Energy loss for electrons

Bethe-Bloch formula needs modification

Incident and target electron have same mass m_e

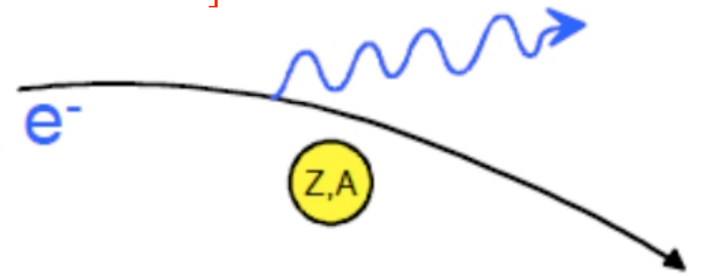
Scattering of identical, undistinguishable particles

$$-\left\langle \frac{dE}{dx} \right\rangle_{Ionization} \propto \ln(E)$$

Dominating process for $E_e >$ 10-30 MeV is not anymore ionization but

Bremsstrahlung: photon emission by an electron accelerated in Coulomb field of nucleus [radiative losses]

$$-\left\langle \frac{dE}{dx} \right\rangle_{Brems} \propto \frac{E}{m^2}$$



energy loss proportional to $1/m^2$ ➙ mainly relevant for electrons (or ultra-relativistic muons)

Bremsstrahlung

$$-\left\langle \frac{dE}{dx} \right\rangle_{Brems} = \frac{E}{X_0}$$

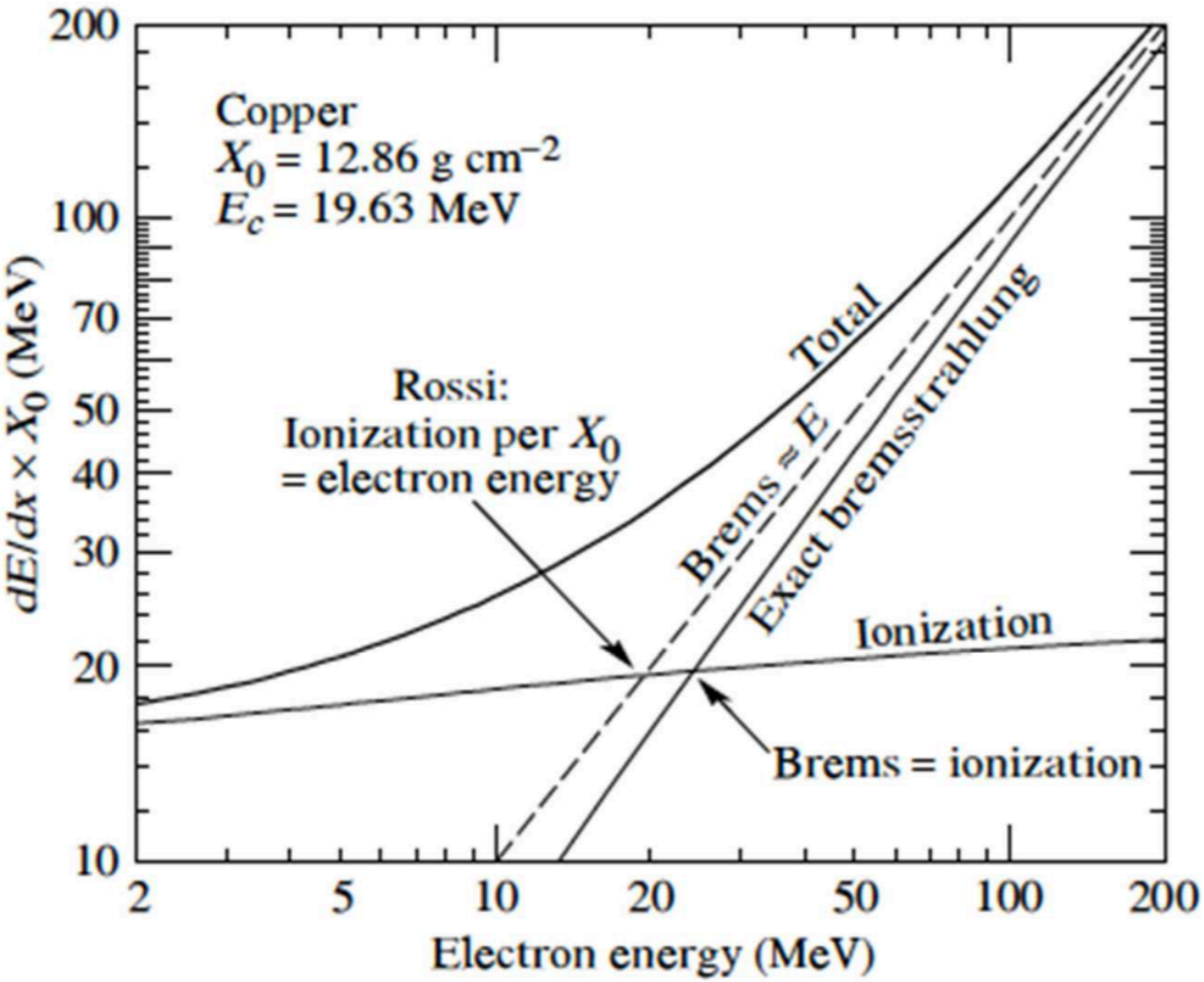
X_0 = radiation length in [g/cm^2]

$$\boxed{X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}}$$

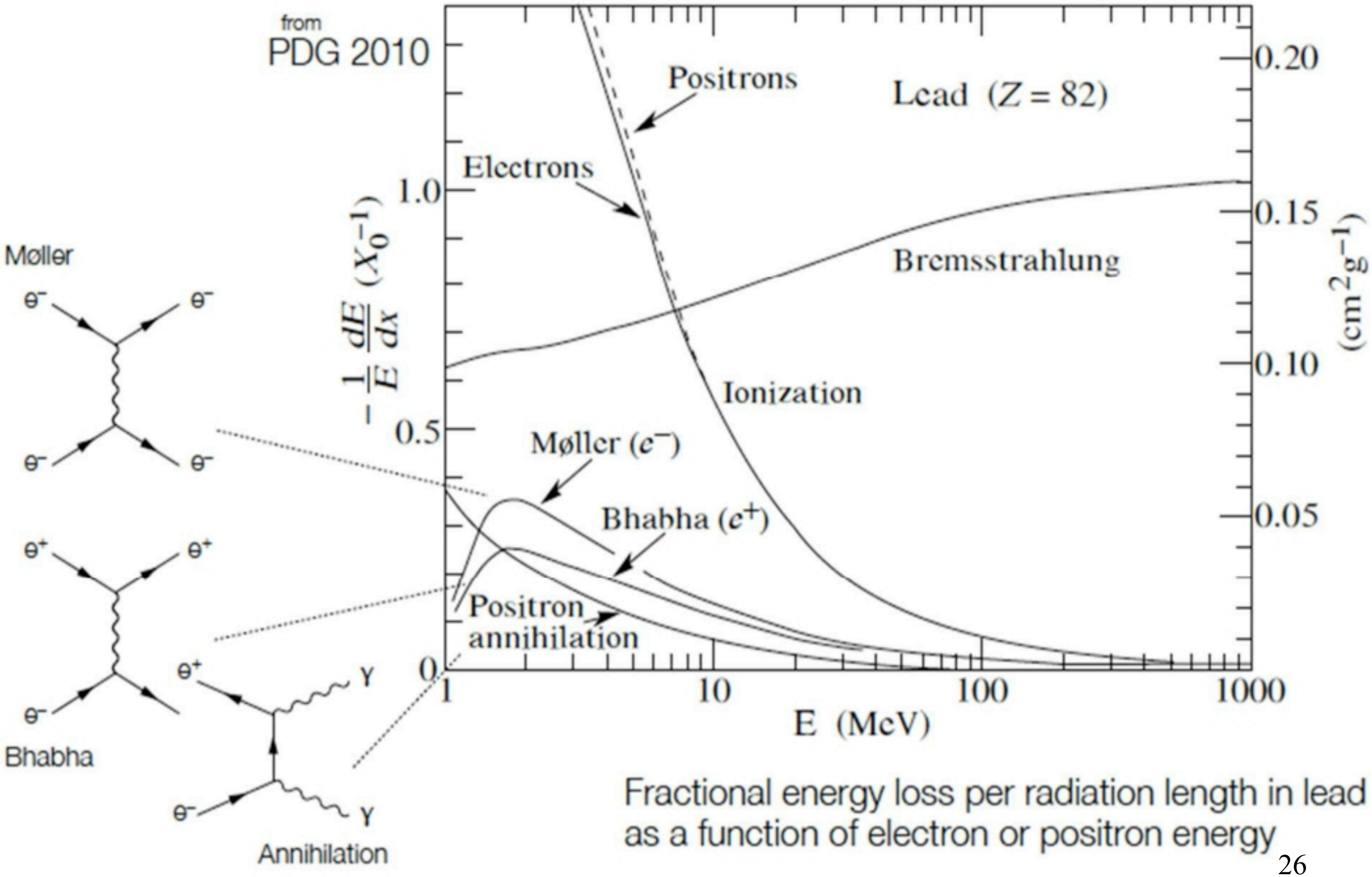
After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy (63%)

