2. Particle interactions with matter

type of interactions for charged and neutral particles

Difference “scale” of processes for electromagnetic and strong interactions

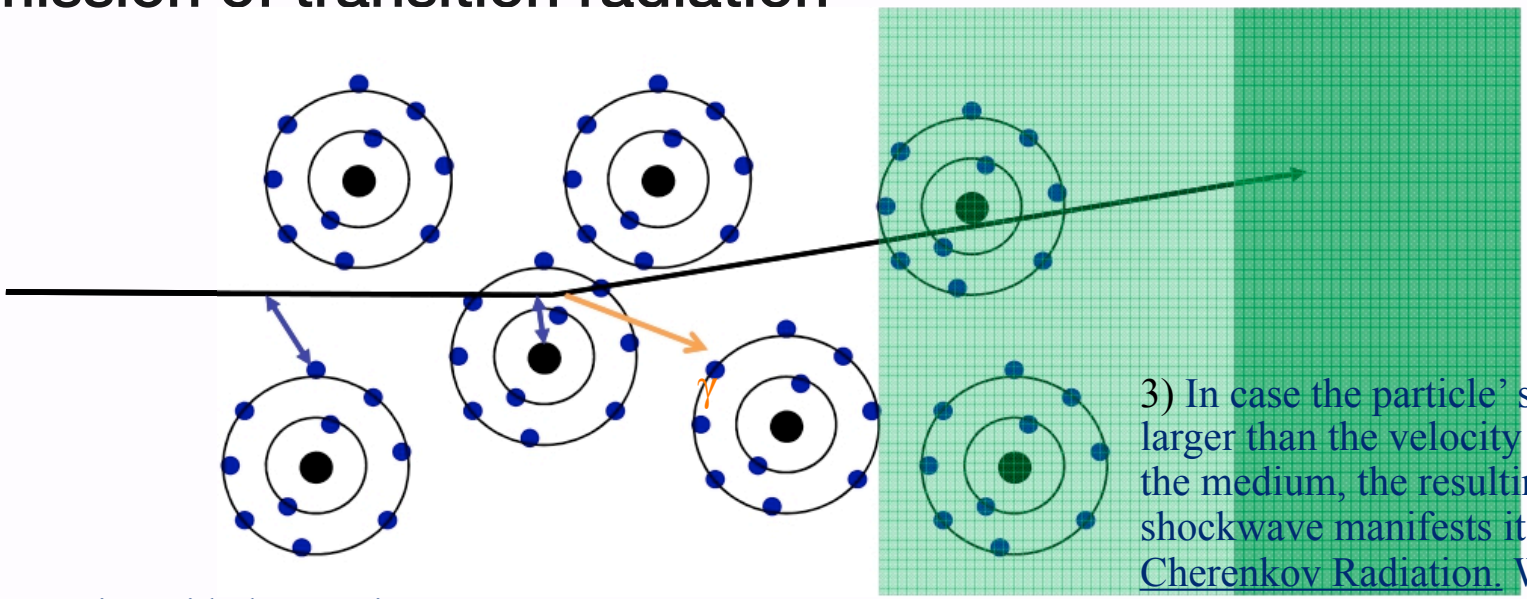
- Detection of charged particles(Ionization, Bremsstrahlung, Cherenkov …)
- Detection of γ -rays (Photo/Compton effect, pair production)
- Detection of neutrons (strong interaction)
- Detection of neutrinos (weak interaction)

Mind: a phenomenological treatment is given, no emphasis on derivation of the formulas, but on the meaning and implication for detector design.

Interactions of charged particles

Three type of electromagnetic interactions:

1. Ionization (of the atoms of the traversed material)
2. Emission of Cherenkov light
3. Emission of transition radiation



1) Interaction with the atomic electrons. The incoming particle loses energy and the atoms are excited or ionized

2) Interaction with the atomic nucleus. The particle is deflected (scattered) causing multiple scattering of the particle in the material. During this scattering a Bremsstrahlung photon can be emitted.

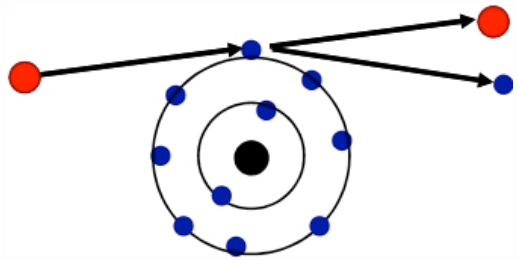
3) In case the particle' s velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called Transition radiation.

