$$\Delta \bar{x} = \sqrt{\frac{\sum_{i=1}^{n} (\bar{x} - x_i)^2}{n(n-1)}}$$

Für eine Messreihe mit nur 5 Messungen gilt:

i	$F(N)$
1	57
2	53
3	52
4	57
5	58

$$\bar{x} = \frac{\sum_{i=n}^{n} x_i}{n} = \frac{(57 + 53 + 52 + 57 + 58)}{5}$$

$$\bar{x} = 55{,}4 \ N$$

Wie groß ist der relativer Fehler des Mittelwerts?

x_i	$(\bar{x} - x_i)$	$(\bar{x} - x_i)^2$
57	$-1{,}6$	2,56
53	2,4	5,76
52	3,4	11,56
57	$-1{,}6$	2,56
58	$-2{,}6$	6,76

$$\sum_{i=1}^{n} (\bar{x} - x)^2 = 2{,}56 + 5{,}76 + 11{,}56 + 2{,}56 + 6{,}76$$

$$= 29{,}2$$

$$\frac{\sum_{i=1}^{n} (\bar{x} - x)^2}{n(n-1)} = \frac{29{,}2}{(5 \times 4)} = \frac{29{,}2}{20} = 1{,}46$$

$$\sqrt{\frac{\sum_{i=1}^{n} (\bar{x} - x)^2}{n(n-1)}} = \Delta \bar{x} = \sqrt{1{,}46} = 1{,}2083$$ absoluter Fehler

Relativer Fehler

$$\frac{\Delta \bar{x}}{\bar{x}} \times 100 = \frac{1{,}2083}{55{,}4} \times 100 = 2{,}18\,\% \approx 2{,}2\,\%$$

Ein Temperatur werde fünf Mal gemessen

$t_1 = 18{,}9\,^\circ C \quad t_2 = 21{,}5\,^\circ C, \quad t_3 = 19{,}4\,^\circ C \quad t_4 = 18{,}5\,^\circ C \quad t_5 = 21\,^\circ C$

Wie groß ist der absoluter Fehler des Mittelwerts?

$$\Delta\bar{x} = \sqrt{\frac{\sum_{i=1}^{n} (\bar{x}-x^2}{n(n-1)}}$$

$$\bar{x} = \frac{18{,}9 + 21{,}5 + 19{,}4 + 18{,}5 + 21}{5}$$

$$\bar{x} = \frac{99{,}3}{5} = 19{,}9\,^\circ C$$

x_i	$(\bar{x}-x)$	$(\bar{x}-x^2)$
18,9	1	1
21,5	−1,6	2,56
19,4	0,4	0,16
18,5	1,4	1,96
21	−1,1	1,21

$$\sum_{i=1}^{n} (\bar{x}-x^2) = 1 + 2{,}56 + 0{,}16 + 1{,}96 + 1{,}21$$

$$= 6{,}89$$

$$\frac{\sum_{i=1}^{n} (\bar{x}-x)^2}{n(n-1)} = \frac{6{,}89}{5 \times 4} = \frac{6{,}89}{20}$$

$$= 0{,}3445$$

$$\sqrt{\frac{\sum_{i=1}^{n} (\bar{x}-x)^2}{n(n-1)}} = \sqrt{0{,}3445}$$

$$= 0{,}59\,^\circ C$$

einheit auch wichtig für absoluter Fehler

Ableitung

Example B-20

$y(x) = \sin 2x \cdot \cos x$

this is a multiplication. so $y = u \cdot v$ $y' = u \cdot v' + v \cdot u'$

so lets work each of them out:

$u = \sin 2x$ $v = \cos x$

$u' = (\cos 2x)2$ $v' = -\sin x$

this has to be further differentiated as it is not simply x

$w = 2x$ $w' = 2$

chain rule says we multiply $(\cos 2x)2$

put this all together:

$$\frac{dy}{dx} = \underset{u}{(\sin 2x)}\underset{v'}{(-\sin x)} + \underset{v}{(\cos x)}\underset{u'}{(\cos 2x)2}$$

now if $x = \pi$

$$\frac{dy}{dx} = (\sin 2\pi)(-\sin \pi) + (\cos \pi)(\cos 2\pi)2$$

$$= (0)(-0) + (-1)(1)2$$

$$= -2$$

Umformungen / Umrechnungen

$x \cdot \sqrt[3]{\frac{y^2}{x}} + \sqrt{a^3} = \sqrt[3]{x^2 y^2} + a\sqrt{a}$

lets split it up.

$\sqrt{a^3} = a\sqrt{a}$

$a^{3/2} = a^{2/2} \cdot a^{1/2}$ ✓

what about the next bit

$x\left(\sqrt[3]{y^2/x}\right) \to x \frac{y^{2/3}}{x^{1/3}}$

$\Rightarrow x^{3/3}\, y^{2/3}\, x^{-\frac{1}{3}} \to x^{2/3}\, y^{2/3} \to \sqrt[3]{x^2 y^2}$

$a(\lg 10)^2 = 100a$? ✗

$a(1)^2 \neq 100a$

1.57 rad $=$ 90°

$90° \mathrel{\hat{=}} \frac{\pi}{2} = \frac{3.14\ldots}{2} \approx 1.57$ rad ✓

$(-a^2)^3 - (3x)^4 = -a^6 - 81x^4$ ✓

$(-a^2)^3 = -a^6$ ✓

$(3x)^4 = 3^4 x^4 = 81x^4$

$3a^2b^3 + 9a^2b^2 + 15ab = 3ab(ab^2 + 3ab + 5)$ ✓

$10^{\lg x} = e^{\ln x}$ $\quad x = x$ ✓

$\log\frac{(a+b)}{10} + \log 3 + \log\frac{1}{3} = \log(a+b) - 1$ ✓

$\log\frac{(a+b)}{10} = \log(a+b) - \overset{1}{\cancel{\log 10}}$

$\log 3 + \log\frac{1}{3} = \log 3 + \overset{0}{\cancel{\log 1}} - \log 3$

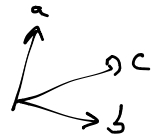
$= 0$

we're left with $\log(a+b) - 1 + 0$

$\log\left(\sqrt[3]{a}\sqrt{b}\right) = \frac{1}{3}\log a + \frac{1}{2}\log b$ ✓

$\log\left(a^{1/3} b^{1/2}\right) = \log a^{1/3} + \log b^{1/2} = \frac{1}{3}\log a + \frac{1}{2}\log b$

Vectors



What values of $\vec{a}, \vec{b}, \vec{c}$ and a b c satisfy the following state mea.

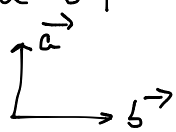
$\vec{a} + \vec{b} = \vec{c}$ $\quad a + b = c$

must be parallel $\longrightarrow \vec{a}$ $\longrightarrow \vec{b}$ $\quad \longrightarrow \vec{c}$

$\vec{a} - \vec{b} = \vec{c}$ $\quad a + b = c$

$\longrightarrow \vec{a}$ $\longleftarrow \vec{b}$ $\quad \longrightarrow \vec{c}$

$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$



$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

$\longrightarrow \vec{a} \longrightarrow \vec{b}$

$|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2}$ length of the sum of two vectors $= \sqrt{a^2 + b^2 \ldots}$ by def.