E_c = critical energy

$$\boxed{\left.\frac{dE}{dx}(E_c)\right|_{Brems} = \left.\frac{dE}{dx}(E_c)\right|_{Ion}}$$



Total energy loss for electrons



Fractional energy loss per radiation length in lead as a function of electron or positron energy

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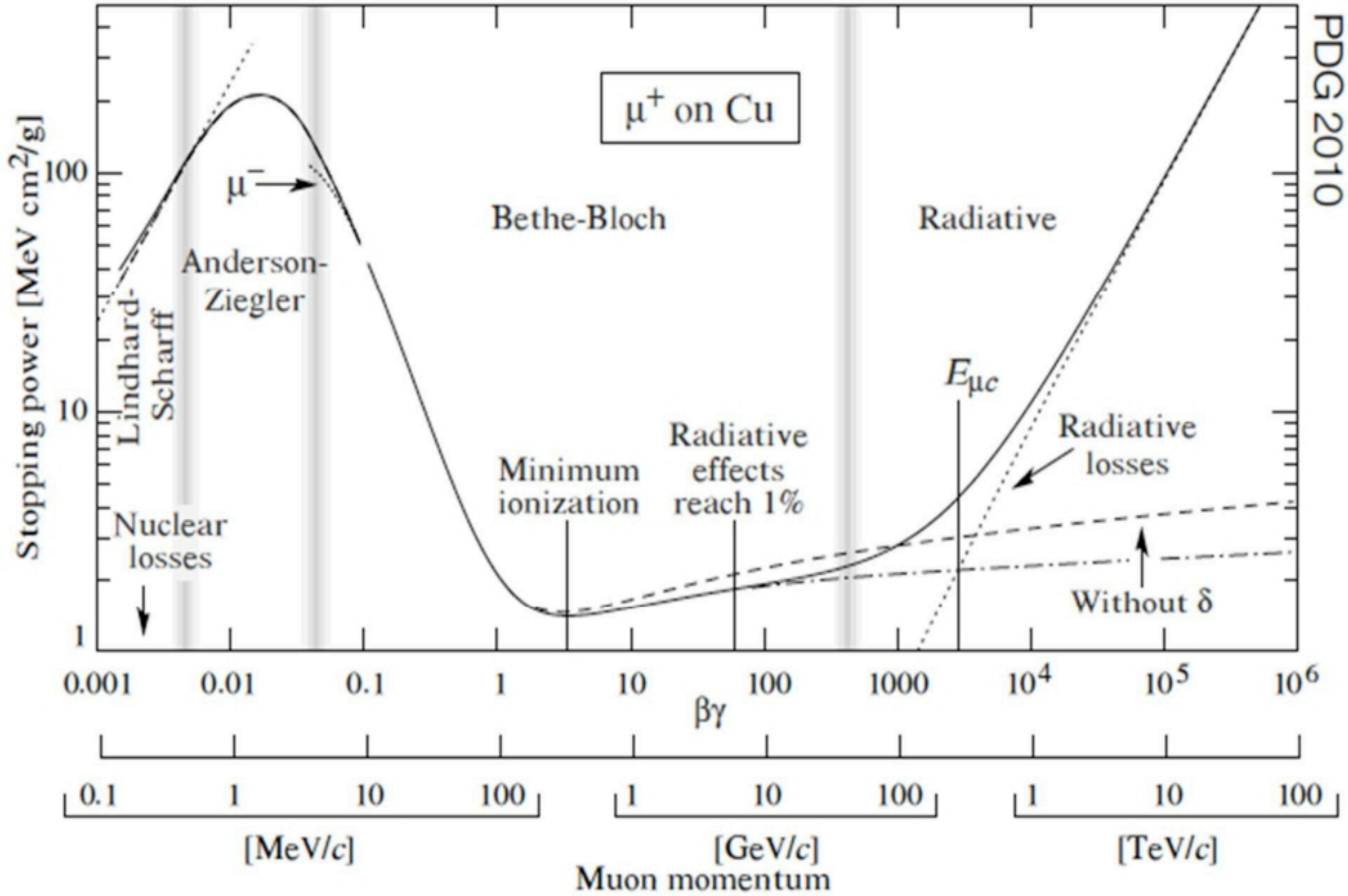
Energy loss summary



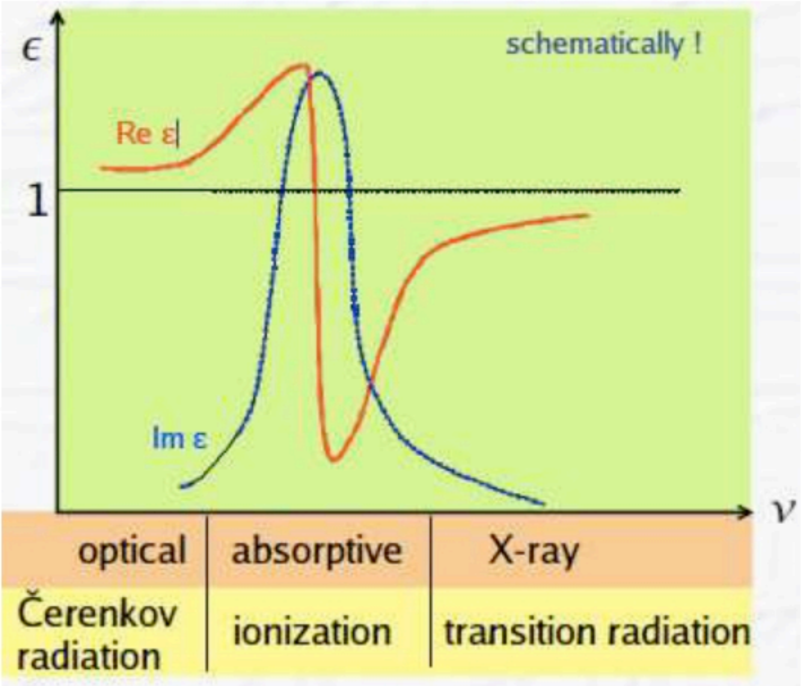
Energy Loss by Photon Emission

Ionization is one way of energy loss
emission of photons is another...

Optical behavior of medium is characterized by the (complex) dielectric constant $\boldsymbol{\varepsilon}$

Re $\sqrt{\boldsymbol{\varepsilon}}$ = n Refractive index

Im $\boldsymbol{\varepsilon}$ = k Absorption parameter



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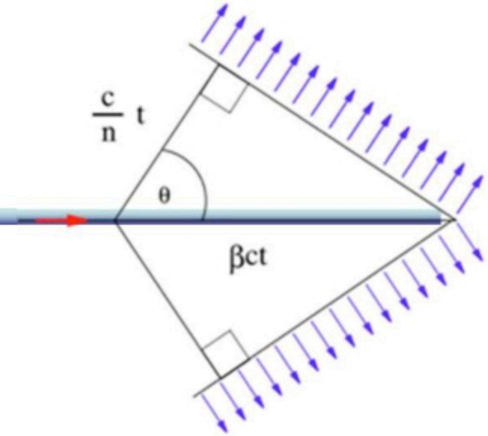
Cherenkov radiation



Polarization of atoms by charged particle
No destructive interference if v >speed of light in medium
Velocity of the particle: v
Velocity of light in a medium of refractive index n: c/n