Bethe - Bloch formula

energy loss of a heavy particle through many scatterings on electrons in material

electrons at rest, β = initial velocity of heavy particle,

T = energy transfer to electron, 4-momentum transfer:



$$Q^2 = -(e - e')^2 = 2m_e c^2 T$$

Rutherford cross section in rest frame of electron:

$$\frac{d\sigma}{dQ^2} = 4\pi\,\alpha^2\,Z^2\,(\hbar c)^2 \frac{1}{\beta^2}\frac{1}{Q^4}$$

with electron spin, recoil: Mott cross section

$$\frac{d\sigma}{dT} = \underbrace{2\pi\,\alpha^2\,Z^2\,(\hbar c)^2 \frac{1}{\beta^2\,m_e c^2}\frac{1}{T^2}}_{\text{Rutherford}} \left(1 - \beta^2 \frac{T}{T_{max}}\right)$$

Energy loss of heavy particle after scattering (Tmin from ionization)

$$-\left\langle \frac{dE}{dx} \right\rangle = n_e \int_{T_{min}}^{T_{max}} T\,\frac{d\sigma}{dT}\,dT \qquad -\left\langle \frac{dE}{dx} \right\rangle = 2\pi\,\alpha^2\,Z^2\,(\hbar c)^2 \frac{1}{\beta^2\,m_e c^2} n_e \left(\ln\frac{T_{max}}{T_{min}} - \beta^2\right)$$

Bethe - Bloch formula

Valid for heavy charged particles ($m_{incident} >> m_e$), e.g. proton, k, π, μ

$$-\left\langle \frac{dE}{dx} \right\rangle = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e c^2 \beta^2 \gamma^2}{I^2} W_{max}\right) - 2\beta^2 - \delta(\beta\gamma) - \frac{C}{Z} \right]$$

$2\pi N_a r_e^2 m_e c^2$ = =0.1535 MeV cm²/g

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

Fundamental constants

r_e=classical radius of electron
m_e=mass of electron
N_a=Avogadro's number
c =speed of light

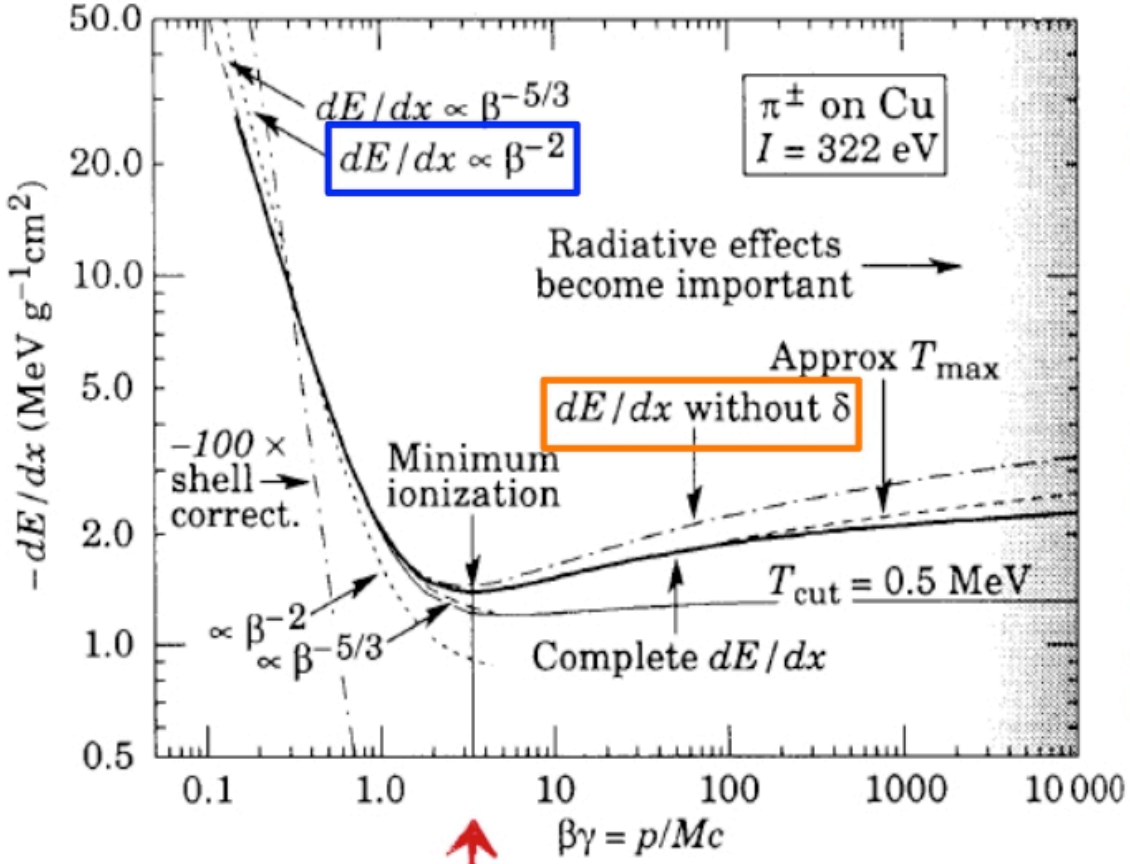
Absorber medium

I = mean ionization potential
Z = atomic number of absorber
A = atomic weight of absorber
ρ = density of absorber
δ = density correction
C = shell correction

