

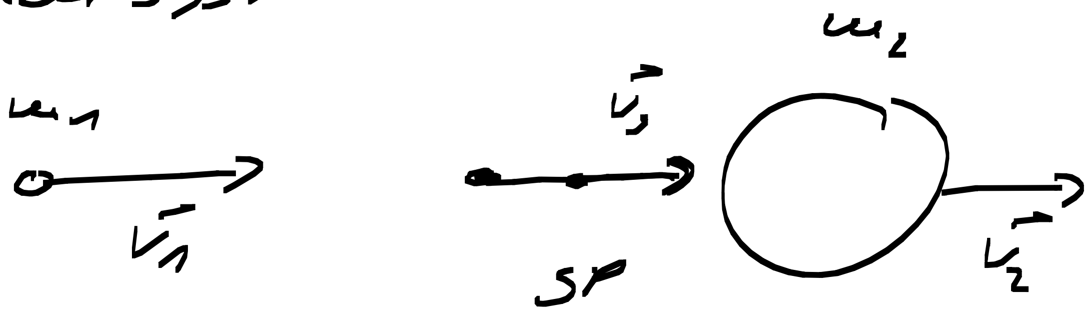
$$\vec{v}_2^{(S)} = \vec{v}_2 - \vec{v}_S$$

→ Gesamtimpuls im SP-System:

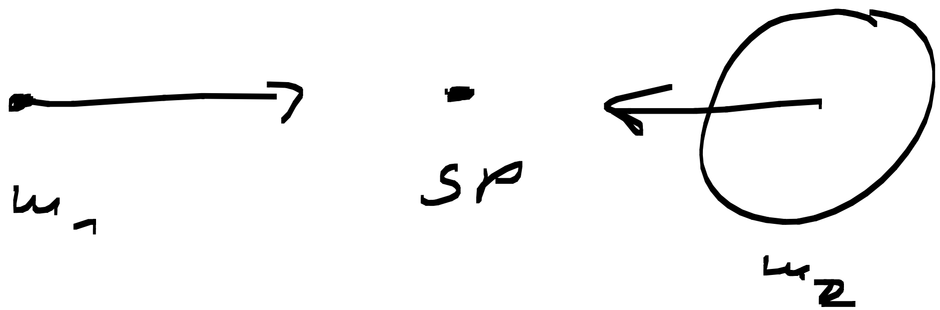
$$= \sum m_i \vec{v}_i^{(S)} = m \underbrace{\vec{v}_S^{(S)}}_{=0} = 0$$

$$\Rightarrow \boxed{\vec{p}_2^{\,(s)} = -\vec{p}_1^{\,(s)}}$$

im Laborsystem:

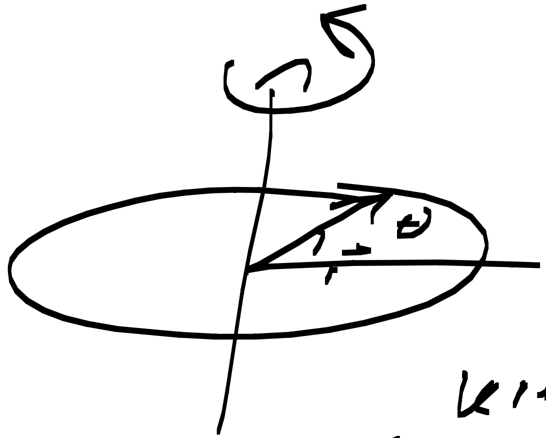


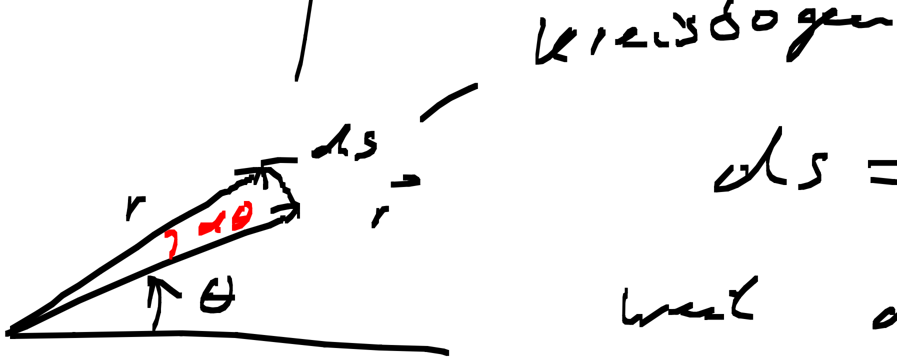
im SP-System:



Drehbewegungen

→ Bsp: rotierende Scheibe





$$ds = r \cdot d\theta$$

weil $$ds = \frac{d\theta}{2\pi} \cdot \underbrace{2\pi r}_{\text{Umfang}}$$

mit $[d\theta] = \text{rad}$

$$2\pi\,[\text{rad}] \mathrel{\hat{=}} 360^\circ$$

↳ Winkelgeschw.: $\omega \equiv \frac{d\theta}{dt}$ > 0 gegen / < 0 im Uhrzeigersinn

Winkelbeschl. $\alpha \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

↳ analog zu linearer Bewegung

$$\omega = \omega_0 + \alpha \cdot t$$

$$\theta = \theta_0 + \omega_0 \cdot t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$