Threshold condition for Cherenkov light emission: $v_{th} \geq \frac{c}{n} \Rightarrow \beta_{th} \geq \frac{1}{n}$

for water $\theta_{cmax} = 42^\circ$
for neon at 1 atm $\theta_{cmax} = 11\,\mathrm{mrad}$

$$-\left\langle \frac{dE}{dx} \right\rangle_{Cherenkov} \propto z^2 \sin^2\theta_c \qquad \cos\theta = \frac{1}{n\beta}$$

Energy loss by Cherenkov radiation very small w.r.t. ionization (< 1%)

Typically O(1-2 keV / cm) or O(200-1000) visible photons / cm

Visible photons:
E = 1 - 5 eV; λ = 300 - 600 nm

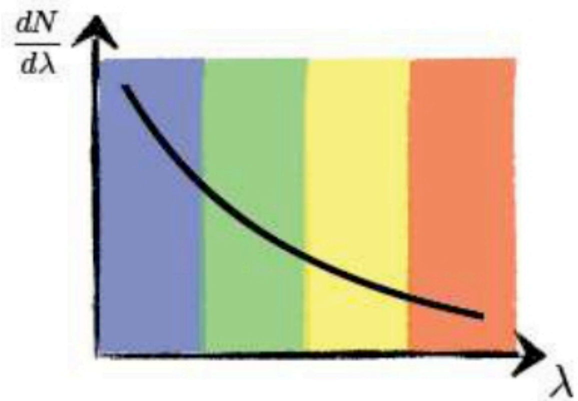
Cherenkov radiation

In a Cherenkov detector the produced photons are measured

Number of emitted photons per unit of length:

- wavelength dependence ~ $1/\lambda^2$

$$\frac{d^2N}{d\lambda dx} = \frac{2\pi\alpha z^2}{\lambda^2}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{2\pi\alpha z^2}{\lambda^2}\sin^2\theta_C$$



Integrate over sensitivity range: [for typical Photomultiplier]
$$\frac{dN}{dx} = \int_{350\ \mathrm{nm}}^{550\ \mathrm{nm}} d\lambda \frac{d^2N}{d\lambda dx}$$

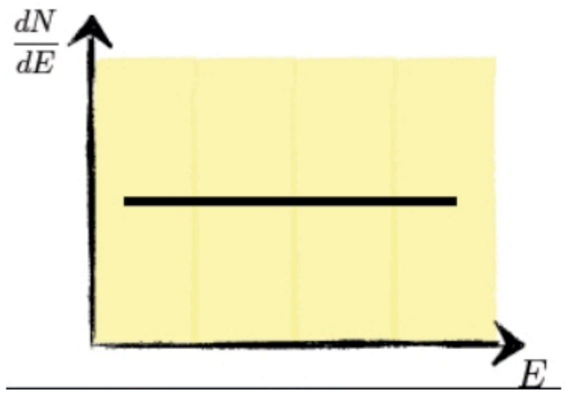
$$= 475\, z^2 \sin^2\theta_C \ \mathrm{photons/cm}$$

- energy dependence ~ constant

$$\frac{d^2N}{dE dx} = \frac{z^2\alpha}{\hbar c}\left(1 - \frac{1}{\beta^2 n^2(\lambda)}\right) = \frac{z^2\alpha}{\hbar c}\sin^2\theta_C$$

$$\approx \mathrm{const}$$

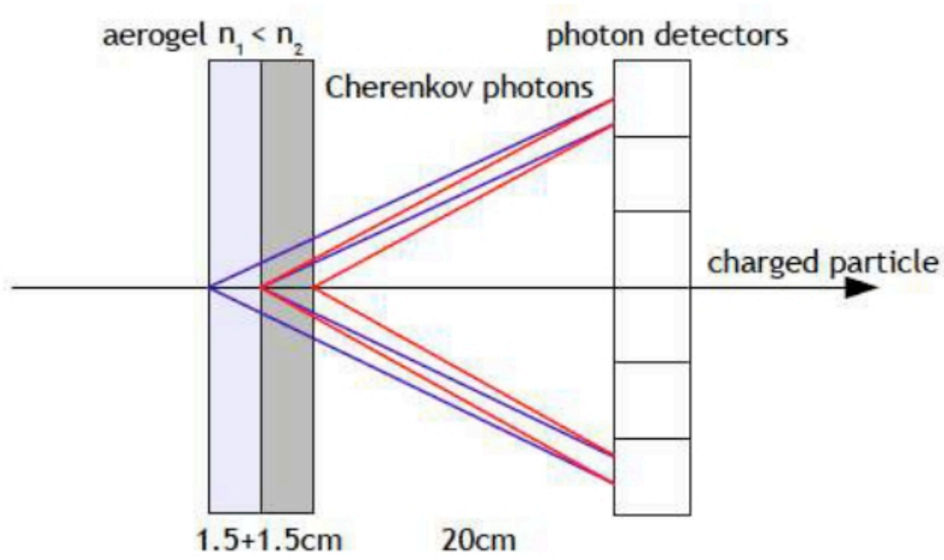
$$\frac{d^2N}{dE dx} = 370 \sin^2\theta_C \ \mathrm{eV}^{-1}\ \mathrm{cm}^{-1}$$

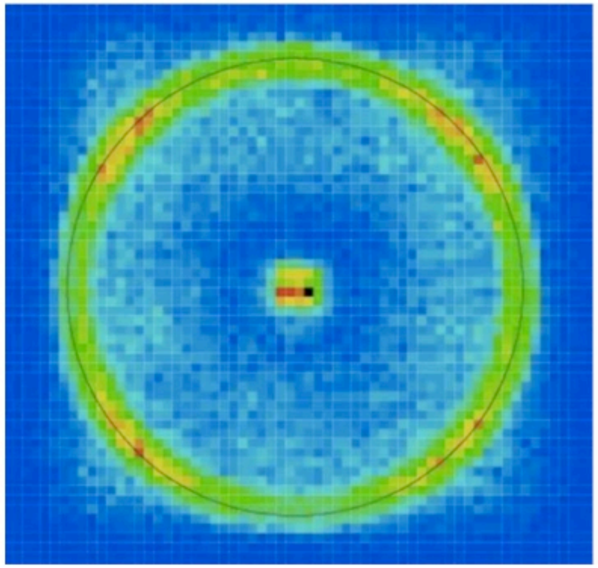


Detection of Cherenkov radiation

Parameters of Typical Radiator

Medium	n	β_{thr}	θ_{max} [β=1]	N_{ph} [eV^{-1} cm^{-1}]
Luft	1.000283	0.9997	1.36	0.208
Isobutan	1.00127	0.9987	2.89	0.941
Wasser	1.33	0.752	41.2	160.8
Quartz	1.46	0.685	46.7	196.4





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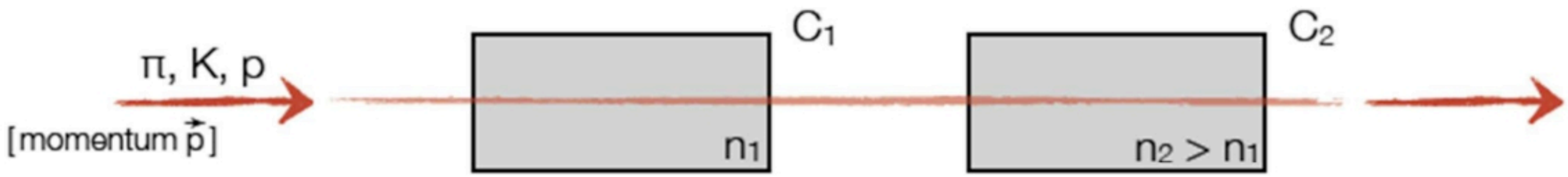
Cherenkov radiation - application

Threshold detection:

Observation of Cherenkov radiation → $\beta > \beta_{thr}$



Choose n_1, n_2 in such a way that for:

n_2 : $\beta_\pi, \beta_K > 1/n_2$ and $\beta_p < 1/n_2$

n_1 : $\beta_\pi > 1/n_1$ and $\beta_K, \beta_p < 1/n_1$

Note:
e always visible in
Cherenkov counters

Light in C_1 and C_2	→	identified pion
Light in C_2 and not in C_1	→	identified kaon
Light neither in C_1 and C_2	→	identified proton