Incident particle

z = charge of incident particle
β = v/c of incident particle
γ = $(1-\beta^2)^{-1/2}$
W_{max}= max. energy transfer in one collision

Understanding Bethe - Bloch



Minimum ionizing particles (MIP): $\beta\gamma$ = 3-4

dE/dx falls ~ β^{-2}; kinematic factor
[precise dependence: ~ $\beta^{-5/3}$]

dE/dx rises ~ $\ln(\beta\gamma)^2$; relativistic rise
[rel. extension of transversal E-field]

Saturation at large ($\beta\gamma$) due to density effect (correction δ)
[polarization of medium]

Units: MeV g^{-1} cm^2

MIP looses ~ 13 MeV/cm
[density of copper: 8.94 g/cm^3]

$\beta\gamma$ = 3-4

$$\frac{dE}{dx} \propto \frac{Z^2}{\beta^2} \ln\left(a\beta^2\gamma^2\right)$$

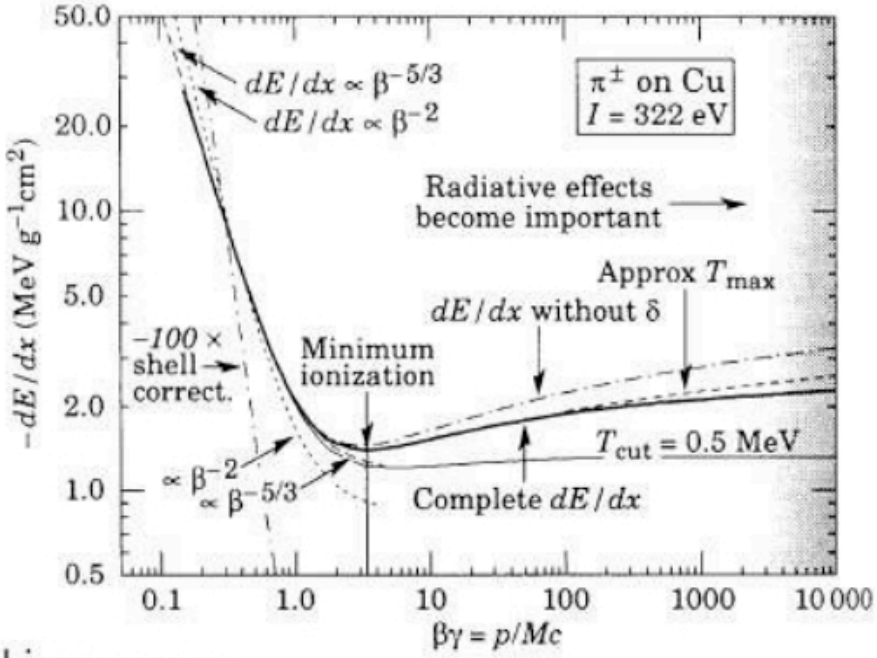
Understanding Bethe - Bloch

$1/\beta^2$-dependence:

Remember:

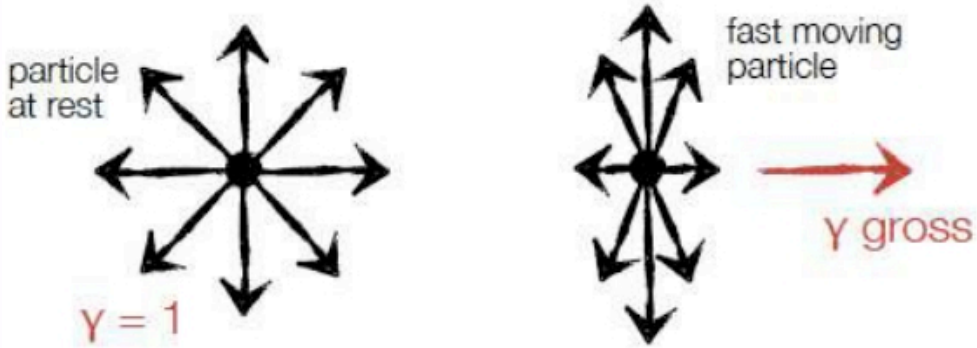
$$\Delta p_\perp = \int F_\perp dt = \int F_\perp \frac{dx}{v}$$

i.e. slower particles feel electric force of atomic electron for longer time ...



Relativistic rise for $\beta\gamma > 4$:

High energy particle: transversal electric field increases due to Lorentz transform; $E_y \rightarrow \gamma E_y$. Thus interaction cross section increases ...



