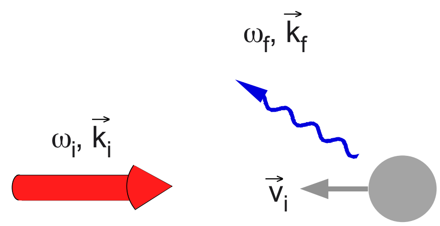
Energy and Momentum Transfer of Radiation Pressure



Consider an atom moving with a velocity $\vec{v}_i$ in a laser beam of frequency ω_i and wave vector $\vec{k}_i$. Now transfer into the rest frame. The photon absorbed by the atom is thus shifted in frequency by the Doppler effect to have the energy $\hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right)$. The resting atom thereby gets a momentum kick $\hbar\vec{k}_i$ such that its center of mass velocity after absorbing the photon is $\hbar\vec{k}_i / m$ and its kinetic center of mass energy is $\left(\hbar\vec{k}_i\right)^2 / 2m$. This energy is paid for by the photon energy $\hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right)$ received. Hence, the internal energy of the atom is $\hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right) - \frac{\left(\hbar\vec{k}_i\right)^2}{2m}$.

Next we transfer again to the rest frame and consider the emission of a photon with wave vector $\vec{k}_f$ and frequency ω_f by the resting atom. This emission will lead to a momentum kick $-\hbar\vec{k}_f$ such that its center of mass velocity after emitting the photon is $-\hbar\vec{k}_f / m$ and its kinetic center of mass energy is $\left(\hbar\vec{k}_f\right)^2 / 2m$. This kinetic energy must result from the internal energy $\hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right) - \frac{\left(\hbar\vec{k}_i\right)^2}{2m}$, such hat the energy of the emitted photon is $\hbar\omega_f = \hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right) - \frac{\left(\hbar\vec{k}_i\right)^2}{2m} - \frac{\left(\hbar\vec{k}_f\right)^2}{2m}$.

Finally, we return to the laboratory frame again, where the velocity of the atom is $\vec{v}_i + \frac{1}{m}\hbar\vec{k}_i$. This leads to a Doppler shift $\hbar\vec{k}_f\left(\vec{v}_i + \frac{1}{m}\hbar\vec{k}_i\right)$, which has to be added to $\hbar\omega_f$ thus leading to $\hbar\omega_f = \hbar\left(\omega_i - \vec{k}_i\vec{v}_i\right) - \frac{\left(\hbar\vec{k}_i\right)^2}{2m} - \frac{\left(\hbar\vec{k}_f\right)^2}{2m} + \hbar\vec{k}_f\left(\vec{v}_i + \frac{1}{m}\hbar\vec{k}_i\right)$.

Therefore, we obtain:

$$\begin{aligned}\hbar\omega_f - \hbar\omega_i &= -\hbar\vec{k}_i\vec{v}_i - \frac{\left(\hbar\vec{k}_i\right)^2}{2m} - \frac{\left(\hbar\vec{k}_f\right)^2}{2m} + \hbar\vec{k}_f\left(\vec{v}_i + \frac{1}{m}\hbar\vec{k}_i\right) \\ &= \hbar\left(\vec{k}_f - \vec{k}_i\right)\vec{v}_i - \frac{\hbar^2\left(\vec{k}_f - \vec{k}_i\right)^2}{2m}\end{aligned}$$