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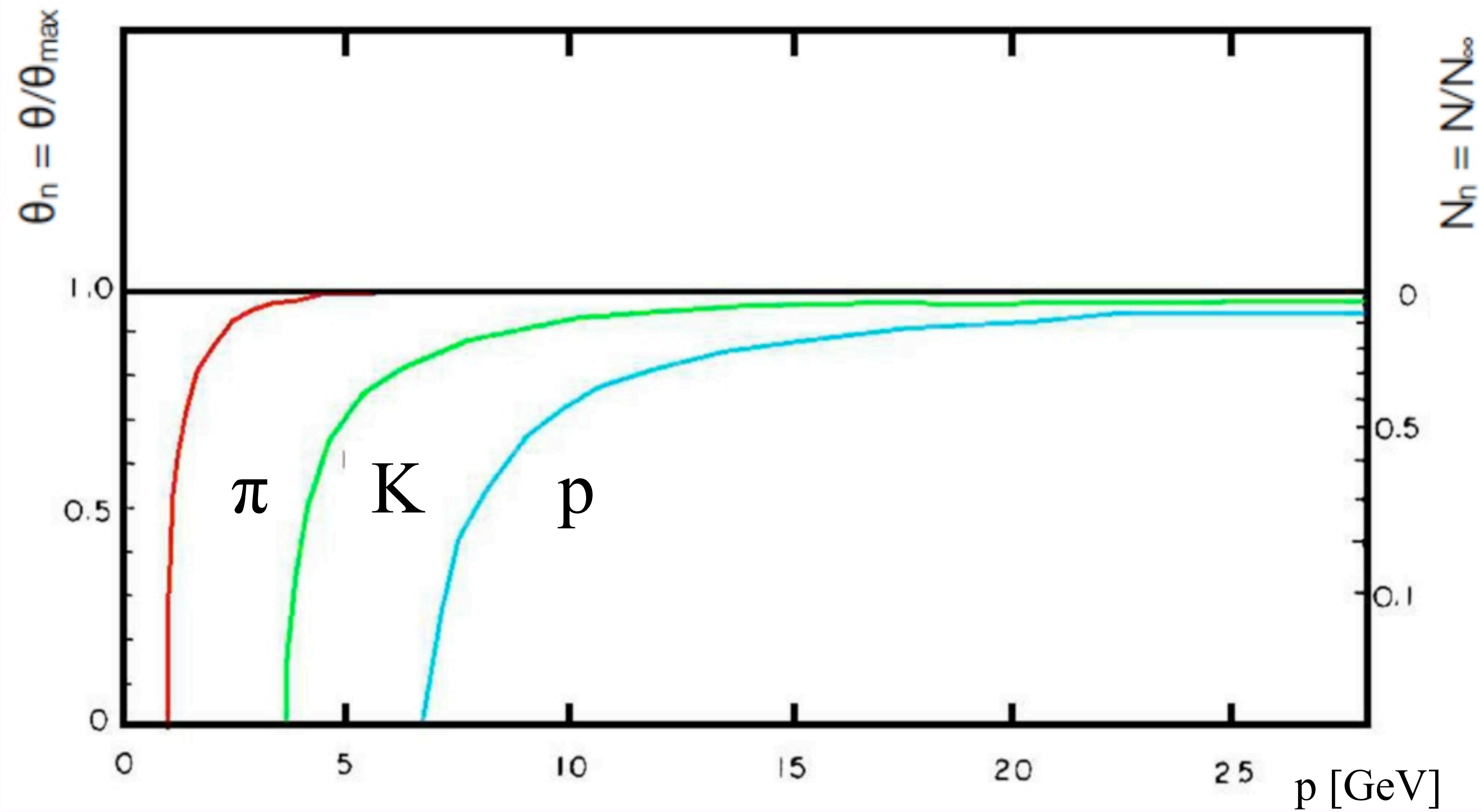
Cherenkov Radiation – Momentum Dependence



Cherenkov angle θ and number of photons N grow with β

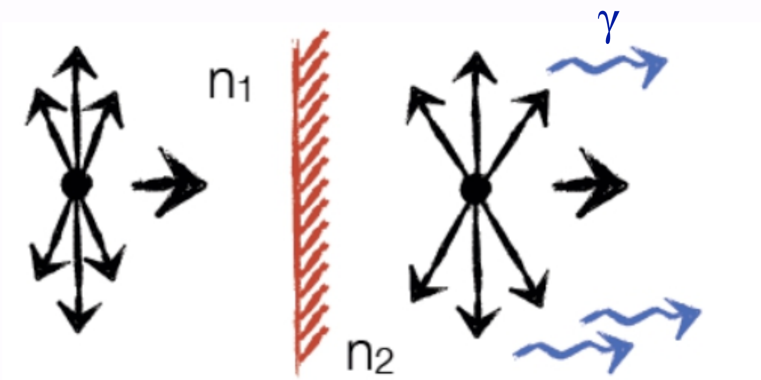
Asymptotic value for $\beta = 1$: $\cos\theta_{max} = 1/n$; $N_\infty = x \cdot 370\ /\ \mathrm{cm}\ (1 - 1/n^2)$

Transition Radiation

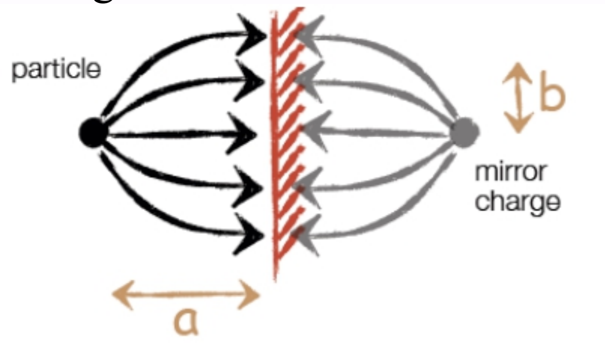
Transition radiation occurs if a relativist particle (large γ) passes the boundary between two media with different refraction indices ($n_1 \neq n_2$)

[predicted by Ginzburg and Frank 1946; experimental confirmation 70ies]

Effect can be explained by re-arrangement of electric field:



A charged particle approaching a boundary created a magnetic dipole with its mirror charge

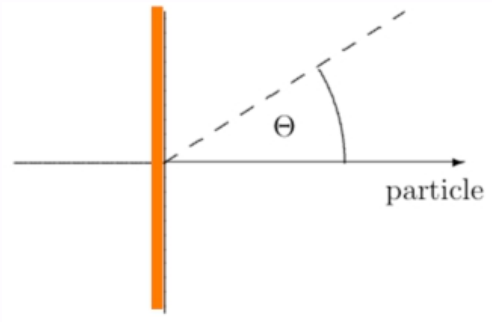


The time-dependent dipole field causes the emission of electromagnetic radiation

Energy radiated from a single boundary:

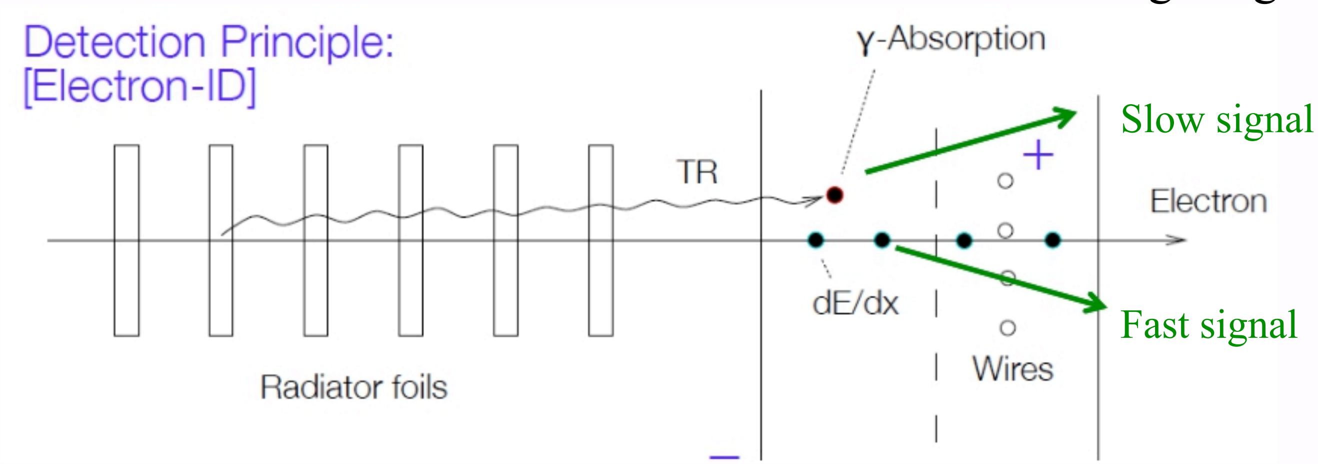
$$S = \frac{1}{3}\alpha z^2 \gamma \hbar \omega_P \quad (\hbar\omega_P \approx 20\,eV)$$