Corrections:

low energy : shell corrections

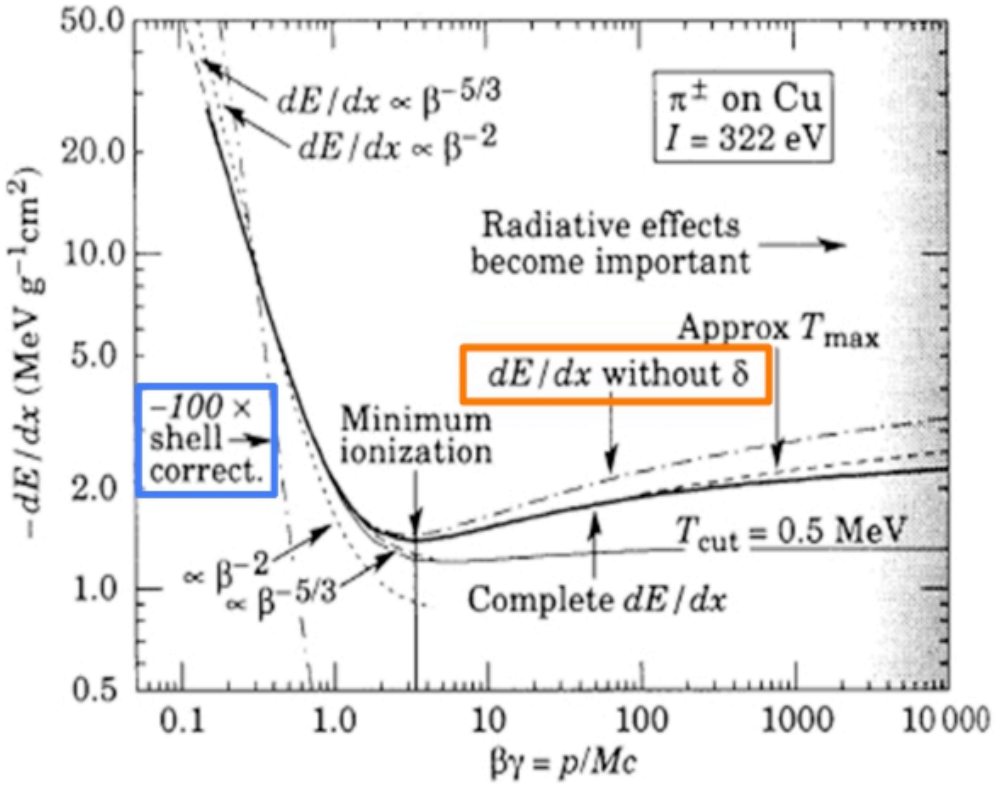
high energy : density corrections

Understanding Bethe - Bloch

Density correction
[saturation at high energy]
Density dependent polarization effect
...

Shielding of electrical field far from particle path; effectively cuts of the long range contribution ...
More relevant at high γ

Shell correction [small effect]
For small velocity assumption that electron is at rest breaks down,
Capture process is possible



