SS 2012

Exercise Sheet 3, Theoretical Quantum and Atom Optics

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To be returned on 30/04/2012, in the tutorials

Exercise 5. Coherent States

The coherent states $\{|\alpha\rangle, \alpha \in \mathbb{C}\}$ are an overcomplete non-orthogonal “basis“ of the Fock space:

(a) Show that the orthogonality relation $\langle\alpha|\alpha'\rangle = \delta(\alpha - \alpha')$ does not hold for coherent states by explicitly computing the scalar product $\langle\alpha|\alpha'\rangle$.

(b) Prove the following completeness relation of the coherent states:

$$\int \mathrm{d}^2\alpha \, |\alpha\rangle \langle\alpha| = \pi.$$

(c) Combining the above results, show that for a given coherent state $|\beta\rangle$ there are at least two different ways of expanding it in terms of the coherent states $\{|\alpha\rangle, \alpha \in \mathbb{C}\}$.

Hints:
For the calculations expand the coherent state in Fock states $|\alpha\rangle = \sum_n |n\rangle \langle n|\alpha\rangle$.
Use $\alpha = a\mathrm{e}^{\mathrm{i}\phi}$ and $\mathrm{d}^2\alpha = a\mathrm{d}a\mathrm{d}\phi$ before performing the integration and use the definition of the gamma function to solve the integral.

5 Points

Exercise 6. Squeeze operator, squeezed states

Consider the squeeze operator

$$S(\xi) = \exp\left(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi(a^\dagger)^2\right),$$

where $\xi = re^{i\theta}$ is a complex number.

(a) Show that $S^\dagger(\xi) = S(-\xi)$. Further conclude that $S(-\xi) = S^{-1}(\xi)$ by showing that $S(\xi)S(-\xi) = 1$.

(b) Now consider the squeezed coherent state $|\alpha, \xi\rangle = S(\xi)D(\alpha)|0\rangle$. Explicitly calculate the expectation values $\langle a\rangle = \langle\alpha, \xi|a|\alpha, \xi\rangle$ and $\langle a^2\rangle = \langle(a^\dagger)^2\rangle^*$.

(c) Y_1 and Y_2 are the rotated quadrature variables, see the lecture. Explicitly calculate the variances $(\triangle Y_1)^2$ and $(\triangle Y_2)^2$ and show that $\triangle Y_1 \triangle Y_2 = \frac{1}{4}$.

5 Points