Transition Radiation



- Typical emission angle: $\Theta = 1/\gamma$
- Energy of radiated photons: $\sim \gamma$
- Number of radiated photons: $\sim z^2$
- Effective threshold: $\gamma > 1000$

Use stacked assemblies of low Z material with many transitions +
a detector with high Z gas



Note: Only X-ray (E>20keV) photons can traverse the many radiators without being absorbed

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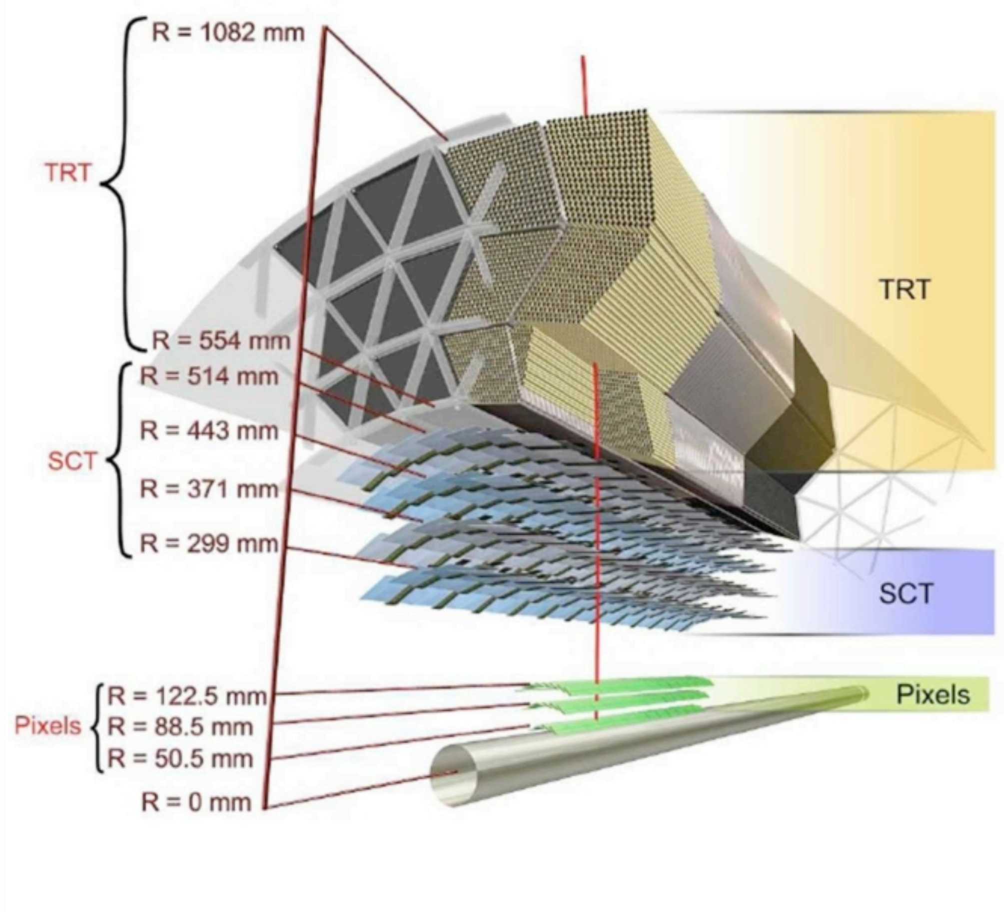
Transition radiation detectors



- straw tubes with xenon-based gas mixture
- 4 mm in diameter, equipped with a 30 μm diameter gold-plated W-Re wire

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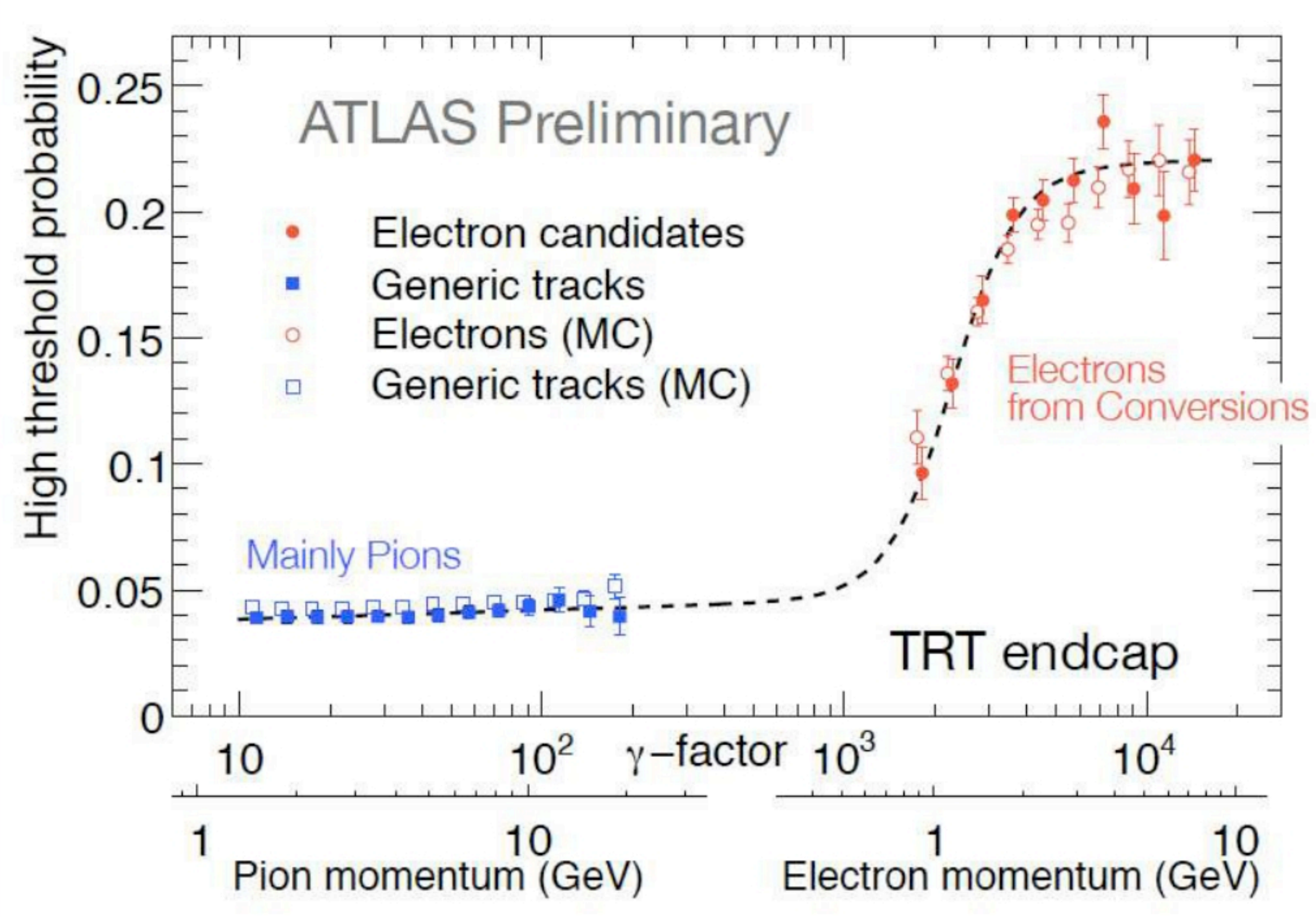
Transition radiation detector (ATLAS)



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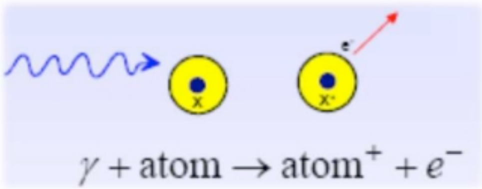
Interactions of photons with matter

Characteristic for interactions of photons with matter:

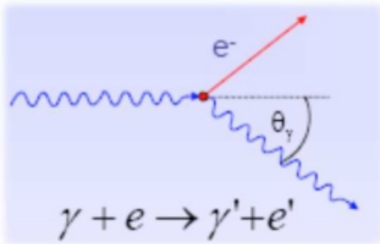
A photon is removed from the beam after one single interaction