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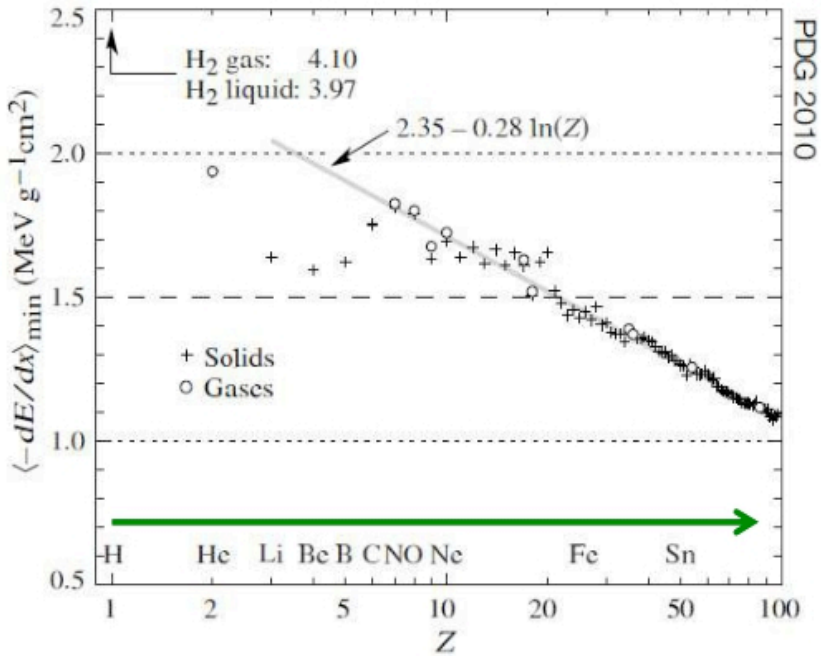
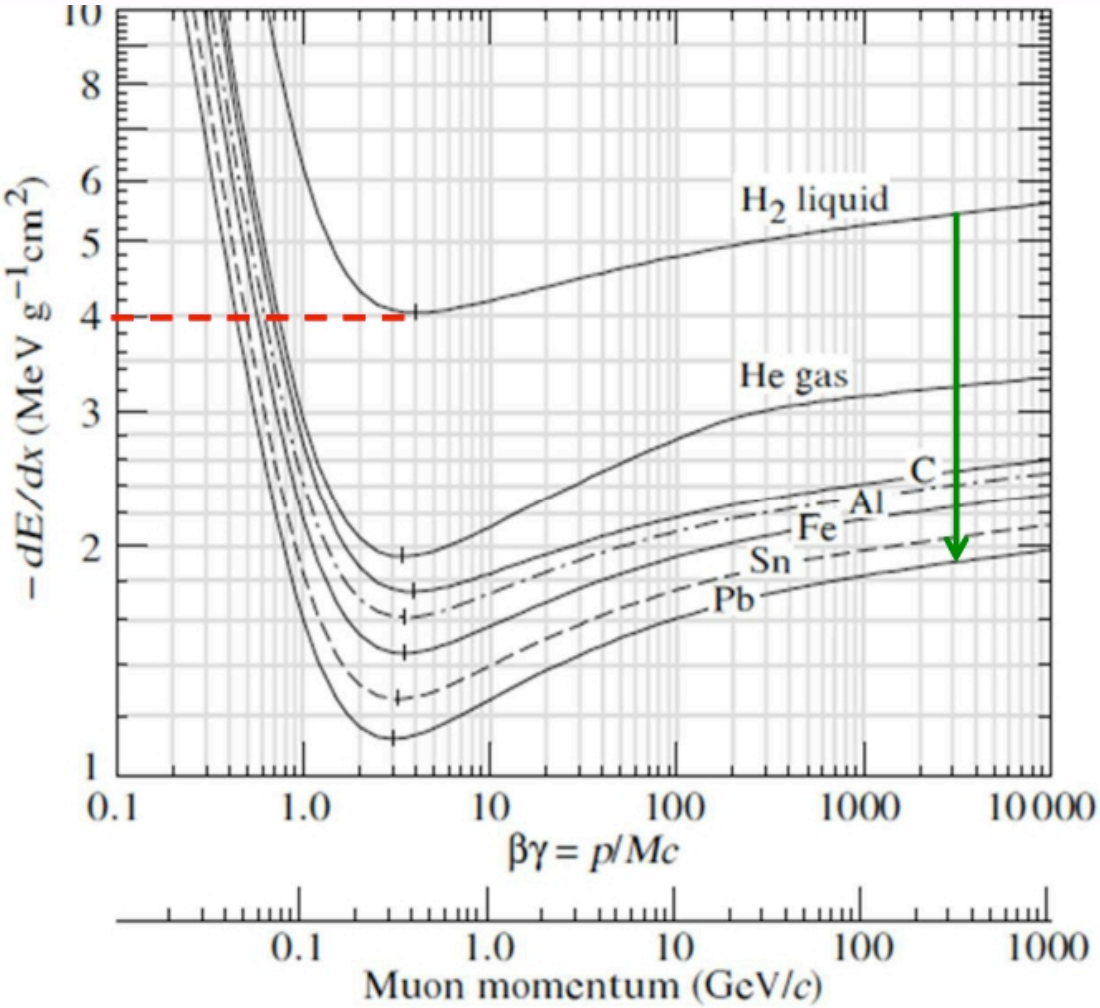
Understanding Bethe - Bloch

Minimum ionization:
ca. 1 - 2 MeV/g cm-2
[H 2 : 4 MeV/g cm-2]





some numbers

Minimum ionization:
ca. 1 - 2 MeV/g cm^{-2}

i.e. for a material with $\rho = 1$ g/cm^3 dE/dx = 1-2 MeV/cm

Example :
Iron: Thickness = 100 cm; $\rho = 7.87$ g/cm^3
$dE \approx 1.4$ MeV g^{-1} cm^2 * 100 cm * 7.87g/cm^3 = 1102 MeV