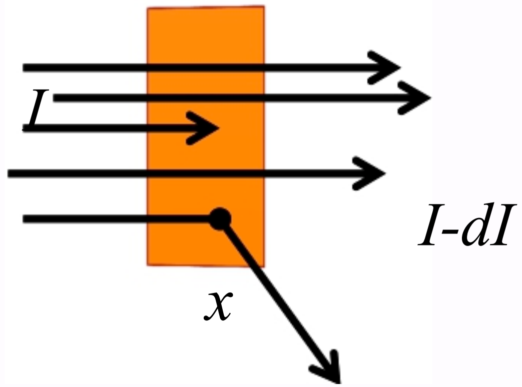
either because of total absorption or scattering

1) Photoelectric Effect

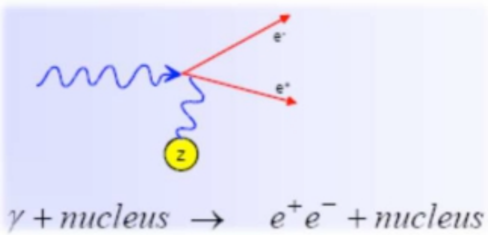


2) Compton Scattering





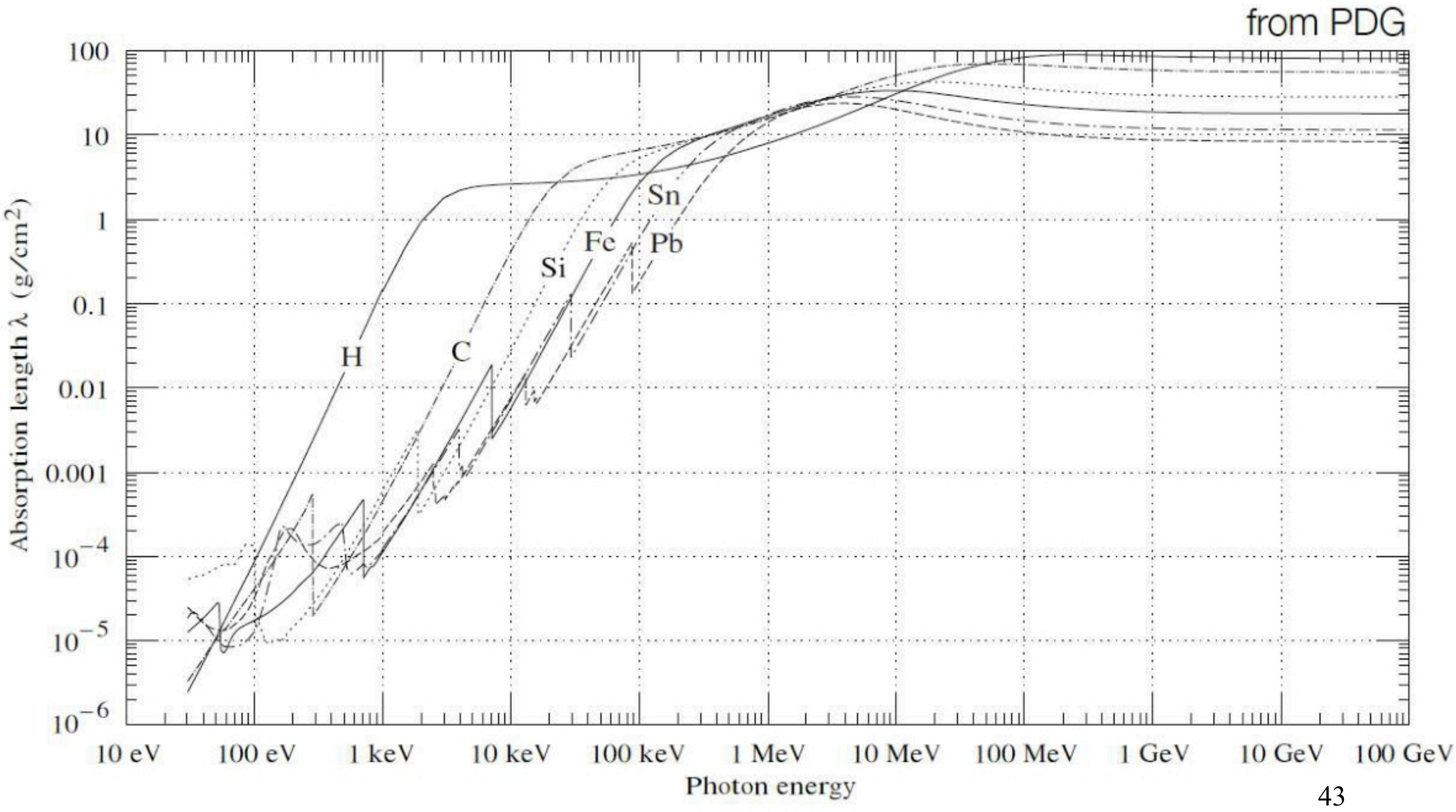
3) Pair Production



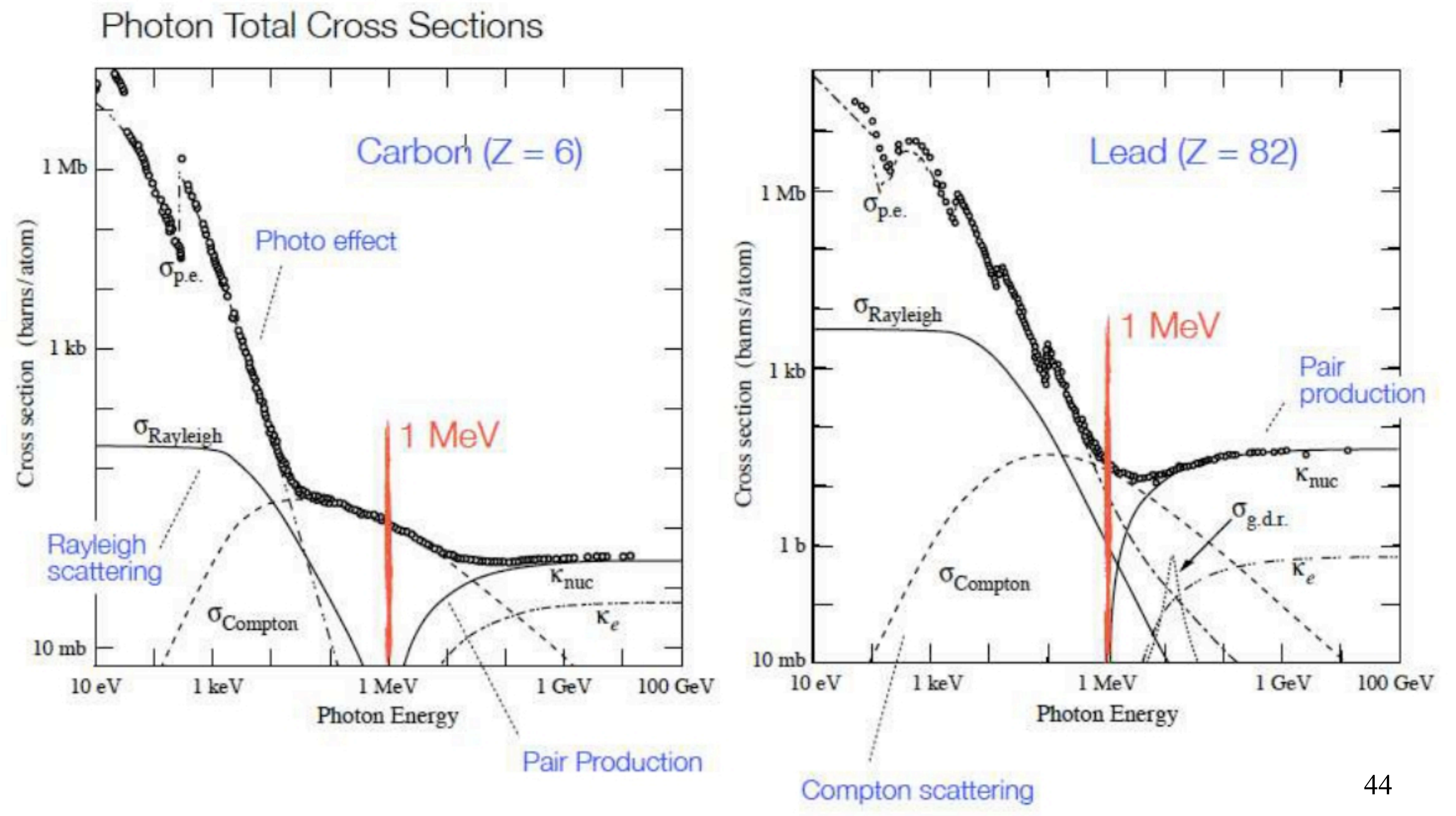
$$I(x) = I_0 e^{-\mu x}, \quad \mu = \frac{N}{A}\sum_{i=1}^{3}\sigma_i$$