A 1 GeV Muon can traverse 1m of Iron

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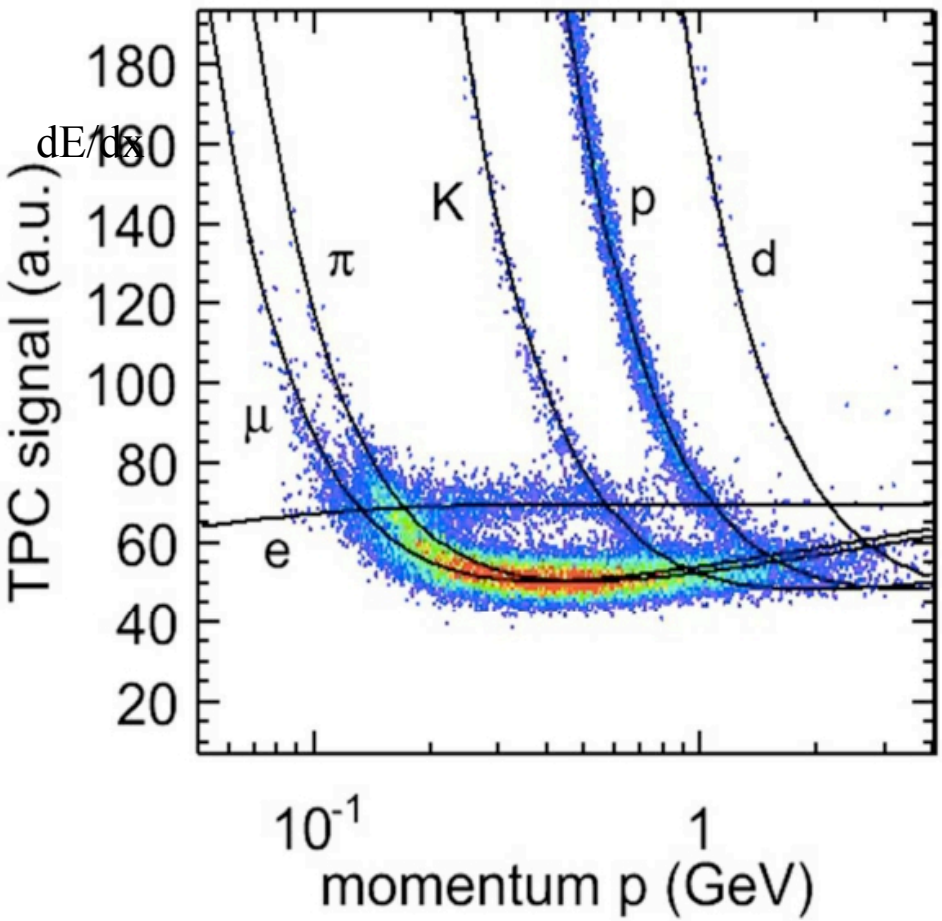
dE / dx for particle identification



The energy loss as a function of momentum $p = mc\beta\gamma$ is dependent on the particle mass

By measuring the particle momentum (deflection in a magnetic field) and the energy loss one gets the mass of the particle, i.e. particle ID

(at least in a certain energy region)

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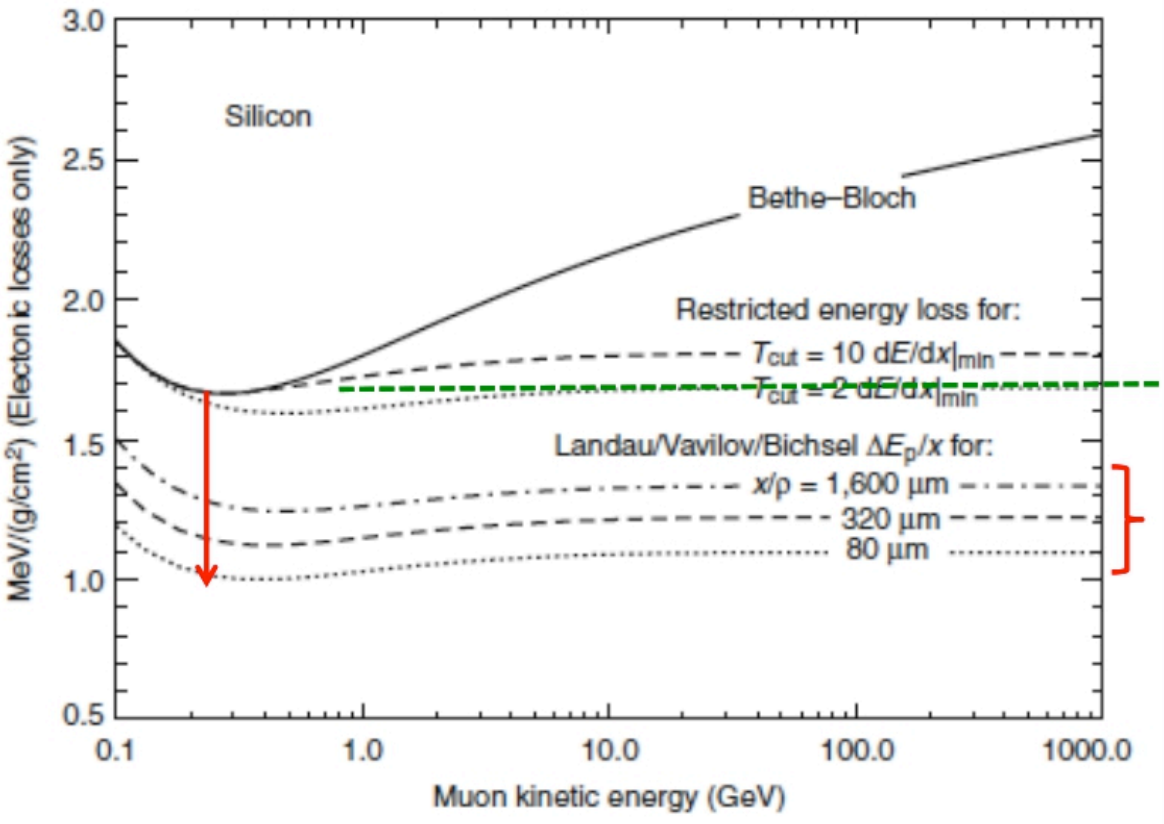
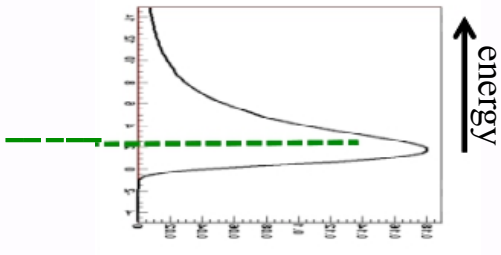
Dependence on absorber thickness