$\lambda = 1/\mu$ Mean free path / absorption length

Interactions of photons with matter



Interactions of photons with matter



Photoelectric effect

From energy conservation:

$$E_e = E_\gamma - E_N = h\nu - I_b$$

I_b = Nucleus binding energy
introduces strong Z dependence

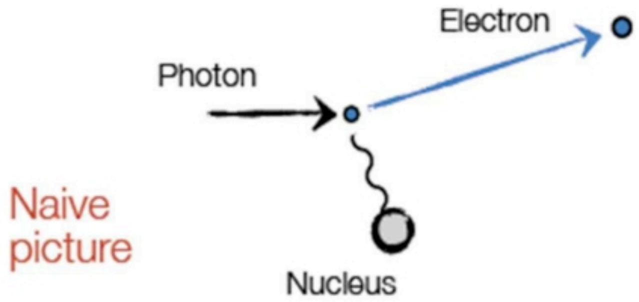
Cross-section largest for $E_\gamma \approx$ K-shell energy
Strongest E dependence for $I_0 < E_\gamma < m_e c^2$

$$\sigma_{ph} = \alpha\pi a_B Z^5 (I_0 / E_\gamma)^{7/2}$$

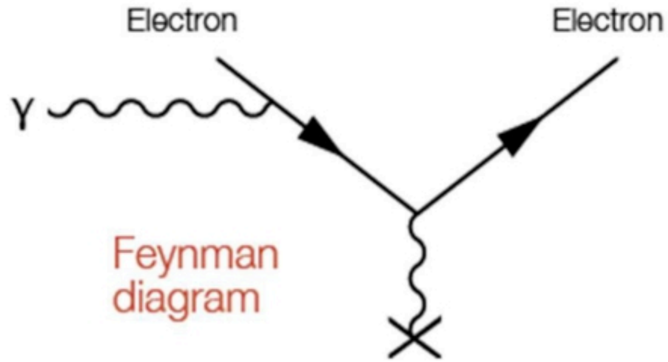
$$a_B = 0.53\ \text{Å}$$
$$I_0 = 13.6\ \text{eV}$$

E-dependence softer for $E_\gamma > m_e c^2$

$$\sigma_{ph} = 2\pi r_e^2 \alpha^4 Z^5 (mc)^2 / E_\gamma$$



γ + atom → e^- + atom$^+$



Compton scattering

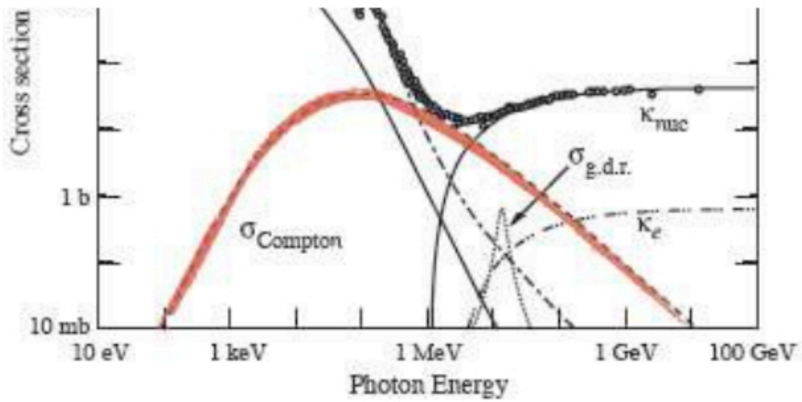
Best known electromagnetic process
(Klein–Nishina formula)

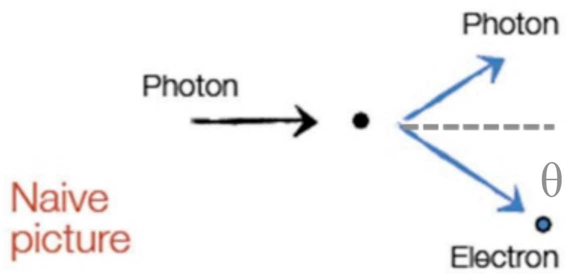
for $E_\lambda << m_e c^2$ $\boxed{\sigma_c \propto \sigma_{Th}(1 - 2\varepsilon + \frac{26}{5}\varepsilon^2)}$

Thompson cross-section:
$\sigma_{Th} = 8\pi/3\ r_e^2 = 0.66$ barn

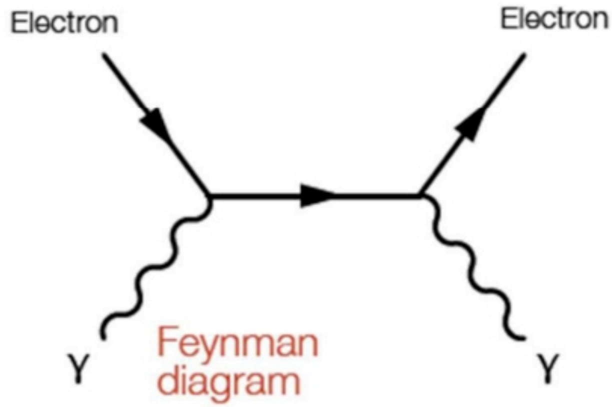
$$\varepsilon = \frac{E_\lambda}{m_e c^2}$$

for $E_\lambda >> m_e c^2$ $\boxed{\sigma_c \propto \frac{\ln \varepsilon}{\varepsilon} Z}$





$\gamma + e^- \rightarrow (\gamma)' + (e^-)'$



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Compton scattering

From E and p conservation get the energy of the scattered photon

$$E_\gamma{}' = \frac{E_\gamma}{1+\varepsilon(1-\cos\theta)} \qquad \varepsilon = \frac{E_\lambda}{m_e c^2}$$

Kinetic energy of the outgoing electron:

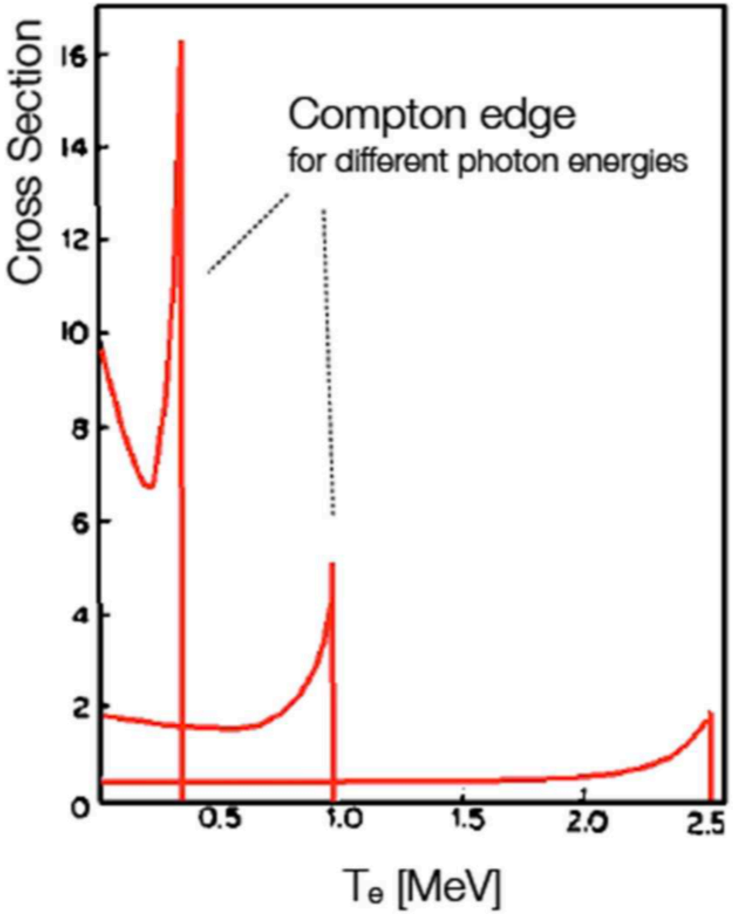
$$T_e = E_\gamma - E_\gamma{}' = E_\gamma \frac{\varepsilon(1-\cos\theta)}{1+\varepsilon(1-\cos\theta)}$$