- Bethe-Bloch equation describes the mean energy loss
- layer of material with thickness x

➜ energy distribution of the δ-electrons and the fluctuations of their number (nδ) cause fluctuations of the energy losses ΔE



The energy loss Δ E in a layer of material is distributed according to the Landau function:



For a realistic thin silicon detector $n_\delta \lesssim$ 1-10, fluctuations do not follow the Landau distribution

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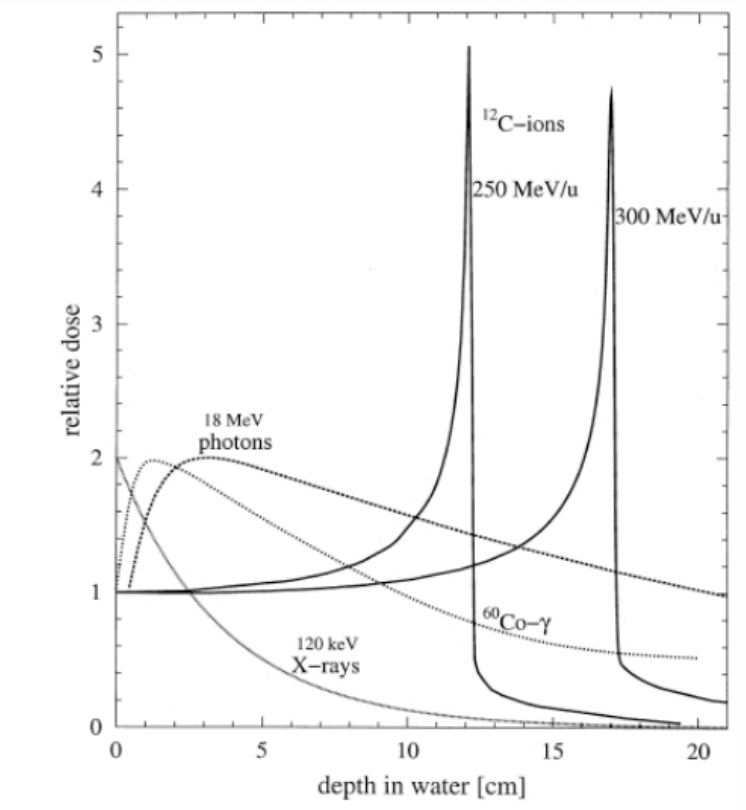
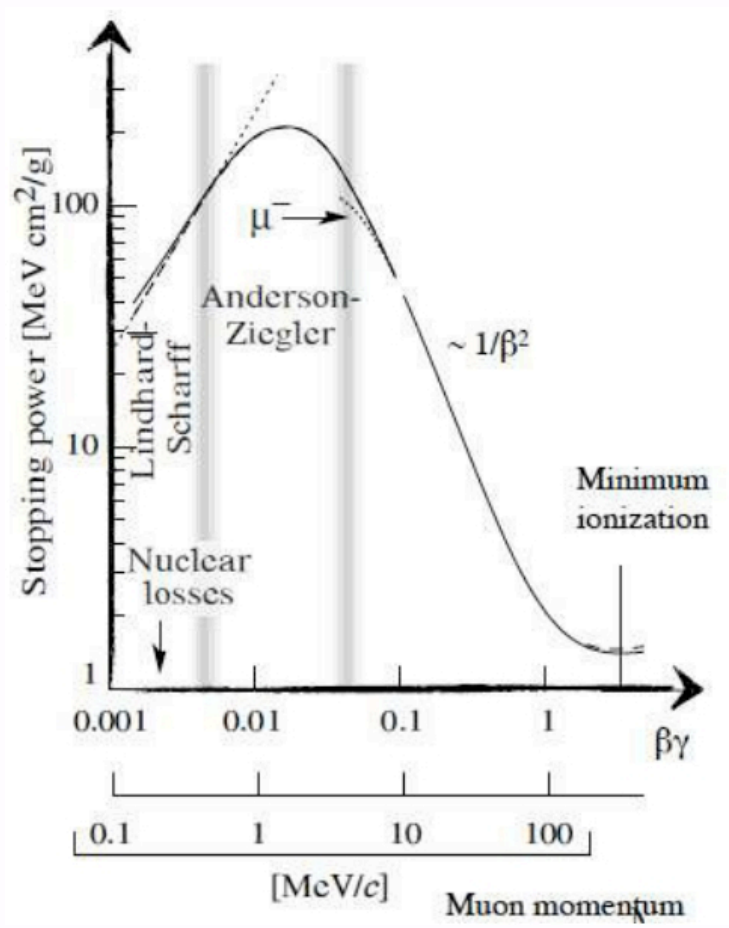
Energy loss at small momenta

- energy loss increases at small βγ
- particles deposit most of their energy at the end of their track

Bragg peak

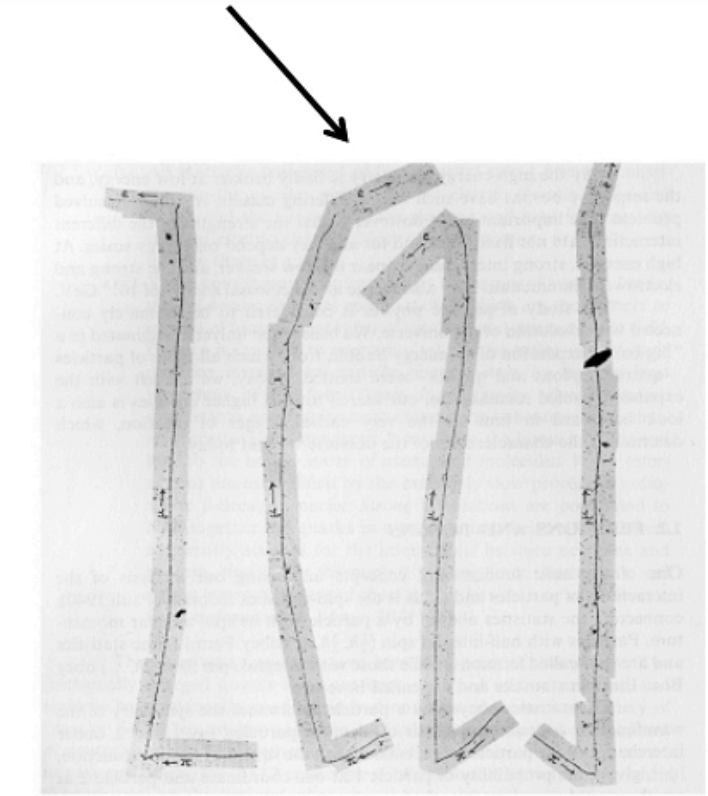




Important effect for tumor therapy

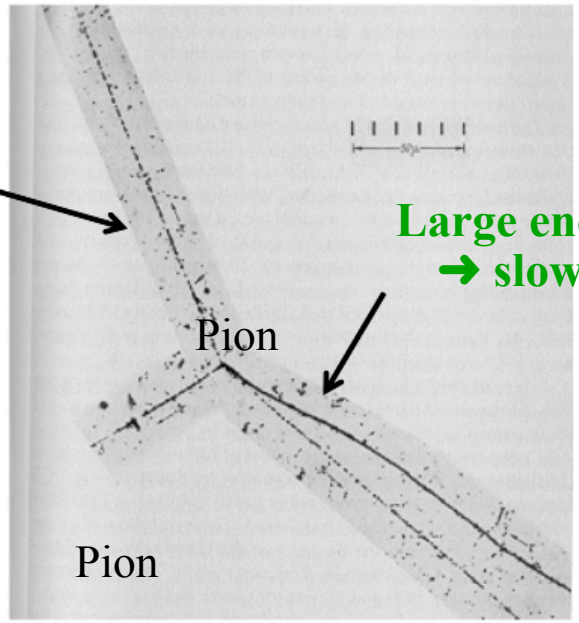
Energy loss at small momenta

Small energy loss
➜ fast particle



Discovery of muon and pion

Small energy loss
➜ fast particle



Large energy loss
➜ slow particle

Pion Kaon

Mean particle range

Integrate over energy loss

from the total energy T to zero

$$R(T) = \int_0^T \left[-\frac{dE}{dx} \right]^{-1} dE$$

More often use empirical formula

Example:

Proton with p = 1 GeV
Target: lead with ρ = 11.34 g/cm^3

R/M = 200 g cm^{-2} GeV^{-1}
→ R = 200/11.34/1 cm ~ 20 cm