Max. electron recoil energy for $\theta = \pi$:

$$\boxed{T_{\max} = E_\gamma \frac{2\varepsilon}{1+2\varepsilon}}$$

Transfer of complete γ-energy via Compton scattering not possible:

$$\Delta E = E_\gamma - T_{\max} = E_\gamma \frac{1}{1+2\varepsilon}$$

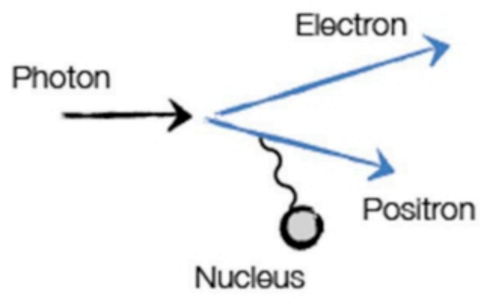


Important for single photon detection; if photon is not completely absorbed a minimal amount of energy is missing (Compton rejection in PET)

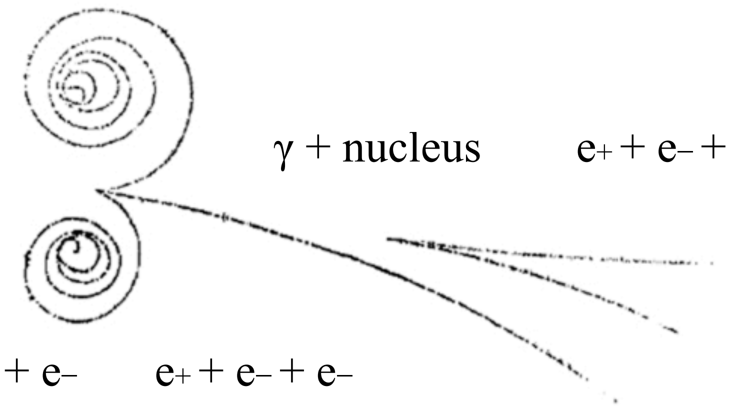
Pair production

Minimum energy required for this process
2 m_e + Energy transferred to the nucleus

$$E_\gamma \geq 2m_e c^2 + \frac{2m_e c^2}{m_{Nucleus}}$$

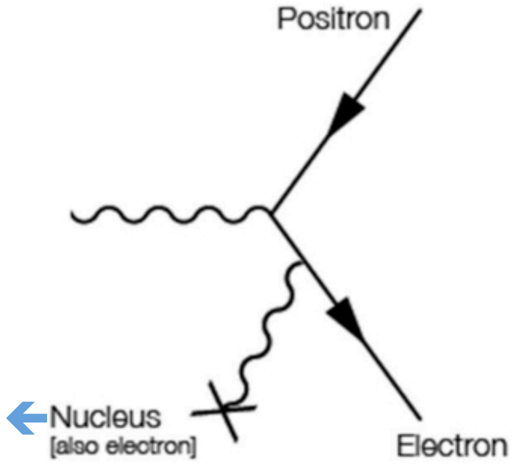


γ + atom $\rightarrow$ $e^+ + e^-$



γ + nucleus $\quad$ $e_+ + e_- +$ nucleus'

$\gamma + e_- \quad e_+ + e_- + e_-$



Necessary to satisfy energy and momentum conservation

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Pair production

for $E_\lambda >> m_e c^2$

$$\sigma_{\text{pair}} = 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{1/3}} - \frac{1}{54} \right) \quad [\text{cm}^2/\text{atom}]$$

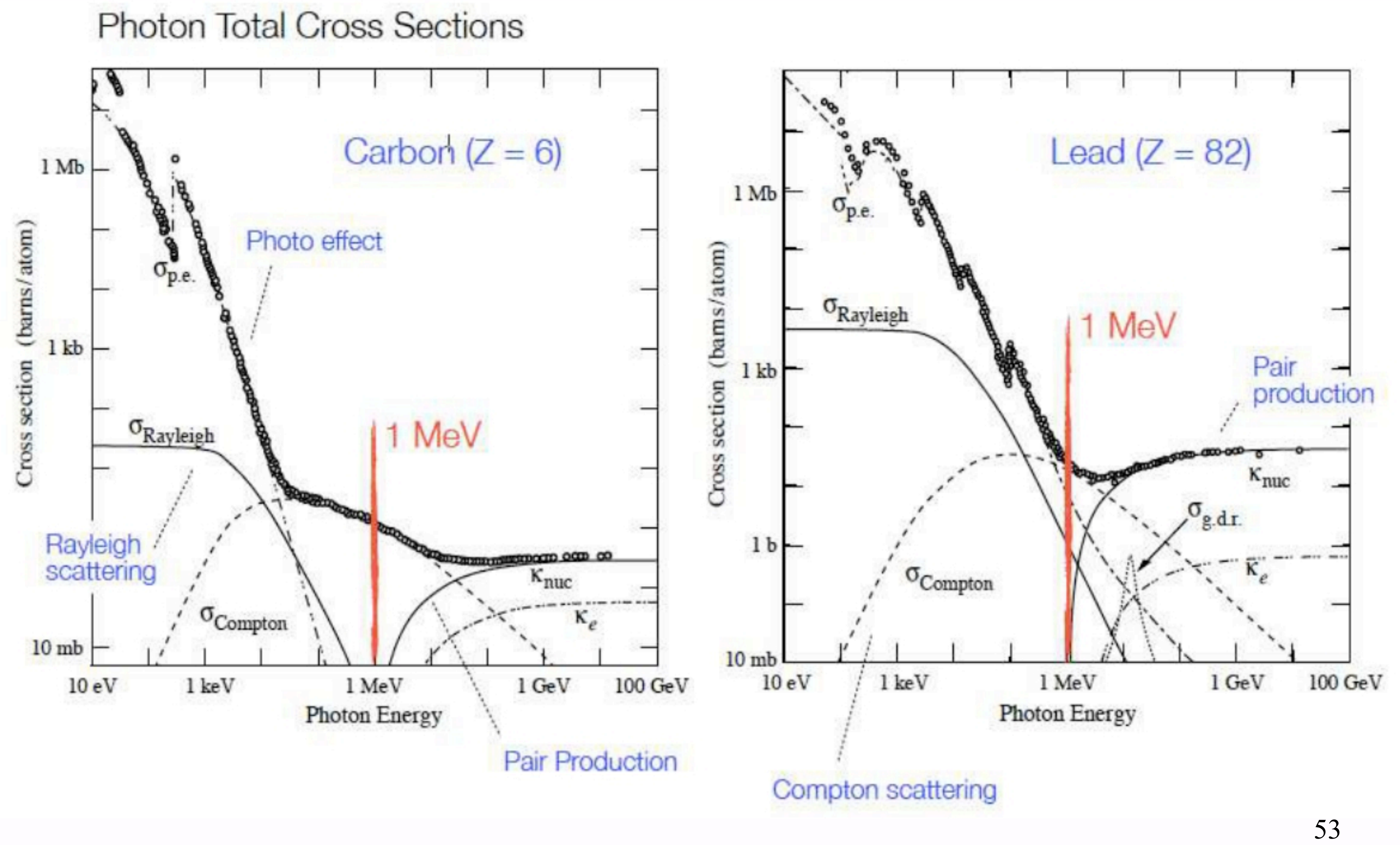
Using as for Bremsstrahlung the radiation length

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln \frac{183}{Z^{1/3}}}$$

$$\boxed{\sigma_{pair} = \frac{7}{9} \frac{N_A}{A} \cdot \frac{1}{X_0}}$$

	ρ [g/cm^3]	X_0 [cm]
H_2 [fl.]	0.071	865
C	2.27	18.8
Fe	7.87	1.76
Pb	11.35	0.56
Luft	$1.2 \cdot 10^{-3}$	$30 \cdot 10^{3}$

Interactions of photons with matter



Electromagnetic interactions